Polarized deep inelastic diffractive *ep* scattering: Operator approach

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Polarized inclusive deep-inelastic diffractive scattering is dealt with in a quantum field theoretic approach. The process can be described in the general framework of nonforward scattering processes using the light-cone expansion in the generalized Bjorken region applying the generalized optical theorem. The diffractive structure functions $g_1^{D(3)}$ and $g_2^{D(3)}$ are calculated in the twist-2 approximation and are expressed by diffractive parton distributions, which are derived from pseudoscalar two-variable operator expectation values. In this approximation the structure function $g_2^{D(3)}$ is obtained from $g_1^{D(3)}$ by a Wandzura-Wilczek relation similar as for deep inelastic scattering. The evolution equations are given. We also comment on the higher twist contributions in the light-cone expansion.

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I. INTRODUCTION

Unpolarized deep inelastic diffractive lepton-nucleon scattering was observed at the DESY electron-proton collider HERA some years ago [1]. In the region of hard diffractive scattering this process is described by structure functions which are represented by diffractive parton distributions. They depend on two scaling variables x and $x_{\mathbb{P}}$ and are different from the parton densities of deep inelastic scattering. New diffractive parton densities are expected to occur in polarized deep inelastic diffractive lepton nucleon scattering. They can be measured at potential future polarized ep facilities capable of probing the kinematic range of small x, cf. [2]. Dedicated future experimental studies of this process can reveal the helicity structure of the nonperturbative color-neutral exchange of diffractive scattering with respect to the quark and gluon structure and how the nucleon spin is viewed under a diffractive exchange. At short distances the problem can be clearly separated into a part, which can be described within perturbative QCD, and another part which is thoroughly nonperturbative. In this paper we use the light-cone expansion to describe the process of polarized diffractive deep-inelastic scattering similar to a recent study for the unpolarized case [3]. While the scaling violations of the process can be calculated within perturbative QCD, the polarized diffractive two-variable parton densities are nonperturbative and can be related to expectation values of (non)local operators. Their Mellin moments with respect to the variable $\beta = x/x_{\mathbb{P}}$ may, in principle, be calculated on the lattice and one may try to understand the ratios of these moments and those for the related deep-inelastic process with respect to their scaling violations as being measurable in future experiments.

In this paper we describe the process of polarized deep-inelastic diffractive scattering, which is a nonforward process in its hadronic variables, at large spacelike momentum transfer q^2. In this approach there is no need to refer to any specific mechanism of color-singlet exchange. It is completely sufficient to select the process by a rapidity gap between the final state proton and the other diffractively produced hadrons, which is sufficiently large. The operator formulation allows straightforwardly the description of also higher twist operators in the light cone expansion, which is potentially more involved in other scenarios [4], to which we agree on the level of twist-2.

We first derive the Lorentz structure of the process for the general kinematics, before we specify to the case of $-t = -(p_2 - p_1)^2, M^2 \ll -q^2$ which is often met in experiment. The diffractive parton densities are derived on the level of the twist-2 operators. In this approximation the scattering cross sections are described by two structure functions $g_1^{D(3)}(x,Q^2,x_{\mathbb{P}})$ and $g_2^{D(3)}(x,Q^2,x_{\mathbb{P}})$ for pure electromagnetic scattering.[1] Also in the present case it turns out that the structure functions are related by the Wandzura-Wilczek relation [7]. Analogously to the unpolarized case, Ref. [3], the anomalous dimensions ruling the evolution of the polarized diffractive parton densities turn out to be those for deep-inelastic forward scattering.

II. LORENTZ STRUCTURE

The process of deep-inelastic diffractive scattering is described by the diagram in Fig. 1. The differential scattering cross section for single-photon exchange is given by

$$d^5\sigma_{\rm diffr} = \frac{1}{2(s-M^2)}\frac{1}{4}dPS^{(3)}\sum_{\rm spins}\frac{e^4}{Q^2}L_{\mu\nu}W^{\mu\nu}. \quad (1)$$

Here $s=(p_1+l)^2$ is the c.m.s. energy of the process squared and M denotes the nucleon mass. The phase space $dPS^{(3)}$ depends on five variables since one final state mass varies. They can be chosen as Bjorken $x = Q^2/(W^2+Q^2-M^2)$, the

[1] The exchange of electro-weak gauge bosons requires at least five structure functions [5]. QED radiative corrections to the process were given in [6].

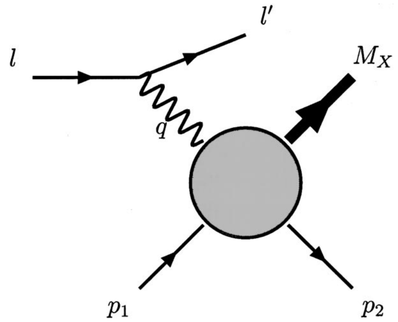

FIG. 1. The virtual photon-hadron amplitude for diffractive ep scattering.

photon virtuality $Q^2=-q^2, t=(p_1-p_2)^2$, a variable describing the nonforwardness with respect to the incoming proton direction,

$$x_{\mathrm{P}}=-\frac{2\eta}{1-\eta}=\frac{Q^2+M_X^2-t}{Q^2+W^2-M^2}\geqslant x, \tag{2}$$

demanding $M_X^2>t$ and where

$$\eta=\frac{q\cdot(p_2-p_1)}{q\cdot(p_2+p_1)}\epsilon\left[-1,\frac{-x}{2-x}\right], \tag{3}$$

and Φ the angle between the lepton plane $\mathbf{p}_1\times\mathbf{l}$ and the hadron plane $\mathbf{p}_1\times\mathbf{p}_2$,

$$\cos\Phi=\frac{(\mathbf{p}_1\times\mathbf{l})\cdot(\mathbf{p}_1\times\mathbf{p}_2)}{|\mathbf{p}_1\times\mathbf{l}||\mathbf{p}_1\times\mathbf{p}_2|}. \tag{4}$$

$W^2=(p_1+q)^2$ and $M_X^2=(p_1+q-p_2)^2$ denote the hadronic mass squared and the square of the diffractive mass, respectively. The process of hard diffractive scattering is characterized by a large rapidity gap of the order $\Delta y \sim \ln(1/x_{\mathrm{P}})$ [8]. As we will show below it is *this property* which is sufficient for our treatment below and no reference to a special kind of a nonperturbative color-neutral exchange is needed.[2]

Unpolarized deep inelastic diffractive scattering was considered in a previous paper [3] in detail. Here we focus on the polarized part only, which can be measured in terms of a polarization asymmetry:

$$A(x,Q^2,x_{\mathrm{P}},S_\mu)=\frac{d^5\sigma(S_\mu)-d^5\sigma(-S_\mu)}{d^5\sigma(S_\mu)+d^5\sigma(-S_\mu)}. \tag{5}$$

S_μ is the spin vector of the incoming proton with $S=S_1$ and $S\cdot p_1=0$. Since the cross sections are linear functions in the initial-state state proton spin-vector, the denominator projects on the even part and the numerator on the odd part in S_μ.

We consider the case of single photon exchange, which is projected by the polarized contribution

$$L_{\mu\nu}^{\mathrm{pol}}=2i\varepsilon_{\mu\nu\rho\sigma}l^\rho q^\sigma \tag{6}$$

to the leptonic tensor. Since the electromagnetic current is conserved, the strong interactions conserve parity and are even under time reversal,[3] and the Hadronic tensor has to be hermitic due to Eq. (6), the following relations hold [12]:

Current conservation: $q^\mu W_{\mu\nu}(q,p_1,S_1,p_2,S_2)$

$$=W_{\mu\nu}(q,p_1,S_1,p_2,S_2)\,q^\nu=0, \tag{7}$$

P invariance: $W_{\mu\nu}(\overline{q},\overline{p}_1,-\overline{S}_1,\overline{p}_2,-\overline{S}_2)$

$$=W^{\mu\nu}(q,p_1,S_1,p_2,S_2), \tag{8}$$

T invariance: $W_{\mu\nu}(\overline{q},\overline{p}_1,\overline{S}_1,\overline{p}_2,\overline{S}_2)$

$$=[W^{\mu\nu}(q,p_1,S_1,p_2,S_2)]^*, \tag{9}$$

Hermiticity: $W_{\mu\nu}(q,p_1,S_1,p_2,S_2)$

$$=[W_{\nu\mu}(q,p_1,S_1,p_2,S_2)]^*, \tag{10}$$

with $\overline{a}_\mu=a^\mu$. Constructing the hadronic tensor we seek a structure which is linear in the initial proton spin. Upon noting that

$$\varepsilon^{\mu\nu\alpha\beta}=-\varepsilon_{\mu\nu\alpha\beta} \tag{11}$$

the spin pseudovector $S_{1\mu}$ has to occur together with the Levi-Civita pseudotensor. The most general asymmetric hadronic tensor, which obeys Eqs. (7)–(10), is[4]

$$\begin{aligned}W_{\mu\nu}={}&i[\hat{p}_{1\mu}\hat{p}_{2\nu}-\hat{p}_{1\nu}\hat{p}_{2\mu}]\varepsilon_{p_1,p_2,q,S}\frac{W_1}{M^6}+i[\hat{p}_{1\mu}\varepsilon_{\nu Sp_1q}\\&-\hat{p}_{1\nu}\varepsilon_{\mu Sp_1q}]\frac{W_2}{M^4}+i[\hat{p}_{2\mu}\varepsilon_{\nu Sp_1q}-\hat{p}_{2\nu}\varepsilon_{\mu Sp_1q}]\frac{W_3}{M^4}\\&+i[\hat{p}_{1\mu}\varepsilon_{\nu Sp_2q}-\hat{p}_{1\nu}\varepsilon_{\mu Sp_2q}]\frac{W_4}{M^4}+i[\hat{p}_{2\mu}\varepsilon_{\nu Sp_2q}\\&-\hat{p}_{2\nu}\varepsilon_{\mu Sp_2q}]\frac{W_5}{M^4}+i[\hat{p}_{1\mu}\hat{\varepsilon}_{\nu p_1p_2S}-\hat{p}_{1\nu}\hat{\varepsilon}_{\mu p_1p_2S}]\frac{W_6}{M^4}\\&+i[\hat{p}_{2\mu}\hat{\varepsilon}_{\nu p_1p_2S}-\hat{p}_{2\nu}\hat{\varepsilon}_{\mu p_1p_2S}]\frac{W_7}{M^4}+i\varepsilon_{\mu\nu qS}\frac{W_8}{M^2}.\end{aligned} \tag{12}$$

[2]Indeed, the literature offers a large host of different Pomeron models, cf. [9], to describe these processes. The fact that many of the descriptions yield similar results at equally large rapidity gaps and the same kinematic variables supports our observation.

[3]Here we disregard potential contributions due to strong CP violation [10], because of the smallness of the θ parameter, $|\theta|<3\times10^{-9}$ [11].

[4]A subset of this structure based on p, q and S was considered in Ref. [13].

It is constructed out of the four-vectors q,p_1,p_2 and $S=S_1$. Terms with a genuine structure $\propto M^2/q^2$ are not considered. Here we use the abbreviations

$$\hat{V}_\mu = V_\mu - q_\mu \frac{q\cdot V}{q^2}, \tag{13}$$

$$\hat{\varepsilon}_{\mu v_1 v_2 v_3} = \varepsilon_{\mu v_1 v_2 v_3} - \varepsilon_{q v_1 v_2 v_3}\frac{q_\mu}{q^2}, \tag{14}$$

$$\tilde{\varepsilon}_{\mu\nu v_1 v_2} = \varepsilon_{\mu\nu v_1 v_2} - \varepsilon_{q\nu v_1 v_2}\frac{q_\mu}{q^2} - \varepsilon_{\mu q v_1 v_2}\frac{q_\nu}{q^2}. \tag{15}$$

The Schouten relation [14] in either of the forms

$$X_\mu \varepsilon_{\nu\rho\sigma\tau} = X_\nu \varepsilon_{\mu\rho\sigma\tau} + X_\rho \varepsilon_{\nu\mu\sigma\tau} + X_\sigma \varepsilon_{\nu\rho\mu\tau} + X_\tau \varepsilon_{\nu\rho\sigma\mu} \tag{16}$$

$$g_{\lambda\mu}\varepsilon_{\nu\rho\sigma\tau} = g_{\lambda\nu}\varepsilon_{\mu\rho\sigma\tau} + g_{\lambda\rho}\varepsilon_{\nu\mu\sigma\tau} + g_{\lambda\sigma}\varepsilon_{\nu\rho\mu\tau} + g_{\lambda\tau}\varepsilon_{\nu\rho\sigma\mu} \tag{17}$$

is used to eliminate other possible structures. Particularly, the spin vector S_μ may always be contracted with the Levi-Civita symbol, along with it it has to occur due to parity conservation. Because $S\cdot p_1=0$ two other structures are eliminated using

$$q\cdot p_1 \tilde{\varepsilon}_{\mu\nu S p_1} = p_1\cdot p_1 \varepsilon_{\nu\mu q S} - [\hat{p}_{1\mu}\varepsilon_{\nu p_1 q S} - \hat{p}_{1\nu}\varepsilon_{\mu p_1 q S}] \tag{18}$$

$$q\cdot p_1 \tilde{\varepsilon}_{\mu\nu S p_2} = p_1\cdot p_2 \varepsilon_{\nu\mu q S} - [\hat{p}_{1\mu}\varepsilon_{\nu p_2 q S} - \hat{p}_{1\nu}\varepsilon_{\mu p_2 q S}]. \tag{19}$$

The structure functions W_i are real functions and are given by

$$W_i = W_i(x,Q^2,x_{\mathrm{P}},t). \tag{20}$$

Let us consider the limit in which target masses can be neglected and t is very small. In this case the proton momenta become proportional: $p_2=zp_1$ with

$$z = 1 - x_{\mathrm{P}} = \frac{1+\eta}{1-\eta}. \tag{21}$$

Correspondingly the hadronic tensor simplifies to

$$W_{\mu\nu} = i\varepsilon_{\mu\nu q S}\frac{W_8}{M^2} + i[\hat{p}_{1\mu}\varepsilon_{\nu S p_1 q} - \hat{p}_{1\nu}\varepsilon_{\mu S p_1 q}]\frac{W_9}{M^4}, \tag{22}$$

and contains only two structure functions, where

$$W_9 = W_2 + (1-x_{\mathrm{P}})[W_3+W_4] + (1-x_{\mathrm{P}})^2 W_5. \tag{23}$$

One may wish to rewrite Eq. (22) further into the form which is similar to that used in polarized deep-inelastic scattering:

$$W_{\mu\nu} = i\varepsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda S^\sigma}{p_1\cdot q} g_1(x,Q^2,x_{\mathrm{P}}) + i\varepsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda(p_1\cdot q S^\sigma - S\cdot q p_1^\sigma)}{(p_1\cdot q)^2} g_2(x,Q^2,x_{\mathrm{P}}). \tag{24}$$

This again is achieved by using the Schouten relation, Eq. (16), noting that

$$\hat{p}_{1\mu}\varepsilon_{\nu S p_1 q} - \hat{p}_{1\nu}\varepsilon_{\mu S p_1 q} = -\frac{(S\cdot q)(q\cdot p_1)}{q^2}\varepsilon_{\mu\nu q p_1} + \frac{(q\cdot p_1)^2}{q^2}\varepsilon_{\mu\nu q S}. \tag{25}$$

The relation between the structure functions $W_{8,9}$ and $g_{1,2}$ is

$$g_1 = \frac{q\cdot p_1}{M^2} W_8 \tag{26}$$

$$g_2 = \frac{(q\cdot p_1)^3}{q^2 M^4} W_9. \tag{27}$$

Due to the dependence of the structure functions on x_{P} or η, Eq. (2), the process is *nonforward* with respect to the protons, although the algebraic structure of the hadronic tensor is the same as in the forward case. Finally the generalized Bjorken limit is carried out:

$$2p_1\cdot q = 2M\nu \to \infty, \quad p_2\cdot q \to \infty, \quad Q^2 \to \infty \quad \text{with } x \text{ and } x_{\mathrm{P}} = \text{fixed}, \tag{28}$$

which leads to Eq. (24) using Eqs. (26), (27). For the scattering cross sections we consider the cases of longitudinal and transverse target polarization for which the initial state hadron spin vectors are given by

$$S_\parallel = (0,0,0,M) \tag{29}$$

$$S_\perp = M(0,\cos\gamma,\sin\gamma,0), \tag{30}$$

and γ is the spin direction in the plane orthogonal to the 3-momentum $\vec{p}_1$. In the limit $p_2=zp_1$ and $M^2,t=0$ the Φ integral becomes trivial in the case of longitudinal nucleon polarization, while it is kept as a differential variable for transverse polarization:

$$\frac{d^3\sigma_{\mathrm{diffr}}(\lambda,\pm S_\parallel)}{dx\,dQ^2\,dx_{\mathrm{P}}} = \mp 4\pi s\lambda\frac{\alpha^2}{Q^4} \times\left[y\left(2-y-\frac{2xyM^2}{s}\right)x g_1(x,Q^2,x_{\mathrm{P}}) - 4xy\frac{M^2}{s}g_2(x,Q^2,x_{\mathrm{P}})\right] \tag{31}$$

$$\frac{d^4\sigma_{\rm diffr}(\lambda,\pm S_\perp)}{dx dQ^2 d\Phi dx_{\rm P}} = \mp 4 s\lambda \sqrt{\frac{M^2}{s}}\frac{\alpha^2}{Q^4} \times \sqrt{xy\left[1-y-\frac{xyM^2}{s}\right]}\cos(\gamma-\Phi) \times [yxg_1(x,Q^2,x_{\rm P})+2xg_2(x,Q^2,x_{\rm P})], \quad (32)$$

where $y=q\cdot p_1/l\cdot p_1$ and λ denotes the degree of longitudinal lepton polarization.[5]

III. THE COMPTON AMPLITUDE

We first consider the operator given by the renormalized and time-ordered product of two electromagnetic currents

$$\hat{T}_{\mu\nu}(x) = RT\left[J_\mu\left(\frac{x}{2}\right)J_\nu\left(-\frac{x}{2}\right)S\right] = -e^2\frac{\tilde{x}^\lambda}{2\pi^2(x^2-i\epsilon)^2}RT\left[\overline{\psi}\left(\frac{\tilde{x}}{2}\right)\gamma^\mu\gamma^\lambda\gamma^\nu \times \psi\left(-\frac{\tilde{x}}{2}\right)-\overline{\psi}\left(-\frac{\tilde{x}}{2}\right)\gamma^\mu\gamma^\lambda\gamma^\nu\psi\left(\frac{\tilde{x}}{2}\right)\right]S. \quad (33)$$

Here, $\tilde{x}$ denotes a lightlike vector corresponding to x,

$$\tilde{x} = x+\frac{\zeta}{\zeta^2}[\sqrt{x\cdot\zeta^2-x^2\zeta^2}-x\cdot\zeta], \quad (34)$$

and ζ is a subsidiary vector. Following Refs. [16,17] the operator $\hat{T}_{\mu\nu}$ can be expressed in terms of a vector and an axial-vector operator decomposing

$$\gamma_\mu\gamma_\lambda\gamma_\nu = [g_{\mu\lambda}g_{\nu\rho}+g_{\nu\lambda}g_{\mu\rho}-g_{\mu\nu}g_{\lambda\rho}]\gamma^\rho - i\varepsilon_{\mu\nu\lambda\rho}\gamma^5\gamma^\rho. \quad (35)$$

We will consider only the contribution of the latter one, since this yields the polarized part,

$$\hat{T}^{\rm pol}_{\mu\nu}(x) = ie^2\frac{\tilde{x}^\lambda}{2\pi^2(x^2-i\epsilon)^2}\varepsilon_{\mu\nu\lambda\sigma}O_5^\sigma\left(\frac{\tilde{x}}{2},-\frac{\tilde{x}}{2}\right), \quad (36)$$

with $\varepsilon_{\mu\nu\lambda\sigma}$ the Levi-Civita symbol. The bilocal axial-vector light-ray operator is

$$O_5^\alpha\left(\frac{\tilde{x}}{2},-\frac{\tilde{x}}{2}\right) = \frac{i}{2}RT\left[\overline{\psi}\left(\frac{\tilde{x}}{2}\right)\gamma_5\gamma^\alpha\psi\left(-\frac{\tilde{x}}{2}\right) + \overline{\psi}\left(-\frac{\tilde{x}}{2}\right)\gamma_5\gamma^\alpha\psi\left(\frac{\tilde{x}}{2}\right)\right]S. \quad (37)$$

[5] In the case of longitudinal nucleon polarization polarized diffractive scattering was discussed neglecting the contribution due to the structure function g_2 in [15].

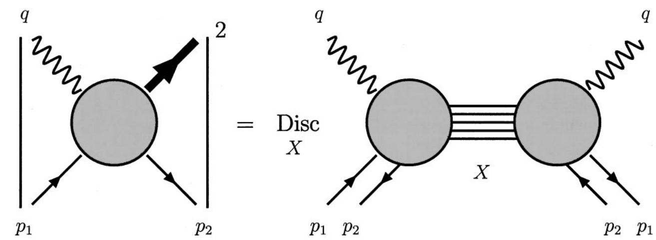

FIG. 2. Mueller's optical theorem.

The polarized part of the Compton operator $\hat{T}^{\rm pol}_{\mu\nu}$ is related to the diffractive scattering cross section using Mueller's generalized optical theorem [20] (Fig. 2), which moves the final state proton into an initial state antiproton.

The polarized part of the Compton amplitude is obtained as the expectation value

$$T^{\rm pol}_{\mu\nu}(x) = \langle p_1,S_1,-p_2,S_2|\hat{T}_{\mu\nu}|p_1,S_1,-p_2,S_2\rangle, \quad (38)$$

which is forward with respect to the direction defined by the state $\langle p_1,-p_2|$. The twist-2 contributions to the expectation values of the operator (37) are obtained

$$\langle p_1,S_1,-p_2,S_2|O_5^{A,\mu}(\kappa_+\tilde{x},\kappa_-\tilde{x})|p_1,S_1,-p_2,S_2\rangle = \int_0^1 d\lambda\,\partial_x^\mu\langle p_1,S_1,-p_2,S_2|O_5^A(\lambda\kappa_+x,\lambda\kappa_-x) \times|p_1,S_1,-p_2,S_2\rangle|_{x=\tilde{x}} \quad (39)$$

as partial derivative of the expectation values of

$$O_5^A(\kappa_+x,\kappa_-x) = x^\alpha O^A_{5,\alpha}(\kappa_+x,\kappa_-x), \quad (40)$$

the corresponding pseudoscalar operator. The index $A=q,G$ labels the quark or gluon operators; cf. [16]. From now on we keep only the spin vector of the initial-state proton and sum over that of the final-state proton. The pseudoscalar twist-2 quark operator matrix element has the following representation[6] due to the overall symmetry in x:

$$\langle p_1,S_1,-p_2|O^q(\kappa_+x,\kappa_-x)|p_1,S_1,-p_2\rangle = xS\int Dz\,e^{-i\kappa_-xp_z}f_5^q(z_+,z_-) \quad (41)$$

with $S\equiv S_1$, $\kappa_-=1/2$ and where all the trace terms were subtracted, see [16,22]. $f_5^A(z_+,z_-)$ denotes the scalar two-variable distribution amplitudes and the measure Dz is

$$Dz = dz_+dz_-\theta(1+z_++z_-)\theta(1+z_+-z_-) \times\theta(1-z_++z_-)\theta(1-z_+-z_-). \quad (42)$$

Here, we decomposed the vector p_z as

$$p_z = p_-z_-+p_+z_+ = p_-\vartheta+\pi_-z_+, \quad (43)$$

[6] For parametrizations of similar hadronic matrix elements see e.g. [21].

with $z_{1,2}$ momentum fractions along $p_{1,2}$ and $p_\pm = p_2 \pm p_1$, $z_\pm = (z_2 \pm z_1)/2$ and

$$\vartheta = z_- + \frac{1}{\eta} z_+, \quad \pi_- = p_+ - \frac{1}{\eta} p_-, \tag{44}$$

with $q \cdot \pi_- \equiv 0$. In the limit $M^2, t \sim 0$, in which we work from now on, the vector π_- even vanishes.

The Fourier transform of the Compton amplitude is given by [17]

$$\begin{aligned} T^{\mathrm{pol}}_{\mu\nu}&(p_1, p_2, S, q) \\ &= \int d^4 x e^{iqx} T_{\mu\nu}(x) \\ &= 4 i \varepsilon_{\mu\nu\lambda\sigma} \int Dz \left[\frac{Q_z^\lambda S^\sigma}{Q_z^2 + i\varepsilon} - \frac{1}{2} \frac{p_z^\sigma S^\lambda}{Q_z^2 + i\varepsilon} \right. \\ &\quad \left. + \frac{Q_z \cdot S}{(Q_z^2 + i\varepsilon)^2} p_z^\sigma Q_z^\lambda \right] F_5(z_+, z_-), \end{aligned} \tag{45}$$

with $Q_z = q - p_z/2$ and

$$\overline{u(p_1)} \gamma_5 \gamma_\lambda u(p_1) = 2 S_\lambda. \tag{46}$$

The function $F_5(z_+, z_-)$ is related to the polarized distribution function $f_5(z_+, z_-)$ by

$$\begin{aligned} F_5(z_+, z_-) = \int_0^1 \frac{d\lambda}{\lambda^2} f_5\left(\frac{z_+}{\lambda}, \frac{z_-}{\lambda} \right) \\ \times \theta(\lambda - |z_+|) \theta(\lambda - |z_-|). \end{aligned} \tag{47}$$

We rewrite the denominators by

$$\frac{1}{Q_z^2 + i\varepsilon} = -\frac{1}{q p_-} \frac{1}{(\vartheta - 2\beta + i\varepsilon)}, \tag{48}$$

defining

$$\beta = \frac{x}{x_{\mathbb{P}}} = \frac{q^2}{2 q \cdot p_-}. \tag{49}$$

The conservation of the electromagnetic current is easily seen

$$q^\mu T_{\mu\nu}(p_1, p_2, S, q) = q^\nu T_{\mu\nu}(p_1, p_2, S, q). \tag{50}$$

It follows because the contraction with q^α leads to Levi-Civita symbols being contract with the same 4-vector. By

$$\begin{aligned} \hat{F}(\vartheta, \eta) &= \int Dz F(z_+, z_-) \delta(\vartheta - z_- - z_+/\eta) \\ &= \int_\vartheta^{-\mathrm{sgn}(\vartheta)/\eta} \frac{dz}{z} \hat{f}(z, \eta) \end{aligned} \tag{51}$$

we change to the variable ϑ, the main momentum fraction in the subsequent representation. Equation (51) is the preform of the Wandzura-Wilczek integral [7]. It emerges seeking the representation of vector-valued distributions (47) in terms of scalar distributions; cf. [16,17]. In most of the applications these integrals remain. An exception is the Callan-Gross relation, see Refs. [17,3], where all these integrals cancel and only scalar distribution functions remain. Here the distribution function $\hat{f}_5(z, \eta)$ is related to $f_5(z_+, z_-)$ by

$$\hat{f}_5(z, \eta) = \int_{\eta(1+z)}^{\eta(1-z)} d\rho\, \theta(1-\rho) \theta(\rho+1) f_5(\rho, z - \rho/\eta), \tag{52}$$

with $\rho = z_+/\eta$.

The Compton amplitude takes the following form:

$$\begin{aligned} T^{\mathrm{pol}}_{\mu\nu}(p_-, S, q) = 4 i \varepsilon_{\mu\nu\lambda\sigma} \int_{+1/\eta}^{-1/\eta} d\vartheta \left\{ \frac{q^\lambda S^\sigma}{Q_z^2 + i\varepsilon} \right. \\ \left. - q \cdot S \frac{\vartheta q^\lambda p_-^\sigma}{(Q_z^2 + i\varepsilon)^2} \right\} \int_\vartheta^{-\mathrm{sgn}(\vartheta)/\eta} \frac{dz}{z} \hat{f}_5(z, \eta). \end{aligned} \tag{53}$$

The ϑ integral in Eq. (53) can be simplified using the identities

$$\begin{aligned} \int_{+1/\eta}^{-1/\eta} & d\vartheta \frac{\vartheta^k}{(\vartheta - 2\beta + i\varepsilon)^2} \int_\vartheta^{\mathrm{sgn}(\vartheta)/\eta} \frac{dz}{z} \hat{f}_5(z, \eta) \\ &= \int_{+1/\eta}^{-1/\eta} d\vartheta \frac{k \vartheta^{k-1}}{(\vartheta - 2\beta + i\varepsilon)} \int_\vartheta^{\mathrm{sgn}(\vartheta)/\eta} \frac{dz}{z} \hat{f}_5(z, \eta) \\ &\quad - \int_{+1/\eta}^{-1/\eta} d\vartheta \frac{\vartheta^{k-1} \hat{f}_5(\vartheta, \eta)}{(\vartheta - 2\beta + i\varepsilon)}. \end{aligned} \tag{54}$$

As we work in the approximation of $M^2, t \ll |q^2|$ the vector p_- obeys the representation

$$p_- = -x_{\mathbb{P}} p_1. \tag{55}$$

Using these variables the Compton amplitude reads

$$\begin{aligned} T^{\mathrm{pol}}_{\mu\nu}(p_1, S, q) = -4 i \varepsilon_{\mu\nu\lambda\sigma} \int_{+1/\eta}^{-1/\eta} \frac{d\vartheta}{\vartheta - 2\beta + i\varepsilon} \\ \times \left\{ \left[\frac{q^\lambda S^\sigma}{q \cdot p_1} + \frac{q \cdot S}{(q \cdot p_1)^2} q^\lambda p_1^\sigma \right] \right. \\ \times \int_\vartheta^{-\mathrm{sgn}(\vartheta)/\eta} \frac{dz}{z} \hat{\hat{f}}_5(z, \eta) \\ \left. - \frac{q \cdot S}{(q \cdot p_1)^2} q^\lambda p_1^\sigma \hat{\hat{f}}_5(\vartheta, \eta) \right\}. \end{aligned} \tag{56}$$

Here,

$$\hat{\hat{f}}_5(z, \eta) = \frac{1}{x_{\mathbb{P}}} \hat{f}_5(z, \eta). \tag{57}$$

Taking the absorptive part one obtains[7]

$$W_{\mu\nu}^{\rm pol}=\frac{1}{2\pi}{\rm Im}\, T_{\mu\nu}^{\rm pol}(p_1,p_2,q)$$

$$=i\varepsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda S_1^\sigma}{q\cdot p_1}\mathsf{G}_1(\beta,\eta,Q^2)$$

$$+i\varepsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda(p_1\cdot qS^\sigma-S\cdot qp_1^\sigma)}{(p_1\cdot q)^2}\mathsf{G}_2(\beta,\eta,Q^2), \quad (58)$$

where

$$\mathsf{G}_1(\beta,\eta,Q^2)=\sum_{q=1}^{N_f} e_q^2[\Delta f_q^D(\beta,Q^2,x_{\rm P}) + \Delta\overline{f}_q^D(\beta,Q^2,x_{\rm P})] \equiv g_1^{D(3)}(x,Q^2,x_{\rm P}), \quad (59)$$

$$\mathsf{G}_2(\beta,\eta,Q^2)=-\mathsf{G}_1(\beta,\eta,Q^2) + \int_\beta^1\frac{d\beta'}{\beta'}\mathsf{G}_1(\beta',\eta,Q^2) \equiv g_2^{D(3)}(x,Q^2,x_{\rm P}), \quad (60)$$

with N_f the number of flavors, choosing the factorization scale $\mu^2=Q^2$. As we were working in the twist-2 approximation, the Wandzura-Wilczek relation (60) describes $\mathsf{G}_2(\beta,\eta,Q^2)$.

To derive the representation for the diffractive parton densities Δf_q^D, Eq. (59), we consider the symmetry relation for the polarized distribution functions $F^A(z_1,z_2)$, Ref. [17],

$$F_5^A(z_1,z_2)=F_5^A(-z_1,-z_2). \quad (61)$$

It translates into

$$\hat{F}_5^A(\vartheta,\eta)=\hat{F}_5^A(-\vartheta,\eta), \quad (62)$$

and, cf. Eq. (51),

$$\hat{\hat{f}}_5^A(\vartheta,\eta)=\hat{\hat{f}}_5^A(-\vartheta,\eta). \quad (63)$$

The polarized diffractive quark and anti-quark densities are given by

$$\sum_{q=1}^{N_f} e_q^2\Delta f_q^D(\beta,Q^2,x_{\rm P})=\hat{\hat{f}}_5(2\beta,\eta,Q^2)$$
$$\sum_{q=1}^{N_f} e_q^2\Delta\overline{f}_q^D(\beta,Q^2,x_{\rm P})=\hat{\hat{f}}_5(-2\beta,\eta,Q^2). \quad (64)$$

Unlike in the deep-inelastic case, where the scaling variable $x\epsilon[0,1]$, the support of the distributions $\Delta f_q^D(\beta,Q^2,x_{\rm P})$ is $x\epsilon[0,x_{\rm P}]$.

We express the diffractive parton densities in terms of the distribution function $f_5(z_+,z_-)$ directly

$$\Delta\hat{\hat{f}}_5(\pm 2\beta,\eta,Q^2)=\frac{1}{x_{\rm P}}\int_{-(x_{\rm P}\pm 2x)/(2-x_{\rm P})}^{-(x_{\rm P}\mp 2x)/(2-x_{\rm P})}d\rho \times f_5(\rho,\pm 2\beta+\rho(2-x_{\rm P})/x_{\rm P};Q^2). \quad (65)$$

The latter relations are needed to compare experimental quantities with those which might be obtained measuring the corresponding operators on the lattice.

Finally, we would like to make a remark on the evolution of the diffractive parton densities being derived above. In a previous paper [3] the corresponding evolution equations for unpolarized diffractive scattering have been derived in detail. Also here one may start with the general formalism for non-forward scattering, see e.g. [16], and discuss the evolution of the scalar operators. The evolution equations are independent of the parameter κ_+ emerging in the anomalous dimensions $\gamma_5^{AB}(\kappa_+,\kappa_-,\kappa'_+,\kappa'_-;\mu^2)$ which therefore may be set to zero. Moreover, the all-order rescaling relation

$$\gamma^{AB}(\kappa_+,\kappa_-,\kappa'_+,\kappa'_-;\mu^2) =\sigma^{d_{AB}}\gamma^{AB}(\sigma\kappa_+,\sigma\kappa_-,\sigma\kappa'_+,\sigma\kappa'_-), \quad (66)$$

holds, with $d_{AB}=2+d_A-d_B$, $d_q=1,d_G=2$. A straightforward calculation leads to the evolution equation for the polarized (singlet) diffractive parton densities $f_5^A(\vartheta,\eta;\mu^2)$ in the momentum fraction ϑ

$$\mu^2\frac{d}{d\mu^2}f^A(\vartheta,\eta;\mu^2)=\int_\vartheta^{-{\rm sgn}(\vartheta)/\eta}\frac{d\vartheta'}{\vartheta'}P_5^{AB}\left(\frac{\vartheta}{\vartheta'},\mu^2\right) \times f_B(\vartheta',\eta;\mu^2). \quad (67)$$

The splitting functions P_5^{AB} are the *forward* splitting functions [18],[8] which are independent of η respectively $x_{\rm P}$. Taking the absorptive part the usual evolution equations are obtained, with the difference that the evolution takes place in the variable β. The nonforwardness η or $x_{\rm P}$ behave as plain parameters:

$$\mu^2\frac{d}{d\mu^2}f_A^D(\beta,x_{\rm P};\mu^2)=\int_\beta^1\frac{d\beta'}{\beta'}P_{5,A}^B\left(\frac{\beta}{\beta'};\mu^2\right) \times f_B^D(\beta',x_{\rm P};\mu^2). \quad (68)$$

We expressed the Compton amplitude with the help of the light-cone expansion at short distances and applied this representation to the process of deep-inelastic diffractive scattering using Mueller's generalized optical theorem. This representation is *not* limited to leading twist operators but can be extended to all higher twist operators. The corresponding evolution equations for the higher twist hadronic matrix elements depend on more than one momentum fraction ϑ_i, which have a less trivial connection to the outer kinematical variables similar to the case of deep-inelastic scattering [23].

[7] The "imaginary part" concerns that of the Schwartz distribution, Eq. (48). Because of the relations, Eqs. (9),(10), an overall i emerges in the hadronic tensor.

[8] For the nonforward anomalous dimensions see [19].

The construction is similar to the above and applies as well for the generalized optical theorem. The evolution of the associated parton correlation functions is for the same reason as follows.

IV. CONCLUSIONS

The differential cross section of polarized deep-inelastic ep-diffractive scattering for pure photon exchange is described by eight structure functions. They depend on the four kinematic variables, x, Q^2, x_{P} and t. In the limit of small values of t and neglecting target masses two structure functions contribute. In the generalized Bjorken range and the presence of a sufficiently large rapidity gap the scaling violations of hard diffractive scattering can be described within perturbative QCD. In this range, processes which are dominated by light-cone contributions are described. The scattering amplitude can be rewritten using Mueller's generalized optical theorem moving the outgoing diffractive proton into an incoming anti-proton. In this kinematical domain diffractive scattering *is* deep-inelastic scattering off a state $\langle p_1, S_1, -p_2|$. Nonforward techniques may be used to describe this process. In this way the two-variable polarized amplitudes turn into the polarized diffractive parton densities, which depend on one momentum fraction and a parameter η, which describes the nonforwardness, and is directly related to the variable x_{P}. For the absorptive part the scaling variable can be expressed by the variable β, which also is the variable on which the evolution kernels act in the twist-2 contributions, whereas x_{P} remains as a simple parameter of the process. In the limit $t, M^2 \to 0$ the twist-2 contributions to the two structure functions $g_{1,2}^{D(3)}(x, Q^2, x_{\mathrm{P}})$ are related by a Wandzura-Wilczek relation in the variable $\beta = x/x_{\mathrm{P}}$. The approach followed in the present paper for twist-2 operators can be synonymously extended to higher twist-operators in the kinematic domain of the general Bjorken limit.

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