Improved analysis for the baryon masses to order $\Lambda_{\mathrm{QCD}}/m_Q$ from QCD sum rules

Dao-Wei Wang
Department of Applied Physics, National University of Defense Technology, Hunan 410073, China

Ming-Qiu Huang
Department of Applied Physics, National University of Defense Technology, Hunan 410073, China and CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China

Cheng-Zu Li
Department of Applied Physics, National University of Defense Technology, Hunan 410073, China
(Received 17 January 2002; published 15 May 2002)

We use the QCD sum rule approach to calculate the masses of the Λ_Q and Σ_Q baryons to $\Lambda_{\mathrm{QCD}}/m_Q$ order within the framework of heavy quark effective theory. We compare the direct approach and the covariant approach to this problem. Two forms of current have been adopted in our calculation and their effects on the results are discussed. Numerical results obtained in both the direct and covariant approaches are presented. The splitting between spin 1/2 and 3/2 doublets derived from our calculation is $\Sigma_Q^{*2}-\Sigma_Q^2 \simeq 0.35\pm 0.03\ \mathrm{GeV}^2$, which is in good agreement with experiment.

DOI: 10.1103/PhysRevD.65.094036 PACS number(s): 14.20.$-$c, 11.55.Hx, 12.38.Lg, 12.39.Hg

I. INTRODUCTION

Important progress in the theoretical description of hadrons containing a heavy quark has been achieved with the development of the heavy quark effective theory (HQET) [1–3]. Based on the spin-flavor symmetry of QCD, exactly valid in the infinite m_Q limit, this framework provides a systematic expansion of heavy hadron spectra and both the strong and weak transition amplitudes in terms of the leading contribution, plus corrections decreasing as powers of $1/m_Q$. HQET has been applied successfully to learn about the properties of mesons and baryons made of both heavy and light quarks.

The effective Lagrangian of the HQET, up to order $1/m_Q$, can be written as

$$\mathcal{L}_{\mathrm{eff}}=\overline{h}_v iv\cdot Dh_v+\frac{1}{2m_Q}\mathcal{K}+\frac{1}{2m_Q}\mathcal{S}+\mathcal{O}(1/m_Q^2), \qquad (1)$$

where $h_v(x)$ is the heavy quark field in effective theory. Apart from the leading contribution, the Lagrangian density contains to $\mathcal{O}(1/m_Q)$ accuracy two additional operators $\mathcal{K}$ and $\mathcal{S}$. $\mathcal{K}=\overline{h}_v(iD^{\perp})^2h_v$ is the nonrelativistic kinetic energy operator and $\mathcal{S}=\frac{1}{2}[\alpha_s(m_Q)/\alpha_s(\mu)]^{3/\beta_0}\overline{h}_v\sigma_{\mu\nu}g_sG^{\mu\nu}h_v$ is the chromomagnetic interaction. Here $(D^{\perp})^2=D_\mu D^\mu-(v\cdot D)^2$, with $D_\mu=\partial_\mu-igA_\mu$ the covariant derivative, and $\beta_0=11-\frac{2}{3}n_f$ is the first coefficient of the β function.

The matrix elements of the operators $\mathcal{K}$ and $\mathcal{S}$ in Eq. (1) play a most significant role in many phenomenological applications such as the spectroscopy of heavy hadrons [4] and the description of inclusive decay rates [5]. For the ground-state Λ_Q and Σ_Q baryons, one defines two hadronic parameters λ_1 and λ_2 as

$$\langle B(v)|\mathcal{K}|B(v)\rangle=\lambda_1,$$

$$\langle B(v)|\mathcal{S}|B(v)\rangle=d_M\lambda_2, \qquad (2)$$

where d_M is zero for Λ_Q and $-\frac{1}{2}$,1 for Σ_Q^*,Σ_Q baryons, respectively. The constant d_M characterizes the spin-orbit interaction of the heavy quark and the gluon field. Therefore, the mass of heavy baryons up to order $1/m_Q$ corrections can be written in a compact form:

$$M=m_Q+\overline{\Lambda}-\frac{1}{2m_Q}(\lambda_1+d_M\lambda_2), \qquad (3)$$

where the parameter $\overline{\Lambda}$ is the energy of the light degrees of freedom in the infinite mass limit. Thus the splitting of the spin 1/2 and 3/2 doublets is

$$\Sigma_Q^{*2}-\Sigma_Q^2=\frac{3}{2}\lambda_2. \qquad (4)$$

The hadronic parameters λ_1 and λ_2 are nonperturbative ones that should be either determined phenomenologically from experimental data or estimated in some nonperturbative approach. A viable approach is the QCD sum rules [6] formulated in the framework of HQET [7]. This method allows us to relate hadronic observables to QCD parameters via the operator product expansion (OPE) of the correlator. In the case of heavy mesons, these two matrix elements and thus masses were calculated completely first by Ball and Braun [8] and later by Neubert [9] taking a different approach. The masses of excited meson states were calculated up to $1/m_Q$ order in [10]. For the case of heavy baryons, there are several attempts to calculate the baryonic matrix elements of $\mathcal{K}$ and $\mathcal{S}$. Using HQET sum rules, Colangelo *et al.* have derived the value of λ_1 for Λ_Q baryons [11]. Furthermore, the baryonic parameters λ_1 and λ_2 for the ground-state baryons were calculated in [12] by evaluating the two-point correlation functions. The mass parameters of the lowest lying excited heavy baryons have also been determined recently in [13]. In the present work we shall calculate the baryonic parameters λ_1 and λ_2 for ground-state Λ_Q and Σ_Q baryons using QCD sum rules in the HQET. Following Ball and Braun [8] and Neubert's [9] work done for the meson, we adopt these two approaches, named the direct approach and covariant approach, to evaluate the three-point correlators and obtain the values of baryonic parameters. It is of interest to compare the two methods in the analysis.

The remainder of this paper is organized as follows. In Sec. II A we introduce the interpolating currents for baryons and briefly present the two-point sum rules. The direct Laplace sum rules analysis for the matrix elements is presented in Sec. II B. Another feasible approach (covariant approach) with this aim can be found in Sec. II C. Section III is devoted to numerical results and our conclusions. Some comments are also available in Sec. III.

II. DERIVATION OF THE SUM RULES FOR λ_1 AND λ_2

A. Heavy baryonic currents and two-point sum rules

The basic points in the application of QCD sum rules to problems involving heavy baryons are to choose a suitable interpolating current in terms of quark fields and to define the corresponding vacuum-to-baryon matrix element. As is well known, the form of interpolating currents for baryons with given spin and parity is not unique [14–16]; the choice of which one is just a question of predisposition. The most generally used form of the heavy baryon current can be written as [15]

$$j^v = \epsilon_{abc}(q_1^{Ta} C\Gamma \tau q_2^b)\Gamma' h_v^c, \tag{5}$$

in which C is the charge conjugation matrix, τ is the flavor matrix, which is antisymmetric for Λ_Q baryons and symmetric for $\Sigma_Q^{(*)}$ baryons, Γ and Γ' are some gamma matrices, and a,b,c denote the color indices. Γ and Γ' can be chosen covariantly as

$$\Gamma = \gamma_5, \quad \Gamma' = 1 \tag{6}$$

for the Λ_Q baryon, and

$$\Gamma = \gamma_\mu, \quad \Gamma' = (\gamma_\mu + v_\mu)\gamma_5 \tag{7}$$

for the Σ_Q baryon, and

$$\Gamma = \gamma_\nu, \quad \Gamma' = -g_{\mu\nu} + \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3}(\gamma_\mu v_\nu - \gamma_\nu v_\mu) + \frac{2}{3}v_\nu v_\mu \tag{8}$$

for Σ_Q^* baryon. Also the choice of Γ is not unique. We can insert a factor $\not{v}$ before Γ defined by Eqs. (6)–(8). The current given by Eqs. (6)–(8) is denoted as j_1^v and that with $\not{v}$ insertion as j_2^v, which are two independent current representations.

The baryonic coupling constants in HQET are defined as follows:

$$\langle 0|j^v|\Lambda(v)\rangle = F_\Lambda u,$$

$$\langle 0|j^v|\Sigma(v)\rangle = F_\Sigma u,$$

$$\langle 0|j^v|\Sigma^*(v)\rangle = \frac{1}{\sqrt{3}}F_{\Sigma^*}u^\alpha, \tag{9}$$

where u is the spinor and u_α is the Rarita-Schwinger spinor in the HQET, respectively. The coupling constants F_Σ and F_Σ^* are equivalent since Σ_Q and Σ_Q^* belong to the doublet with the same spin parity of the light degrees of freedom.

The QCD sum rule determination of these coupling constants can be done by analyzing the two-point function

$$i\int dx e^{ik\cdot x}\langle 0|T\{j^v(x)\overline{j^v}(0)\}|0\rangle = \frac{1+\not{v}}{2}\mathrm{Tr}[\tau\tau^+]\Pi(\omega), \tag{10}$$

where k is the residual momentum and $\omega = 2v\cdot k$. It is straightforward to obtain the two-point sum rule:

$$F_\Lambda^2 e^{-2\bar{\Lambda}_\Lambda/T} = \frac{3T^6}{2^5\pi^4}\delta_5(\omega_c/T) + \frac{T^2}{2^7\pi^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\delta_1(\omega_c/T) + \frac{1}{6}\langle\bar{q}q\rangle^2,$$

$$F_\Sigma^2 e^{-2\bar{\Lambda}_\Sigma/T} = \frac{9T^6}{2^5\pi^4}\delta_5(\omega_c/T) - \frac{T^2}{2^7\pi^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\delta_1(\omega_c/T) + \frac{1}{2}\langle\bar{q}q\rangle^2. \tag{11}$$

The functions $\delta_n(\omega_c/T)$ arise from the continuum subtraction and are given by

$$\delta_n(x) = \frac{1}{n!}\int_0^x dt t^n e^{-t} = 1 - e^{-x}\sum_{k=0}^{n}\frac{x^k}{k!}. \tag{12}$$

The second term of the last equation is assigned to the continuum mode, which can be much larger than the ground-state contributions for the typical value of parameter T due to the high dimensions of the spectral densities.

B. The direct approach

In order to evaluate the matrix elements λ_1 and λ_2 we consider the three-point correlation functions with $\mathcal{K}$ and $\mathcal{S}$ inserted directly between two interpolating currents at zero recoil as below:

$$i^2\int dx\int dy e^{ik\cdot x - ik'\cdot y}\langle 0|T\{j^v(x)\mathcal{K}(0)\overline{j^v}(y)\}|0\rangle = \frac{1+\not{v}}{2}\mathrm{Tr}[\tau\tau^+]T_K(\omega,\omega'),$$

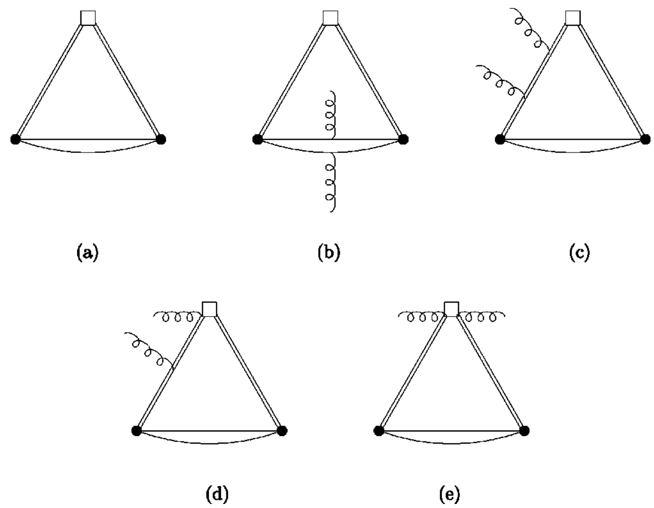

FIG. 1. Nonvanishing diagrams for the kinetic energy: (a) perturbative contribution, (b)–(e) gluon condensate. The kinetic energy operator is denoted by a white square, the interpolating baryon currents by black circles. Heavy-quark propagators are drawn as double lines. Diagrams (b)–(e) are calculated in the Fock-Schwinger gauge. The lower right vertices of those diagrams are set to the origin in coordinate space.

$$i^2\int dx\int dy e^{ik\cdot x-ik'\cdot y}\langle 0|T\{j^v(x)\mathcal{S}(0)\overline{j^v}(y)\}|0\rangle$$

$$=d_M\frac{1+\not v}{2}\mathrm{Tr}[\tau\tau^+]T_S(\omega,\omega'), \tag{13}$$

where the coefficients $T_K(\omega,\omega')$ and $T_S(\omega,\omega')$ are analytic functions in the "off-shell energies" $\omega=2v\cdot k$ and $\omega'=2v\cdot k'$ with discontinuities for positive values of these variables. Saturating the three-point functions with a complete set of baryon states, one can isolate the part of interest, the contribution of the lowest lying baryon states associated with the heavy-light currents, as one having poles in both the variables ω and ω' at the value $\omega=\omega'=2\bar\Lambda$:

$$T_K(\omega,\omega')=4\frac{\lambda_1F^2}{(2\bar\Lambda-\omega)(2\bar\Lambda-\omega')}+\cdots,$$

$$T_S(\omega,\omega')=4\frac{\lambda_2F^2}{(2\bar\Lambda-\omega)(2\bar\Lambda-\omega')}+\cdots, \tag{14}$$

where the ellipses denote the contribution of higher resonances. In the theoretical calculation of the correlator it is convenient to choose the residual momenta k and k' parallel to v, such that $k_\mu=(\omega/2)v_\mu$ and $k'_\mu=(\omega'/2)v_\mu$.

The leading contribution to the matrix element of kinetic energy is of order 1, whereas that to the chromomagnetic interaction is of order α_s. Confining ourselves to taking into account these leading contributions of perturbation and the operators with dimension $D\leqslant 6$ in the OPE, the relevant diagrams in our calculation are shown in Fig. 1 and Fig. 2. The calculation of the diagram (a) in Fig. 2 is the most tedious one. It can be computed using Feynman parametrization and the integral representation of the propagators, which is the standard technique [18,19]. The factorization approximation has been used to reduce the four-quark condensates to $\langle\bar qq\rangle^2$ in the calculation.

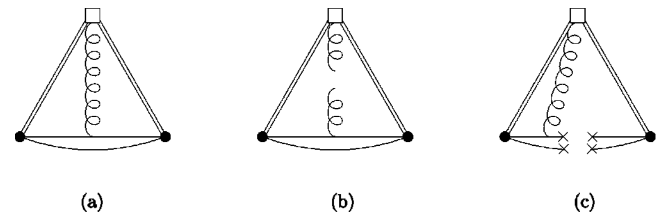

FIG. 2. Nonvanishing diagrams for the chromomagnetic interaction: (a) perturbative contribution, (b) gluon-condensate, (c) quark-condensate. The chromomagnetic interaction (velocity-changing current) operator is denoted by a white square, the interpolating baryon currents by black circles.

On the theoretical side the correlators $T_K(\omega,\omega')$ and $T_S(\omega,\omega')$ can be cast into the form of integrals of double spectral densities as

$$T_K(\omega,\omega')=\int\int\frac{ds}{s-\omega}\frac{ds'}{s'-\omega'}\rho_K(s,s'),$$

$$T_S(\omega,\omega')=\int\int\frac{ds}{s-\omega}\frac{ds'}{s'-\omega'}\rho_S(s,s'), \tag{15}$$

where the double spectral density functions are

$$\rho_K^{\Lambda,1}(s,s')=-\frac{3^3}{2^4\pi^4 7!}s^7\delta(s-s')$$

$$-\frac{7}{2^6\pi^2 3!}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle s^3\delta(s-s'),$$

$$\rho_K^{\Sigma,1}(s,s')=-\frac{3^2 11}{2^4\pi^4 7!}s^7\delta(s-s')$$

$$-\frac{11}{2^6\pi^2 3!}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle s^3\delta(s-s'),$$

$$\rho_K^{\Lambda,2}(s,s')=-\frac{3^2 5}{2^4\pi^4 7!}s^7\delta(s-s')$$

$$+\frac{1}{2^6\pi^2 3!}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle s^3\delta(s-s'),$$

$$\rho_K^{\Sigma,2}(s,s')=-\frac{3^2}{\pi^4 7!}s^7\delta(s-s')$$

$$-\frac{19}{2^6\pi^2 3!}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle s^3\delta(s-s'),$$

$$\rho_S^{\Sigma}(s,s')=\frac{\alpha_s}{2^3\pi^5}\bigg[\Theta(s-s')\int_0^{s'}dx(s-x)(s'-x)x^3+\Theta(s'-s)\int_0^{s}dx(s-x)(s'-x)x^3\bigg]-\frac{1}{48\pi^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle s^3\delta(s-s')+\frac{4\alpha_s}{3\pi}\langle\bar{q}q\rangle^2[s\,\delta(s')+s'\,\delta(s)]. \tag{16}$$

The unitary normalization of the flavor matrix $\mathrm{Tr}[\tau\tau^+]=1$ has been applied to get these densities. Here we use the numbers 1,2 to denote the results corresponding to the different choices of currents j_1^v and j_2^v. Those we do not discriminate with numeric superscripts indicate that with or without $\not v$ insertion the results are identical. Following Refs. [20–22], we then introduce new variables $\omega_+=\frac{1}{2}(\omega+\omega')$ and $\omega_-=\omega-\omega'$, perform the integral over ω_-, and employ quark-hadron duality to equate the remaining integral over ω_+ up to a "continuum threshold" ω_c to the Borel transform of the double-pole contribution in Eq. (14). Then following the standard procedure we resort to the Borel transformation $B_\tau^\omega, B_{\tau'}^{\omega'}$ to suppress the contributions of the excited states. Considering the symmetries of the correlation functions it is natural to set the parameters τ, τ' to be the same and equal to $2T$, where T is the Borel parameter of the two-point functions. We end up with the set of sum rules

$$-4\lambda_1^{\Lambda,1}F^2e^{-2\bar{\Lambda}_\Lambda/T}=\frac{3^3T^8}{(2\pi)^4}\delta_7(\omega_c/T)+\frac{7T^4}{2^6\pi^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\delta_3(\omega_c/T),$$

$$-4\lambda_1^{\Lambda,2}F^2e^{-2\bar{\Lambda}_\Lambda/T}=\frac{3^25T^8}{(2\pi)^4}\delta_7(\omega_c/T)-\frac{T^4}{2^6\pi^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\delta_3(\omega_c/T),$$

$$-4\lambda_1^{\Sigma,1}F^2e^{-2\bar{\Lambda}_\Sigma/T}=\frac{3^211T^8}{(2\pi)^4}\delta_7(\omega_c/T)+\frac{11T^4}{2^6\pi^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\delta_3(\omega_c/T),$$

$$-4\lambda_1^{\Sigma,2}F^2e^{-2\bar{\Lambda}_\Sigma/T}=\frac{3^2T^8}{\pi^4}\delta_7(\omega_c/T)+\frac{19T^4}{2^6\pi^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\delta_3(\omega_c/T),$$

$$4\lambda_2F^2e^{-2\bar{\Lambda}_\Sigma/T}=\frac{12}{\pi^4}\frac{\alpha_s}{\pi}T^8\delta_7(\omega_c/T)-\frac{T^4}{8\pi^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\delta_3(\omega_c/T)+\frac{32T^2\alpha_s}{3\pi}\langle\bar{q}q\rangle^2\delta_1(\omega_c/T). \tag{17}$$

It is worth noting that the next-to-leading order α_s corrections have not been included in the sum rule calculations. However, the baryonic parameter obtained from the QCD sum rules actually is a ratio of the three-point correlator to the two-point correlator results. While both of these correlators are subject to large perturbative QCD corrections, it is expected that their ratio is not much affected by these corrections because of cancellation. On the other hand, we have only calculated the diagonal sum rules by using the same type of interpolating current in the correlator. As to the nondiagonal sum rules, the only nonvanishing contributions in the OPE of the correlator are terms with odd numbers of dimensions; thus the perturbative term gives no contribution. The resulting sum rules are dominated by the quark-gluon condensates. It is expected that the nondiagonal sum rules will give no more information than diagonal ones. This has been proved to be true in the analysis of Ref. [17].

C. The covariant approach

In the previous subsection we have completed the task of determining the matrix elements for both the operators of kinetic energy and chromomagnetic interaction by direct calculation of three-point correlation functions. In fact, there exists a field-theory analog of the virial theorem [23,24] in consideration of the restrictions the equation of motion and the heavy quark symmetry impose on baryons, which relates the kinetic energy and chromointeraction to each other and ensures the intrinsic smallness of the kinetic energy explicitly. In this subsection we shall follow Neubert's procedure [9] and take those restrictions into account to deduce a new result for the kinetic energy (the chromomagnetic interaction is identical).

The main idea of that procedure is that the coefficients of the covariant decomposition of the bilinear matrix elements, the so-called invariant functions, can be related to the kinetic energy and chromomagnetic interaction at zero recoil. Following the discussion in [4,25] we have the general decomposition (see the Appendix)

$$\langle\Lambda|\bar{h}_v\sigma_{\mu\nu}ig_sG^{\mu\nu}h_{v'}|\Lambda'\rangle=\phi_1(v',v)(v'_\mu v_\nu-v_\mu v'_\nu)\bar{u}\sigma^{\mu\nu}u' \tag{18}$$

for the Λ_Q baryon, in which u is the spinor in HQET, and

$$\langle\Sigma|\bar{h}_v\sigma_{\mu\nu}ig_sG^{\mu\nu}h_{v'}|\Sigma'\rangle=\phi^{\mu\nu}_{\alpha\beta}(v',v)\bar{\Psi}^\alpha\sigma^{\mu\nu}\Psi'^\beta \tag{19}$$

for the Σ_Q baryon, where $\phi^{\mu\nu}_{\alpha\beta}$ bears the decomposition

$$\begin{aligned}\phi^{\mu\nu}_{\alpha\beta}=&\phi_1(g_{\mu\alpha}g_{\nu\beta}-g_{\mu\beta}g_{\nu\alpha})\\&+\phi_2(g_{\mu\beta}v_\nu v'_\alpha-g_{\nu\beta}v_\mu v'_\alpha+g_{\nu\alpha}v_\beta v'_\mu-g_{\mu\alpha}v_\beta v'_\nu)\\&+\phi_3(g_{\alpha\mu}v_\nu v_\beta-g_{\nu\alpha}v_\mu v_\beta+g_{\nu\beta}v'_\alpha v'_\mu-g_{\mu\beta}v'_\alpha v'_\nu)\\&+\phi_4(v'_\mu v_\nu-v'_\nu v_\mu)g_{\alpha\beta}+\phi_5(v'_\mu v_\nu-v'_\nu v_\mu)v'_\alpha v_\beta,\end{aligned}\tag{20}$$

in which we use the covariant representation of the doublets $\Psi_\mu=u_\mu+(1/\sqrt{3})(v_\mu+\gamma_\mu)u$ with restrictions $\not{v}u=u$, $v_\mu u_\mu=0$, and $\gamma_\mu u_\mu=0$. The normalization of these coefficients at zero recoil is $\phi_1(1)=-\frac{1}{3}\lambda_1$ for the Λ_Q baryon and

$$\begin{aligned}&\pm\lambda_2=2\phi_1(1),\\&\pm\lambda_1=\phi_0(1)=\phi_1(1)-2[\phi_2(1)-\phi_3(1)]-3\phi_4(1)\end{aligned}\tag{21}$$

for the Σ_Q baryon. The foregoing minus sign corresponds to the Σ_Q baryon and plus sign to the Σ_Q^* baryon.

Let us now derive the Laplace sum rules for the invariant functions $\phi_i(w)$. The analysis proceeds in complete analogy to that of the Isgur-Wise function. We shall only briefly sketch the general procedure and refer for details to Refs. [20,21]. We consider, in the HQET, the three-point correlation function of the local operator appearing in Eq. (19) with two interpolating currents for the ground-state heavy baryons:

$$\begin{aligned}&i^2\int dxdye^{ik\cdot x-ik'\cdot y}\\&\quad\times\langle0|\mathrm{T}\{j^v(x),\overline{h}_v ig_s\Gamma G^{\mu\nu}h_{v'}(0),\overline{j}^{v'}(y)\}|0\rangle\\&\quad=\Phi^{\mu\nu}_{\alpha\beta}(v',v,k',k)\Gamma'_\alpha\frac{1+\not{v}}{2}\Gamma\frac{1+\not{v}'}{2}\overline{\Gamma}'_\beta,\end{aligned}\tag{22}$$

where k and k' are the residual momenta. The Dirac structure of the correlation function, as shown in the third line, is a consequence of the Feynman rules of the HQET. $\Phi^{\mu\nu}_{\alpha\beta}$ obeys a decomposition analogous to Eq. (20), with coefficient functions $\Phi_i(\omega,\omega',w)$ that are analytic in the "residual energy" $\omega=2v\cdot k$ and $\omega'=2v'\cdot k'$, with discontinuities for positive values of these variables. These functions also depend on the velocity transfer $w=v\cdot v'$.

The lowest lying states are the ground-state baryons $B(v)$ and $B'(v')$ associated with the heavy-light currents. They lead to a double pole located at $\omega=\omega'=2\overline{\Lambda}$. The residue of this double pole is proportional to the invariant functions $\phi_i(w)$. We find

$$\Phi_i^{\text{pole}}(\omega,\omega',w)=\frac{4s_c\phi_i(w)F^2}{(\omega-2\overline{\Lambda})(\omega'-2\overline{\Lambda})},\tag{23}$$

where s_c is the structure constant, 1 for Λ_Q, $-(2+w)/9$ for Σ_Q, and 1 for Σ_Q^* baryons. In the deep Euclidean region the correlation function can be calculated perturbatively because of asymptotic freedom. Following the standard procedure, we write the theoretical expressions for Φ_i as double dispersion integrals and perform a Borel transformation in the variables ω and ω', then set the associated Borel parameters equal: $\tau=\tau'\equiv2T$. Just as in the direct approach, we introduce new variables $\omega_+=\frac{1}{2}(\omega+\omega')$ and $\omega_-=\omega-\omega'$, perform the integral over ω_-, and get the Laplace sum rules at zero recoil:

$$\begin{aligned}8s_cF^2\phi_1e^{-2\overline{\Lambda}_\Sigma/T}=&\frac{4}{\pi^4}\frac{\alpha_s}{\pi}T^8\delta_7(\omega_c/T)\\&-\frac{1}{24\pi^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\delta_3(\omega_c/T)\\&+\frac{32\alpha_s}{9\pi}\langle\overline{q}q\rangle^2T^2\delta_1(\omega_c/T),\\8s_cF^2\phi_1e^{-2\overline{\Lambda}_\Sigma/T}=&8s_cF^22(\phi_2-\phi_3)e^{-2\overline{\Lambda}_\Sigma/T},\\8s_cF^2\phi_4e^{-2\overline{\Lambda}_\Sigma/T}=&\frac{2}{\pi^4}\frac{\alpha_s}{\pi}T^8\delta_7(\omega_c/T),\\8s_cF^2\phi_5e^{-2\overline{\Lambda}_\Sigma/T}=&-\frac{1}{\pi^4}\frac{\alpha_s}{\pi}T^8\delta_7(\omega_c/T)\end{aligned}\tag{24}$$

for the Σ_Q baryon, and

$$8F^2\phi_1e^{-2\overline{\Lambda}_\Lambda/T}=-\frac{2}{\pi^4}\frac{\alpha_s}{\pi}T^8\delta_7(\omega_c/T)\tag{25}$$

for the Λ_Q baryon. After some simple algebra we find

$$\begin{aligned}-4\lambda_1F^2e^{-2\overline{\Lambda}_\Sigma/T}=&\frac{9}{\pi^4}\frac{\alpha_s}{\pi}T^8\delta_7(\omega_c/T),\\4\lambda_2F^2e^{-2\overline{\Lambda}_\Sigma/T}=&\frac{12}{\pi^4}\frac{\alpha_s}{\pi}T^8\delta_7(\omega_c/T)\\&-\frac{T^4}{8\pi^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\delta_3(\omega_c/T)\\&+\frac{32T^2\alpha_s}{3\pi}\langle\overline{q}q\rangle^2\delta_1(\omega_c/T)\end{aligned}\tag{26}$$

for the Σ_Q baryon, and

$$-4\lambda_1F^2e^{-2\overline{\Lambda}_\Lambda/T}=-\frac{3T^8}{\pi^4}\frac{\alpha_s}{\pi}\delta_7(\omega_c/T)\tag{27}$$

for the Λ_Q baryon. The minus sign of the Λ_Q baryon result may seem bizarre; in Sec. III we will return to dwell on this point.

III. NUMERICAL RESULTS AND CONCLUSIONS

In order to get the numerical results, we divide our three-point sum rules by two-point functions to obtain λ_1 and λ_2 as functions of the continuum threshold ω_c and the Borel

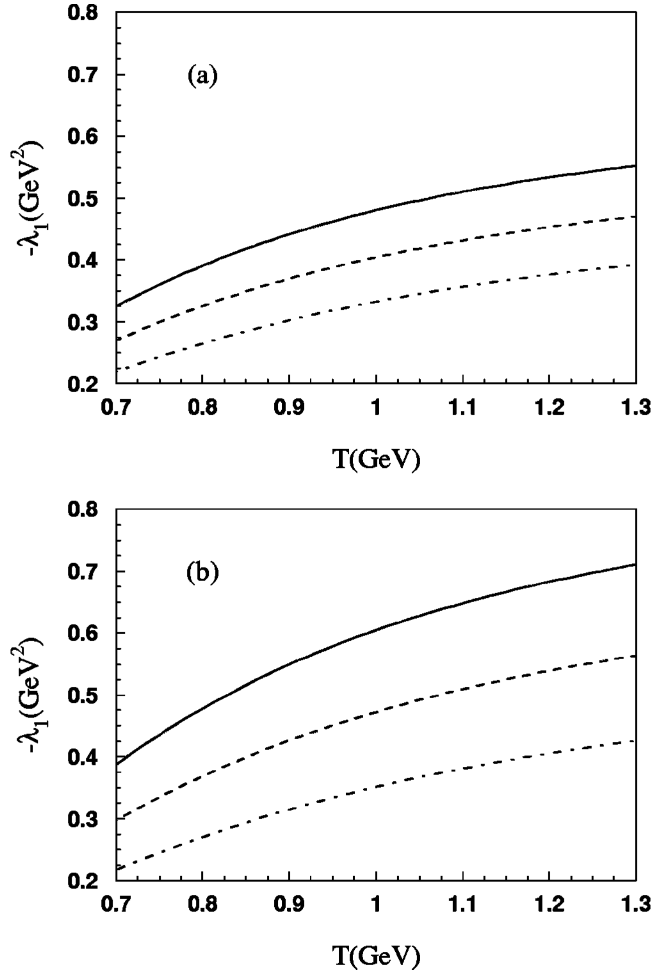

FIG. 3. Sum rules for Λ_Q baryons: (a) for j_1^v, (b) for j_2^v. The dash-dotted, dashed, and solid curves correspond to the threshold $\omega_c = 2.4$, 2.6, and 2.8 GeV, respectively. The working region is $T = 0.8$–1.2 GeV.

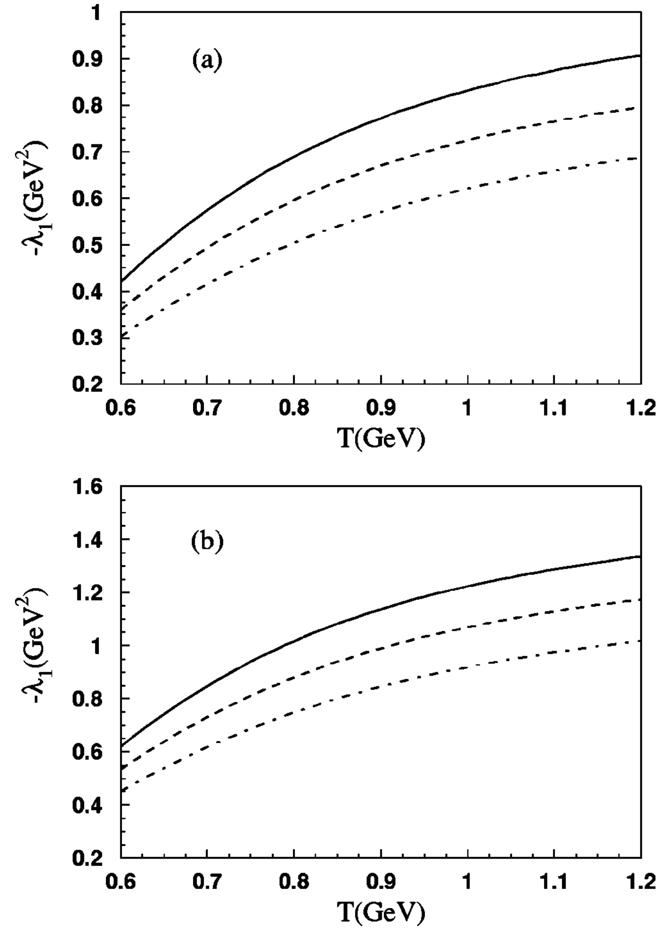

FIG. 4. Sum rules of the kinetic energy for Σ_Q baryons: (a) for j_1^v, (b) for j_2^v. The dash-dotted, dashed, and solid curves correspond to the threshold $\omega_c = 2.9$, 3.1, and 3.3 GeV, respectively. The working region is $T = 0.7$–1.1 GeV.

parameter T. This procedure can eliminate the systematic uncertainties and cancel the parameter $\bar{\Lambda}$. As for the condensates, we adopt the standard values

$$\langle \bar{q}q \rangle = -(0.23\ \text{GeV})^3,$$

$$\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle = 0.012\ \text{GeV}^4. \tag{28}$$

From two-point sum rules one knows that there exist stable windows in the ranges $0.8 < T < 1.2$ GeV and $\omega_c = 2.2$–2.7 GeV for Λ_Q baryons, and $0.7 < T < 1.1$ GeV and $\omega_c = 2.6$–3.3 GeV for Σ_Q baryons. The stability window for three-point functions starts almost from values of the Borel parameter at 0.7 GeV and stretches almost to $T \to \infty$. It is known that stability at lager Borel parameters could not give any valuable information, since in this region the sum rule is strongly affected by the continuum model. The usual criterion that both the higher order power corrections and the continuum contribution should not be very large restricts the working region considerably. In the case of the three-point functions, the results are severely smeared by the continuum contributions for high dimension of spectral densities and thus it is very difficult to ensure that the contribution of the continuum mode is small. The working region of the three-point functions should be determined by the stable region of the two-point functions. So it does not necessarily coincide with the stable windows of the three-point functions [8]. Thus we find that our working region for three-point functions is $T = 0.8$–1.2 GeV for Λ_Q baryons and $T = 0.7$–1.1 GeV for Σ_Q baryons. The results for λ_1 in the direct approach with two different choices of currents are shown in Fig. 3 for Λ_Q baryons and Fig. 4 for Σ_Q baryons. Results for the kinetic energy of Λ_Q and Σ_Q baryons obtained by the covariant approach are presented in Fig. 5. The forms of the chromomagnetic interaction obtained by both approaches do not differ from each other, so we plot that unique curve in one figure, Fig. 6. For the Λ_Q baryon we obtain the residual mass $\bar{\Lambda}_\Lambda = 0.8 \pm 0.1$ GeV and

$$-\lambda_1^1 = 0.4 \pm 0.1\ \text{GeV}^2,$$

$$-\lambda_1^2 = 0.5 \pm 0.1\ \text{GeV}^2 \tag{29}$$

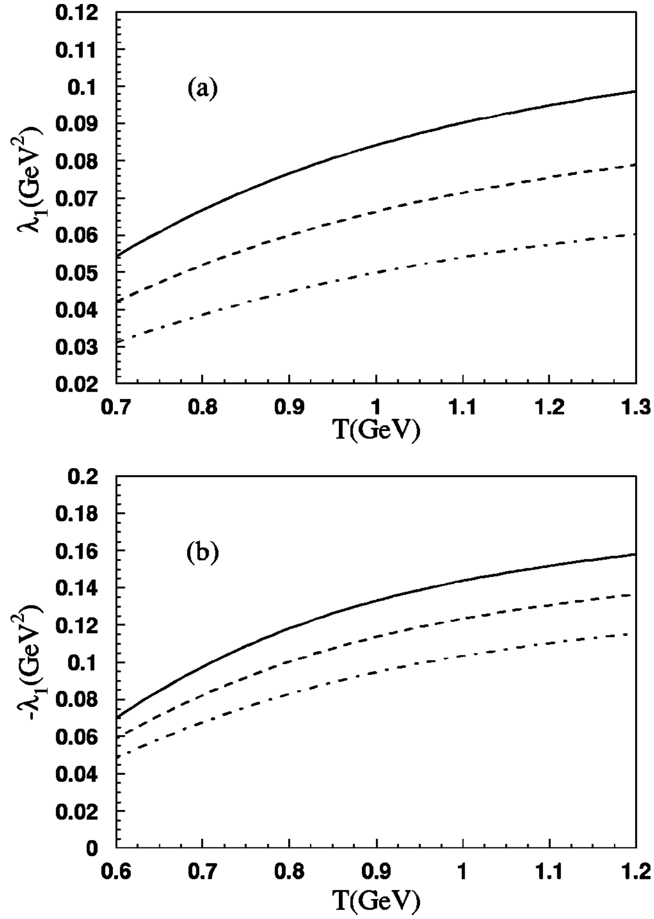

FIG. 5. Covariant sum rules of the kinetic energy: (a) for Λ_Q baryons, (b) for Σ_Q baryons. The dash-dotted, dashed, and solid curves correspond to $\omega_c=2.4$, 2.6, and 2.8 GeV for Λ_Q baryons, and $\omega_c=2.9$, 3.1, and 3.3 GeV for Σ_Q baryons. The working region is $T=0.8$–1.2 GeV for Λ_Q baryons and $T=0.7$–1.1 GeV for Σ_Q baryons.

in the direct approach, where the superscripts denote the different choices of currents, and

$$-\lambda_1=-0.08\pm0.02\ \ \mathrm{GeV}^2 \tag{30}$$

in the covariant approach. For the Σ_Q baryon the effective mass we obtained is $\overline{\Lambda}_\Sigma=1.0\pm0.1$ GeV and

$$-\lambda_1^1=0.7\pm0.2\ \ \mathrm{GeV}^2,$$

$$-\lambda_1^2=1.0\pm0.2\ \ \mathrm{GeV}^2 \tag{31}$$

in the direct approach, where the superscripts also denote different currents, and

$$-\lambda_1=0.11\pm0.03\ \ \mathrm{GeV}^2 \tag{32}$$

in the covariant approach. For the chromomagnetic interaction for Σ baryons the results read

$$\lambda_2=0.23\pm0.02\ \ \mathrm{GeV}^2. \tag{33}$$

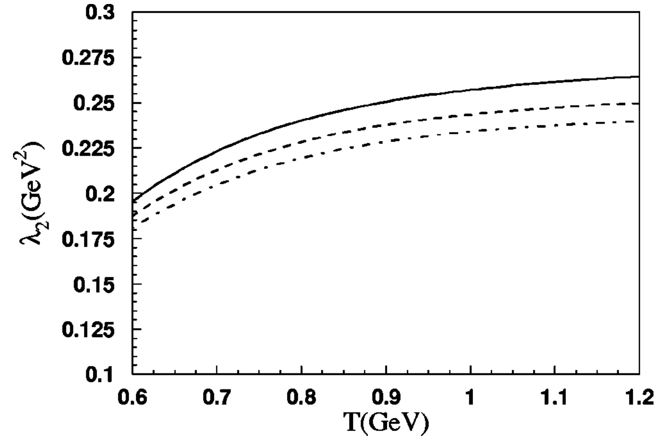

FIG. 6. Sum rules for the chromomagnetic interaction. The dash-dotted, dashed, and solid curves correspond to $\omega_c=2.9$, 3.1, and 3.3 GeV. The working region is $T=0.7$–1.1 GeV.

Then we get the splitting of the spin 1/2 and spin 3/2 doublets as

$$\Sigma_Q^{*2}-\Sigma_Q^2=\frac{3}{2}\lambda_2=0.35\pm0.03\ \ \mathrm{GeV}^2. \tag{34}$$

All errors quoted before are due to the variation of the Borel parameter T and the continuum threshold ω_c. When scaled up to the bottom quark mass scale there will be a factor ~0.8 approximately due to the renormalization group improvement.

As for the effects on the correlation function of the different choices of the interpolating currents we may assert some facts and inferences. From the preceding numerical results it is clear that the interpolating currents with the $\not{v}$ insertion give a considerable larger result for the kinetic energy than those without the insertion. Nevertheless, the two-point sum rules do not differ with the different currents [12,15]. From our calculation it is explicit that the sum rules associated with chromointeraction insertion are identical. In our covariant calculation we find that the invariant functions do differ from each other generally, but interestingly they coincide at zero recoil; thus the chromointeraction and kinetic energy do not take different forms. Naively, we can tell that the disparity of the two forms of the kinetic energy obtained in our direct calculation mainly comes from the Lorenz structural differences of the two interpolating currents. It may be noted that the derivative operator acts differently on the currents with or without $\not{v}$ insertion; thus with the insertion it is easier for the continuum contamination to go into the correlation functions. Exclusion of the continuum contribution is urgently needed it heavily smeared the results of both the direct and covariant approaches. It is this continuum contamination that makes the prediction of the kinetic energy more intriguing. All previous theoretical calculations with the QCD sum rule approach or lattice calculation give various results and can differ from each other by several times [11,12,8,9,22,26]. The current experimental data are not enough to judge which one is right; what we can get is some restrictions on the kinetic energy [27] or a rough estimate of the kinetic energy extracted from experimental data with some assumptions [28]. As demonstrated in Refs. [24,29] using the toy model of a harmonic oscillator, the main origin of the discrepancy between the direct and covariant approaches is the continuum smeared contribution. In the direct approach the first excited contribution plays an important role. If we want to suppress this contribution, we go to such a large Borel parameter that the power corrections blow up. For acceptable Borel parameters, we get an overestimated sum rule for the the kinetic energy. In the covariant approach (via the virial theorem) the situation is especially bad. The excited contribution consists of two components—the diagonal transitions and off-diagonal ones—and each one is large, but they have opposite relative signs. For highly excited states the sign-alternating terms are smeared to zero after summation. However, the first two terms do not cancel with each other and screen the ground-state contribution. Thus a lower estimated result will be obtained. The minus sign before the kinetic energy of the Λ_Q baryon in the covariant approach may be seen as a manifestation of this assertion. Due to the unknown weight, we cannot annihilate those contributions by weighted averaging just as in quantum mechanics. But we may safely take the results of the direct and covariant approaches as the lower bound and the higher bound of the kinetic energy parameter, respectively. Then, following [30], taking the mean value of the direct and covariant approaches results in a rough estimate. The result thus obtained is

$$-\overline{\lambda_1^1}\simeq 0.18\pm 0.06\ \ \mathrm{GeV}^2,$$

$$-\overline{\lambda_1^2}\simeq 0.24\pm 0.06\ \ \mathrm{GeV}^2 \tag{35}$$

for Λ_Q baryons and

$$-\overline{\lambda_1^1}\simeq 0.39\pm 0.12\ \ \mathrm{GeV}^2,$$

$$-\overline{\lambda_1^2}\simeq 0.54\pm 0.12\ \ \mathrm{GeV}^2 \tag{36}$$

for Σ_Q baryons. Taking all results obtained the mass of the ground state baryon is on hand. From m_{Λ_c} and m_{Λ_b} [31], we determine the heavy quark masses $m_c\simeq 1.41\pm 0.16$ GeV and $m_b\simeq 4.77\pm 0.12$ GeV. In the determination we have taken the average of the results obtained from two interpolating currents to be the physical pole masses of the heavy quarks because the difference of the corresponding mass does not exceed the error bar. These values give the following results:

$$m_{\Sigma_c}\simeq 2.47\pm 0.20\ \ \mathrm{GeV},$$

$$m_{\Sigma_c^*}\simeq 2.59\pm 0.20\ \ \mathrm{GeV},$$

$$m_{\Sigma_b}\simeq 5.79\pm 0.13\ \ \mathrm{GeV},$$

$$m_{\Sigma_b^*}\simeq 5.82\pm 0.13\ \ \mathrm{GeV}, \tag{37}$$

with interpolating current j_v^1 and

$$m_{\Sigma_c}\simeq 2.52\pm 0.20\ \ \mathrm{GeV},$$

$$m_{\Sigma_c^*}\simeq 2.64\pm 0.20\ \ \mathrm{GeV},$$

$$m_{\Sigma_b}\simeq 5.80\pm 0.13\ \ \mathrm{GeV},$$

$$m_{\Sigma_b^*}\simeq 5.83\pm 0.13\ \ \mathrm{GeV} \tag{38}$$

with interpolating current j_v^2. The spin average of the doublets is free of the chromointeraction contribution and thus free of the uncertainties involved in the calculation of λ_2. Averaging over the doublets we have the quantity

$$\frac{1}{3}(M_{\Sigma_Q}+2M_{\Sigma_Q^*})=m_Q+\overline{\Lambda}_\Sigma+\frac{1}{2m_Q}\overline{\lambda}_1,$$

which is more reliable. For the c quark case, it is 2.55 ± 0.20 GeV with current j_v^1 and 2.60 ± 0.20 GeV with current j_v^2. For the b quark case it is 5.81 ± 0.13 GeV with current j_v^1 and 5.83 ± 0.13 GeV with current j_v^2. Experimentally $M_{\Sigma_c}=2453\pm 0.2$ MeV [31]. There is experimental evidence for Σ_c^* at $M_{\Sigma_c^*}=2519\pm 2$ MeV [32]. If we take this value for Σ_c^*, we have $\frac{1}{3}(M_{\Sigma_c}+2M_{\Sigma_c^*})=2497\pm 1.4$ MeV, which is in reasonable agreement with the theoretical prediction. For lack of experimental data the corresponding quantity for the bottom quark will be checked in the future. If we take the preceding masses of the charmed Σ baryons the splitting thus reduced is 0.33 GeV^2 and our theoretical splitting is in good agreement with the experimental data.

As the kinetic energy of the Λ_Q baryon can be related to the spectrum via the kinetic energy of the meson[1] as [33]

$$(m_{\Lambda_b}-m_{\Lambda_c})-(\overline{m}_B-\overline{m}_D)$$
$$=[\lambda_1(\Lambda_b)-\lambda_1(B)]\left(\frac{1}{2m_c}-\frac{1}{2m_b}\right)+\mathcal{O}(1/m_Q^2), \tag{39}$$

where $\overline{m}_B=\frac{1}{4}(m_B+3m_B^*)$ and $\overline{m}_D=\frac{1}{4}(m_D+3m_D^*)$ denote the spin-averaged meson masses, the difference between the kinetic energy of the B meson and that of the Λ_b baryon can be extracted as

$$\lambda_1(\Lambda_b)-\lambda_1(B)=0.01\pm 0.02\ \ \mathrm{GeV}^2, \tag{40}$$

which is consistent with the value obtained in Ref. [33]. Resorting to the recent experimental data for the mesonic kinetic energy parameter obtained in the inclusive semileptonic B decays [34], $-\lambda_1=0.24\pm 0.11$ GeV^2, one can thus get the value of baryonic kinetic energy as

$$-\lambda_1(\Lambda_b)=0.23\pm 0.13\ \ \mathrm{GeV}^2, \tag{41}$$

[1]The relation between μ_π^2 in Ref. [33] and λ_1 in this paper is $\mu_\pi^2=-\lambda_1$.

which is in reasonable agreement with our theoretical prediction given in Eq. (35).

In conclusion, we have calculated the $1/m_Q$ corrections to the heavy baryon masses from the QCD sum rules within the framework of the HQET. Two approaches have been adopted in the evaluation of the three-point correlators. Our final results read

$$M_{\Sigma_Q}=m_Q+\bar{\Lambda}_\Sigma+\frac{1}{2m_Q}(0.16\pm0.12\ \ \mathrm{GeV}^2),$$

$$M_{\Sigma_Q^*}=m_Q+\bar{\Lambda}_\Sigma+\frac{1}{2m_Q}(0.51\pm0.12\ \ \mathrm{GeV}^2) \tag{42}$$

for interpolating current without $\not{v}$ insertion and

$$M_{\Sigma_Q}=m_Q+\bar{\Lambda}_\Sigma+\frac{1}{2m_Q}(0.31\pm0.12\ \ \mathrm{GeV}^2),$$

$$M_{\Sigma_Q^*}=m_Q+\bar{\Lambda}_\Sigma+\frac{1}{2m_Q}(0.66\pm0.12\ \ \mathrm{GeV}^2) \tag{43}$$

for interpolating current with $\not{v}$ insertion. The $1/m_Q$ corrections are small. We have taken the mean value of the direct and covariant approaches as the rough estimate of the kinetic energy parameter λ_1. Our theoretical predictions are in agreement with the recent experimental data. For a more precise treatment of the kinetic energy, a more sophisticated technique to distinguish the smearing continuum contribution is in urgent need of development.

ACKNOWLEDGMENTS

This work was supported in part by the National Natural Science Foundation of China under Contract No. 19975068.

APPENDIX: THE DECOMPOSITION OF THE BILINEAR MATRIX ELEMENT

In this appendix, we present the decomposition of the bilinear matrix element. There exists a decomposition of the Λ_Q baryon; we present it here merely for completeness and convention.

First, let us consider the bilinear matrix element over the Λ_Q baryons

$$\langle\Lambda|\bar{h}_v(-i\overleftarrow{D}_\mu)\Gamma^{\mu\nu}iD_\nu h_{v'}|\Lambda'\rangle=\psi_{\mu\nu}(v',v)\bar{u}\Gamma^{\mu\nu}u'. \tag{A1}$$

The coefficients obey the symmetric relation $\psi_{\mu\nu}(v',v)=\psi^*_{\nu\mu}(v,v')$. It is convenient to write the coefficient ψ in a sum of symmetric and antisymmetric parts $\psi_{\mu\nu}=\frac{1}{2}[\psi^A_{\mu\nu}+\psi^S_{\mu\nu}]$ which can be presented covariantly as

$$\psi^A_{\mu\nu}=\psi^A_1(v'_\mu v_\nu-v_\mu v'_\nu),$$
$$\psi^S_{\mu\nu}=\psi^S_1 g_{\mu\nu}+\psi^S_2(v+v')_\mu(v+v')_\nu$$
$$+\psi^S_3(v-v')_\mu(v-v')_\nu. \tag{A2}$$

The HQET equation of motion implies that $v'_\nu\psi_{\mu\nu}=0$ from which we can obtain the relations between those coefficients:

$$\psi^S_1+(1+y)\psi^S_2+(1-y)\psi^S_3+y\psi^A_1=0,$$
$$(1+y)\psi^S_2+(y-1)\psi^S_3-\psi^A_1=0, \tag{A3}$$

with

$$\overline{hi\vec{D}_\mu}\Gamma h'+\bar{h}iD_\mu\Gamma h'=i\partial_\mu(\bar{h}\Gamma h'). \tag{A4}$$

Bear in mind that we can get

$$\langle\Lambda|\bar{h}_v\Gamma^{\mu\nu}iD_\mu iD_\nu h_{v'}|\Lambda'\rangle$$
$$=\psi_{\mu\nu}(v',v)\bar{u}\Gamma^{\mu\nu}u'+\bar{\Lambda}(v'-v)_\mu\xi_\nu\bar{u}\Gamma^{\mu\nu}u', \tag{A5}$$

using the x dependence of the state in HQET $|B(x)\rangle=e^{-i\bar{\Lambda}v\cdot x}|B(0)\rangle$. The ξ_ν are defined as

$$\langle\Lambda|\bar{h}_v\Gamma^\nu iD_\nu h_{v'}|\Lambda'\rangle=\xi_\nu\bar{u}\Gamma^\nu u'. \tag{A6}$$

Similarly, we can define the matrix elements for the operators of kinetic energy and chromointeraction over baryon states with different velocities:

$$\langle\Lambda|\bar{h}_v\sigma_{\mu\nu}iG^{\mu\nu}h_{v'}|\Lambda'\rangle=\phi_1(v'_\mu v_\nu-v_\mu v'_\nu)\bar{u}\sigma^{\mu\nu}u', \tag{A7}$$

$$\langle\Lambda|\bar{h}_v(iD^\perp)^2\Gamma h_{v'}|\Lambda'\rangle=\phi_0\bar{u}\Gamma u'. \tag{A8}$$

Once defined, the ψ_i can be expressed via two ϕ_i:

$$\psi^A_1=\phi_1-\bar{\Lambda}^2\xi\frac{y-1}{y+1},$$

$$\psi^s_1=\phi_0+y\phi_1+\bar{\Lambda}^2\xi\frac{y-1}{y+1},$$

$$\psi^s_3=\frac{(1+2y)\phi_1+\phi_0}{2(y-1)}-\frac{y}{2(y+1)}\bar{\Lambda}^2\xi,$$

$$\psi^s_2=\frac{\psi^A_1-(y-1)\psi^S_3}{1+y}, \tag{A9}$$

the normalizations of ϕ_0,ϕ_1 are $\phi_0(1)=\lambda_1,\phi_1(1)=-\frac{1}{3}\phi_0(1)$, and thus we get the desired result.

Generalization can be made to higher spin states such as Σ_Q baryons. The procedure is almost the same. The only difference lies in the decomposition of the matrix element. Hence we will give the forms of decomposition and the final desired relation; we will not dwell on the details. The covariant representation of the doublet is $\Psi_\mu=u_\mu+(1/\sqrt{3})(v_\mu+\gamma_\mu)u$. The matrix element is

$$\langle\Sigma|\bar{h}_v(-i\vec{D}_\mu)\Gamma^{\mu\nu}iD_\nu h_{v'}|\Sigma'\rangle$$
$$=\psi^{\alpha\beta}_{\mu\nu}(v',v)\bar{\Psi}_\alpha\Gamma^{\mu\nu}\Psi'_\beta, \tag{A10}$$

in which the coefficients obey the symmetric relation $\psi^{\alpha\beta}_{\mu\nu}(v,v')=\psi^{\beta\alpha}_{\nu\mu}(v',v)$. Adopting the same symmetric and antisymmetric decomposition of the coefficients as that in the Λ_Q baryon case, we have

$$\begin{aligned}\psi^{\alpha\beta,A}_{\mu\nu}=&\psi^A_1(g_{\mu\alpha}g_{\nu\beta}-g_{\mu\beta}g_{\nu\alpha})+\psi^A_2(g_{\mu\beta}v_\nu v'_\alpha-g_{\nu\beta}v_\mu v'_\alpha\\&+g_{\nu\alpha}v_\beta v'_\mu-g_{\mu\alpha}v_\beta v'_\nu)+\psi^A_3(g_{\alpha\mu}v_\nu v_\beta-g_{\nu\alpha}v_\mu v_\beta\\&+g_{\nu\beta}v'_\alpha v'_\mu-g_{\mu\beta}v'_\alpha v'_\nu)+\psi^A_4(v'_\mu v_\nu-v'_\nu v_\mu)g_{\alpha\beta}\\&+\psi^A_5(v'_\mu v_\nu-v'_\nu v_\mu)v'_\alpha v_\beta,\end{aligned}$$

$$\begin{aligned}\psi^{\alpha\beta,S}_{\mu\nu}=&\psi^S_1 g_{\alpha\beta}g_{\mu\nu}+\psi^S_2(g_{\mu\alpha}g_{\nu\beta}+g_{\mu\beta}g_{\nu\alpha})+\psi^S_3(g_{\mu\beta}v_\nu v'_\alpha\\&+g_{\nu\beta}v_\mu v'_\alpha+g_{\nu\alpha}v_\beta v'_\mu+g_{\mu\alpha}v_\beta v'_\nu)+\psi^S_4(g_{\alpha\mu}v_\nu v_\beta\\&+g_{\nu\alpha}v_\mu v_\beta+g_{\nu\beta}v'_\alpha v'_\mu+g_{\mu\beta}v'_\alpha v'_\nu)+\psi^S_5(v'\\&-v)_\mu(v'-v)_\nu v'_\alpha v_\beta+\psi^S_6(v'-v)_\mu(v'-v)_\nu g_{\alpha\beta}\\&+\psi^S_7(v'+v)_\mu(v'+v)_\nu v'_\alpha v_\beta+\psi^S_8(v'+v)_\mu(v'\\&+v)_\nu g_{\alpha\beta}+\psi^S_9 v_\beta v'_\alpha g_{\mu\nu}.\end{aligned}\quad\text{(A11)}$$

Introducing other universal parameters in the leading order

$$\langle\Sigma|\overline{h}_v\Gamma^\nu iD_\nu h_{v'}|\Sigma'\rangle=\xi^{\alpha\beta}_\nu(v,v')\overline{\Psi}_\alpha\Gamma^{\mu\nu}\Psi'_\beta,$$

$$\langle\Sigma|\overline{h}_v\Gamma^\nu(-iD_\nu)h_{v'}|\Sigma'\rangle=\overline{\xi}^{\beta\alpha}_\nu(v',v)\overline{\Psi}_\alpha\Gamma^{\mu\nu}\Psi'_\beta \quad\text{(A12)}$$

as usual, $\xi^{\alpha\beta}_\nu(v,v')$ can be decomposed into the general form

$$\begin{aligned}\xi^{\alpha\beta}_\nu(v,v')=&\xi_1(v+v')_\nu g_{\alpha\beta}+\xi_2(v'-v)_\nu g_{\alpha\beta}\\&+\xi_3(v+v')_\nu v_\beta v'_\alpha+\xi_4(v'-v)_\nu v_\beta v'_\alpha\\&+\xi_5 v'_\alpha g_{\beta\nu}+\xi_6 v_\beta g_{\alpha\nu}.\end{aligned}\quad\text{(A13)}$$

The equation of motion implies that $v'^\nu\xi^{\alpha\beta}_\nu=0$ and $v'^\nu\psi^{\alpha\beta}_{\mu\nu}=0$ from which we can derive the relations

$$\begin{aligned}&w\psi^A_3-\psi^A_2+\psi^S_3+\psi^S_4=0,\\&w\psi^A_2-\psi^A_1-\psi^A_3+\psi^S_2+w\psi^S_3+\psi^S_4=0,\\&w\psi^A_4+\psi^S_1+(1-w)\psi^S_6+(1+w)\psi^S_8=0,\\&\psi^A_2+w\psi^A_5+\psi^s_3+(1-w)\psi^S_5+(1+w)\psi^S_7+\psi^S_9=0,\\&(1+w)\psi^S_8-\psi^A_4-(1-w)\psi^S_6=0,\\&\psi^S_4+(w-1)\psi^S_5-\psi^A_3-\psi^A_5+(1+w)\psi^S_7=0,\end{aligned}\quad\text{(A14)}$$

and

$$(1+w)\xi_1+(1-w)\xi_2=0,$$

$$(1+w)\xi_3+(1-w)\xi_4+\xi_6=0.\quad\text{(A15)}$$

Taking the difference of the two terms in Eqs. (A12) and using Eq. (A4) we can reach

$$\begin{aligned}\xi_1&=\frac{w-1}{w+1}\frac{c_1}{2}\overline{\Lambda},\\\xi_3&=\frac{w-1}{w+1}\frac{c_2}{2}\overline{\Lambda}-\xi_6,\\\xi_2&=\frac{c_1}{2}\overline{\Lambda},\\\xi_4&=\frac{c_2}{2}\overline{\Lambda},\\\xi_5&=\xi_6,\end{aligned}\quad\text{(A16)}$$

where c_1,c_2 parametrize the matrix element

$$\langle\Sigma|\overline{h}_v\Gamma h_{v'}|\Sigma'\rangle=(c_1 g_{\alpha\beta}+c_2 v_\beta v'_\alpha)\overline{\Psi}^\alpha\Gamma\Psi'^\beta.\quad\text{(A17)}$$

The matrix elements for the kinetic energy and chromomagnetic interaction are defined similarly to those for the Λ_Q baryon:

$$\langle\Sigma|\overline{h}_v(iD^\perp)^2\Gamma h_{v'}|\Sigma'\rangle=(\phi_0 g_{\alpha\beta}+\overline{\phi}_0 v_\beta v'_\alpha)\overline{\Psi}^\alpha\Gamma\Psi'^\beta,\quad\text{(A18)}$$

$$\langle\Sigma|\overline{h}_v\sigma_{\mu\nu}iG^{\mu\nu}h_{v'}|\Sigma'\rangle=\phi^{\alpha\beta}_{\mu\nu}\overline{\Psi}_\alpha\sigma^{\mu\nu}\Psi'_\beta,\quad\text{(A19)}$$

where $\phi^{\alpha\beta}_{\mu\nu}$ bear the same decomposition as $\psi^{\alpha\beta}_{\mu\nu}$. They have simple relations between each other:

$$\begin{aligned}\phi_1&=\psi^A_1,\\\phi_2&=\psi^A_2-\xi_6\overline{\Lambda},\\\phi_3&=\psi^A_3-\xi_6\overline{\Lambda},\\\phi_4&=\psi^A_4-2\xi_1\overline{\Lambda},\\\phi_5&=\psi^A_5-2\xi_3\overline{\Lambda},\\\phi_0&=2\psi^s_1+\psi^s_2+(1-w)\psi^s_6+(1+w)\psi^s_8\\&\quad+2(1-w)\xi_2\overline{\Lambda},\\\overline{\phi}_0&=2\psi^s_3+(1-w)\psi^s_5+(1+w)\psi^s_7+2(1-w)\xi_4\overline{\Lambda},\end{aligned}\quad\text{(A20)}$$

The normalization condition is that $\phi_1(1)=A\lambda_2$, $\phi_0(1)=B\lambda_1$ where A is $-1/2,1/2$ and B is $1,-1$ for Σ^*_Q,Σ_Q, respectively. At zero recoil λ_1 can be expressed via ϕ_0:

$$\phi_0(1)=\phi_1(1)-2[\phi_2(1)-\phi_3(1)]-3\phi_4(1).\quad\text{(A21)}$$

[1] B. Grinstein, Nucl. Phys. **B339**, 253 (1990); E. Eichten and B. Hill, Phys. Lett. B **234**, 511 (1990); A. F. Falk, H. Georgi, B. Grinstein, and M. B. Wise, Nucl. Phys. **B343**, 1 (1990); F. Hussain, J. G. Körner, K. Schilcher, G. Thompson, and Y. L. Wu, Phys. Lett. B **249**, 295 (1990); J. G. Körner and G. Thompson, *ibid.* **264**, 185 (1991).
[2] M. Neubert, Phys. Rep. **245**, 259 (1994).
[3] J. G. Körner, M. Krämer, and D. Pirjol, Prog. Part. Nucl. Phys. **33**, 787 (1994).
[4] A. F. Falk and M. Neubert, Phys. Rev. D **47**, 2965 (1993); **47**, 2982 (1993).
[5] I. I. Bigi, N. G. Uraltsev, and A. I. Vainshtein, Phys. Lett. B **293**, 430 (1992); I. I. Bigi, M. Shifman, N. G. Uraltsev, and A. I. Vainshtein, Phys. Rev. Lett. **71**, 496 (1993); A. V. Manohar and M. B. Wise, Phys. Rev. D **49**, 1310 (1994); M. Luke and M. J. Savage, Phys. Lett. B **321**, 88 (1994).
[6] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979); **B147**, 448 (1979); V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Fortschr. Phys. **32**, 11 (1984).
[7] For recent review of the QCD sum rule method, see P. Colangelo and A. Khodjamirian, *At the Frontier of Particle Physics/Handbook of QCD*, Boris Ioffe Festschrift, edited by M. Shifman (World Scientific, Singapore, 2001), pp. 1495–1576.
[8] P. Ball and V. M. Braun, Phys. Rev. D **49**, 2472 (1994).
[9] M. Neubert, Phys. Lett. B **389**, 727 (1996).
[10] Y.-B. Dai, C.-S. Huang, M.-Q. Huang, and C. Liu, Phys. Lett. B **390**, 350 (1997); Y.-B. Dai, C.-S. Huang, and M.-Q. Huang, Phys. Rev. D **55**, 5719 (1997).
[11] P. Colangelo, C. A. Dominguez, G. Nardulli, and N. Paver, Phys. Rev. D **54**, 4622 (1996).
[12] Y.-B. Dai, C.-S. Huang, C. Liu, and C.-D. Lü, Phys. Lett. B **371**, 99 (1996).
[13] J. P. Lee, C. Liu, and H. S. Song, Phys. Lett. B **476**, 303 (2000); C.-S. Huang, A. Zhang, and S.-L. Zhu, *ibid.* **492**, 288 (2000).
[14] B. L. Ioffe, Nucl. Phys. **B188**, 317 (1981); **B191**, 591(E) (1981); Z. Phys. C **18**, 67 (1983).
[15] A. G. Grozin and O. I. Yakovlev, Phys. Lett. B **285**, 254 (1992); **291**, 441 (1992).
[16] M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, M. A. Pisarev, and A. G. Rusetsky, Phys. Rev. D **61**, 114010 (2000).
[17] S. Groote and J. G. Korner, Phys. Rev. D **55**, 3016 (1997); S. Groote, J. G. Korner, and O. I. Yakovlev, *ibid.* **54**, 3447 (1996).
[18] M. Neubert, Phys. Rev. D **47**, 4063 (1993).
[19] P. Colangelo, F. De Fazio, and N. Paver, Phys. Rev. D **58**, 116005 (1998).
[20] M. Neubert, Phys. Rev. D **45**, 2451 (1992); **46**, 3914 (1992).
[21] Y.-B. Dai, C.-S. Huang, M.-Q. Huang, and C. Liu, Phys. Lett. B **387**, 379 (1996).
[22] M. Neubert, Phys. Rev. D **46**, 1076 (1992).
[23] M. Neubert, Phys. Lett. B **322**, 419 (1994); D. S. Hwang, C. S. Kim, and W. Namgung, *ibid.* **406**, 117 (1997).
[24] I. I. Bigi, M. Shifman, and N. Uraltselv, Annu. Rev. Nucl. Part. Sci. **47**, 591 (1997).
[25] T. Mannel, W. Roberts, and Z. Ryzak, Phys. Lett. B **271**, 421 (1991).
[26] V. Giménez, G. Martinelli, and C. T. Sachrajda, Nucl. Phys. **B486**, 227 (1997).
[27] T. Mannel, Phys. Rev. D **50**, 428 (1994); H. Davoudiasl and A. K. Leibovich, Phys. Lett. B **403**, 323 (1997); Z. Ligeti and Y. Nir, Phys. Rev. D **49**, 4331 (1994); I. Bigi, A. G. Grozin, M. Shifman, N. G. Uraltsev, and A. Vainshtein, Phys. Lett. B **339**, 160 (1994); C. G. Boyd, B. Grinstein, and A. V. Manohar, Phys. Rev. D **54**, 2081 (1996).
[28] K. K. Jeong and C. S. Kim, Phys. Rev. D **59**, 114019 (1999); M. Gremm, A. Kapustin, Z. Ligeti, and M. B. Wise, Phys. Rev. Lett. **77**, 20 (1996).
[29] B. Blok and M. Shifman, Phys. Rev. D **47**, 2949 (1993); I. I. Bigi, M. A. Shifman, N. G. Uraltsev, and A. I. Vainshtein, Phys. Rev. D **52**, 196 (1995).
[30] V. Braun, ''Recent Development of QCD Sum Rules in Heavy Flavor Physics,'' hep-ph/9911206.
[31] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998).
[32] CLEO Collaboration, G. Brandenburg *et al.*, Phys. Rev. Lett. **78**, 2304 (1997).
[33] M. Neubert and C. T. Sachrajda, Nucl. Phys. **B483**, 339 (1997).
[34] CLEO Collaboration, D. Cronin-Hennessy *et al.*, Phys. Rev. Lett. **87**, 251808 (2001).