Strong-weak *CP* hierarchy from nonrenormalization theorems

Gudrun Hiller*
SLAC, Stanford University, Stanford, California 94309

Martin Schmaltz†
Department of Physics, Boston University, Boston, Massachusetts 02215
(Received 5 February 2002; published 8 May 2002)

We point out that the hierarchy between the measured values of the Cabibbo-Kobayashi-Maskawa (CKM) phase and the strong *CP* phase has a natural origin in supersymmetry with spontaneous *CP* violation and low-energy supersymmetry breaking. The underlying reason is simple and elegant: in supersymmetry the strong *CP* phase is protected by an exact nonrenormalization theorem while the CKM phase is not. We present explicit examples of models that exploit this fact and discuss corrections to the nonrenormalization theorem in the presence of supersymmetry breaking. This framework for solving the strong *CP* problem has generic predictions for the superpartner spectrum and for *CP* and flavor violation, and predicts a preferred range of values for electric dipole moments.

DOI: 10.1103/PhysRevD.65.096009 PACS number(s): 11.30.Er, 12.10.Dm, 12.60.Jv

I. INTRODUCTION

Despite its impressive phenomenological success the standard model has serious shortcomings which should be understood as pointers toward physics beyond the standard model. One such shortcoming is the puzzling hierarchy between the *CP* violating phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix and the strong *CP* phase $\bar{\theta}$. This "strong *CP* problem" [1] has recently become more severe as results from the *B* factories now clearly favor a unitarity triangle with three large angles [2,3], implying that the complex phase in the CKM matrix is of order 1. In contrast, the strong *CP* phase, which is the only other *CP* violating parameter in the standard model, has been experimentally bounded to be tiny, $\bar{\theta} \lesssim 10^{-10}$ from measurements of electric dipole moments of the neutron and ^{199}Hg [4–6].

In the standard model this hierarchy between the two *CP* violating phases is puzzling because the phases have a common origin: the Yukawa couplings of the quarks. The CKM matrix is the unitary transformation matrix which takes one from the basis with a diagonal up quark Yukawa matrix Y_u to the basis with a diagonal down quark Yukawa matrix Y_d. An irremovable large phase in the CKM matrix implies at least one irremovable large phase in the Yukawa matrices. This then requires a fine-tuning of the strong *CP* phase to one part in 10^{10} because $\bar{\theta}$ depends on the phases in the Yukawa matrices via

$$\bar{\theta} = \theta - \arg\det Y_u - \arg\det Y_d . \qquad (1)$$

Here, we denote the physical (rephase invariant) theta angle by $\bar{\theta}$ to distinguish it from the basis dependent unphysical "bare" θ.

Several resolutions of the puzzle have been proposed. The axion mechanism [7] promotes $\bar{\theta}$ to a field. QCD dynamics gives this field a potential with a minimum at zero. Experimental searches for the axion have come up empty handed, and—when combined with constraints from cosmology and astrophysics—they have reduced the allowed parameter space to a narrow window [2]. Another proposed solution, a vanishing up quark mass [8], is on the verge of being ruled out by using partially quenched chiral perturbation theory to compare lattice calculations to experiment [9].

There are also proposals based on specific models which we may classify as "high-scale solutions" [10–14], the most famous of which is the Nelson-Barr mechanism [10]. These models use a symmetry (parity or *CP*) to enforce $\bar{\theta}=0$ at high scales. But in the standard model both *P* and *CP* are badly broken, and it becomes a challenging and cumbersome model building task to design realistic models that predict $\bar{\theta}<10^{-10}$ also at low energies after including all renormalization effects. While some of the models in the literature work, they lack the appeal of the axion and $m_u=0$ solutions which attempt to solve the strong *CP* problem with symmetries at low energies and are therefore relatively robust against changes in the high-energy theory and renormalization.

Recently, we pointed out [15] that by combining spontaneous *CP* violation with supersymmetry one can construct viable high-scale solutions in which $\bar{\theta}$ is automatically insensitive to radiative corrections and new high-energy physics. Our proposal makes use of the fact that in supersymmetry the strong *CP* phase $\bar{\theta}$ is not renormalized because of a nonrenormalization theorem [16]. This makes the task of building a successful model much easier. One only needs to make sure that $\bar{\theta}$ is zero at the tree level. Loop corrections are automatically absent if supersymmetry breaking occurs at energies much below the spontaneous *CP* violation.

In our previous publication [15] we briefly introduced our framework and presented an example model. The basic ingredients of the framework are spontaneous *CP* violation,

*Email address: ghiller@slac.stanford.edu
†Email address: schmaltz@bu.edu

supersymmetric (SUSY) nonrenormalization theorems, and flavor and *CP* preserving SUSY breaking such as gauge mediation.

In this paper we discuss our mechanism in more detail and provide a number of arguments and calculations to corroborate the claims made in [15]. In particular, in Sec. II we review our general framework. In Sec. III we show that a sufficiently large CKM phase can be generated from wave function renormalization. We also review the supersymmetric nonrenormalization theorem for $\bar{\theta}$. Section IV is devoted to explicit models; we discuss a Nelson-Barr model in which the CKM phase is generated at the tree level. We further present a model in which the CKM phase vanishes at tree level but is generated at the loop level from strongly coupled *CP* violating dynamics. In Sec. V we discuss the spectrum of supersymmetry breaking masses that is required for a successful implementation of our scheme. In Sec. VI we determine the expected size of $\bar{\theta}$ from radiative corrections in the standard model and from supersymmetry breaking. Sections VII and VIII contain our predictions, summary, and conclusions. In Appendixes A–D we define our notation, show that a large CKM phase from wave function renormalization requires strong coupling, and present calculational details regarding the renormalization and nonrenormalization of $\bar{\theta}$.

II. THE FRAMEWORK

In this section we summarize the basic ingredients of our framework. More details on each will be given in the following sections.

We require *CP* and supersymmetry to be exact at high energies. At such energies our theory is therefore described by a supersymmetric Lagrangian with coupling constants which can be chosen real. We will think of this Lagrangian as an effective Lagrangian valid up to a cutoff scale which we call $M_{\rm Pl}$ for convenience. But this scale could be any other high scale of new physics such as the grand unified theory (GUT) scale or the string scale. Our Lagrangian also contains higher-dimensional operators suppressed by the cutoff. Such operators are also required to be supersymmetric and *CP* preserving.

Since the standard model is neither SUSY nor *CP* symmetric, both symmetries must be spontaneously broken. We denote the scales at which the symmetry breaking is mediated to the minimal supersymmetric standard model (MSSM) fields by M_{CP} and $M_{\rm SUSY}$, respectively. Note that this is a somewhat unconventional definition for $M_{\rm SUSY}$. To be completely clear, in gauge mediation superpartner masses are proportional to $F/M_{\rm SUSY}$ in our notation, and in minimal supergravity we would have $M_{\rm SUSY}=M_{\rm Pl}$.

In order for our mechanism to work we require that $M_{CP} \gg M_{\rm SUSY}$ as shown in Fig. 1. Therefore the theory is still supersymmetric at M_{CP} and the well-known nonrenormalization theorems apply. In particular, the strong *CP* phase $\bar{\theta}$ is not renormalized. This makes building models of spontaneous *CP* violation that solve the strong *CP* problem relatively easy. We only need to require vanishing of $\bar{\theta}=0$ at the tree level; the nonrenormalization theorem guarantees that

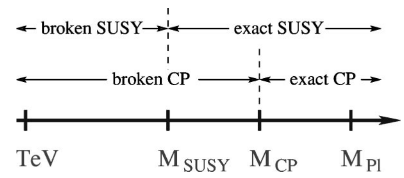

FIG. 1. SUSY and *CP* breaking scales in our framework. Figure not to scale.

this remains true after quantum corrections. However, the CKM phase is renormalized so that a nonvanishing $\Phi_{\rm CKM}$ can be obtained from quantum corrections as in our example model of Ref. [15] or already at the tree level as in the models of Nelson and Barr.

At the much lower scale $M_{\rm SUSY}$ Kaehler potential couplings of MSSM fields to the supersymmetry breaking sector are generated. These couplings turn into soft supersymmetry breaking masses once the SUSY breaking fields are replaced by their vacuum expectation values. It is important that these couplings to the SUSY breaking sector do not yet exist at the scale M_{CP}. This is because they would be renormalized and would pick up phases from the *CP* violating dynamics. We discuss this issue in more detail in Sec. V.

At scales below $M_{\rm SUSY}$ the theory is simply the MSSM with soft masses. Thus the low-energy *CP* violating parameters can be determined using the well-known renormalization group equations of the MSSM. This renormalization generates only negligibly small contributions to $\bar{\theta}$ if the soft SUSY breaking parameters are real and flavor universal. We review the arguments that prove this in gauge mediated supersymmetry breaking in Sec. VI.

III. CKM PHASE FROM WAVE FUNCTIONS

In this section we show explicitly that wave function renormalization does not contribute to $\bar{\theta}$. This is crucial to our mechanism because wave function renormalization is not constrained by $N=1$ supersymmetry. However, and this is important for our model of *CP* violation from wave functions in Sec. IV, wave function renormalization of the quarks contributes to the CKM phase. We show that a large CKM phase can be generated entirely from renormalization of the quark kinetic terms if the Z matrices appearing in the renormalization deviate from the unit matrix by order 1. Finally we discuss the (non)renormalization of $\bar{\theta}$ and $\Phi_{\rm CKM}$ in supersymmetry.

To begin, consider the following Lagrangian containing the kinetic terms of the standard model (SM) quarks and their Yukawa couplings:

$$\mathcal{L}_{\rm kinetic}=\bar{Q} i \not{D} Z_Q Q+\bar{D} i \not{D} Z_d D+\bar{U} i \not{D} Z_u U, \tag{2}$$

$$-\mathcal{L}_{\rm Yukawa}=\bar{Q}\hat{Y}_u H_u U+\bar{Q}\hat{Y}_d H_d D. \tag{3}$$

We use two-component spinor notation; Q are the SU(2)-doublet quarks, D and U are SU(2) singlets. Z_i denote wave function renormalization factors which in general are complex, Hermitian, and positive definite 3×3 matrices. Such matrices can always be written as the square of other positive definite Hermitian matrices $Z_i=(T_i)^{-2}$. Thus we can always change from this most general basis to canonical fields by a Hermitian basis change $Q\to T_Q Q$, $U\to T_u U$, and $D\to T_d D$, which leads to new Yukawa matrices

$$Y_u=T_Q\hat{Y}_u T_u, \quad Y_d=T_Q\hat{Y}_d T_d. \tag{4}$$

It is important to note that this basis change does not shift θ. This is most easily seen by writing $T=U^\dagger S U$ with unitary U and real diagonal S. Rescaling the quark fields by the real matrix S does not change θ and potential contributions from U and $U^\dagger$ cancel.

It is now easy to see that the contribution to $\bar{\theta}$ from quark masses [see Eq. (1)] vanishes if the only phases in the quark sector are in the Z_i. This follows because

$$\arg\det Y_{u/d}=\arg\det T_Q+\arg\det\hat{Y}_{u/d}+\arg\det T_{u/d}=0 \tag{5}$$

by Hermiticity of T and reality of $\hat{Y}_{u/d}$.

Note that the phases contained in the Z factors are physical and lead to a nonvanishing CKM phase. In fact, arbitrary quark masses and CKM matrices can be obtained as can be seen from the example $\hat{Y}_u=\hat{Y}_d=T_Q=1$, $T_u\propto\mathrm{diag}(m_u,m_c,m_t)$, and $T_d\propto V_{\mathrm{CKM}}\,\mathrm{diag}(m_d,m_s,m_b)V^\dagger_{\mathrm{CKM}}$.

We note one more result here which is important for the model in Sec. IV B. In order to generate an order 1 CKM phase from wave function factors the Z's cannot be close to the unit matrix. In other words, if—for example—nontrivial δZ's (with $Z=1+\delta Z$) are generated dynamically from loops, then this dynamics needs to be strongly coupled so that $\delta Z\sim O(1)$. If the δZ's are small, then a large *CP* violating phase cannot be generated. While this is plausible, it turns out to be difficult to prove. A somewhat pedestrian derivation is given in Appendix B.

To summarize, what we have discussed above outlines a possible strategy for solving the strong *CP* problem: if one can construct a model with vanishing bare θ, real Yukawa matrices, but complex Hermitian wave function factors Z_i, then $\bar{\theta}$ vanishes even for large *CP* violation in the CKM matrix.[1]

However, the above is not yet a solution to the strong *CP* problem. In the presence of *CP* violating dynamics reality of the Yukawa matrices in Eq. (3) is not enforced by any symmetries, and it is in general just as miraculous as a vanishing strong *CP* phase $\bar{\theta}$. But we will show in the following section that supersymmetry and its nonrenormalization theorems can naturally give complex phases in the kinetic terms and real Yukawa matrices.

A. Supersymmetry

As we will now explain, the situation improves dramatically in the presence of supersymmetry. This is essentially because in supersymmetry $\bar{\theta}$ and the Yukawa matrices $\hat{Y}$ are holomorphic quantities which are protected by nonrenormalization theorems. However, the wave function factors Z_i stem from the Kaehler potential and are renormalized. Thus, if it can be arranged in a model that $\bar{\theta}$ remains zero at the tree level, then the nonrenormalization theorem guarantees this also at the quantum level. The CKM phase is renormalized and can be generated either at the tree level or by loops.

More explicitly, a supersymmetric Lagrangian can be written as

$$\mathcal{L}=\int d^4\theta\, K+\int d^2\theta\, W+W_{\mathrm{gauge}}, \tag{6}$$

where K is the Kaehler potential, W the superpotential, and W_{gauge} contains the gauge kinetic terms. Matter fermion masses are given by second variations of the superpotential $\hat{M}_{ij}\equiv\partial^2 W/\partial\phi_i\partial\phi_j$ times wave function renormalization factors from the Kaehler potential. The wave function renormalization is determined from the kinetic terms $Z_{ij}\equiv\partial^2 K/\partial\phi_i\partial\phi_j^*$. Since Z_{ij} is positive definite and Hermitian it can be written as the square of a nonsingular Hermitian matrix $Z=T^{-2}$.

Chiral superfields in the original basis are related to fields in the canonical basis by $\phi_k\to T_{ki}\phi_i$, and the general expression for properly normalized fermion masses is

$$M_{ij}=-\frac{1}{2}T_{ik}\hat{M}_{kl}T_{jl}. \tag{7}$$

It follows that the contribution to $\bar{\theta}$ from $\arg\det M$ vanishes if the couplings [and vacuum expectation values (VEVs)] in W are real. This remains true for arbitrary complex Kaehler potential couplings.

In the MSSM, we have

$$\hat{M}_u=\hat{Y}_u v_u, \quad \hat{M}_d=\hat{Y}_d v_d, \tag{8}$$

where v_u and v_d are the VEVs of the up- and down-type MSSM Higgs fields, respectively. The quark mass matrices are defined in terms of $\hat{M}_{u,d}$ with products of wave function factors $T_{Q,u,d}$ as in the nonsupersymmetric case.

We are now ready to discuss the nonrenormalization of $\bar{\theta}=\theta-\arg\det M$. We showed above that $\arg\det M$ is not renormalized. To understand the renormalization of θ it is convenient to define the superfield

$$\tau=\frac{1}{g^2}+i\frac{\theta}{8\pi^2} \tag{9}$$

and work in a basis in which the gauge-kinetic term is $\int d^2\theta 1/4\tau W_\alpha W^\alpha$, and where no wave function renormalization is performed. In this basis τ is renormalized at one loop only [17,18]:

$$\tau(\mu)=\tau(\mu_0)-\frac{b_0}{8\pi^2}\log(\mu/\mu_0). \tag{10}$$

[1] We have not yet shown that this is stable under radiative corrections. We will deal with this in Sec. VI where we discuss renormalization of $\bar{\theta}$.

Here μ and μ_0 are real renormalization scales and b_0 is the one-loop β function coefficient. Taking the imaginary part on both sides shows that θ is also not renormalized.

So far, we have ignored mass thresholds. A superfield with a mass m between μ and μ_0 should be integrated out at the scale m. This gives a shift $\delta\tau = -t_2/8\pi^2 \log(m)$, and if m is complex we have $\theta \to \theta - t_2 \arg m$. Here t_2 is the Dynkin index in the color representation of the field which was integrated out ($t_2 = 1$ for a quark). This is exactly what is needed for $\bar{\theta}$ to be invariant, because the massive field should not be included in the $\arg\det M$ term in the definition of $\bar{\theta}$ in the low-energy theory.[2] We discuss the nonperturbative generalization of this nonrenormalization theorem in Appendix C.

To end this section, we wish to clarify a potential confusion stemming from the possibility of redefining the phase of the gluino field via an anomalous R symmetry transformation. In the absence of a gluino mass this appears to allow rotating away the θ angle. However, in order for supersymmetry breaking to generate a gluino mass as required for phenomenology, the R symmetry has to be broken in the theory. If this breaking is spontaneous then the theory has an R axion, and we have rediscovered the axion solution to the strong *CP* problem (with its associated phenomenological constraints). If the breaking is explicit θ cannot be rotated away.

IV. THE *CP* VIOLATING SECTOR

In this section we discuss the requirements on the sector of the theory that is responsible for breaking *CP*. We also give two examples and emphasize the trouble with models without SUSY.

The job of the *CP* violating sector is to produce a CKM phase of order 1 while avoiding a *tree level* contribution to $\bar{\theta}$. Quantum corrections to $\bar{\theta}$ are automatically taken care of by the nonrenormalization theorem and low-energy SUSY breaking as discussed in Secs. III and V.

There are two possible strategies for generating the CKM phase. The first was proposed long ago by Nelson and by Barr. In their scenario, the ordinary quarks mix with ultraheavy vectorlike quarks via complex couplings. As we will review in the next subsection this mixing generates a CKM phase at the tree level while a clever choice of Yukawa couplings (or equivalently of field content and global symmetries) forbids the tree level contribution to $\bar{\theta}$. The other possibility was proposed in our recent publication [15]. In our scenario, *CP* violation couples to the MSSM only at the loop level. This automatically guarantees a vanishing $\bar{\theta}$, and the CKM phase must be generated from loops that renormalize the quark wave functions. We review this scenario in the second subsection.

Spontaneous *CP* violation requires some fields to have potentials that are minimized at complex vacuum expectation values. For simplicity we will omit the specific potentials. They are not difficult to construct even though the Lagrangian is real because of the underlying *CP* invariance. A simple example is given by the superpotential

$$W = \Xi(\Sigma^2 + \mathcal{E}^2) \tag{11}$$

with singlet chiral superfields Ξ and Σ, whose scalar potential forces a complex VEV for $\Sigma = \pm i\mathcal{E}$.

A. Nelson-Barr

In Nelson-Barr models [10] *CP* violation is communicated to the quarks already at the tree level. The nontrivial model building feat is to arrange the superpotential such that $\bar{\theta} = 0$ at the tree level. A relatively simple choice is to add a vectorlike fourth singlet down quark. The superfields $D_4 + \bar{D}_4$ have $(\mathrm{SU}(3),\mathrm{SU}(2))_{\mathrm{U}(1)}$ quantum numbers $(\bar{3},1)_{1/3} + (3,1)_{-1/3}$ and couple in the superpotential to the MSSM fields and three complex VEVs Σ_i as follows:

$$W = Q_i \hat{M}_{ij} D_j, \quad i,j = 1-4. \tag{12}$$

Abusing notation, we define $Q_4 \equiv \bar{D}_4$, and the mass matrix $\hat{M}$ is

$$\hat{M} = \begin{pmatrix} \hat{Y}_d H_d & 0 \\ r\Sigma & \mu \end{pmatrix}. \tag{13}$$

Here $\mu \gg M_{\mathrm{weak}}$ contributes to the mass of the vectorlike fermions, and all couplings and μ are real because of the underlying *CP* symmetry. This form of the Lagrangian can be enforced by additional global symmetries. A similar mixing could also be introduced in the up sector and everything is straightforwardly extended to a GUT.

One can easily verify that $\arg\det\hat{M} = 0$, and therefore $\bar{\theta} = 0$ at the tree level as desired. The CKM matrix is the mismatch of the basis in which

$$Y_u Y_u^\dagger = \hat{Y}_u \hat{Y}_u^T$$

and

$$Y_d Y_d^\dagger = \hat{Y}_d \left(1 - \frac{a a^\dagger}{|a|^2 + \mu^2}\right) \hat{Y}_d^T \tag{14}$$

are diagonal. Here we have defined the three-vector $a^\dagger = (r_1\Sigma_1, r_2\Sigma_2, r_3\Sigma_3)$ and used the approximation $\hat{Y}_d H_d \ll r\Sigma \sim \mu$ to compute the down quark Yukawa matrix. We see that if the VEVs Σ_i are complex and $a_i \gtrsim \mu$, the down quark matrix has large phases giving an unsuppressed CKM phase as desired.

As discussed in previous sections, supersymmetry guarantees that $\bar{\theta}$ remains zero at the loop level as well. If we had considered this model without SUSY as originally proposed

[2] Note that we have been somewhat cavalier with the Dynkin indices in the definition of $\bar{\theta}$. The correct definition contains a factor of $t_2(R_i)$ for each of the different representations R_i of colored fermions in the theory.

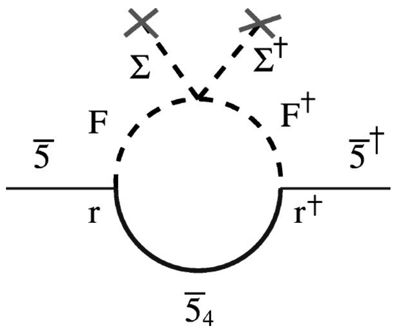

FIG. 2. Figure of wave function renormalization for the $\overline{5}_i$ following from the superpotential in Eq. (15).

by Nelson, then we would have to worry about loops involving the heavy fermions and Σ fields which contribute to $\overline{\theta}$. In order to make the Nelson-Barr models safe without SUSY one needs to take the couplings r_i very small ($\sim 10^{-3}$–10^{-4}) while simultaneously tuning $r_i \Sigma_i \sim \mu$.

B. CKM phase from loops

In this section we review an example model which was presented in our first paper [15]. In this model the CKM phase stems from wave function renormalization factors Z_i of the quarks. The Z_i factors arise from loops of heavy superfields with complex masses.

Here we will describe an SU(5) GUT version of the model. In addition to the usual three generations of $\overline{5}_i + 10_i$ matter fields we also require a vectorlike $\overline{5}_4 + 5_4$ (models with one or several $10 + \overline{10}$ or both $\overline{5} + 5$ and $10 + \overline{10}$ are, of course, also possible). Furthermore, we have the usual Higgs fields H_u and H_d and three generations of gauge singlet superfields $F_i + \overline{F}_i$. The superpotential contains the usual MSSM couplings as well as

$$W_{\rm CP} = r_{ij} \overline{5}_i F_j 5_4 + s \Sigma_{ij} F_i \overline{F}_j + M 5_4 \overline{5}_4 . \tag{15}$$

M, r, and s are real, the indices i,j run over 1–3, and the matrix Σ_{ij} is assumed to have complex entries from spontaneous *CP* breaking.[3]

To determine the low-energy *CP* violation we integrate out the massive fields F and $5_4 + \overline{5}_4$. The low-energy superpotential that derives from Eq. (15) vanishes when the equations of motion for the F's and 5_4 are inserted, but the diagram of Fig. 2 generates a noncanonical complex kinetic term for $\overline{5}_i$. Note that for *CP* violation to be mediated to the MSSM fields the *CP* breaking sector needs to violate flavor, otherwise the resulting kinetic term for $\overline{5}_i$ would be diagonal and real. For $\Sigma > M$ the Feynman diagram is easily evaluated. Its *CP* violating part involving the VEV of Σ is finite and can be expanded to give

$$\delta Z_{\overline{5}} \sim \frac{1}{16\pi^2} r^\dagger \frac{\Sigma^\dagger \Sigma}{M_{CP}^2} r \tag{16}$$

where M_{CP} is the scale at which spontaneous *CP* breaking is mediated to the quarks, $M_{CP}^2 \sim s^2 {\rm tr}[\Sigma^\dagger \Sigma]$.

As discussed in Sec. III, a sufficiently large phase in the CKM matrix can be generated only when wave function renormalization is large, which requires $r \sim 4\pi$. This implies that the one-loop approximation is not reliable. Therefore, it is most useful to parametrize the wave function coefficient by an arbitrary Hermitian matrix $Z_{\overline{5}}$.

As it stands, this model is incomplete because of the large Yukawa coupling r. The problem is that if the scale of *CP* violation is below the Planck scale then the Yukawa coupling runs to values of order 1 within one e-folding even if it is 4π at the Planck scale. A large Yukawa coupling at the lower scale M_{CP} can be arranged by letting the F_i's and $\overline{5}_4 + 5_4$ interact with a new strong gauge group. It is easy to modify this model to include these interactions. We present such a model in Appendix C, where we also show that the relevant nonperturbative effects in r and the new strong gauge coupling can be determined exactly and do not contribute to $\overline{\theta}$.

C. Reproducing the quark masses and CKM matrix

We make some general remarks on models with *CP* violation from kinetic terms. Since wave function renormalization factors are required to be large (and not computable in perturbation theory) and flavor violating, their effects on quark masses and mixing angles are important. This suggests two different basic scenarios (models which interpolate between the two extreme cases are of course also possible).

(A) The hierarchical structure of the Yukawa couplings is generated at a scale above M_{CP}, and the wave function renormalization is only responsible for generating the necessary phases. In the process, the strong dynamics necessarily changes at least some of the mixing angles completely, but the quark mass hierarchy is essentially unchanged.

(B) Flavor and *CP* violation have a common origin. At scales above M_{CP} the Yukawa couplings are either universal ($\hat{Y}^i_j \propto \delta^i_j$) because of non-Abelian flavor symmetries, or have "random" $O(1)$ entries (flavor anarchy), and the entire flavor structure including the hierarchy stems from the wave function renormalization factors T. Models in which flavor originates from wave function renormalization have been built by Nelson and Strassler [19]. Their models, when adapted to incorporate our mechanism, generate flavor and solve the strong *CP* problem.

D. The trouble with models without SUSY

Note that the necessity of strong coupling $r \sim 4\pi$ underlines why supersymmetry is so important to our approach: Non-SUSY models of *CP* violation induced by noncanonical

[3]A more minimal *CP* violating sector with only two F's and no $\overline{F}$ would work as well. Note also that we have vanishing VEVs for F and $\overline{F}$, with a nonvanishing and complex $r\langle \overline{F} \rangle \sim M$ this model would essentially be Nelsons [2].

kinetic terms have been discussed in the literature, with the new sector coupling only to the doublet quarks [12], or to the singlets [13]. Without SUSY no nonrenormalization theorem protects the colored fermion masses from *CP* violating vertex corrections, which occur at some—possibly high—loop level. However, because of the required large coupling for r, arbitrarily high loop diagrams can still violate the bound on $\bar{\theta}$. Turning the argument around, electric dipole moments (EDM) data put severe constraints on the model parameters, in particular the coupling r. For example, because of a vertex correction at four loops the authors of Ref. [12] were forced to take $r<1$, and therefore their model cannot produce large CKM *CP* violation. Like the one(s) in [13], it is superweak and therefore ruled out. In general, this is the fate of non-SUSY models with *CP* violation from kinetic terms; supersymmetry and its nonrenormalization theorems appear to be necessary ingredients for this mechanism to yield realistic models with a large CKM phase.

V. SUSY BREAKING

In this section we discuss the constraints on SUSY breaking and communication which follow from our solution to the strong *CP* problem. The nonrenormalization theorems guarantee $\bar{\theta}=0$ with exact SUSY. However, after SUSY breaking $\bar{\theta}$ is renormalized, and we find that avoiding large contributions from loops including superpartners forces the SUSY breaking masses to be highly degenerate. Furthermore, flavor preserving SUSY parameters, such as the gaugino masses and $B\mu$, are required to be real to a high accuracy. We also argue that low-energy SUSY breaking models such as gauge mediation are most compatible with our mechanism. This is because the *CP* violating dynamics renormalizes the SUSY breaking masses and spoils the necessary degeneracies if soft masses are already present at the high scale M_{CP}. We give a more detailed discussion of the renormalization of $\bar{\theta}$ in gauge mediation in Sec. VI and Appendix D, where we also give more references.

To begin, note that in the MSSM Eq. (1) is generalized to

$$\bar{\theta}=\theta-\arg\det Y_u-\arg\det Y_d-3\arg(v_u v_d)-3\arg m_{\tilde{g}}. \tag{17}$$

This immediately implies a strong constraint on SUSY breaking parameters as the gluino mass and the Higgs VEV have to be real to one part in 10^{10}. The reality of the Higgs VEVs translates into constraints on parameters in the Higgs potential. In particular, a complex $B\mu$ induces complex VEVs already at the tree level. We discuss further constraints on the Higgs potential in Sec. VI B.

The phases of all other flavor-blind MSSM parameters are constrained because they feed into colored fermion masses through radiative corrections from the diagrams of Fig. 3. We summarize these constraints as

$$\arg m_{\tilde{g}}, \arg B\mu<10^{-10}, \quad \arg A_0, \arg \mu<10^{-8},$$
$$\arg m_{\tilde{\gamma},\tilde{Z},\tilde{W}}<10^{-7}, \tag{18}$$

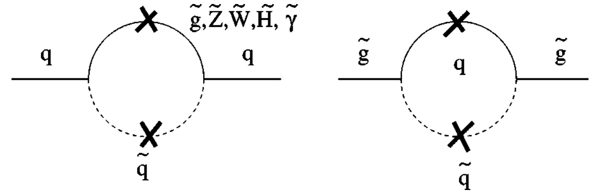

FIG. 3. Lowest-order SUSY diagrams contributing to $\bar{\theta}$. A cross denotes a left-right mass insertion.

where A_0 denotes the proportionality constant of the A terms $A=A_0 Y$. We note that these constraints are much more stringent than the bounds on soft phases from direct contributions to EDMs which only require phases to be smaller than order 10^{-2}.

The diagrams in Fig. 3 also lead to strong constraints on both real and imaginary parts of flavor violating soft masses. The contributions to $\bar{\theta}$ are proportional to traces over flavor violating quantities such as $\mathrm{Im\,tr}[Y_x^\dagger A_x]$, $\mathrm{Im\,tr}[Y_x^{-1} A_x]$, and $\mathrm{Im\,tr}[Y_x^{-1} m_{\tilde{q}}^2 A_x m_{\tilde{x}}^2]$ where $x=u,d$ (see Appendix D). The most natural way to satisfy the bounds on $\bar{\theta}$ is to assume proportionality and degeneracy

$$A_{u,d}\propto Y_{u,d}, \quad m^2_{\tilde{q},\tilde{u},\tilde{d}}\propto 1. \tag{19}$$

Deviations from Eq. (19), parametrized as δA and δm^2, are very constrained (see Appendix D). For generic deviations which are not "aligned" with the Yukawa matrices, the constraints on some of the matrix elements are as strong as

$$\frac{\delta A}{m_0}<10^{-13}, \quad \frac{\delta m^2}{(m_0)^2}<10^{-6}, \tag{20}$$

where m_0 denotes the average superpartner mass scale. These bounds apply at m_0. They require a much higher degree of "flavor blindness" from the mechanism of supersymmetry breaking and mediation than flavor changing neutral current (FCNC) bounds.

Note also that the contributions to $\bar{\theta}$ from loops with flavor violation in superpartner masses do not decouple in the limit of heavy superpartners. Thus the bounds (20) apply equally for heavier superpartners. This is in contrast to the case of FCNCs.

A. Why do we need $M_{CP}>M_{\mathrm{SUSY}}$?

For example, minimal supergravity (MSUGRA) is not compatible with our solution to the strong *CP* problem; this can be seen as follows. The Kaehler potential relevant for squark masses in MSUGRA is

$$\int d^4\theta\, Z_{ij} Q_i^\dagger Q_j + \frac{S^\dagger S}{M_{\mathrm{Pl}}^2} X_{ij} Q_i^\dagger Q_j \tag{21}$$

for the quark SU(2) doublets, and similar terms for the singlets. If we assume a SUSY breaking expectation value for the F component of the superfield S scalar masses result

$$(m^2_{\tilde{q}})_{ij}=(TXT)_{ij}\frac{F^*F}{M^2_{\rm Pl}}, \qquad (22)$$

where $Z=T^{-2}$ as in Sec. III. Of course, we can always work in a basis where $T=1$ at $M_{\rm Pl}$, but in general X will not be proportional to the unit matrix in this same basis. Partial alignment $X\simeq Z$ can be achieved by imposing non-Abelian flavor symmetries [20], but residual nondegeneracies are expected to violate the bounds Eq. (20) by orders of magnitude [21]. This is the usual flavor problem of MSUGRA.

In our scenario, the situation for a SUSY breaking mechanism where superpartner masses are generated at scales above M_{CP} is even worse. This is because the flavor and *CP* violating (*CPX*) dynamics at M_{CP} renormalizes Z and X in Eq. (21) differently. So, even if we had somehow arranged $X=Z$ at $M_{\rm Pl}$, this alignment would be spoiled at scales below M_{CP}.[4] To illustrate this point we give the one-loop renormalization of the right-handed down squark masses in the second model of the previous section. Ignoring all coupling constants except r_{ij}, this is $m^2(\mu)/m^2(M_{\rm Pl})\sim(\mu/M_{\rm Pl})^{r^\dagger r/16\pi^2}$, which is completely nonuniversal when $r\sim 4\pi$.

We conclude that SUSY models with spontaneous *CP* violation require a mechanism of SUSY breaking and mediation in which the scalar masses are generated below M_{CP}. We will therefore discuss gauge mediation as a compatible SUSY breaking mechanism in more detail in the next section.

Of course, any other mechanism of SUSY breaking which generates universal scalar masses at low scales is compatible with our scheme. A preliminary look at gaugino mediation [23] with the *CPX* dynamics at $M_{\rm Pl}$ on the visible sector brane suggests that gaugino mediation is also compatible with the constraints Eq. (20).

VI. RENORMALIZATION OF $\bar{\theta}$ IN GAUGE MEDIATION

In this section we summarize results on the renormalization of $\bar{\theta}$ in the MSSM with gauge mediated supersymmetry breaking (GMSB). That gauge mediation is compatible with solutions to the strong *CP* problem based on spontaneous *CP* violation has been known for some time [24]. We quote here only the most important results, with further details provided in Appendix D.

In GMSB, the superpartner masses arise from loop diagrams involving the SM gauge interactions and messenger particles with SUSY violating masses. The dominant contributions to the scalar masses from these diagrams have loop momenta of order of the messenger mass; at higher energies the scalar masses are power suppressed. Thus by separating the messenger scale (which we have been calling $M_{\rm SUSY}$) and *CPX* scale $M_{CP}>M_{\rm SUSY}$, one can suppress the dangerous renormalization of the scalar masses from the *CPX* sector.

The leading contributions to $\bar{\theta}$ in GMSB can be divided into two classes which we discuss in turn: contributions that arise in the effective theory below M_{CP} from renormalizable interactions and are relatively model independent and contributions from higher-dimensional operators suppressed by the scale M_{CP} which are model dependent but can always be made small by taking $M_{CP}\gg M_{\rm SUSY}$.

At the renormalizable level the only flavor violating couplings in the effective theory below M_{CP} are the Yukawa couplings. The gauge mediated soft SUSY violating masses are approximately given by $A_x=0$ and $m^2_{\tilde{x}}=(m_0)^2$. Using the flavor symmetries one can then show that the renormalization of $\bar{\theta}$ from SUSY breaking can always be written in terms of the Hermitian matrices $h_x=Y_xY_x^\dagger$ in the combination det$[h_u,h_d]$, the Jarlskog invariant (see Appendix A). The leading contribution is $\delta\bar{\theta}\sim 10^{-29}\tan^6\beta$, which is smaller than the leading finite SM renormalization $\delta\bar{\theta}\sim 10^{-19}$ (see Appendix D).

The other class of contributions to $\bar{\theta}$ involves higher-dimensional operators generated from integrating out the strong *CPX* dynamics at M_{CP}. For example,

$$\int d^4\theta\frac{D^\dagger DD^\dagger D}{M^2_{CP}} \quad \text{and} \quad \left(\frac{\alpha_s}{4\pi}\right)^2\frac{X^\dagger X}{M^2_{CP}}D^\dagger D. \qquad (23)$$

Here D is the right-handed down quark superfield and X is the superfield whose F component is the source of supersymmetry breaking. The first of these operators is generated from the *CPX* violating dynamics directly, whereas the second comes from computing the two-loop gauge mediation diagram for scalar masses but restricting the loop momenta to be above the scale M_{CP}. Because of the strongly coupled *CPX* dynamics at M_{CP} the coefficients of these operators are not calculable and are flavor off diagonal. Both operators lead to contributions to δm^2 that are proportional to $(M_{\rm SUSY}/M_{CP})^2$. The bounds given in Eq. (20) then constrain

$$\frac{M_{\rm SUSY}}{M_{CP}}\lesssim 10^{-3}. \qquad (24)$$

Combining this with the fact that the gauge mediation scale is bounded from below by roughly 10^4 GeV, we learn that $M_{CP}\gtrsim 10^7$ GeV, well out of reach of any current or planned accelerator. Both scales are unknown *a priori* so that we cannot predict the size of $\bar{\theta}$.

We discuss implications for superpartner masses and electric dipole moments in Sec. VII. For a brief compilation of values of $\bar{\theta}$ induced by renormalization, see Table I in Sec. VIII.

A. Anomaly mediation

Even though SUSY breaking and mediation are at high scales in anomaly mediation (AMSB) [22], the superpartner masses at the weak scale are determined by supersymmetric

[4]The only known exception to this is anomaly mediation [22], where the special form of supersymmetry breaking proportional to the conformal anomaly enforces $X=Z$ at all scales. Therefore, anomaly mediation works very nicely with our scenario; we briefly discuss it in Sec. VI A.

low-energy couplings. They are ultraviolet insensitive and therefore independent of the *CPX* dynamics. The resulting soft terms are approximately flavor universal, and contributions to $\bar{\theta}$ are similar to the contributions from renormalizable couplings in gauge mediation, negligibly small. A complete model of course requires a solution to the problem of negative slepton masses. Any solution that retains the uv insensitivity and flavor universality is compatible with our framework. A nice example is given by [25].

B. Contributions from higher-dimensional operators

In this section we discuss a number of different corrections to $\bar{\theta}$ which are model dependent. They include higher-dimensional operators in the superpotential, corrections to the gauge kinetic functions, Kaehler potential terms that renormalize the superpotential after SUSY breaking, and higher-derivative operators. All of these operators may arise from quantum gravity dynamics suppressed by the Planck scale, but some may also arise from *CPX* dynamics and are therefore only suppressed by M_{CP}.

(i) *Higher-dimensional operators in the superpotential*. We showed in Sec. III A that in the absence of SUSY breaking the only contributions to $\bar{\theta}$ can come from the superpotential. Therefore the most dangerous couplings are direct couplings of *CP* violating VEVs to the MSSM or any colored fields in the superpotential. In order for our mechanism to work we must assume that there are no such couplings at the renormalizable level. For example, we cannot have the couplings $\Sigma H_u H_d$ or $\Sigma T\bar{T}$. Both of these couplings can easily be forbidden by a symmetry under which Σ transforms and the MSSM fields are neutral. At the nonrenormalizable level we may have

$$\int d^2\theta\left(\frac{\Sigma}{M_{\rm Pl}}\right)^k W_\alpha W^\alpha+\left(\frac{\Sigma}{M_{\rm Pl}}\right)^l \mu H_u H_d + \left(\frac{\Sigma}{M_{\rm Pl}}\right)^m Q\hat{Y}_u U H_u. \qquad (25)$$

Each of these operators, if present, would give a contribution to $\bar{\theta}$ which is proportional to powers of $M_{CP}/M_{\rm Pl}$ and could be important if M_{CP} is large and the exponents k, l, m are small. Again, these superpotential operators are strongly constrained by symmetries and even in the absence of symmetries superpotentials need not be generic because of the nonrenormalization theorems. For example, in our model in Sec. IV B, one can define a U(1) symmetry under which only Σ and $\bar{F}$ are charged and which forbids all these terms.

(ii) *Kaehler potential terms involving SUSY breaking*. Higher-dimensional operators in the Kaehler potential which couple the MSSM fields to fields with SUSY breaking VEVs can give rise to superpotential terms proportional to SUSY breaking. For example, a Kaehler potential term $X^\dagger/M_{\rm Pl}^2 Q H_d D$ with complex coefficient gives rise to a superpotential Yukawa coupling with a coefficient $F/M_{\rm Pl}^2$. The same operator with $M_{\rm Pl}$ replaced by M_{CP} is suppressed by powers of SM gauge couplings over $16\pi^2$ if SUSY breaking and *CPX* dynamics are not strongly coupled to each other. None of these operators are dangerous if the SUSY breaking scale is sufficiently low.

(iii) *Higher derivative Kaehler terms*. Kaehler potential terms involving the covariant derivative D_α suppressed by $M_{\rm Pl}$ or M_{CP} generate effective $d^2\theta$ terms with ordinary derivatives. For example,

$$\int d^4\theta \frac{QH_d(D_\alpha)^2 D}{M_{CP}^2} \rightarrow \int d^2\theta\, QH_d \frac{\Box}{M_{CP}^2} D. \qquad (26)$$

These operators can have different flavor structure from the Yukawa couplings and at one loop give a flavor nonuniversal renormalization of the soft SUSY breaking scalar masses that are suppressed by $(M_{\rm SUSY}/M_{CP})^2$. We find the same bound as from the higher-dimensional operators in Eq. (23): $M_{\rm SUSY}/M_{CP} \lesssim 10^{-3}$.

(iv) *Phases in the SUSY breaking sector*. If Σ couples directly to the SUSY breaking sector, then one has to worry about generating a complex SUSY breaking VEV F. A phase in F contributes directly to $\bar{\theta}$ via the gluino mass. It is easy to see that couplings of Σ in the superpotential of the dynamical SUSY breaking sector lead to complex F. We therefore need to forbid such couplings; this can be arranged in the same way as superpotential couplings of Σ to the MSSM fields can be forbidden. Phases in the Kaehler potential are less dangerous because of the Hermiticity of the Kaehler potential. At tree level and in looking at simple toy models we found $F \propto \det Z_{\rm SUSY}$, which is real. Here, $Z_{\rm SUSY}$ is a wave function renormalization factor in the Kaehler potential of the SUSY breaking sector.

A more general analysis of phases in SUSY breaking sectors including loop corrections is desirable but beyond the scope of this paper. In any case, such phases can always be avoided by separating the SUSY breaking and *CPX* sectors. For example, if the SUSY breaking sector does not carry the global flavor symmetries of Σ, then couplings of Σ to the SUSY breaking sector have to be of the form $\mathrm{Tr}(\Sigma^\dagger\Sigma)$ and are therefore real.

(v) *Phases in the MSSM Higgs sector*. We already showed that phases in μ or $B\mu$ are strongly constrained. $m_{H_u}^2$ and $m_{H_d}^2$ and the supersymmetric quartic couplings are automatically real, but one might worry about phases from higher-dimensional operators in the Kaehler potential for H_u and H_d. For example,

$$\int d^4\theta \frac{c}{M^2} H_u^2 H_u^\dagger H_d + \text{H.c.}, \qquad (27)$$

with complex c and $M=M_{\rm Pl}$ or $M=M_{CP}$ leads to complex phases in the Higgs VEVs which are suppressed by $(M_{\rm weak}/M)^2$. This is harmless even for the lowest possible values of M_{CP} and unsuppressed coupling constant c.

VII. PREDICTIONS

Our framework requires tight constraints on the flavor (and *CP*) structure of the SUSY breaking soft terms which have various testable consequences. We predict [15] the following.

(1) Supersymmetry.

(2) Minimal flavor violation; i.e., there are no significant new sources of flavor violation beyond the Yukawa couplings at energies near the weak scale. This has well-known implications for *B* physics [26,27].

(3) No measurable new *CP* violation in the quark sector beyond the SM, in particular no new *CP* violation in the *B* system. For example, $\sin 2\beta$ is large as in the SM [26]. We might expect the phases in the lepton mixing matrices to be large in analogy with the quarks.

(4) Almost degenerate first and second generation scalars of each gauge quantum number. Generic violations of quark mass universality are very tightly constrained [see Eq. (20)]. However, by aligning squark masses with quark masses

$$m_{\tilde{q}}^2 = m_0^2(1 + c_u Y_u Y_u^\dagger + c_d Y_d Y_d^\dagger),$$
$$m_{\tilde{x}}^2 = m_0^2(1 + c_x Y_x^\dagger Y_x), \tag{28}$$

the renormalization of $\bar{\theta}$ remains small ($<10^{-10}$) as can be seen from Eqs. (D4)–(D7), even though this ansatz allows for more flavor violation than Eq. (19). In this ansatz the (real) coefficients $c_{u,d}$ are not expected to be arbitrarily large since at some point the contribution to third generation superpartners becomes very large. Imposing $c_i < 1/Y_3^2$, where Y_3 denotes the Yukawa of the top, bottom, and tau, gives $\Delta m < 1$ GeV for the difference between the first and second generation scalars. This is a prediction which should be tested at a linear collider. We stress that this degeneracy holds independent of the SUSY breaking mechanism. It follows only from demanding that the radiative corrections to $\bar{\theta}$ not be too large (and a reasonable constraint on the c_i). Note that this also bypasses possible FCNC problems since the resulting off-diagonal squark masses obey

$$\frac{\Delta m_{\tilde{q}}^2}{m_0^2}(12,13,23) \lesssim (V_{ub}V_{cb}^*, V_{ub}V_{tb}^*, V_{cb}V_{tb}^*)$$
$$\lesssim (10^{-5}, 10^{-2}, 10^{-1}) \tag{29}$$

where $\Delta m_{\tilde{q}}^2(ij)$ denotes the mixing between the ith and jth generations. These values are within the experimental bounds [28].

(5) At the renormalizable level, the radiatively induced strong *CP* phase is of the order $\bar{\theta} \simeq 10^{-19}$. However, depending on the model dependent ratios $M_{\rm SUSY}/M_{CP}$ and $M_{CP}/M_{\rm Pl}$, the strong phase $\bar{\theta}$ can be as large as 10^{-10}. Thus, the corresponding hadron electric dipole moments can be close to the experimental bound and might be measured soon [29].

(6) A weak electric dipole moment d_f is a contribution to the five-dimensional operator $d_f(i/2)\bar{f}\sigma_{\mu\nu}\gamma_5 f F^{\mu\nu}$. The EDMs for quarks and leptons arise from one-loop diagrams like Fig. 3 with an external photon attached wherever possible. In the general MSSM with arbitrary phases the experimental bound from the electron EDM $d_e < 1.8\times 10^{-27}$ e cm [2], and from the neutron EDM $d_n < 6.3\times 10^{-26}$ e cm [2] require the phases of μ, A terms, and gaugino masses to be less than 10^{-2}, e.g., [30]. Since the dipole moments are linear in the soft SUSY phases we conclude that the phases which are constrained by Eqs. (18) and (20) give weak quark and lepton EDMs which are at least five orders of magnitude below their experimental bounds. Note that improvements of the experimental EDM limits further strengthen the bounds Eq. (18); thus weak EDMs are always smaller than strong EDMs in our framework.

(7) Large flavor preserving phases in the soft terms with their associated "SUSY *CP* problem" have no place in our framework; see the bounds in Eq. (18). This simplifies in particular the phenomenological analysis of the Higgs potential.

VIII. SUMMARY AND CONCLUDING REMARKS

We presented a new theory of *CP* symmetry with supersymmetry and spontaneous *CP* violation. *CP* symmetry is assumed to break spontaneously and *CP* violation is communicated to the MSSM fields at the scale M_{CP}. SUSY breaking is communicated to the MSSM at the lower scale $M_{\rm SUSY}$. With these ingredients, a natural solution to the strong *CP* problem arises, because at the scale of *CP* violation the strong *CP* phase $\bar{\theta}$ is protected by a nonrenormalization theorem of the unbroken supersymmetry. At lower energies SUSY is broken and the nonrenormalization theorem does not apply, but we showed that the generated $\bar{\theta}$ is much smaller than the experimental bound if SUSY breaking is sufficiently flavor universal. Because of the nonrenormalization theorem at high scales a successful model for the *CP* violating sector needs to ensure $\bar{\theta}=0$ only at the tree level, which is easy to arrange. The CKM phase is generated either at the tree level as in Nelson-Barr models or else at the loop level from wave function renormalization as we proposed in [15].

We have explicitly shown that low-scale gauge mediation with $M_{\rm SUSY} < M_{CP}$ is compatible with our framework, but other mechanisms can also be implemented. A model independent constraint is that SUSY breaking has to be *CP* conserving and either flavor universal or else flavor aligned as in Eq. (28). A summary of values of $\bar{\theta}$ in some theories discussed in this paper is compiled in Table I.

From the low-energy point of view, our theory is the MSSM with minuscule flavor violation and no significant phases beyond those already present in the SM. The only possible deviation from this picture is that higher-dimensional operators may bring the nucleon EDMs into experimental reach. Our proposal requires supersymmetry, and the strong constraints on the superpartner spectrum from the renormalization of $\bar{\theta}$ automatically also nullify the SUSY phases and FCNC problems. We have pointed out many of the testable signatures for *B* physics and collider and nucleon EDM experiments. Note that our proposal does not require light superpartners; by low-scale SUSY breaking we mean that its *mediation* to the MSSM occurs below M_{CP}.

In our paper we have given two explicit examples of *CP* violation, but we stress that our solution to the strong *CP* problem can be incorporated in a much larger class of models because our main tool, the nonrenormalization of $\bar{\theta}$ in

TABLE I. Magnitude of $\overline{\theta}$ from renormalization starting from $\overline{\theta}_{\rm tree}=0$ in some theories discussed in text. Here, SM denotes the standard model and $\rm MSSM_{gen}$ a generic minimal supersymmetric model. In the minimal supersymmetric model denoted as $\rm MSSM_{flav}$, flavor violation is minimal, i.e., not bigger than in the SM. This suppresses large radiative corrections to $\overline{\theta}$ that are present in $\rm MSSM_{gen}$. Note that the MSSM with gauge mediated SUSY breaking (GMSB) belongs to the class $\rm MSSM_{flav}$. The last column corresponds to contributions from higher-dimensional operators (HDO) in GMSB. Now the size of $\overline{\theta}$ depends on the hierarchy between the scale of SUSY breaking $\rm M_{SUSY}$ and the scale of spontaneous CP violation M_{CP}. The last line shows the contributions to $\overline{\theta}$ from renormalization group equation (RGE) running in the SM and $\rm MSSM_{flav}$.

	SM	$\rm MSSM_{gen}$	$\rm MSSM_{flav}$	$\rm HDO_{GMSB}$
$\overline{\theta}$	$\sim 2\times 10^{-19}$	$\frac{\alpha_s}{4\pi}$	$\sim 2\times 10^{-19}$	$\left(\frac{\alpha_s}{4\pi}\right)^2\left(\frac{M_{\rm SUSY}}{M_{CP}}\right)^2$
$\delta\overline{\theta}_{\rm RGE}$	10^{-30}		$10^{-29}\tan(\beta)^6$	

SUSY, is general. It would be interesting to combine our theory of CP with a theory of flavor, e.g., with [19]. This is because a necessary ingredient in our CP violating sectors is flavor violation. Thus there may be elegant models in which both goals are achieved at once. Such a model could also include grand unification.

Finally, we briefly comment on cosmological issues. The spontaneous breaking of CP symmetry leads to the formation of domain walls. Such domain walls are potentially problematic because they can overclose the universe. However, in our theory the scale of CP breaking is sufficiently high that several possible mechanisms (including inflation) exist to avoid this problem. Baryogenesis can occur in a number of different ways, such as CP asymmetrical decays of GUT scale or M_{CP} scale particles, the Affleck-Dine mechanism, or leptogenesis.

ACKNOWLEDGMENTS

We thank Bill Bardeen, Wilfried Buchmüller, Koichi Hamaguchi, David E. Kaplan, Ann Nelson, Michael Peskin, and Tsutomu Yanagida for stimulating questions and discussions. This work was supported by the Department of Energy, Contracts DE-AC03-76SF00515 (G.H.) and DE-AC02-76CH03000 (M.S.).

APPENDIX A: NOTATION

We settle here our notation of quark masses and the CKM mixing matrix $V_{\rm CKM}$:

$$M_u=\mathrm{diag}(m_u,m_c,m_t),$$

$$M_d=\mathrm{diag}(m_d,m_s,m_b), \tag{A1}$$

$$M_u=V_uY_uU_u^\dagger v_u,\quad M_d=V_dY_dU_d^\dagger v_d. \tag{A2}$$

We will also use the normalized mass matrices

$$\hat{M}_u=\mathrm{diag}\left(\frac{m_u}{m_t},\frac{m_c}{m_t},1\right),\quad \hat{M}_d=\mathrm{diag}\left(\frac{m_d}{m_b},\frac{m_s}{m_b},1\right). \tag{A3}$$

Here, the unitary matrices $U_{u,d}$, $V_{u,d}$ diagonalize the Yukawa matrices $Y_{u,d}$, which are given in the basis with canonical kinetic terms:

$$Y_uY_u^\dagger v_u^2=V_u^\dagger M_u^2V_u, \tag{A4}$$

$$Y_dY_d^\dagger v_d^2=V_d^\dagger M_d^2V_d, \tag{A5}$$

$$V_{CKM}=V_uV_d^\dagger. \tag{A6}$$

The amount of weak CP violation in the SM is given by the Jarlskog determinant

$$\det[h_u,h_d]\mathrm{SM}=2iJ(m_t^2-m_c^2)(m_t^2-m_u^2)(m_c^2-m_u^2)\times(m_b^2-m_s^2)(m_b^2-m_d^2)(m_s^2-m_d^2)/v^{12}. \tag{A7}$$

Here, $v=174$ GeV, $h_u=Y_uY_u^\dagger$, $h_d=Y_dY_d^\dagger$, $J=s_{12}s_{13}s_{23}c_{12}c_{13}^2c_{23}\sin\phi_{\rm CKM}$, and $s_{ij}=\sin\phi_{ij}$, $c_{ij}=\cos\phi_{ij}$, where ϕ_{ij} and $\phi_{\rm CKM}$ are the angles and phase of the CKM matrix in the Particle Data Group (PDG) parametrization. Numerically, $J\approx 2\times 10^{-5}$ [2].

APPENDIX B: THE CKM PHASE

In this appendix, we show that the heavy sector has to couple strongly to the SM fermions to yield an $\mathcal{O}(1)$ CKM phase from CP violation in quark kinetic terms. In particular, the ansatz $Z^{-1/2}=1+\varepsilon H$, where H is Hermitian and has order 1 entries, leads to the observed pattern of quark masses, mixing, and CP violation only if the parameter $\varepsilon \gtrsim 1$.

To begin, we note that if the initial Yukawa matrices do not have the right (hierarchical) eigenvalues, then large rescaling is required from the wave function renormalization, which implies $\varepsilon\gtrsim 1$ (we give a proof for this below). Thus, we only have to exclude the possibility that the Yukawa matrices $\hat{Y}$ already have approximately the correct eigenvalues to correspond to the SM quark masses but that the CP phase (and possibly also the mixing angles) are generated from wave function renormalization with small ε. Without loss of generality, we work in a basis in which $\hat{Y}_u\approx M_u/v_u$ is diagonal. It is furthermore general to choose $\hat{Y}_d\approx OM_d/v_d$ where O is a general orthogonal (real) matrix. Finally, since we are concerned only with determining the CKM matrix, we are free to rescale $\hat{Y}_u$ and $\hat{Y}_d$ such that the largest eigenvalue in each is approximately equal to 1.

The CKM matrix is then the unitary transformation between the basis in which the following two matrices h_u,h_d are diagonal:

$$h_u=\frac{1}{\sqrt{Z_Q}}\hat{M}_u\frac{1}{Z_u}\hat{M}_u\frac{1}{\sqrt{Z_Q}},\quad h_d=\frac{1}{\sqrt{Z_Q}}O\hat{M}_d\frac{1}{Z_d}\hat{M}_dO^T\frac{1}{\sqrt{Z_Q}}. \tag{B1}$$

Now we assume that ε is small and show that one cannot generate a sufficient amount of *CP* violation. First, note that $V_{\rm CKM}=O$ if all $Z_i=1$. Anticipating this to still be approximately true when the Z_i differ from 1 perturbatively, we rotate h_d by O so that its unperturbed component is already diagonal. We now have

$$V_{\rm CKM}=V_uO(V_d^O)^\dagger \tag{B2}$$

where V_u diagonalizes h_u and V_d^O diagonalizes

$$h_d^O=\left(O^T\frac{1}{\sqrt{Z_Q}}O\right)\hat{M}_d\frac{1}{Z_d}\hat{M}_d\left(O^T\frac{1}{\sqrt{Z_Q}}O\right). \tag{B3}$$

In order to determine the eigenvalues and unitary matrices V_u and V_d^O, we use standard nondegenerate perturbation theory familiar from quantum mechanics (see, e.g., [31]). First, we parametrize $(Z_Q)^{-1/2}=1+\varepsilon H$ and $Z_{u,d}^{-1}=1+\varepsilon J_{u,d}$. To linear order in ε we have

$$h_u=\hat{M}_u^2+\varepsilon\Delta_u,\quad h_d^O=\hat{M}_d^2+\varepsilon\Delta_d^O, \tag{B4}$$

where

$$\Delta_u=\{H,\hat{M}_u^2\}+\hat{M}_uJ_u\hat{M}_u,$$

$$\Delta_d^O=\{H^O,\hat{M}_d^2\}+\hat{M}_dJ_d\hat{M}_d, \tag{B5}$$

and $H^O=O^THO$. Here, the unperturbed "Hamiltonians" $\hat{M}_u^2$ and $\hat{M}_d^2$ are already diagonal. The perturbed eigenvalues to order ε are then

$$(\hat{M}_u^2)_i+\varepsilon(\Delta_u)_{ii}=(\hat{M}_u^2)_i[1+\varepsilon(2(H_u^O)_{ii}+(J_u)_{ii})], \tag{B6}$$

and a similar expression for the down sector. Thus we see that the renormalizations of individual quark masses are multiplicative; this implies, e.g., that there are no corrections to m_u proportional to m_t. Here, we discovered this property to linear order in ε; it is straightforward to extend this analysis to higher order. We have computed the corrections up to second order and also verified our results numerically without expanding in ε. This verifies our claim that large corrections to masses can only come from nonperturbatively large ε.

The unitary matrices that diagonalize h_u and h_d^O are

$$(V_u)_{ij}=\delta_{ij}+\varepsilon\left.\frac{(\Delta_u)_{ij}}{(\hat{M}_u^2)_i-(\hat{M}_u^2)_j}\right|_{i\neq j}, \tag{B7}$$

$$(V_d^O)_{ij}=\delta_{ij}+\varepsilon\left.\frac{(\Delta_d^O)_{ij}}{(\hat{M}_d^2)_i-(\hat{M}_d^2)_j}\right|_{i\neq j}. \tag{B8}$$

Since contributions to the CKM angles from the different terms above are additive in perturbation theory [i.e., $\Pi_i(1+\varepsilon_i)=1+\Sigma_i\varepsilon_i$], we discuss each of them in turn.

Nonvanishing J_d (contributions from J_u are smaller) in Eq. (B8) lead to complex corrections to the CKM matrix elements of order

$$\delta V_{ub}\sim\varepsilon\frac{m_d}{m_b},\quad \delta V_{cb}\sim\varepsilon\frac{m_s}{m_b},\quad \Delta V_{us}\sim\varepsilon\frac{m_d}{m_s}. \tag{B9}$$

This is most significant for δV_{cb} and gives $\phi_{\rm CKM}\lesssim\varepsilon(m_s/m_b)/V_{cb}\sim\varepsilon$.

The case of nontrivial Z_Q (i.e., nonvanishing H) is slightly more complicated. Assuming that the matrix H has entries of order 1, and choosing the angles in O similar to the experimental values in $V_{\rm CKM}$, we find for the Jarlskog invariant (see Appendix A)

$$J\lesssim2\varepsilon(\theta_{12}\theta_{23}-\theta_{13})\theta_{13}\theta_{12}, \tag{B10}$$

where θ_{ik} are the angles of O in the parametrization of the PDG [2]. We extract $\sin\phi_{\rm CKM}$ by dividing by the angles. This yields the bound

$$\sin\phi_{\rm CKM}\lesssim2\varepsilon|V_{ub}|/|V_{cb}| \tag{B11}$$

which is too small since data imply $\sin\phi_{\rm CKM}\sim\mathcal{O}(1)$.

A comment on the usefulness of our expansion in ε is in order. There are many small parameters in the problem with the potential danger of factors such as m_t/m_u ruining the expansion. We believe that such factors do not occur. This is manifest to order ε from our expressions above, and we have verified it explicitly to second order. Furthermore, extensive numerical study [32] has shown that our results are not affected by higher-order corrections in ε: large departures from canonical kinetic terms are required if we want to generate sufficient CKM *CP* violation from wave function renormalization.

APPENDIX C: STRONG INTERACTIONS AT M_{CP}

The model of Sec. IV B is incomplete because renormalization from the Planck scale to M_{CP} drives the Yukawa coupling r to values that are too small to give sufficient *CP* violation in the quark kinetic terms. The model can be fixed by introducing a new gauge group SU(N) under which $\overline{5}_4$ and F transform in the fundamental representation and 5_4 and $\overline{F}$ are antifundamentals. The superpotential (15) remains invariant. The SU(N) theory has eight flavors (five from $\overline{5}_4+5_4$ and three from $F+\overline{F}$) and its gauge coupling becomes strong in the ir for $N\geqslant3$. The strong gauge interactions then also drive the Yukawa coupling r to large values as can be seen from the sign of the beta function (schematically, ignoring coefficients)

$$16\pi^2\frac{d}{d(\log\mu)}r=r(r^2-g_N^2) \tag{C1}$$

where g_N is the coupling of the new strong SU(N).

At the scale M_{CP}, the F's and 5_4's are massive. Integrating them out leads to noncanonical CP violating kinetic terms for the right-handed down quarks (and lepton doublets), vanishing $\overline{\theta}$, and no new superpotential couplings to all orders in perturbation theory as described in Sec. VI B.

But what about nonperturbative effects that could arise from the strong SU(N) dynamics? These effects can be deduced from Seiberg's solution of supersymmetric QCD [18,33]. Most important here are the matching relations for the strong interaction scale across mass thresholds. After integrating out F's and 5_4's the SU(N) gauge theory is flavorless and confined. Gaugino condensation generates a superpotential $W=\Lambda_{\rm ir}^3=[\Lambda_{\rm uv}^{3N-8}M^5\det(\Sigma)]^{1/N}$ where M and Σ are defined in Sec. IV B, and $\Lambda_{\rm uv/ir}$ is the "QCD" scale of the one-loop SU(N) beta function below/above M_{CP}. This superpotential is complex, but it does not couple to any MSSM fields and is therefore harmless. We should also worry about direct nonperturbative contributions to θ of the GUT SU(5) group. These can be determined from the SU(5) scale matching. The phase of the scale of the SU(5) group at the high scale $\Lambda_{5\rm uv}^{8-N}$ vanishes because of CP invariance. This is the statement that $\theta=0$ at the Planck scale. At lower scales, after integrating out F's and 5_4's, the phase is determined by scale matching: $\Lambda_{5\rm ir}^{8}=\Lambda_{5\rm uv}^{8-N}M^N$. This is also real. At even lower scales the dynamics of the SU(N) theory and the SU(5) are completely decoupled so that no further scale matching for the SU(5) theory is required. This proves that $\overline{\theta}=0$ in the effective supersymmetric theory below M_{CP} even after including nonperturbative dynamics in the strongly coupled SU(N) and the coupling r.

APPENDIX D: RADIATIVELY GENERATED STRONG *CP* PHASE

We start with a discussion of contributions to $\overline{\theta}$ from the renormalization of quark masses in the SM. Corrections can be written as $m=m_0(1+x)$ and we will use $\arg\det(1+x)=\mathrm{Im\,tr}[x]$ for small x. Using the flavor symmetries, it is easy to show that corrections to $\overline{\theta}$ can always be written as the imaginary part of traces over the Hermitian matrices $h_u=Y_uY_u^\dagger$ and $h_d=Y_dY_d^\dagger$ (here we work in the basis with canonical kinetic terms). The lowest-order nonvanishing contribution to $\overline{\theta}$ arises at sixth order in $h_{u,d}$. It is related to the Jarlskog determinant (see Appendix A) by

$$2\,\mathrm{Im\,tr}[h_uh_dh_u^2h_d^2]=\det[h_u,h_d]. \tag{D1}$$

Expressions involving n powers of h arise from diagrams with at least n loops. Alternatively, they arise in a stepwise linear approximation to the RGEs with at least n steps [30]. This defines our power counting: one Higgs loop or one RGE step each gives $h_{u,d}/(16\pi^2)$. Higher orders in n are suppressed and one can show that Eq. (D1) is indeed the trace with the largest imaginary part. But at six loops a cancellation occurs between diagrams where up and down quarks are interchanged because $\mathrm{Im\,tr}[h_uh_dh_u^2h_d^2]+(u\leftrightarrow d)=0$. An extra loop with a photon splits the isospin symmetry. Thus, the RGE induced correction to $\overline{\theta}$ in the SM is [34,30]

$$\theta_{\rm SM}^{\rm RGE}\approx\frac{\alpha}{4\pi}\left(\frac{\Delta t}{16\pi^2}\right)^6\det[h_u,h_d]\mathrm{SM} \tag{D2}$$

which is approximately $\theta_{\rm SM}^{\rm RGE}\approx10^{-30}$ for $\Delta t=\log M_{\rm Pl}/M_Z$.

The largest contribution to $\overline{\theta}$ in the SM arises from the finite and strongly GIM suppressed four-loop cheburashka diagram [35]

$$\theta_{\rm SM}^{\rm finite}=-\frac{7}{9}\frac{\alpha_s}{4\pi}\left(\frac{\alpha W}{4\pi}\right)^2\frac{m_s^2m_c^2}{m_W^4}J\ln\frac{m_t^2}{m_b^2}\ln^2\frac{m_b^2}{m_c^2}\times\left(\ln\frac{m_c^2}{m_s^2}+\frac{2}{3}\ln\frac{m_b^2}{m_c^2}\right), \tag{D3}$$

which gives $\theta_{\rm SM}^{\rm finite}\approx2\times10^{-19}$ using $\alpha_s=0.2$ and $J=2\times10^{-5}$, and is consistent with earlier estimates [34].

In the MSSM, the leading divergent diagrams that renormalize $\overline{\theta}$ cancel because of the SUSY nonrenormalization theorem. However, there are new finite contributions from one-loop quark and gluino mass corrections which involve supersymmetry breaking. The diagrams for gluino and quark mass renormalization are proportional to soft A_x terms and soft masses[5] $m_{\tilde{x}}^2$, $m_{\tilde{q}}^2$ and yield ($x=u,d$)

$$\theta_{\tilde{g}}^A\approx\frac{\alpha_s}{4\pi}\frac{v_x^2}{m_0^3}\mathrm{Im\,tr}[Y_xA_x^\dagger], \tag{D4}$$

$$\theta_{\tilde{g}}^m\approx\frac{\alpha_s}{4\pi}\frac{v_x^3v_y}{m_0^8}\mathrm{Im\,tr}[h_xY_xm_{\tilde{x}}^2Y_x^\dagger m_{\tilde{q}}^2], \tag{D5}$$

and similar expressions for the quark mass contributions,

$$\theta_{\tilde{q}}^A\approx\frac{\alpha_s}{4\pi}\frac{1}{m_0}\mathrm{Im\,tr}[Y_x^{-1}A_x], \tag{D6}$$

$$\theta_{\tilde{q}}^m\approx\frac{\alpha_s}{4\pi}\frac{v_y}{m_0^4v_x}\mathrm{Im\,tr}[Y_x^{-1}m_{\tilde{q}}^2Y_xm_{\tilde{x}}^2]. \tag{D7}$$

Here m_0 is an effective average soft mass, v_x are the Higgs VEVs, and $y\neq x$. The size of the induced $\overline{\theta}$ depends crucially on the flavor structure of the soft breaking parameters. Arbitrary A terms and soft masses can violate the experimental bound on $\overline{\theta}$ by many orders of magnitude. On the other hand, for soft terms that satisfy exact proportionality and degeneracy as in Eq. (19) these contributions to $\overline{\theta}$ vanish. However, proportionality and degeneracy are not stable under renormalization. The RGEs for the soft terms [36] involve products of h_u and h_d. Inserting the renormalized soft masses into the one-loop diagrams Fig. 3, and using arguments very similar to the SM discussion above, one finds [30]

[5]We use the soft Lagrangian as $-\mathcal{L}_{\rm soft}\supset\tilde{Q}A_uH_u\tilde{U}+\tilde{Q}A_dH_d\tilde{D}+(1/2)m_{\tilde{g}}\tilde{g}\tilde{g}+B\mu H_uH_d+\mathrm{c.c.}+\tilde{Q}^\dagger m_{\tilde{q}}^2\tilde{Q}+\tilde{U}^\dagger m_{\tilde{u}}^2\tilde{U}+\tilde{D}^\dagger m_{\tilde{d}}^2\tilde{D}$; see, e.g., [36].

$$\theta_{\rm SUSY}^{\rm RGE} \approx \frac{\alpha_s}{4\pi}\left(\frac{\Delta t}{16\pi^2}\right)\frac{v_u^2}{m_0^2}\tan^6\beta\,\det[h_u,h_d]{\rm SM}. \quad \text{(D8)}$$

This gives $\theta_{\rm SUSY}^{\rm RGE} \approx 10^{-29}$–$10^{-19}$ for $\tan\beta$ ranging from 1 to 50.

Thus in the MSSM with strictly proportional and universal soft terms at a high scale (e.g., $M_{\rm SUSY}$ in GMSB) the contributions from diagrams involving superpartners are smaller than the finite diagram in the SM. Diagrams that are similar to the leading SM contribution Eq. (D3) but involve superpartners or charged Higgs bosons are suppressed by the heavier superpartner and Higgs boson masses and are therefore smaller than Eq. (D3).

Let us work out the constraints on A terms and soft masses if we allow for additional flavor violating contributions. We parametrize the departure from proportionality and degeneracy as δA, δm^2. From Eq. (D6) follows immediately for the A terms

$$\text{Im tr}\left[Y^{-1}\frac{\delta A}{m_0}\right] \leqslant 10^{-8}. \quad \text{(D9)}$$

We need nonuniversality for both soft masses in Eq. (D7) for a nonzero contribution to $\bar{\theta}$. For example, our power counting discussed previously gives $m_{\tilde{q}}^2 \simeq m_0^2[1+h_x/(16\pi^2)]$ and thus

$$\text{Im tr}\left[Y_y^{-1}h_x Y_y\frac{\delta m^2}{m_0^2}\right] \leqslant 10^{-6}. \quad \text{(D10)}$$

These bounds are generally much more severe than the bounds from FCNCs (see, e.g., [28]). The constraints on the smallest elements of δA and δm^2 are quoted in Eq. (20).

[1] For reviews, see M. Dine, hep-ph/0011376; R. D. Peccei, in "*CP* Violation," edited by C. Jarlskog, Report No. DESY-88-109, pp. 503–551.

[2] Particle Data Group, D. E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).

[3] A. Ali and D. London, Eur. Phys. J. C **18**, 665 (2001); M. Ciuchini *et al.*, J. High Energy Phys. **07**, 013 (2001); S. Mele, hep-ph/0103040; A. Hocker *et al.*, hep-ph/0112295.

[4] R. J. Crewther, P. Di Vecchia, G. Veneziano, and E. Witten, Phys. Lett. **88B**, 123 (1979); **91B**, 487(E) (1980); V. Baluni, Phys. Rev. D **19**, 2227 (1979); M. Pospelov and A. Ritz, Nucl. Phys. **B573**, 177 (2000).

[5] P. G. Harris *et al.*, Phys. Rev. Lett. **82**, 904 (1999).

[6] M. V. Romalis, W. C. Griffith, and E. N. Fortson, Phys. Rev. Lett. **86**, 2505 (2001).

[7] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. **38**, 1440 (1977); Phys. Rev. D **16**, 1791 (1977); S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978); F. Wilczek, *ibid.* **40**, 279 (1978).

[8] G. 't Hooft, Phys. Rev. Lett. **37**, 8 (1976).

[9] A. G. Cohen, D. B. Kaplan, and A. E. Nelson, J. High Energy Phys. **11**, 027 (1999); S. R. Sharpe and N. Shoresh, Nucl. Phys. B (Proc. Suppl.) **83**, 968 (2000); UKQCD Collaboration, A. C. Irving *et al.*, Phys. Lett. B **518**, 243 (2001); D. R. Nelson, G. T. Fleming, and G. W. Kilcup, Nucl. Phys. B (Proc. Suppl.) **106**, 221 (2002).

[10] A. Nelson, Phys. Lett. **136B**, 387 (1984); S. M. Barr, Phys. Rev. Lett. **53**, 329 (1984).

[11] M. A. Beg and H.-S. Tsao, Phys. Rev. Lett. **41**, 278 (1978); R. N. Mohapatra and G. Senjanovic, Phys. Lett. **79B**, 283 (1978); R. N. Mohapatra and A. Rasin, Phys. Rev. Lett. **76**, 3490 (1996); C. Hamzaoui and M. Pospelov, Phys. Rev. D **65**, 056002 (2002); B. Holdom, *ibid.* **61**, 011702(R) (2000).

[12] H. Georgi and S. L. Glashow, Phys. Lett. B **451**, 372 (1999).

[13] D. Bowser-Chao, D. Chang, and W. Keung, hep-ph/9811258.

[14] S. L. Glashow, hep-ph/0110178.

[15] G. Hiller and M. Schmaltz, Phys. Lett. B **514**, 263 (2001).

[16] J. Ellis, S. Ferrara, and D. V. Nanopoulos, Phys. Lett. **114B**, 231 (1982).

[17] V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B229**, 381 (1983); **B260**, 157 (1985) [Yad. Fiz. **42**, 1499 (1985)]; N. Arkani-Hamed and H. Murayama, J. High Energy Phys. **06**, 030 (2000).

[18] K. Intriligator and N. Seiberg, Nucl. Phys. B (Proc. Suppl.) **45BC**, 1 (1996).

[19] A. E. Nelson and M. J. Strassler, J. High Energy Phys. **09**, 030 (2000).

[20] P. Pouliot and N. Seiberg, Phys. Lett. B **318**, 169 (1993); D. B. Kaplan and M. Schmaltz, Phys. Rev. D **49**, 3741 (1994); A. Pomarol and D. Tommasini, Nucl. Phys. **B466**, 3 (1996).

[21] M. Dine, R. G. Leigh, and A. Kagan, Phys. Rev. D **48**, 2214 (1993).

[22] L. Randall and R. Sundrum, Nucl. Phys. **B557**, 79 (1999); G. F. Giudice, M. A. Luty, H. Murayama, and R. Rattazzi, J. High Energy Phys. **12**, 027 (1998).

[23] D. E. Kaplan, G. D. Kribs, and M. Schmaltz, Phys. Rev. D **62**, 035010 (2000); Z. Chacko, M. A. Luty, A. E. Nelson, and E. Ponton, J. High Energy Phys. **01**, 003 (2000); M. Schmaltz and W. Skiba, Phys. Rev. D **62**, 095005 (2000); **62**, 095004 (2000).

[24] S. M. Barr and G. Segre, Phys. Rev. D **48**, 302 (1993); S. M. Barr, *ibid.* **56**, 1475 (1997); G. F. Giudice and R. Rattazzi, Phys. Rep. **322**, 419 (1999).

[25] N. Arkani-Hamed, D. E. Kaplan, H. Murayama, and Y. Nomura, J. High Energy Phys. **02**, 041 (2001).

[26] A. J. Buras and R. Buras, Phys. Lett. B **501**, 223 (2001).

[27] G. Buchalla, G. Hiller, and G. Isidori, Phys. Rev. D **63**, 014015 (2001); A. Ali, E. Lunghi, C. Greub, and G. Hiller, hep-ph/0112300.

[28] J. S. Hagelin, S. Kelley, and T. Tanaka, Nucl. Phys. **B415**, 293 (1994); F. Gabbiani, E. Gabrielli, A. Masiero, and L. Silvestrini, *ibid.* **B477**, 321 (1996).

[29] R. Golub and K. Lamoreaux, Phys. Rep. **237**, 1 (1994).

[30] M. Dugan, B. Grinstein, and L. Hall, Nucl. Phys. **B255**, 413 (1985).

[31] For example, C. Cohen-Tannoudji, B. Diu, and F. Laloe, *Quantum Mechanics* (Wiley, New York, 1977).
[32] FORTRAN or MATHEMATICA codes STRONGCKMFITTER 7.58B are available from the authors by request.
[33] M. Graesser and B. Morariu, Phys. Lett. B **429**, 313 (1998).
[34] J. Ellis and M. K. Gaillard, Nucl. Phys. **B150**, 141 (1979).
[35] I. B. Khriplovich, Phys. Lett. B **173**, 193 (1986).
[36] S. P. Martin and M. T. Vaughn, Phys. Rev. D **50**, 2282 (1994).