Gluon induced contributions to WZ and $W\gamma$ production at NNLO

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We calculate the contribution of the partonic processes $gg \to WZq\bar{q}$ and $gg \to W\gamma q\bar{q}$ to WZ and $W\gamma$ pair production at hadron colliders, including anomalous triple gauge-boson couplings. We use the helicity method and include the decay of the W and Z bosons into leptons in the narrow-width approximation. In order to integrate over the $q\bar{q}$ final state phase space we use an extended version of the subtraction method to NNLO and remove collinear singularities explicitly. Because of the large gluon density at low x, the gluon induced terms of vector-boson pair production are expected to be the dominant NNLO QCD correction, relevant at CERN LHC energies. However, we show that due to a cancellation they turn out to provide a rather small contribution, anticipating good stability for the perturbative expansion.

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I. INTRODUCTION

A direct test of the non-Abelian nature of the weak interactions can be made through the study of vector-boson pair production. These processes already involve triple gauge-boson couplings at the tree level and, therefore, offer a promising framework for their study. In the standard model, the triple gauge-boson couplings are completely fixed by gauge symmetry. However, these couplings can be modified by new physics occurring at a higher energy scale. Given that the triple gauge-boson couplings have been much less precisely measured than, for example, the couplings of gauge bosons to fermions, it is natural to search for new physics by looking for anomalous triple gauge-boson couplings.

Trilinear gauge-boson couplings have been studied at the CERN e^+e^- collider LEP2 [1] and the Fermilab Tevatron [2]. It should be emphasized that the studies at hadron colliders are complementary to studies at electron-positron colliders. While bounds on anomalous couplings obtained from LEP2 data are more stringent than those from hadron colliders, the latter offer the possibility to study these couplings at a higher center of mass energy, where the effects of anomalous couplings are enhanced. The amount of available data from hadron colliders will dramatically increase with run II at the Tevatron and even more so with the CERN Large Hadron Collider (LHC). This will allow the study of triple gauge-boson couplings to be taken one step further. Of course, all possible vector-boson pair production processes will be analyzed. However, in this paper we concentrate on $W^\pm\gamma$ and $W^\pm Z$ pair production.

In order to get reliable theoretical predictions for vector-boson pair production it is important to include higher-order corrections. One-loop QCD corrections, treating the vector bosons as stable particles, were computed for $W^\pm\gamma$ production [3] and $W^\pm Z$ production [4] ten years ago. Since vector bosons are identified through their decay products it is essential to include the decay of the bosons into fermions. This allows arbitrary cuts to be added to the kinematics of the final states and therefore facilitates the comparison of theoretical predictions with experimental measurements. The calculation of these processes in the narrow-width approximation but retaining spin information via decay-angle correlations only in the real part of the amplitudes has been published in [5]. Anomalous couplings for the $W^\pm\gamma$ [6] and $W^\pm Z$ [7] processes have also been included. Finally, analytic results of the one-loop amplitudes for vector-boson pair production with the subsequent decay into lepton pairs in the narrow-width approximation have been presented in [8]. In [9,10] these amplitudes have been used to obtain numerical results including all decay-angle correlations. Steps beyond the inclusion of $\mathcal{O}(\alpha_s)$ corrections in the narrow-width approximation have been made in [11], where nondoubly resonant diagrams have been included and in [12], where one-loop logarithmic electroweak corrections have been calculated.

The size of the one-loop QCD corrections depends crucially on the physical quantity under investigation. Usually they are of the order of a few 10%, but in some particularly interesting cases, for example the high p_T tail of transverse momentum distributions they can increase the leading order cross section by several 100%. The reason for such huge corrections is that the next-to-leading order correction to vector-boson pair production consists of two parts. First, there is the one-loop correction to the leading order partonic process $q\bar{q} \to VV$. Second, there are new partonic channels, $qg \to VVq$, that contribute to the cross section at NLO. Even though these processes are suppressed with respect to the leading order partonic process $q\bar{q} \to VV$ by the strong coupling constant α_s, they can easily be more important numerically, since they are enhanced by the gluonic parton distribution function. This is particularly the case for the LHC [or even a much higher energy Very Large Hadron Collider or (VLHC)], where gluon induced processes become increasingly dominant. Unfortunately, the kinematical regimes where these contributions are largest are exactly those where effects from anomalous couplings are expected to manifest themselves. Therefore, it is important to gain a better control of the theoretical predictions in these kinematical regimes.

One way to improve the situation is to impose a jet veto. This suppresses the effect of the new partonic channels that are opened at higher order [7,9,10]. On the other hand, such a veto also reduces the amount of data considerably. Another possibility is to include even higher order terms. Of course, a full next-to-next-to-leading-order (NNLO) calculation of vector-boson pair production is not to be expected in the near future. In [13] a prescription to approximately evaluate higher-order QCD corrections to $W^+\gamma$ production has been proposed. Based on this approximation the order α_s^2 corrections have been calculated. A full NNLO calculation would have to include two-loop corrections to the partonic subprocesses $q\bar{q}\to VV$, one-loop corrections to $qg\to VVq$ and, finally, (amongst others) processes with gg in the initial state. Motivated by the observation that loop corrections to partonic processes tend to be of the expected moderate size and huge corrections are mainly due to the opening of new channels, in this paper we include all NNLO terms that are maximally enhanced by the gluon distribution functions.

In general, there are two classes of such contributions. First, there is the process $gg\to VV$. These processes have been studied previously [14]. The amplitude for this process vanishes at the tree level but may be nonzero at one loop. In the case of $W^\pm\gamma$ and $W^\pm Z$ pair production, however, the amplitude for this process vanishes at all orders, due to charge conservation. Second, there are the processes $gg\to VVq\bar{q}$. The amplitudes for these processes have been calculated in [15] and have been used to compute cross sections for the production of a vector boson pair together with two jets. We extend this work by not requiring any jets to be observed in the final state. Hence, the amplitudes $gg\to VVq\bar{q}$ have to be integrated over the $q\bar{q}$ final state phase space. This results in infrared singularities that will have to be absorbed in the parton distribution functions. Furthermore, we also extend the previous work by adding anomalous couplings.

We find that the contribution of the gluon induced processes to WZ and $W\gamma$ production is smaller than may have been anticipated, due to a sign change in the hard scattering part. We also find that this NNLO term is affected less by anomalous couplings than the corresponding LO and NLO terms.

II. CALCULATION

Vector boson pair production has been calculated up to NLO in the helicity method. However, in an NLO calculation for $pp\to WZ$, whilst the process $\bar{q}q\to WZ$ is included at NLO, the process $qg\to WZq$ is only calculated at LO. This means that we only have a leading order calculation in the circumstances where gluon-induced subprocesses are important, as they are expected to be at the LHC due to the large gluon density at low x. Therefore we want to include the next order in α_s, the process $gg\to WZq\bar{q}$ (and similarly for $W\gamma$).

For WZ production we will use the labelling $g_1g_2l_3\bar{\nu}_4\bar{l}'_5l'_6q_7\bar{q}_8$, where g_1 and g_2 are the incoming gluons, leptons 3 and 4 are the decay products of the W and 5 and 6 are from the Z. Clearly, $W\gamma$ will have one less particle but this is a trivial modification. It is also straightforward to transform the calculation for other vector bosons.

In calculating $gg\to WZ$, we use a version of the subtraction method [16] to cancel the infrared singularities analytically and evaluate the finite remainders numerically. We show the finite parts of the cross section in Eqs. (8), (11) and (15).

A generic differential cross section is written as

$$d\sigma_{AB}(p_A,p_B)=\sum_{ab}\int dx_1dx_2f_{a/A}(x_A)f_{b/B}(x_B)\times d\hat{\sigma}_{ab}(x_Ap_A,x_Bp_B), \tag{1}$$

where A and B are hadrons while a and b are partons and $f_{a/A}$ and $f_{b/B}$ are the parton distribution functions. The subtracted partonic cross section, $d\hat{\sigma}$, is finite for any infrared safe observable.

In a general case, in order to obtain $d\hat{\sigma}$, soft and final state collinear singularities have to be cancelled with the singularities coming from virtual corrections. The remaining initial state collinear singularities have to be factored out into the parton distribution functions. In an extension of the subtraction method to NNLO, this demands the subtraction of all possible singularities as well as their analytical integration in the corresponding limits. Up to now, such a general framework has not been developed. Only a limited number of analytical calculations to NNLO accuracy have been performed.

The main difficulty for the implementation of a subtraction procedure at NNLO is the appearance of several new (multiple parton) kinematical configurations where soft and/or collinear singularities arise. These are: the triple collinear case (three partons become collinear), the double soft case (two partons are soft), the soft-collinear case (one parton is soft and the other collinear) and the double collinear case (two pairs of partons are independently collinear). The single soft and collinear singularities, which appear also at NLO, have to be added to that list.

Even though the behavior of tree and one-loop amplitudes in those limits has been recently obtained [17], it is not yet clear how to unify all singularities in a single subtraction term, or how to subtract all of them independently while avoiding double counting and allowing the analytical integration of the subtracted terms to be performed.

Our case, however, is particularly simple. Since we include only gg induced processes at NNLO there are no final state collinear singularities (see below about final state singularities in the $W\gamma$ case). There are also no soft singularities, since there are no gluons (or $q\bar{q}$ pairs) in the final state. Therefore, the only singularities that arise are the single collinear ones, which can be treated in a similar way as a typical

NLO calculation, and the double collinear ones, i.e., the collinear splitting of each initial state gluon into a $q\bar{q}$ pair, a "genuine" NNLO contribution.

In order to absorb the remaining initial state collinear singularities we use

$$f_{a/d}=\delta_{ad}\delta(1-x)-\frac{\alpha_s}{2\pi}\frac{1}{\bar{\epsilon}}P_{ad}(x,0)-\left(\frac{\alpha_s}{2\pi}\right)^2 Q_{ad}(x). \tag{2}$$

This is the NNLO expression in the modified numerical subtraction scheme ($\overline{\text{MS}}$) and as usual we define $1/\bar{\epsilon}\equiv 1/\epsilon-\gamma_E+\log 4\pi$. $P_{ad}(x,0)$ is the one-loop Altarelli-Parisi splitting function for $\varepsilon=0$ (4 dimensions). The $\mathcal{O}(\alpha_s^2)$ term Q_{ad}, including contributions from the two-loop splitting functions, does not actually contribute in our case.

Comparing subtracted and unsubtracted terms and using Eq. (2), we obtain the relation

$$\begin{aligned} d\hat{\sigma}^{(2)}_{gg}(k_1,k_2)=&\,d\sigma^{(2)}_{gg}(k_1,k_2)+\frac{\alpha_s}{2\pi}\int dx_1\left(\frac{1}{\bar{\epsilon}}P_{qg}(x_1)\right)d\sigma^{(1)}_{qg}(x_1k_1,k_2)+\frac{\alpha_s}{2\pi}\int dx_1\left(\frac{1}{\bar{\epsilon}}P_{\bar{q}g}(x_1)\right)d\sigma^{(1)}_{\bar{q}g}(x_1k_1,k_2)\\ &+\frac{\alpha_s}{2\pi}\int dx_2\left(\frac{1}{\bar{\epsilon}}P_{qg}(x_2)\right)d\sigma^{(1)}_{gq}(k_1,x_2k_2)+\frac{\alpha_s}{2\pi}\int dx_2\left(\frac{1}{\bar{\epsilon}}P_{\bar{q}g}(x_2)\right)d\sigma^{(1)}_{g\bar{q}}(k_1,x_2k_2)\\ &+\left(\frac{\alpha_s}{2\pi}\right)^2\int dx_1dx_2\left(\frac{1}{\bar{\epsilon}}P_{qg}(x_1)\right)\left(\frac{1}{\bar{\epsilon}}P_{\bar{q}g}(x_2)\right)d\sigma^{(0)}_{q\bar{q}}(x_1k_1,x_2k_2)+\left(\frac{\alpha_s}{2\pi}\right)^2\int dx_1dx_2\left(\frac{1}{\bar{\epsilon}}P_{\bar{q}g}(x_1)\right)\\ &\times\left(\frac{1}{\bar{\epsilon}}P_{qg}(x_2)\right)d\sigma^{(0)}_{\bar{q}q}(x_1k_1,x_2k_2), \end{aligned} \tag{3}$$

where k_1,k_2 are the momenta of the incoming gluons 1 and 2. The superscript (2) denotes the NNLO term, while (1) is NLO and (0) is leading order. In Eq. (3) all terms on the right hand side are separately divergent but the sum is finite.

Once we have this expression, we can write out the counterterms in terms of energy and angle variables and cancel the poles explicitly. Following [16] we use the parametrization

$$k_1=\frac{\sqrt{s_{12}}}{2}(1,\vec{0},1) \tag{4}$$

$$k_2=\frac{\sqrt{s_{12}}}{2}(1,\vec{0},-1) \tag{5}$$

$$k_i=\frac{\sqrt{s_{12}}}{2}\xi_i(1,\sqrt{1-y_i^2}\vec{e}_T,y_i), \tag{6}$$

where k_i is the momentum of an outgoing (anti)quark, $\vec{e}_T$ is a unit vector in transverse momentum space, $y_i=\cos\theta_i$ is the angle variable $-1\leqslant y_i\leqslant 1$ and ξ_i is the rescaled energy variable $0\leqslant\xi_i\leqslant 1$. Performing part of the phase-space integration analytically we can rewrite Eq. (3) as a sum of three separately finite pieces

$$d\hat{\sigma}^{(2)}_{gg}=d\sigma^{(\text{fin},8)}+d\sigma^{(\text{fin},7)}+d\sigma^{(\text{fin},6)}, \tag{7}$$

with terms $d\sigma^{(\text{fin},8,7,6)}$ denoting the 8,7 and 6-parton finite parts. The explicit form of $d\sigma^{(\text{fin},8)}$ is given by

$$\begin{aligned} d\sigma^{(\text{fin},8)}=&\,(1-y_7^2)(1-y_8^2)\mathcal{M}^{(8)}_{gg}\\ &\times(k_1,k_2,\{k_i\}_{3,6},k_7,k_8)\\ &\times\frac{s_{12}^2}{64(2\pi)^6}\xi_7\xi_8\mathcal{P}(y_7)\mathcal{P}(y_8)\\ &\times d\xi_7 d\xi_8 dy_7 dy_8 d\varphi_7 d\varphi_8 d\Phi_{(3-6)}, \end{aligned} \tag{8}$$

where $d\Phi_{(3-6)}$ denotes the phase-space integration over the vector-boson decay products. $\mathcal{M}^{(8)}_{gg}$ denotes the squared amplitude summed (averaged) over helicities, including the flux factor $1/(2s_{12})$. $\mathcal{P}(y_i)$ is given by

$$\mathcal{P}(y_i)\equiv\frac{1}{2}\left[\left(\frac{1}{1-y_i}\right)_{\delta_I}+\left(\frac{1}{1+y_i}\right)_{\delta_I}\right]. \tag{9}$$

The distributions $(1/(1\pm y_i))_{\delta_I}$ have been introduced in [16] and are defined for an arbitrary test function $f(y_i)$ through

$$\left\langle\left(\frac{1}{1\pm y_i}\right)_{\delta_I},f(y_i)\right\rangle=\int_{-1}^{1}dy_i\frac{f(y_i)-f(\mp 1)\,\theta(\mp y_i-1+\delta_I)}{1\pm y_i}. \tag{10}$$

The two $\mathcal{P}$ distributions in Eq. (8) perform the subtraction of both double and single collinear singularities of NNLO amplitudes, leaving a (numerically) integrable remnant.

The 7-parton finite part reads

$$d\sigma^{(\mathrm{fin},7)}=\frac{\alpha_s}{2\pi}(\mathcal{L}_7P^{<}_{qg}(1-\xi_7)-P'^{<}_{qg}(1-\xi_7))(1-y_8^2)\{\mathcal{M}^{(7)}_{qg}((1-\xi_7)k_1,k_2,\{k_i\}_{3,6},k_8)+\mathcal{M}^{(7)}_{gq}(k_1,(1-\xi_7)k_2,\{k_i\}_{3,6},k_8)\}$$
$$\times\frac{s_{12}}{8(2\pi)^3}\xi_8\mathcal{P}(y_8)d\xi_7d\xi_8dy_8d\varphi_8d\Phi_{(3-6)}+\frac{\alpha_s}{2\pi}(\mathcal{L}_8P^{\leq}_{qg}(1-\xi_8)-P'^{<}_{qg}(1-\xi_8))(1-y_7^2)$$
$$\times\{\mathcal{M}^{(7)}_{qg}((1-\xi_8)k_1,k_2,\{k_i\}_{3,6},k_7)+\mathcal{M}^{(7)}_{gq}(k_1,(1-\xi_8)k_2,\{k_i\}_{3,6},k_7)\}$$
$$\times\frac{s_{12}}{8(2\pi)^3}\xi_7\mathcal{P}(y_7)d\xi_7d\xi_8dy_7d\varphi_7d\Phi_{(3-6)}, \tag{11}$$

where we introduced

$$\mathcal{L}_7=\log\frac{s_{12}\delta_I\xi_7^2}{2\mu^2},\qquad \mathcal{L}_8=\log\frac{s_{12}\delta_I\xi_8^2}{2\mu^2}, \tag{12}$$

and split up the unregularized Altarelli-Parisi function into the 4-dimensional piece, $P^{<}_{ad}$, and the piece proportional to ε, $P'^{<}_{ad}$. For completeness we give the explicit form

$$P^{<}_{qg}(1-\xi_i)=\frac{1}{2}(1-2\xi_i+2\xi_i^2) \tag{13}$$

$$P'^{<}_{qg}(1-\xi_i)=\xi_i(\xi_i-1). \tag{14}$$

In this case, the single collinear singularities of the NLO amplitudes are subtracted by the appearance of a single $\mathcal{P}$ distribution.

Finally, the 6-parton finite piece is given by

$$d\sigma^{(\mathrm{fin},6)}=\left(\frac{\alpha_s}{2\pi}\right)^2(\mathcal{L}_7P^{<}_{qg}(1-\xi_7)-P'^{<}_{qg}(1-\xi_7))$$
$$\times(\mathcal{L}_8P^{\leq}_{qg}(1-\xi_8)-P'^{<}_{qg}(1-\xi_8))$$
$$\times\{\mathcal{M}^{(6)}_{qq}((1-\xi_7)k_1,(1-\xi_8)k_2,\{k_i\}_{(3,6)})$$
$$+\mathcal{M}^{(6)}_{qq}((1-\xi_8)k_1,(1-\xi_7)k_2,\{k_i\}_{(3,6)})\}$$
$$\times d\xi_7d\xi_8d\Phi_{(3-6)}. \tag{15}$$

We note that δ_I is an arbitrary quantity and even though $d\sigma^{(\mathrm{fin},8)}$,$d\sigma^{(\mathrm{fin},7)}$ and $d\sigma^{(\mathrm{fin},6)}$ depend on it, this dependence cancels exactly in $d\hat{\sigma}^{(2)}_{gg}$. Hence, making sure that the numerical results are independent of δ_I is a useful check for our Monte Carlo program.

In calculating the cross section as above, we use 6 parton and 7 parton amplitudes as given in [8]. We also require the 8 and 7 parton amplitudes for $gg\to q\bar{q}WZ$ and $gg\to q\bar{q}W\gamma$ respectively. These amplitudes have been calculated previously [15]. We recalculated these amplitudes using the helicity method and including anomalous couplings. For the triple gauge vertex for $W^+_\alpha(p_+)W^-(p_-)_\beta V_\mu(q)$, including the anomalous terms, we use

$$\Gamma^{\alpha\beta\mu}_{WWV}(p_+,p_-,q)=\Bigg((p_+-q)^\alpha g^{\beta\mu}\frac{1}{2}\left(g_1^V+\kappa^V+\lambda^V\frac{p_-^2}{M_W^2}\right)$$
$$+(q-p_-)^\beta g^{\alpha\mu}\frac{1}{2}\left(g_1^V+\kappa^V+\lambda^V\frac{p_+^2}{M_W^2}\right)$$
$$+(p_+-p_-)^\mu\left(-g^{\alpha\beta}\frac{1}{2}q^2\frac{\lambda^V}{M_W^2}\right.$$
$$\left.+\frac{\lambda^V}{M_W^2}q^\alpha q^\beta\right)\Bigg), \tag{16}$$

where V can be Z or γ and all momenta are taken to be outgoing. In the standard model, $g_1^V=\kappa^V=1$ and $\lambda^V=0$. In order to preserve electromagnetic gauge invariance we always set g_1^γ to the standard model value 1.

We also make use of form factors to avoid the violation of unitarity that would otherwise result from this vertex at high energies. The form factors used are the conventional ones:

$$\Delta g_1^V\to\frac{\Delta g_1^V}{(1+\hat{s}/\Lambda^2)^2},\quad \Delta\kappa^V\to\frac{\Delta\kappa^V}{(1+\hat{s}/\Lambda^2)^2},$$
$$\Delta\lambda^V\to\frac{\Delta\lambda^V}{(1+\hat{s}/\Lambda^2)^2} \tag{17}$$

where Λ is the scale of the new physics that causes the anomalous couplings.

Photons with large transverse momentum can be produced in hadronic collisions not only directly, but also, and significantly, from the fragmentation of a final state parton. A full perturbative calculation of a process involving the production of photons should in principle include the calculation of both direct and fragmentation components, since only their sum is physically well defined beyond LO. A calculation of the fragmentation part is not even available at NLO for the process of interest in this paper. Fortunately there is a way to suppress the fragmentation contribution, which actually constitutes a background to the search of anomalous couplings, by requiring the photons to be isolated from the hadrons. The usual way to perform the isolation is to require the transverse hadronic momentum in a cone around the photon to be smaller than a fraction of the transverse momentum of the photon. In this way, the contribution of the fragmentation component can be reduced to the percent level. In this paper, we will use the isolation procedure introduced by Frixione [18] which allows us to completely suppress the fragmentation component. Therefore, we reject all events unless the transverse hadronic momentum deposited in a cone of size R_0 around the momentum of the photon satisfies the following condition

$$\sum_i p_{Ti}\,\theta(R-R_{i\gamma}) \leqslant p_{T\gamma}\left(\frac{1-\cos R}{1-\cos R_0}\right), \tag{18}$$

for all $R \leqslant R_0$, where the "distance" in pseudorapidity and azimuthal angle is defined by $R_{i\gamma} = \sqrt{(\eta_i-\eta_\gamma)^2+(\phi_i-\phi_\gamma)^2}$. In this way, only soft partons can be emitted collinearly to the photon in the direct contribution and, therefore, no final state quark-photon collinear singularity arises. For the purposes of our computation this allows us to perform the calculation of the gg initiated contribution at NNLO without needing to do any subtraction of the corresponding singularity. While the possibility of performing such isolation at the experimental level is under investigation, the choice of this particular method will not alter at all the conclusions of our calculation in the $W\gamma$ channel. Particularly, we have checked that the results at NLO using the procedure in Eq. (18) are very close to the ones obtained performing the usual "cone" isolation procedure [10,19].

In this calculation, contributions from b and t quarks have been neglected. It is assumed that these will be suppressed by the large top quark mass. This might not be a particularly good approximation for large energies, but in view of the smallness of the gluon induced corrections a more detailed treatment does not seem to be justified.

III. NUMERICAL RESULTS

For the numerical results presented in this section we use the Martin-Roberts-Stirling-Thorne 2001 (MRST 2001) parton distribution functions [20] with the one-loop expression for the coupling constant $[\alpha_s(M_Z)=0.119]$. The factorization and renormalization scales are fixed to $\mu_F=\sqrt{\hat{s}}, \mu_R = \sqrt{\tfrac{1}{2}(M_W^2+M_Z^2)+\tfrac{1}{2}(p_{TW}^2+p_{TZ}^2)}$ in the case of W^-Z production and to $\mu_F=\mu_R=\sqrt{M_W^2+p_{T\gamma}^2}$ in the $W^-\gamma$ case. Notice that in order to compute the gg induced contribution we also use the same NLO combination of parton distribution function and coupling constant. The effect of changing to NNLO parton distributions, not fully available yet, is expected to be small and will not alter the conclusion of our analysis.

The masses of the vector bosons have been set to $M_Z = 91.187$ GeV and $M_W = 80.41$ GeV. We do not include any electroweak corrections, but choose the coupling constants α and $\sin^2\theta_W$ in the spirit of the improved Born approximation [21]. For the couplings of the vector bosons with the quarks we use $\alpha=\alpha(M_Z)=1/128$ whereas for the photon coupling we use $\alpha=1/137$. We neglect contributions from initial state top quarks and use the following values for the Cabibbo-Kobayashi-Maskawa matrix elements: $|V_{ud}| = |V_{cs}| = 0.975$ and $|V_{us}| = |V_{cd}| = 0.222$. Note that we do not include the branching ratios for the decay of the vector bosons into leptons.

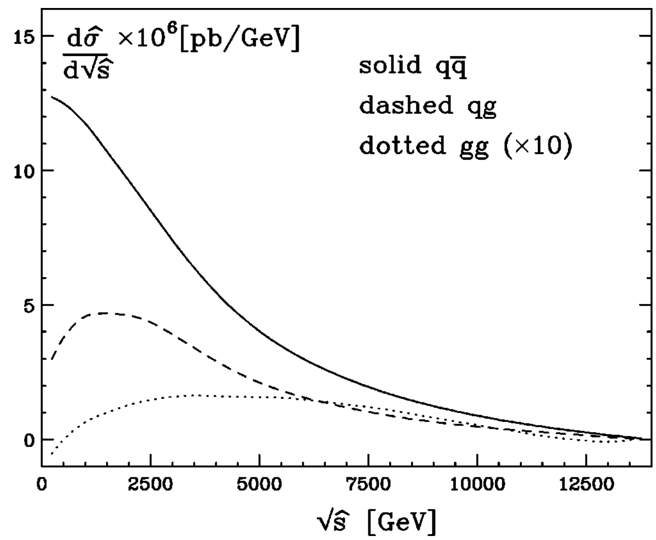

FIG. 1. Partonic cross sections for $W^-\gamma$: $\hat{\sigma}_{q\bar{q}}$, $\hat{\sigma}_{qg}$ and $\hat{\sigma}_{gg}$.

For all plots we use a set of standard cuts. For charged leptons we require $p_T > 20$ GeV and $\eta < 2.5$. In addition, we require a missing transverse momentum $p_T^{\rm miss} > 20$ GeV. The photon transverse momentum cut we use is $p_T^\gamma > 20$ GeV, while for the isolation prescription in Eq. (18) we set $R_0 = 1$.

In Figs. 1 and 2 we show the $\overline{\rm MS}$ subtracted partonic cross sections for the different initial states $q\bar{q}, qg$ and gg. These are produced by binning the results at 14 TeV by $\sqrt{\hat{s}}$. It can be seen that the outcomes for $W\gamma$ and WZ are very similar. For better visibility $\hat{\sigma}_{gg}$ has been increased by a factor 10 in the plot.

As can be observed, there is a reasonably good perturbative convergence at the level of the partonic cross section,

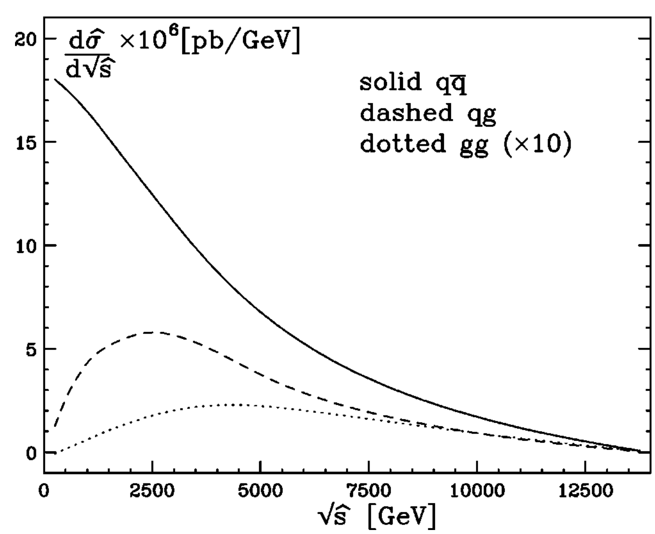

FIG. 2. Partonic cross sections for W^-Z: $\hat{\sigma}_{q\bar{q}}$, $\hat{\sigma}_{qg}$ and $\hat{\sigma}_{gg}$

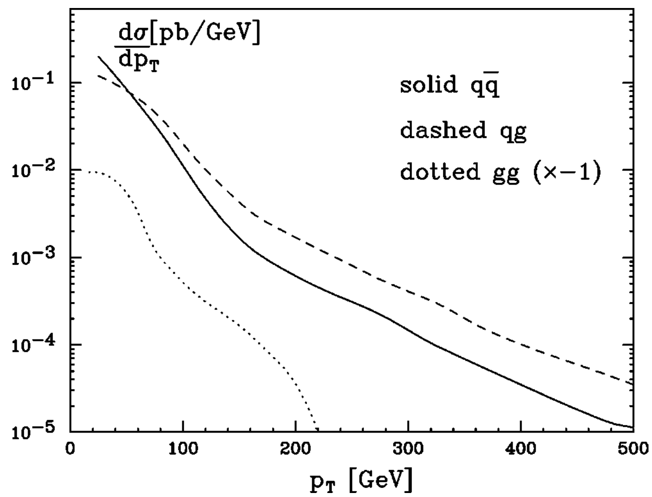

FIG. 3. $W^-\gamma$ production: p_T distribution for LHC, $q\bar{q}$,qg and gg pieces separately.

considering that $\hat{\sigma}_{q\bar{q}}$ is of $\mathcal{O}(1+\alpha_s)$, $\hat{\sigma}_{qg}$ of $\mathcal{O}(\alpha_s)$, and $\hat{\sigma}_{gg}$ of $\mathcal{O}(\alpha_s^2)$, with $\alpha_s\sim 0.1$ for the typical scales of these processes. In the $\overline{\rm MS}$ both the $q\bar{q}$ and qg contributions are positive in the whole range of $\hat{s}$, while the new piece, the gg partonic cross section becomes negative only for small values of $\hat{s}$, close to the threshold for the production of the vector bosons.

The situation changes considerably when the physical hadronic cross section is computed, shown in Figs. 3 and 4. In Fig. 3 we plot the transverse momentum distribution of the photon, whereas in Fig. 4 we see the p_T of the lepton produced by the decay of the W. Again the form of the distributions is very similar for the WZ as compared to the $W\gamma$ case.

As indicated in the Introduction, because of the large qg luminosity, the hadronic contribution due to this initial state becomes even larger than the "leading" $q\bar{q}$ contribution. This is particularly clear at large transverse momentum,

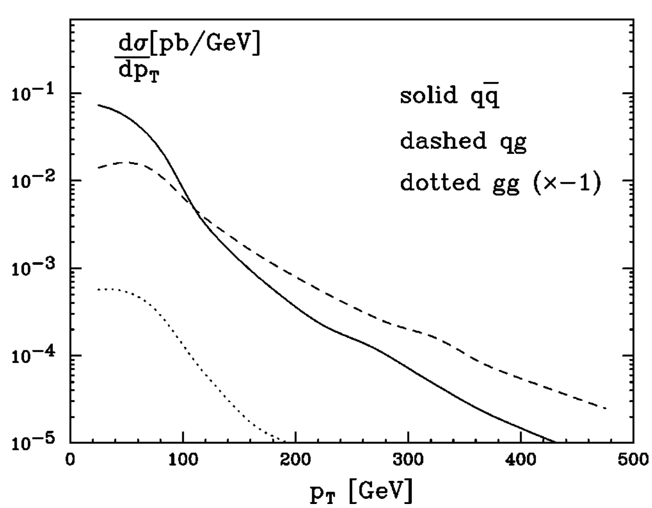

FIG. 4. W^-Z production: p_T distribution for LHC, $q\bar{q}$,qg and gg pieces separately.

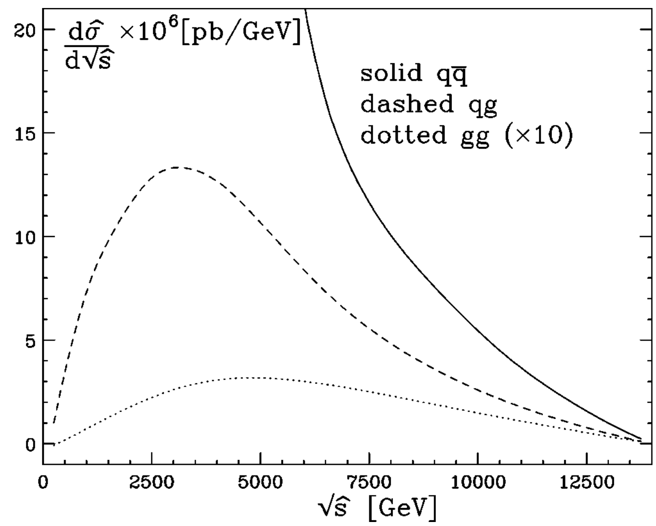

FIG. 5. Partonic cross section $\hat{\sigma}_{q\bar{q}}$, $\hat{\sigma}_{qg}$ and $\hat{\sigma}_{gg}$ for W^-Z production with anomalous couplings as given in Eq. (19).

where the relative increase in the luminosity exceeds the effect of the suppression due to the extra α_s coupling observed at the partonic level. This is not the case for the gg contribution, which remains rather small, and for the observable studied here provides a negative contribution to the hadronic cross section.

The features of both qg and gg contributions to WZ and $W\gamma$ production can be qualitatively understood in the following way: the qg and gg luminosities are steep functions of the momentum fraction carried by the partons, mostly due to the fast increase of the gluon distribution $g(x)$ when x decreases. Even though the hadronic center of mass energy S is very large, the average value of the partonic one $\langle\hat{s}\rangle=\langle x_1x_2\rangle S$ can be considerably smaller, and actually closer to the minimum needed to produce the gauge bosons with the required transverse momentum. Therefore in the convolution between the parton distributions and the partonic cross sections shown in Figs. 1 and 2, the hadronic result will mostly pick up the features of the partonic cross section at low values of $\hat{s}$. In the qg case, the partonic cross section is positive and has its maximum at low $\hat{s}$, giving as a result a large hadronic contribution. In the gg case, the partonic cross section has a change of sign at low values of $\hat{s}$, with the corresponding compensation between negative and positive contributions when convoluting with the parton densities. Therefore the gg hadronic contribution results in a negative and small correction to the NLO cross section.

In order to investigate whether the inclusion of anomalous couplings changes this picture substantially, we show in Fig. 5 the partonic cross sections with the following anomalous couplings:

$$g_1^\gamma=1;\quad g_1^Z=1.13;\quad \kappa^\gamma=1.2;\quad \kappa^Z=1.07;$$
$$\lambda^\gamma=\lambda^Z=0.1. \tag{19}$$

We use the form factors as given in Eq. (17) and we take Λ, the scale of new physics, to be 2 TeV. Figure 5 is given with the same scale as Fig. 2 for comparison. It is clear that, with the values for the couplings that we have chosen, we do not see a significant enhancement of the gluon induced term. In fact, the gluon induced corrections become even less relevant, as the qg and especially the $q\bar{q}$ terms are increased substantially.

In addition to these results, we also considered a hypothetical VLHC or Very Large Hadron Collider, which would run at a center of mass energy of 200 TeV, to see whether the gg part would become significant at the higher energy (cuts etc. all remained the same as previously). It was found that the gg part did not become more important. The contributions from the gg term remained at the 1% level.

IV. CONCLUSIONS

We have presented expressions for the finite cross section for vector-boson pair production from a gluon-gluon initial state. We have then implemented these terms and the appropriate matrix elements into a Monte Carlo integration.

We find the contribution of the gluon induced terms in WZ and $W\gamma$ production to be surprisingly small. Even the addition of anomalous couplings enhances the gg induced term substantially less than the $q\bar{q}$ and qg terms. We have shown that this is due to a change of sign in the hard scattering.

This is a good result from the point of view of experimental predictions of vector-boson pair production, at least in the WZ and $W\gamma$ cases, as the impact of NNLO terms does not appear to be particularly significant. It also seems that the gg term will not be amplified excessively by anomalous couplings, though we have only taken an example of anomalous couplings and have not made a detailed study.

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