Molecular states and 1^{-+} exotic mesons

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This work investigates whether the observed 1^{-+} exotic mesons are molecular states. We first use a potential model to calculate the spectra and lifetimes of the $f_0(980)$ and $a_0(980)$, taken to be loosely bound molecular states of $K\bar{K}$, then apply the same scenario to the 1^{-+} exotic states $\pi_1(1400)$ and $\pi_1(1600)$, assuming them to be $\pi\eta(1295)$ and $\pi\eta(1440)$ molecules, respectively. We derive the effective potential in the framework of field theory at the hadronic level. Our results indicate that the present data on $\pi_1(1400)$ and $\pi_1(1600)$ rule out the specific molecular ansatz. We show that the lifetime of a loosely bound heavy-light molecule with enough angular momentum is fully determined by the lifetimes of its constituent mesons.

I. INTRODUCTION

New gluonic matter, such as glueballs and hybrids, are predicted by the general theory of QCD. But after more than three decades of effort, gluonic excitations have not been discovered unambiguously. The discovery of the $J^{PC}=1^{-+}$ isovector resonances $\pi_1(1400)$ and $\pi_1(1600)$ represents great progress along these lines, because it may imply that new gluonic matter has eventually been observed. The E852 Collaboration at BNL observed $\pi_1(1400)$ in $\eta\pi$ in the reaction $\pi^-p\rightarrow\eta\pi^-p$, with a mass and width of $1370\pm16^{+50}_{-30}$ MeV and $385\pm40^{+65}_{-105}$ MeV, respectively [1]. Independently, $\pi_1(1400)$ has been observed in $\eta\pi$ with $M=1400\pm20\pm20$ MeV and $\Gamma=310\pm50^{+50}_{-30}$ MeV [2], or $M=1360\pm25\pm20$ MeV and $\Gamma=220\pm90$ MeV [3] by the Crystal Barrel Collaboration. The other resonance $\pi_1(1600)$ has been observed by E852 in $\rho\pi$ in $\pi^-p\rightarrow\pi^+\pi^-\pi^-p$ with a mass $1593\pm8^{+29}_{-47}$ MeV and a width of $168\pm20^{+150}_{-12}$ MeV [4], in $\eta'\pi$ in $\pi^-p\rightarrow\eta'\pi^-p$ with $M=1597\pm10^{+45}_{-10}$ MeV and $\Gamma=340\pm40\pm50$ MeV [5], and most recently in a preliminary analysis in $b_1\pi$ in $\pi^-p\rightarrow\omega\pi^-\pi^0p$ with $M=1582\pm10\pm20$ MeV and $\Gamma=289\pm16\pm27$ [6]. The $\pi_1(1600)$ is observed decaying to the same final states by VES [7].

A 1^{-+} meson cannot be a meson ($q\bar{q}$), a gluinoball ($\tilde{g}\tilde{g}$) or a squarkball ($\tilde{q}\bar{\tilde{q}}$), because it is J^{PC} exotic [8]. The observed 1^{-+} mesons therefore should be considered as the hybrid or glueball candidates which are predicted by QCD. Since glueballs are isoscalar this possibility is excluded. The $\pi_1(1600)$ is consistent with expectations for a hybrid meson [9,10], while the $\pi_1(1400)$ is not [9,11]. Discovery of a hybrid is exciting, but identifying the observed 1^{-+} as the long-expected gluon excited states needs careful further study. Possible alternative hypotheses about their structure have been discussed [10]. There was an attempt to explain the $\pi_1(1400)$ as a coupled channel effect dynamically generated in $\pi\eta$ and $\pi\eta'$ scattering [12]. Donnachie and one of us (Page) suggested that the peak at 1400 MeV is a consequence of the interference of a 1600 MeV resonance with a nonresonant Deck-type background with the appropriate phase [13]. So far, this scenario has only been argued to account for the E852 data, and has not been extended to the Crystal Barrel data. Also, aspects of the proposed mechanism have been challenged on theoretical grounds [11,14]. However, it has been argued that there is no need for more than one resonance in the 1.3–1.6 GeV mass region when $\eta\pi, \eta'\pi$ and $\rho\pi$ data are considered [15].

Before one can conclude that the observed 1^{-+} resonances are gluon excited states, even more possibilities should be explored. Among the possible alternatives, we explore the hypothesis of molecular structure. For 1^{-+} quantum numbers there may be many plausible meson-meson bound states and their mixtures. Since the binding energy for molecular states is not very large, the sum of the masses of the constituent mesons should be close to the mass of the exotic state. According to the Particle Data Group [16], one of the favorable combinations are that $\pi_1(1400)$ is composed of $\pi(0^{-+})$ and $\eta(1295)(0^{-+})$ with total mass $m_\pi+m_{\eta(1295)}\approx1430$–$1438$ MeV, and $\pi_1(1600)$ is composed of π and $\eta(1440)$ with total mass $m_\pi+m_{\eta(1440)}\approx1535$–$1620$ MeV. The constituent mesons in these molecules have one unit of relative angular momentum ("P wave"). Since $\Gamma_{\eta(1295)}=53\pm6$ MeV and $\Gamma_{\eta(1440)}=50$–80 MeV are not too wide, the $\eta(1295)$ and $\eta(1440)$ build observable bound states with the pion, i.e., they can be constituents of molecular states with total widths of a few hundred MeV. The possibility has also been discussed elsewhere [17]. In this work, we investigate whether the present data on $\pi_1(1400)$ and $\pi_1(1600)$ support this specific molecular hypothesis. Other possibilities for $\pi_1(1400)$, not discussed in this work, are a b_1 and π with no angular momentum be-

tween them ("S wave"), denoted $(b_1\pi)_S$. Other S- and P-wave molecular possibilities are $(f_1\pi)_S$ and $(K^*K)_P$. For $\pi_1(1600)$ the possibilities are $(f_1(1420)\pi)_S$, $(f_1(1510)\pi)_S$, $(\rho\omega)_P$ and $(\rho(1450)\pi)_P$.

We use a potential model to calculate the spectra and lifetimes of $\pi_1(1400)$ and $\pi_1(1600)$, and then compare them with data to determine whether the molecular hypothesis is reasonable. We derive the potential between the constituent mesons by writing down the elastic scattering amplitude of the mesons in momentum space and then carrying out a Fourier transformation with respect to the 3-momentum of the exchanged mesons to obtain the potential in configuration space [18]. Substituting the potential into the Schrödinger equation, we can obtain the binding energy and wave function at the origin (or its slope) of the bound molecular state. The effective couplings are obtained from data by assuming SU(3) flavor symmetry. To account for off-shell effects in the calculation of the amplitude, certain form factors are introduced. We write down the inelastic scattering amplitude of the mesons in momentum space, from which the lifetime is calculated.

To test validity of the mechanism employed, we first apply it to study $f_0(980)$ and $a_0(980)$, which, in some interpretations, are interpreted as molecular states [19]. The molecular possibility has some support from lattice QCD [20]. As discussed above, we derive the potential between K and $\bar{K}$, calculate the spectra of $f_0(980)$ and $a_0(980)$, obtain all necessary parameters and then calculate their lifetimes. The results are in good agreement with data. Thus we are convinced that the approach we adopt is applicable to 1^{-+} if it is a molecular state.

The paper is organized as follows. After this introduction, we present most of the formulation and some technical details in Sec. II. In Sec. III, we report our numerical results and the last section is devoted to our discussion and conclusion.

II. FORMULATION

A. Exchange mesons

Consideration of composites of two quarks and two antiquarks is complicated by the fact that different color neutral combinations are possible. The first possibility is that no subcomponents of the system is color neutral, commonly referred to as a four-quark state. The second possibility is that one quark-antiquark subcomponent is color neutral, forming a meson, and that the same happens to the remaining subcomponent. This composite of two mesons is referred to as a molecule. The latter possibility will be the subject of this work. In the limit of a large number of colors N_c in QCD there are no four-quark states, but only meson-meson molecules [21]. The molecular picture becomes more accurate if the mesons contained in the molecule are separated far from each other relative to the size of the constituent mesons. This would happen when the binding energy of the two mesons is small relative to the quark-antiquark energy within each individual meson (which approximately equals the mass of the meson when the quarks are light). The theoretical calculation

of such a small binding energy of mesons is complicated because the binding energy is the result of delicate cancellations between attractive and repulsive interactions, each of which individually corresponds to much larger energies than the total binding energy. This follows because individual interactions are governed by the QCD scale Λ_{QCD}, which is much larger than the binding energy. The calculation of the binding energy has been done in lattice QCD [20,22], QCD sum rules [23], quark-antiquark pair exchange models [14,24], four-quark potential [19,25], flux-tube [26] and other [27] models, and molecular models with meson exchange [28]. We adopt the latter. Meson exchange models should be most accurate when the binding energy of the constituents is low. This is because in this case the constituents are typically separated far apart, so that the cost that confinement imposes on exchanging colored objects at large distances means that the exchange is more likely to be a color singlet, i.e., a meson. Consideration of a system with low binding energy, i.e., of two mesons separated far apart, yields the result that exchange mesons with a long range would be most important for the calculation of the binding energy. Long range exchange mesons correspond to low mass m_q exchange mesons, taken to be either the π, σ [also called the $f_0(400-1200)$], η, ρ, ω, η', $f_0(980)$, $a_0(980)$ or ϕ in this work. An exchange meson can be neglected when its range $\sim 1/m_q$ is small with respect to the r.m.s. radius r_{rms} of the molecular state, i.e., when $1/m_q r_{rms}$ is small.

The general formalism for meson exchange is reviewed in Appendix A. For the remainder of this section we describe the theoretical formulation used in our calculations.

Because the momentum of the exchanged meson in the t channel is spacelike while it is timelike in the s channel, the propagator is expressed as

$$\frac{1}{q^2-m_q^2} \quad t \text{ channel,} \tag{1}$$

$$\frac{1}{q^2-m_q^2+i\Gamma_m m_q} \quad s \text{ channel,} \tag{2}$$

where the Breit-Wigner form is taken and Γ_m is the total width of the exchanged meson. At vertices, we use the standard strong coupling forms [29] and the proper Clebsch-Gordon coefficients to manifest the isospin structure of the vertices. The couplings take the values extracted from corresponding decay widths. There are three types of decay modes relevant to this work:

$$0^+\rightarrow 0^-0^- \ (S \text{ wave}), \quad 1^-\rightarrow 0^-0^- \ (P \text{ wave}),$$

$$1^-\rightarrow 0^-1^- \ (P \text{ wave}).$$

The decay widths are

$$\Gamma = \frac{l}{(2J+1)} \frac{1}{8\pi m^2}|M|^2, \tag{3}$$

where l is the magnitude of the 3-momentum of the decay product in the c.m. frame, m and J are the mass and internal angular momentum of the decaying meson, and M is the amplitude, with

(i) $|M|^2 = g_1^2,$ for $0^+ \rightarrow 0^- 0^-$,

(ii) $|M|^2 = g_2^2 (p_1 - p_2)^\mu (p_1 - p_2)^\nu$

$$\times \sum_\lambda \epsilon_\mu^{\lambda *} \epsilon_\nu^\lambda, \quad \text{for } 1^- \rightarrow 0^- 0^-,$$

(iii) $|M|^2 = g_3^2 \epsilon_{\alpha\beta\mu\nu} \epsilon_{\alpha'\beta'\mu'\nu'} P^\alpha P^{\alpha'}$

$$\times (p_1 - p_2)^\mu (p_1 - p_2)^{\mu'}$$

$$\times \sum_{\lambda_1} \epsilon_{\lambda_1}^\beta \epsilon_{\lambda_1}^{\beta' *} \sum_{\lambda_2} \epsilon_{\lambda_2}^\nu \epsilon_{\lambda_2}^{\nu' *} \quad \text{for } 1^- \rightarrow 0^- 1^-,$$

$$\tag{4}$$

where g_i is the corresponding strong coupling, P is the 4-momentum of the decaying meson, p_1, p_2 are the momenta of the outgoing mesons, and ϵ_λ is the polarization vector of the vector meson. We note that all the mesons which are connected to the vertex are on their mass shells. Using Eqs. (4), the coupling constant g_i is calculated from the experimental width. If the experimental width is unavailable, g_i is obtained by assuming SU(3) flavor symmetry for the coupling.

Since the propagators are off-shell, we introduce form factors for the effective couplings g_i at the vertices. The usual form factors are expressed as [28,30]

$$\frac{\Lambda^2 - m_q^2}{\Lambda^2 - q^2}, \tag{5}$$

at each vertex with Λ is an adjustable constant which models the off-shell effects at the vertices due to the internal structure of the mesons.

B. Effective potential between two constituent mesons

Following the standard procedure given in Ref. [18] and using the effective vertices, we can write down the scattering amplitude between the two constituent mesons in momentum space and convert it into configuration space by a Fourier transformation. This procedure was used to derive the effective potential between a quark and an antiquark [14]. We consider all pseudoscalar, scalar and vector meson exchanges in t, s and u channel, and indicate explicit expressions for the processes that need to be included in our calculations. In the center of mass (c.m.) frame of the bound state, with

(i) 0^+ the exchanged meson in t channel,

$$V(\mathbf{q}) = -\frac{g_1^2}{\mathbf{q}^2 + m_q^2} \left(\frac{\Lambda^2 - m_q^2}{\Lambda^2 + \mathbf{q}^2} \right)^2 \frac{1}{4 m_1 m_2}$$

$$\times \left[\left(1 - \frac{\mathbf{p}^2}{2 m_1^2} - \frac{\mathbf{p}^2}{2 m_2^2} \right) - (2 \mathbf{p} \cdot \mathbf{q} - \mathbf{q}^2) \left(\frac{1}{4 m_1^2} + \frac{1}{4 m_2^2} \right) \right],$$

$$\tag{6}$$

(ii) 1^- the exchanged meson in t channel,

$$V(\mathbf{q}) = \frac{g_2^2}{\mathbf{q}^2 + m_q^2} \left(\frac{\Lambda^2 - m_q^2}{\Lambda^2 + \mathbf{q}^2} \right)^2 \left(1 + \frac{\mathbf{p}^2}{m_1 m_2} - \frac{\mathbf{p} \cdot \mathbf{q}}{m_1 m_2} \right.$$

$$\left. + \frac{\mathbf{q}^2}{4 m_1 m_2} \right), \tag{7}$$

(iii) 0^+ the exchanged meson in s channel,

$$V(\mathbf{q}) = \frac{g_1^2 (\Lambda^2 - m_q^2)^2}{4 m_1 m_2} \left[\frac{1}{(a'^2 - m_q^2)(a^2 - \Lambda^2)^2} \right.$$

$$- \frac{b' \mathbf{p}^2}{(a'^2 - m_q^2)^2 (a^2 - \Lambda^2)^2} - \frac{2 b \mathbf{p}^2}{(a'^2 - m_q^2)(a^2 - \Lambda^2)^3}$$

$$\left. - \frac{(b^2 - 2b) \mathbf{p}^2}{2 a^2 (a'^2 - m_q^2)(a^2 - \Lambda^2)^2} \right], \tag{8}$$

where we introduce the variables

$$a^2 = (m_1 + m_2)^2, \tag{9}$$

$$b = \frac{(m_1 + m_2)^2}{m_1 m_2}, \tag{10}$$

$$a'^2 = a^2 + \frac{\Gamma_q^2 m_q^2}{a^2 - m_q^2}, \tag{11}$$

$$b' = b \left(1 - \frac{\Gamma_q^2 m_q^2}{(a^2 - m_q^2)^2} \right), \tag{12}$$

where $\mathbf{q}$ is the 3-momentum that is exchanged. The momenta of constituent mesons with mass m_1 or m_2 in the initial state is respectively denoted by $\mathbf{p}$ and $-\mathbf{p}$. After the Fourier transformation, we obtain the potential forms in configuration space:

(i) 0^+ in t channel,

$$V(\mathbf{r}) = -\frac{g_1^2}{16\pi m_1 m_2}\left[\frac{e^{-m_q r}-e^{-\Lambda r}}{r}+\frac{(m_q^2-\Lambda^2)e^{-\Lambda r}}{2\Lambda}-\frac{m_1^2+m_2^2}{2m_1^2 m_2^2}\frac{e^{-m_q r}-e^{-\Lambda r}}{r}\hat{\mathbf{p}}^2+\frac{m_1^2+m_2^2}{2m_1^2 m_2^2}\frac{(m_q^2-\Lambda^2)e^{-\Lambda r}}{2\Lambda}\hat{\mathbf{p}}^2\right]$$

$$+\frac{ig_1^2(\Lambda^2-m_q^2)^2}{2m_1 m_2}[A(m_q r+1)e^{-m_q r}+B(\Lambda r+1)e^{-\Lambda r}+(C-B\Lambda^2)e^{-\Lambda r}r^2]\left(\frac{1}{4m_1^2}+\frac{1}{4m_2^2}\right)\frac{\hat{\mathbf{p}}\cdot\mathbf{r}}{4\pi r^3}-\frac{g_1^2(\Lambda^2-m_q^2)^2}{4m_1 m_2}\left[\frac{e^{-\Lambda r}}{\Lambda}\right.$$

$$+Am_q^2\frac{e^{-m_q r}}{r}+Bm_q^2\frac{e^{-\Lambda r}}{r}+(C-B\lambda^2)m_q^2\frac{e^{-\Lambda r}}{\Lambda}\bigg]\left(\frac{1}{4m_1^2}+\frac{1}{4m_2^2}\right), \tag{13}$$

where $\hat{\mathbf{p}}$ is the momentum operator and

$$A=\frac{1}{(\Lambda^2-m_q^2)^2}, \quad B=-A, \quad C=\frac{m_q^2-2\Lambda^2}{(\Lambda^2-m_q^2)^2},$$

(ii) 1^- in t channel,

$$V(\mathbf{r})=\frac{g_2^2}{4\pi}\left[\frac{e^{-m_q r}-e^{-\Lambda r}}{r}+\frac{(m_q^2-\Lambda^2)e^{-\Lambda r}}{2\Lambda}+\frac{1}{4m_1 m_2}\frac{m_q^2(e^{-m_q r}-e^{-\Lambda r})}{r}+\frac{1}{4m_1 m_2}\frac{(m_q^2-\Lambda^2)\Lambda e^{-\Lambda r}}{2}\right.$$

$$+\frac{1}{m_1 m_2}\frac{e^{-m_q r}-e^{-\Lambda r}}{r}\hat{\mathbf{p}}^2+\frac{1}{m_1 m_2}\frac{(m_q^2-\Lambda^2)e^{-\Lambda r}}{2\Lambda}\hat{\mathbf{p}}^2\bigg]-\frac{ig_2^2(\Lambda^2-m_q^2)^2}{4\pi m_1 m_2}[A(m_q r+1)e^{-m_q r}+B(\Lambda r+1)e^{-\Lambda r}$$

$$+(C-B\Lambda^2)e^{-\Lambda r}r^2]\frac{\hat{\mathbf{p}}\cdot\mathbf{r}}{4\pi r^3}, \tag{14}$$

(iii) 0^+ in s channel,

$$V(\mathbf{r})=\frac{g_1^2(\Lambda^2-m_q^2)^2}{4m_1 m_2}\left[\frac{1}{(a'^2-m_q^2)(a^2-\Lambda^2)^2}-\frac{b'\hat{\mathbf{p}}^2}{(a'^2-m_q^2)^2(a^2-\Lambda^2)^2}-\frac{2b\hat{\mathbf{p}}^2}{(a'^2-m_q^2)(a^2-\Lambda^2)^3}\right.$$

$$-\frac{(b^2-2b)\hat{\mathbf{p}}^2}{2a^2(a'^2-m_q^2)(a^2-\Lambda^2)^2}\bigg]\delta^3(\mathbf{r}), \tag{15}$$

(iv) 0^+ in u channel,

$$V(\mathbf{r})=-\frac{g_1^2(\Lambda^2-m_q^2)^2}{16\pi r m_1 m_2}\left\{\left[\frac{re^{-dr}}{2d(c^2+d^2)}+\frac{e^{-dr}}{(c^2+d^2)^2}+\frac{\cos(cr)}{(c^2+d^2)^2}\right]-\left[\frac{re^{-dr}}{8d^3(c^2+d^2)}+\frac{r^2e^{-dr}}{8d^2(c^2+d^2)}+\frac{re^{-dr}}{2d(c^2+d^2)^2}\right.\right.$$

$$+\frac{e^{-dr}}{(c^2+d^2)^3}-\frac{\cos(cr)}{(c^2+d^2)^3}\bigg]2b_u\hat{\mathbf{p}}^2-\left[\frac{r\sin(cr)}{2c(c^2+d^2)^2}+\frac{2\cos(cr)}{(c^2+d^2)^3}+\frac{re^{-dr}}{4d(c^2+d^2)^2}+\frac{e^{-dr}}{(c^2+d^2)^3}\right]b_u\hat{\mathbf{p}}^2-\left[\frac{re^{-dr}}{2d(c^2+d^2)}\right.$$

$$+\frac{e^{-dr}}{(c^2+d^2)^2}+\frac{\cos(cr)}{(c^2+d^2)^2}\bigg]\frac{(b_u^2-2b_u)}{2a^2}\hat{\mathbf{p}}^2\bigg\}, \tag{16}$$

where the last potential is only displayed in position space for brevity and some small relativistic contributions to it are neglected. Here

$$a_u^2=(m_2-m_1)^2, \quad b_u=\frac{-(m_2-m_1)^2}{m_1 m_2}, \tag{17}$$

and

$$c^2=a_u^2-m_q^2=(m_2-m_1)^2-m_q^2,$$

$$d^2=\Lambda^2-a_u^2=\Lambda^2-(m_2-m_1)^2. \tag{18}$$

Since the mesons are color singlets, we do not need to introduce a "confinement" potential as for the quark-antiquark system. Substituting the potentials into the Schrö-
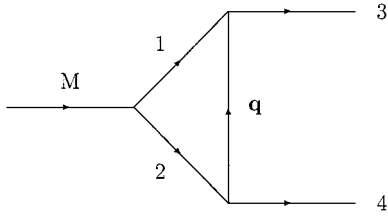
FIG. 1. The decay of meson M. Mesons 1 and 2 are the constituent mesons in the molecular state. Mesons 3 and 4 are the decay products.

dinger equation, we can solve it and obtain the eigenenergy and wave function. As noted previously, our interest is in molecular states with small binding energy. In such a molecule the kinetic energy of the constituent mesons in the bound state is also small, so that relativistic effects can be neglected, and the nonrelativistic Schrödinger equation can be used.

C. Decay width

The decay occurs via the triangle diagram shown in Fig. 1. The case of the t-channel meson exchange is displayed. The s channel is similar. The diagram represents a Bethe-Salpeter (BS) equation where the internal momentum in the loop is integrated out. The two intermediate lines labeled "1" and "2" represent the constituent mesons. In a loosely bound molecule they are very close to their mass shell. Thus, instead of the complicated BS integration, we treat the two constituent mesons as free ones, and they transit into the final states via an inelastic scattering process.

When decay via meson exchange takes place, the amplitude is approximately proportional to the wave function (or its derivative) at the origin. This is due to an integration effect. When we take the BS approach to calculate the amplitude via a loop containing the bound state with an appropriate kernel (the potential we derived above), we immediately find that after integrating out the internal momentum, only a wave function at origin $\psi(0)$ remains when the constituent mesons are in relative S wave. From the formula $|\psi(0)|^2 = 2\mu\langle\psi|(dV/dr)|\psi\rangle$ [31], with μ the reduced mass of the system, it follows that $\psi(0)$ is an average over the entire r region, in other words, effects of all ranges in r are involved in $\psi(0)$. So even though only the wave function at the origin matters, this does not mean that only small distance effects are important. For constituent mesons in P wave we know that the coupling is proportional to the momentum. After integration in the BS equation, it turns into the derivative of the wave function at origin $\psi'(0)$.

Let us briefly discuss the legitimacy of the treatment adopted in the decay calculation. We find that the validity of the approximation used in the BS approach depends on the size of $|\mathbf{p}|/m_q$ being small. From the Heisenberg uncertainty principle, $|\mathbf{p}| \sim 1/r_{rms}$. For a molecular state with small binding energy E_b, the rms radius r_{rms} is relatively large $[E_b \sim 1/(2\mu r_{r.m.s.}^2)]$. Thus $|\mathbf{p}|/m_q$ is small as expected.

The decay can be realized via t and s channels and their contributions must be summed up. For $0^- 0^- \to 0^- 0^-$ scattering, exchanging

(i) 0^+ in t channel,

$$\mathcal{M}_t = \frac{g_1^2}{q^2 - m_q^2}\left(\frac{\Lambda^2 - m_q^2}{\Lambda^2 - q^2}\right)^2, \tag{19}$$

(ii) 1^- in t channel,

$$\mathcal{M}_t = g_2^2\left(\frac{\Lambda^2 - m_q^2}{\Lambda^2 - q^2}\right)^2 (p_1 + p_3)^\mu (p_2 + p_4)^\nu D_{\mu\nu}(q)$$

$$= -\frac{gg'}{q^2 - m_q^2}\left(\frac{\Lambda^2 - m_q^2}{\Lambda^2 - q^2}\right)^2$$

$$\times\left[\frac{2M^2 - m_1^2 - m_2^2 - m_3^2 - m_4^2}{2} + m_1\sqrt{m_4^2 + l^2}\right.$$

$$\left. + m_2\sqrt{m_3^2 + l^2} + \frac{(m_1^2 - m_3^2)(m_2^2 - m_4^2)}{m_q^2}\right], \tag{20}$$

(iii) 0^+ in s channel,

$$\mathcal{M}_s = \frac{g_1^2(q^2 - m_q^2 - i\Gamma_q m_q)}{(q^2 - m_q^2)^2 + \Gamma_q^2 m_q^2}\left(\frac{\Lambda^2 - m_q^2}{\Lambda^2 - q^2}\right)^2, \tag{21}$$

where m_3 and m_4 are the masses of the outgoing mesons in Fig. 1. Besides these three amplitudes, there are those corresponding to $0^- 0^- \to 0^- 1^-$. We omit their explicit expressions.

The decay width is

(i) $\Gamma_{0^+} = \frac{|\psi(0)|^2 l}{16\pi m^3}\left|\sum_{i=t,s,u}\mathcal{M}_i\right|^2$ for 0^+ decay, $\tag{22}$

(ii) $\Gamma_{1^-} = \frac{|\psi'(0)|^2 l}{48\pi m^5}\left|\sum_{i=t,s,u}\mathcal{M}_i\right|^2$ for 1^- decay, $\tag{23}$

where i is summed over the s-, t- and u- channel contributions. $l = (m^4 + m_3^4 + m_4^4 - 2m^2 m_3^2 - 2m^2 m_4^2 - 2m_3^2 m_4^2)^{1/2}/(2m)$ is the 3-momentum of the outgoing mesons in the c.m. frame.

III. NUMERICAL RESULTS

Numerical results are obtained by including the σ as an exchanged scalar meson, and the ω, ρ and ϕ as exchanged vector mesons. There is no low-lying isovector scalar meson, and the exchange of the low-lying pseudoscalar mesons π, η and η' is forbidden by parity. Vector meson exchange in s channel is not allowed by conservation of charge conjugation. For the $\pi_1(1400)$ and $\pi_1(1600)$ systems we also incorporated $f_0(980)$ and $a_0(980)$ exchange.

A. $f_0(980)$ and $a_0(980)$

First we consider $f_0(980)$ and $a_0(980)$ which are taken as bound states of $K\bar{K}$. Since all other parameters are fixed by experimental data and SU(3) flavor symmetry, only the off-shellness parameter Λ is a free parameter. We do not intend to determine its value by fitting the mass spectra of $f_0(980)$ and $a_0(980)$. In a reasonable range of 1 to 10 GeV, we choose a value of Λ which gives reasonable predictions. We have

$$\Lambda = 3.4 \ \text{GeV}.$$

The resultant masses are

$$M_{f_0(980)} = 1013 \ \text{MeV}, \quad M_{a_0(980)} = 996 \ \text{MeV}$$

without zero energy,

$$M_{f_0(980)} = 999 \ \text{MeV}, \quad M_{a_0(980)} = 982 \ \text{MeV}$$

with zero energy,

and should be compared to the experimental data [16]

$$M_{f_0(980)} = 980 \pm 10 \ \text{MeV}, \quad M_{a_0(980)} = 984.8 \pm 1.4 \ \text{MeV}.$$

The calculated values are fairly near to the experimental values, given that Λ is the only free parameter we have for the $K\bar{K}$ system. There is always the possibility to add an additional constant term to the potentials that we derived, called the "zero energy." The term ultimately arises from renormalization when nonperturbative QCD is matched to a meson exchange model. In our final mass estimate we included a zero energy so that the calculated values are in closer agreement with experiment.

The central experimental value of $M_{a_0(980)}$ is a bit larger than that of $M_{f_0(980)}$, but our result is inverted. Our results may be changed as follows. The $a_0(980)$ receives no contribution from s channel σ exchange due to isospin conservation. However, the $f_0(980)$ does. In the case of the $f_0(980)$, the σ intermediate meson constructively interferes with t-channel exchange. However, obtaining coupling constants from widths, as we do, means that we cannot fix the sign of the amplitude. So there is the possibility that the amplitude has an additional -1 between the two channels. This may well change the results. Moreover, if we choose different values of Λ for $f_0(980)$ and $a_0(980)$ and fine-tune other parameters, we can substantially improve our fit of the spectra. Since our aim is to obtain a general picture with the least parameters, we shall not pursue these further refinements. Because parameter changes allow considerable change in binding energy, we do not view our calculated binding energy as accurate. Our concern is with an overall understanding of the spectra and lifetimes, in order to draw qualitative conclusions. The details and subtle deviations are not important.

For both $f_0(980)$ and $a_0(980)$

$$|\psi(0)|^2 = 1.2 \times 10^{-3} \ \text{GeV}^3,$$

showing that the states are loosely bound. Substituting this value into Eq. (22), we obtain

$$\Gamma(f_0(980) \to \pi\pi) = 108 \ \text{MeV}$$

$$s \ \text{and} \ t \ \text{channels interfere constructively,} \tag{24}$$

$$\Gamma(f_0(980) \to \pi\pi) = 86.5 \ \text{MeV}$$

$$s \ \text{and} \ t \ \text{channels interfere destructively,} \tag{25}$$

$$\Gamma(a_0(980) \to \pi\eta) = 36.3 \ \text{MeV}. \tag{26}$$

We display values corresponding to the possibilities of the s and t channels interfering without or with an additional sign between the two channels.

The data indicate that the modes $f_0(980) \to \pi\pi$ and $a_0(980) \to \pi\eta$ are dominant, i.e., these partial widths can approximately be taken as the total widths. The experimental data are

$$\Gamma(f_0(980)) = 40\text{--}100 \ \text{MeV},$$

$$\Gamma(a_0(980)) = 50\text{--}100 \ \text{MeV}.$$

The values we calculated were not fitted to these data, so that the agreement with experiment is noticible.

B. $\pi_1(1400)$ and $\pi_1(1600)$

We now calculate the spectra and the decay widths of the 1^{-+} exotic states, assuming them to be molecular states. As discussed above, we postulate that $\pi_1(1400)$ and $\pi_1(1600)(1^{-+})$ are molecular states of $\pi\eta(1295)$ and $\pi\eta(1440)$. With the potential derived in Sec. II, we obtain the following results.

For $\pi_1(1400)$

$$\Lambda = 1.17 \ \text{GeV}, \quad E_b = -78 \ \text{MeV},$$

$$M_{\pi\eta(1295)} = 1354 \ \text{MeV}, \quad |\psi'(0)|^2 = 3.95 \times 10^{-5} \ \text{GeV}^5.$$

For $\pi_1(1600)$

$$\Lambda = 1.35 \ \text{GeV}, \quad E_b = -14 \ \text{MeV},$$

$$M_{\pi\eta(1440)} = 1521\text{--}1591 \ \text{MeV},$$

$$|\psi'(0)|^2 = 4.18 \times 10^{-5} \ \text{GeV}^5.$$

By contrast to the case for $f_0(980)$ and $a_0(980)$, there are several decay modes for the exotic states. We can only evaluate a few exclusive modes and later by a general argument, we can obtain the order of magnitude of their lifetimes. Below we list several partial widths:

$$\Gamma(\pi_1(1400) \to \pi\eta) = 5.28 \times 10^{-2} \ \text{MeV},$$

$$\Gamma(\pi_1(1400) \to \pi\eta') = 6.80 \times 10^{-2} \ \text{MeV},$$

$$\Gamma(\pi_1(1400) \to \pi^-\rho^0) = 2.86 \times 10^{-2} \ \text{MeV},$$

$$\Gamma(\pi_1(1600) \to \pi\eta) = 4.14 \times 10^{-2} \ \text{MeV},$$

$$\Gamma(\pi_1(1600) \to \pi\eta') = 6.56 \times 10^{-2} \ \text{MeV},$$

$$\Gamma(\pi_1(1600) \to \pi^-\rho^0) = 3.70 \times 10^{-2} \ \text{MeV}, \tag{27}$$

where the calculation was performed for $\pi_{1(\pi^-\eta(1295))}(1400)$ and $\pi_{1(\pi^-\eta(1440))}(1600)$.

In our model, we can estimate the relative decay widths of $\pi_1(1600)$ to $\pi b_1, \pi f_1$ and $\pi\rho$. The difference is due to the effective couplings and the form factor parameter Λ. With isospin symmetry, we have

$$\pi b_1 : \pi f_1 : \pi\rho \approx 2.6 : 2 : 1.$$

One hence obtains various branching ratios for different decay modes, but one can conclude that with the hypothesis that the 1^{-+} exotic states are molecular states, $\pi\eta, \pi\eta', \pi\rho, \pi b_1$ and πf_1 are of the same order of magnitude.

IV. DISCUSSION AND CONCLUSION

In this work we consider molecules of two constituent mesons with meson exchange. We argued in Sec. II A that the picture becomes more accurate if the binding energy of the constituent mesons is small compared to their masses. We also argued that low mass exchanges dominate in a meson exchange model if the binding energy is small compared to the exchange meson mass. In Sec. II B we argued that small binding energy enables the use of a nonrelativistic Schrödinger equation. In Sec. II C we argued that the decay formalism is valid when the binding energy is small with respect to the mass of the exchange meson. To summarize, our formalism should be valid when the binding energy is small relative to the masses of the constituent and exchange meson masses. The lowest mass constituent meson considered in this work is the π, and the lowest mass meson exchanged is found to be the σ.

We investigate the possibility that the observed 1^{-+} exotic states $\pi_1(1400)$ and $\pi_1(1600)$ are molecular states.

Our strategy is to use the potential derived from field theory to calculate the spectra and partial decay widths and see if they can fit the measured data. We derive the potential by calculating the elastic scattering amplitude between the constituent mesons and then carrying out a Fourier transform to convert the amplitude into the potential in configuration space. In our calculations, we obtain the effective couplings from data by assuming SU(3) flavor symmetry. Only the free parameter Λ, which models the off-shell effects at the effective vertices, needs to be determined. The inelastic scattering amplitude in momentum space is used to calculate the lifetimes.

To check the plausibility and validity of the approach, we study $f_0(980)$ and $a_0(980)$ which are commonly considered as molecular states of $K\bar{K}$. The mass spectra and lifetimes of $f_0(980)$ and $a_0(980)$ are in qualitative agreement with data. Thus we are convinced that the potential approach is applicable for evaluating the approximate mass spectrum and lifetimes of molecular states.

With the same scheme, we calculate the decay widths of the observed 1^{-+} states $\pi_1(1400)$ and $\pi_1(1600)$. We assume them to be molecular states of $\pi\eta(1295)$ and $\pi\eta(1440)$, respectively. By fitting the spectra of $\pi_1(1400)$ and $\pi_1(1600)$, we fix the Λ parameter and obtain $|\psi'(0)|^2$ which plays a crucial role in the decay width calculation.

The value of $|\psi'(0)|^2$, and hence the calculated decay widths, are 2–3 orders of magnitude smaller than needed for consistency with the data. This is the main new observation of this work. To understand this we estimate the ratio which governs the width of a P-wave molecule relative to that of an S-wave molecule [Eqs. (22), (23)]

$$R = \frac{|\psi'_P(0)|^2/m^2}{|\psi_S(0)|^2}, \tag{28}$$

where the subscripts P and S stand for the P and S waves, respectively, and m is the mass of the molecular state (whose existence can be understood from dimensional analysis). As a guide, we note that for a Coulomb potential

$$\frac{\psi'_P(0)}{\psi_S(0)} \propto \mu\alpha,$$

where μ is the reduced mass of the system and α is the Coulomb coupling. For the "heavy-light" 1^{-+} molecular states consisting of one heavy meson and one light meson assumed in this work, the reduced mass is close to the mass of the light meson π, which is much smaller than the mass of $\pi_1(1400)$ or $\pi_1(1600)$ (which is close to the mass of the heavy meson). Thus there is a suppression factor $(\mu/m)^2 < 0.01$ in the ratio R. Since the decay widths of $f_0(980)$ and $a_0(980)$ are proportional to $|\psi_S(0)|^2$, but the decay widths of 1^{-+} molecular states (if they exist) are proportional to $|\psi'_P(0)|^2/m^2$, one can easily see that a factor of $10^{-3} - 10^{-2}$ would suppress the 1^{-+} decays.

There is a close correspondence between the exchange mesons we used, and those used in nucleon-nucleon interaction models, e.g., the phenomenologically successful Bonn potential [32]. The exchange mesons are the same except for two cases. First, the η and η' are not employed in the Bonn potential because their couplings to the nucleon are small. (For our molecules, these exchanges are not allowed by parity.) Secondly, the $f_0(980)$ and ϕ are not considered in the Bonn potential because their large $s\bar{s}$ content implies Okubo-Zweig-Iizuka (OZI) rule violating couplings with the nucleon. For our calculations, these mesons have to be considered because the $K\bar{K}$ molecule has OZI rule allowed couplings to $s\bar{s}$ exchange mesons. The $\pi\eta(1295)$ and $\pi\eta(1440)$ molecules can also experience OZI rule allowed $f_0(980)$ exchange, because the $f_0(980)$ is not thought to be an almost pure $s\bar{s}$ state. Unlike the Bonn potential, we do not consider higher order meson exchange in this first orientation.

In our derivations and numerical computations, there are some uncertainties from both experimental and theoretical

sides. For example, the mass of σ meson ranges from 400 to 1200 MeV and its decay width is also uncertain. The model has obvious flaws due to introduction of phenomenological parameters and ignoring the processes at quark level. Therefore we can never expect very accurate predictions within this framework. If an additional factor of $2-3$ exists in the results, it is not surprising. However, since all derivations are based on the quantum field theory in a well motivated approximation, and parameters are fixed by fitting data, order of magnitude estimates should be valid. The only conclusion we can expect is whether the present data can tolerate the molecular state scenario for $\pi_1(1400)$ and $\pi_1(1600)$ within experimental errors.

Since the dissociation rates of the 1^{-+} molecular states calculated in this work are two to three orders of magnitude smaller than the data, the lifetimes of the molecular states are fully determined by the lifetimes of the constituent mesons [π, $\eta(1295)$ and $\eta(1440)$ resonances]. They are not comparable to the measured values of the $\pi_1(1400)$ and $\pi_1(1600)$ lifetimes. So we conclude that the possibility of $\pi_1(1400)$ and $\pi_1(1600)$ respectively being loosely bound $\pi\eta(1295)$ or $\pi\eta(1440)$ molecular states is ruled out by the data.

The decay formalism shows that the dissociation rate of a heavy-light molecule with a binding energy much smaller than the masses of the constituent mesons becomes smaller the higher the relative angular momentum between the constituent mesons is. The explicit calculation suggests that the dissociation rate is already negligible for a P-wave molecule. We hence expect the lifetime of a molecule with one or more units of relative angular momentum to be fully determined by the lifetimes of its constituent mesons. Contrary to the usual expectation for molecules [33], such a molecule will have negligible ''fall-apart'' decay by dissociation to the two mesons that it is composed of. Instead, it will decay via the decay of its constituent mesons. A constituent meson will emit at least two particles in its decay implying that the decay of the molecule is dominated by its decay to three or more particles. This is an unusual decay pattern.

If an experimental state is found with a lifetime equal to the lifetimes of its constituents, a probable explanation is that it is a molecule which has a sufficiently low binding energy so that decay due to dissociation is small. If the constituent mesons need to be in P wave to obtain the desired quantum numbers of the state, one would usually expect a lower-lying S-wave state with different quantum numbers and an increased dissociation rate. This could make the S-wave state so broad that it would be difficult to observe. We hence predict the possibility of longer living orbitally excited molecular states.

APPENDIX: GENERAL FORMALISM

For interactions among mesons, the early theory is the linear σ model and the SU(2) Lagrangian is [34]

$$L = \frac{1}{2}[(\partial_\mu S)^2 - 2m^2 S^2] + \left(\frac{v+S}{2}\right)^2\left[(\partial_\mu \vec{\phi})^2 + \frac{(\vec{\phi}\cdot\partial\vec{\phi})^2}{v^2 - \vec{\phi}^2}\right]$$
$$- \lambda v S^3 - \frac{\lambda}{4}S^4 + \cdots, \tag{A1}$$

where

$$S = \sqrt{\sigma^2 + \vec{\pi}^2} - v, \quad \vec{\phi} = \frac{v\vec{\pi}}{\sqrt{\sigma^2 + \vec{\pi}^2}}, \quad v = \sqrt{\frac{m^2}{\lambda}},$$

and $\vec{\pi}$ and σ are respectively the pion and σ-meson fields.

Taking into account the interaction with vector mesons, one replaces the differentiation symbols with covariant differentiation forms. The effective chiral Lagrangian for pseudoscalar and vector mesons is [35]

$$L = \frac{F^2}{16}\text{Tr}(\nabla_\mu U \nabla^\mu U^\dagger) + \frac{1}{4}m_0^2 \text{Tr}(V_\mu V^\mu), \tag{A2}$$

where the vector meson fields are

$$V_\mu = \lambda^a V_\mu^a = \sqrt{2}\begin{pmatrix} \frac{\rho_\mu^0}{\sqrt{2}} + \frac{\omega_\mu}{\sqrt{2}} & \rho_\mu^+ & K_\mu^{*+} \\ \rho_\mu^- & -\frac{\rho_\mu^0}{\sqrt{2}} + \frac{\omega_\mu}{\sqrt{2}} & K_\mu^{*0} \\ K_\mu^{*-} & \bar{K}_\mu^{*0} & \phi_\mu \end{pmatrix} \tag{A3}$$

with $\lambda^a(a=1,\ldots,8)$ the SU(3) generators. The SU(3) differentiation forms are

$$\nabla_\mu U = \partial_\mu U - ir_\mu U + iUl_\mu,$$

$$\nabla_\mu U^\dagger = \partial_\mu U^\dagger - il_\mu U^\dagger + iU^\dagger r_\mu, \tag{A4}$$

where $l_\mu = v_\mu + a_\mu$ and $r_\mu = v_\mu - a_\mu$ are linear combinations of the external vector v_μ and axial vector a_μ fields. Also

$$U = \exp(i\lambda^a\Phi^a), \tag{A5}$$

with Φ^a the fields of the pseudoscalar meson octet.

From recent studies, the σ meson may be an independent

field, so that we retain the interaction in Eq. (A1). In our final expression, we keep the leading σ-related terms and the leading terms in the pseudoscalar-vector interactions, while we ignore all axial vectors because they are much heavier. We also extend the Lagrangian to include the isoscalars $\eta(1295)$ and $\eta(1440)$. The much simplified Lagrangian reads

$$L_{eff} = g\,\sigma PP + g'_{ijk} v^i_\mu P^j \partial^\mu P^k + \cdots, \qquad (A6)$$

where i,j,k guarantees a proper SU(2) combination, and the ellipsis denotes the smaller contributions which are neglected in our calculations. We note that the symmetry reduces from SU(3) to SU(2).

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