Brane fluctuation and the electroweak chiral Lagrangian

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We use the external field method to study the electroweak chiral Lagrangian of the extra dimension model with brane fluctuation. Under the assumption that the contact terms between the matter of the standard model and Kaluza-Klein (KK) excitations of the bulk gauge fields are heavily suppressed, we use the standard procedure to integrate out the quantum fields of these KK excitations and the equation of motion to eliminate the classic fields of these KK excitations. At the one-loop level, we find that up to the order $O(p^4)$, due to the momentum conservation of the fifth dimension and the gauge symmetry of the zero modes, there is no constraint on the size of the extra dimension. This result is consistent with the decoupling theorem. However, meaningful constraints can come from those operators in $O(p^6)$ which can contribute considerably to some anomalous vector couplings and can be accessible at the 500 GeV linear collider and the CERN Large Hadron Collider.

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I. INTRODUCTION

The extra dimension scenario is one of the interesting candidates for possible new physics beyond the standard model (SM). As we know, for a higher dimensional quantum field theory, there exist several theoretical problems: unitarity violation [1], ultraviolet cutoff dependence, nonrenormalizability [2], and so on. The contribution of the infinite Kaluza-Klein (KK) towers of the bulk fields always violates the unitarity condition of the S matrix and makes it even harder to evaluate loop effects. Reference [3] provided one way to suppress the contribution of KK excitations by considering the power running of gauge coupling constants of non-Abelian gauge groups. References [4,5] provided another ingenious mechanism to suppress the contribution of massive KK excitations by assuming that the 3-brane is flexible. In this mechanism, because of the momentum conservation of fifth dimension, the contact interaction of matter fields localized on the 3-brane and KK excitations of the bulk fields could be exponentially suppressed. Then, at least at the tree level, the contribution of the infinite KK towers can be well regularized. There are papers which discuss the phenomenology of this mechanism [6]. It seems that, due to this suppression mechanism, the constraint on the size of the extra dimensions imposed by the present experimental researches can be considerably relaxed. However, in this brane fluctuation suppression mechanism, those couplings which respect the momentum conservation of the fifth dimension will not be suppressed, say couplings among KK modes in the gauge bosonic part. This part might suffer those aforementioned theoretical problems of higher-dimensional quantum field theory which could not be solved by the brane fluctuation. Then it seems that only string theory can provide radical solutions [7].

Recently, Ref. [8] used the technicolor method (the moose diagram) to deconstruct the extra dimensions and Ref. [9] used the lattice extra dimensions to construct the renormalizable effective theoretical description of the extra dimension models. One of the important features is that the extra components of the bulk vector gauge bosons can act as the Goldstone and Higgs bosons. Based on these two works, there are papers [10] to construct realistic models. We would like to mention that the effective Lagrangian obtained by Refs. [8,9] does not have the contact structure as assumed in [11]

$$g^2|\phi_2|^2\left(W_\mu+\sqrt{2}\sum_{n=1}^{\infty}W_\mu^n\right)^2, \tag{1}$$

where g is the gauge coupling constant, W_μ is the zero mode, and W_μ^n is the nth KK excitations of gauge bosons. Furthermore, it seems that the interaction terms among zero and KK modes have been ignored by these authors.

After taking into account the brane fluctuation given in [4,5], it seems that the contact structure is more likely modified to be

$$|\phi_2|^2\left[(gW_\mu)^2+\sqrt{2}gg_nW_\mu\sum_{n=1}^{\infty}W_\mu^n+\cdots\right], \tag{2}$$

where g_n is the effective coupling constant of the nth KK excitations of gauge bosons to matter of the SM, and its actual form will be given in Sec. II. Assuming that the effective coupling constant g_n is heavily suppressed, we see that the contact term between the Higgs field and KK excitations would be very small.

Then, a common feature in the deconstructing and brane fluctuation extra dimension models is that there could be no large tree level mixing among zero-mode and KK excitations, and the constraint on extra dimensions imposed by the CERN e^+e^- collider LEP and SLAC could be considerably relaxed. If the world is indeed as described by [4,5] and [8,9], it is natural then to wonder whether there still exists a way to find the traces of KK excitations at the low-energy region near the threshold of the first KK modes. Fortunately, the couplings of the gauge bosonic part between the zero mode and KK excitations are not exponentially suppressed and can be large, therefore they can help us to probe KK

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excitations. So in the deconstructing and brane fluctuation extra dimension models, the bosonic part of the bulk gauge fields will act as the main probe to discover the signal of extra dimensions.

The electroweak chiral Lagrangian (EChL) is the model-independent way to describe the spontaneous symmetry breaking of the SU(2)×U(1) symmetry of the standard model [12]. It can be regarded as the effective theory of the underlying theory in its low-energy region after integrating out those heavy degrees of freedom (DOF), where the dynamic degrees of freedom are the particle contents of the SM. The operators in the Lagrangian consist of the low-energy DOF and can be arranged by the momentum expansion, where the external momentum is assumed to be small compared with the mass of the integrated-out particles. These operators can be classified as relevant, marginal, and irrelevant. The relevant and marginal operators, which are normally collected in and referred to as the $O(p^2)$ and $O(p^4)$ part, are the most interesting and heavily studied. The irrelevant operators are normally suppressed by the mass of heavy DOF according to the decoupling theorem [13]. The complete set of operators in $O(p^6)$ has been given by [14]. The coefficients of these operators form the generic theoretical parameter space of all possible new physics at the low-energy scale. The dimension of this parameter space at $O(p^6)$ is quite large. After integrating out those heavy DOFs, a specified underlying theory will occupy a corner of this large parameter space.

There are two drawbacks to the generality of the effective theory: the first one is that the renormalizability of the underlying theory is sacrificed and the couplings of these operators must be determined from experiments. Another one is that the theory is invalid for momentum larger than the scale $\Lambda_{\rm UV}$, and above this scale the unitarity of the S matrix might be explicitly broken down.

Reference [15] used the EChL to study the effects of KK excitations of the graviton and of the dilaton in large extra dimension scenarios. In this paper, we will use the EChL to analyze the effects of KK excitation of gauge bosons of the SM in the small extra dimension scenarios [16]. We will conduct our computation in the background field gauge method. This method has several advantages compared with the standard Feynman diagram method. The computation is manifestly gauge-invariant at every step, there are fewer relevant diagrams, etc., and we find that, under the brane fluctuation suppression assumption and due to the momentum conservation of the fifth dimension and the gauge symmetry of the zero mode, except for contributing to the renormalization of gauge coupling and the wave function, KK excitations have no effect up to $O(p^4)$ and this result is consistent with the decoupling theorem [13]. However, we know that the meaningful contributions of KK excitations can still come from operators higher than $O(p^4)$, say $O(p^6)$.

The paper is organized as follows. We will briefly describe the brane fluctuation in Sec. II and give the gauge boson sector using the external field method Sec. III. We will emphasize some of its features that have been ignored. We will compute the electroweak chiral Lagrangian of KK excitations in Sec. IV by using the path-integral method. We end with a brief discussion and conclusions.

II. THE BRANE FLUCTUATIONS

The total action given by [5] has two parts: (i) the bulk part $S_{\rm bulk}$, where gravity and vector gauge bosons are assumed to propagate in the bulk; (ii) the brane part $S_{\rm brane}$, where fermion and scalar matter are assumed to be localized on the brane. The SM is consisted of the zero mode of gravity and vector gauge bosons, matter fields confined on the brane, and their interactions. New physics include KK excitations of gravity and vector gauge bosons, the Nambu-Goldstone bosons, and their interactions with each other and with the particles of the SM. In the convention of Ref. [5], the bulk part action defined in D dimension takes the form

$$S_{\rm bulk}=\int d^DX\,\det E\left[-\Lambda+\frac{M^{D-2}}{2}R-\frac{1}{4}G^{MR}G^{NS}{\rm tr}(F_{MN}F_{RS})+\cdots\right], \tag{3}$$

where Λ, M, and R are the cosmological constant, the D-dimensional fundamental scale, and the D-dimensional scalar curvature, respectively. F_{MN} are the Yang-Mills field strength defined in D dimensions.

The matter fields on the brane couple to the bulk fields through the induced vielbein and Yang-Mills fields. The d-dimension brane is assumed to embedded in the D-dimension space-time, and its action can be formulated in the following form:

$$S_{\rm brane}=\int d^dx\,\det e\left[-\tau+e^{\mu}_{\alpha}(x)\,\overline{\psi}(x)\,i\,\gamma^{\alpha}\left(\frac{\overleftrightarrow{\nabla}_{\mu}}{2}-ig a_{\mu}(x)\right)\psi(x)-m\,\overline{\psi}(x)\,\psi(x)+\cdots\right], \tag{4}$$

where $\psi(x)$ is a fermion field on the brane which is charged under the Yang-Mills gauge group. The original paper [5] does not consider the scalar case. If we assume there are scalar fields in the theory, we should add their corresponding terms to $S_{\rm brane}$.

In the flat space-time metric, the $S_{\rm brane}$ can be reduced to

$$\int d^dx \det e(-\tau)=\int d^dx\Big[-\tau+\frac{1}{2}\partial^\mu\phi^m(x)\partial_\mu\phi^m(x)+\frac{1}{8\tau}[\partial^\mu\phi^m(x)\partial_\mu\phi^m(x)]^2-\frac{1}{4\tau}[\partial^\mu\phi^m(x)\partial_\nu\phi^m(x)]\times[\partial^\nu\phi^n(x)\partial_\mu\phi^n(x)]+\cdots\Big], \tag{5}$$

where ϕ is the Nambu-Goldstone boson corresponding to the spontaneous breaking of the translation symmetry.

Assuming that the $D-d$ dimensions are compactified, the bulk gauge field can be Fourier expanded in their KK modes:

$$A_M(X^\mu=x^\mu,X^m=Y^m)=\frac{1}{\sqrt{V}}\sum_n A_M^{(n)}(x)e^{in\cdot Y/R}. \tag{6}$$

Then the gauge interaction term on the brane reads

$$\int d^dx\sum_n g\overline{\psi}(x)\gamma^\mu\psi(x)A_\mu^{(n)}(x)\exp\left(\frac{in\cdot\phi(x)}{R\sqrt{\tau}}\right). \tag{7}$$

Considering that the Nambu-Goldstone bosons have their fluctuations, the gauge interaction term should be rewritten as

$$\int d^dx\sum_n g\, e^{-(1/2)(n^2/R^2\tau)\Delta(M^{-1})}\cdot\overline{\psi}(x)\gamma^\mu\psi(x)A_\mu^{(n)}(x):\exp\left(\frac{in\cdot\phi(x)}{R\sqrt{\tau}}\right):, \tag{8}$$

where Δ is the free propagator of ϕ:

$$\Delta(x-y)\equiv\langle\phi(x)\phi(y)\rangle=\frac{-1}{4\pi^2}\frac{1}{(x-y)^2}. \tag{9}$$

The most interesting phenomenon with the brane fluctuation is that the effective coupling g_n of the level n KK mode to the four-dimensional field is suppressed exponentially:

$$g_n\equiv g\cdot e^{-1/2(n/R)^2M^2/f^4}. \tag{10}$$

The origin of this suppression is a recoil effect of the brane. It is this suppression mechanism that causes the constraints on the extra dimensions to be substantially loosened.

According to the analysis of [5,17], although there exists a constraint on the tension of the brane when taking into account the effects of the Nambu-Goldstone boson, it seems that KK excitations might escape our detection.

Fortunately, the couplings in the bosonic part between the zero mode and KK excitations will not be exponentially suppressed, so they can help us to probe KK excitations. Below, we will assume this suppression mechanism for the S^1/Z_2 case,[1] and investigate the effects of KK excitations to the bosonic sector of the SM in the brane-fluctuation extra dimension model. For the sake of simplicity, we omit the gravity part, which should be small when compared with the Yang-Mills (YM) part in the small extra dimension scenarios. In order to compare and contrast with the SM, we assume that there is a Higgs doublet field. To get the electroweak symmetry breaking, the linear Higgs mechanism is assumed. However, considering exponential suppression of the coupling of the matter on the brane and KK excitations, the tree level mixing angle among the zero mode and KK excitations will be neglected.

III. THE GAUGE BOSONIC SECTOR IN THE EXTERNAL FIELD METHOD

To simplify the consideration, we study the 5D compactification on $M_4\times S^1/Z_2$ [16], and we will use the dimension reduction procedure to get the effective theory in 4D. The total action is formulated as

$$S_5=\int d^5x[L_{\rm YM}+L_{\rm contact}\delta(x_5)], \tag{11}$$

$$L_{\rm YM}=-\frac{1}{4}\widetilde{W}_{MN}\widetilde{W}^{MN}-\frac{1}{4}\widetilde{B}_{MN}\widetilde{B}^{MN}-\frac{F^2}{2\xi}+\bar{c}\frac{\delta F}{\delta\widetilde{\alpha}}c, \tag{12}$$

where $M,N=0,1,2,3,5$, $\widetilde{W}_{MN}=\partial_M\widetilde{W}_N-\partial_N\widetilde{W}_M+f\widetilde{W}_M\widetilde{W}_N$, $\widetilde{B}_{MN}=\partial_M\widetilde{B}_N-\partial_N\widetilde{B}_M$, $\widetilde{W}=W(x,x_5)$, and $\widetilde{B}=B(x,x_5)$, and f is the structure constant of the Lie algebra (the group index is suppressed). The $L_{\rm contact}$ contains the contact terms of the SM to the KK excitations except for the

[1]The suppression mechanism given by Ref. [5] is valid for compactified s^1, and it is not very clear whether this assumption can be proper for the S^1/Z_2 case. However, for the sake of simplicity, we show in this paper how to conduct calculation under this assumption in the orbifold compactification case. The computational procedure can be extended to the S^1 compactification straightforwardly.

vector boson field part. The Lagrangian is formally invariant under the gauge transformation in 5D.

In order to get the effective Lagrangian which is manifestly gauge covariant to the symmetry of the SM, we use the background field method [18] and split the vector gauge field $\tilde{V}_M$ into two parts as (here $V=W$ and B, respectively)

$$\tilde{V}_M=\overline{V}_M+\hat{V}_M\,, \tag{13}$$

where $\overline{V}_M$ is the classic part and $\hat{V}_M$ is the quantum fluctuation. In the background field gauge, we have the freedom to choose different gauges for the classic and quantum vector boson field, respectively. For the classic field, we will use the unitary gauge, which means $\overline{V}_5=0$ [this will be more manifest in the deconstructing model [8], where the Goldstone boson field is realized in the nonlinear way, $U=\exp(\int\overline{V}_5 dx^5)$; $U=1$ is the unitary gauge, and this corresponds to $\overline{V}_5=0$] and V_5 will not appear in the Lagrangian. For the quantum field, we will use the R_ξ gauge and the $\hat{V}_5$ does not vanish.

The gauge fixing terms are chosen to be

$$F(W)=\overline{D}^\mu\hat{W}_\mu-\xi_W\partial^5\hat{W}_5, \tag{14}$$

$$F(B)=\partial^\mu\hat{B}_\mu-\xi_B\partial^5\hat{B}_5, \tag{15}$$

where $\overline{D}^\mu=\partial^\mu+gf\overline{A}^\mu$. To write these two gauge fixing term, we have not taken into account the spontaneous symmetry breaking of the SM. The variation of the gauge fixing terms under the gauge transformation is given as

$$\frac{\delta F(W)}{\delta\alpha_W}=\overline{D}^\mu(\overline{D}_\mu+gf\hat{W}_\mu)+\xi_W\partial^5(\partial_5+gf\hat{W}_5), \tag{16}$$

$$\frac{\delta F(B)}{\delta\alpha_B}=\partial^\mu\partial_\mu+\xi_B\partial^5\partial_5\,. \tag{17}$$

By requiring that the field is unchanged under the orbifold transformation, we can decompose vector bosons V as

$$\tilde{V}(\hat{V})_\mu(x,x_5)=\sum_{i=0}^{\infty}\tilde{V}(\hat{V})^i_\mu(x)\cos i\theta_5\,, \tag{18}$$

$$\hat{V}_5(x,x_5)=\sum_{i=1}^{\infty}\hat{V}^i_5(x)\sin i\theta_5\,, \tag{19}$$

where $\theta_5=M_c x_5$, $M_c=2\pi/R_c$, and R_c is the radius of the compactified fifth dimension. To compare and contrast with the SM, below we will omit the index 0 of zero modes and represent $\tilde{W}^0=W$ and $\tilde{B}^0=B$, respectively. W and B are the vector gauge bosons of the SM, respectively. Below we will suppress the bar of the classic background fields.

In order to integrate out the fifth dimension, we decompose the field strength by using Eqs. (18) and (19). The $\tilde{W}_{\mu\nu}$ can be decomposed by cos modes and we have

$$0\ \text{mode:}\quad W_{\mu\nu}+(D_\mu\hat{W}_\nu-D_\nu\hat{W}_\mu)+\frac{1}{2}gf\sum_{i=1}^{\infty}[W^i_\mu W^i_\nu+\hat{W}^i_\mu W^i_\nu+W^i_\mu\hat{W}^i_\nu+\hat{W}^i_\mu\hat{W}^i_\nu], \tag{20}$$

$$n\ \text{mode:}\quad (D_\mu W^n_\nu-D_\nu W^n_\mu+D_\mu\hat{W}^n_\nu-D_\nu\hat{W}^n_\mu)+gf(\hat{W}_\mu W^n_\nu-\hat{W}_\nu W^n_\mu)+gf(\hat{W}_\mu\hat{W}^n_\nu-\hat{W}_\nu\hat{W}^n_\mu)$$

$$+\frac{1}{2}gf\sum_{i=1}^{n-1}(W^i_\mu W^{n-i}_\nu+\hat{W}^i_\mu W^{n-i}_\nu+W^i_\mu\hat{W}^{n-i}_\nu+\hat{W}^i_\mu\hat{W}^{n-i}_\nu) \tag{21}$$

$$+\frac{1}{2}gf\sum_{i=1}^{\infty}(W^i_\mu W^{n+i}_\nu+\hat{W}^i_\mu W^{n+i}_\nu+W^i_\mu\hat{W}^{n+i}_\nu+\hat{W}^{n+i}_\mu\hat{W}^n_\nu+W^{n+i}_\mu W^i_\nu \tag{22}$$

$$+\hat{W}^{n+i}_\mu W^i_\nu+W^{n+i}_\mu\hat{W}^i_\nu+\hat{W}^{n+i}_\mu\hat{W}^i_\nu), \tag{23}$$

where $W_{\mu\nu}=\partial_\mu W_\nu-\partial_\nu W_\mu+gfW_\mu W_\nu$ and $D_\mu=\partial_\mu+gfW_\mu$. For $\tilde{B}_{\mu\nu}$, we have

$$0\ \text{mode:}\quad B_{\mu\nu}+\hat{B}_{\mu\nu}, \tag{24}$$

$$n\ \text{mode:}\quad B^n_{\mu\nu}+\hat{B}^n_{\mu\nu}, \tag{25}$$

where $V_{\mu\nu}=\partial_\mu V_\nu-\partial_\nu V_\mu$, with $V=B,\ \hat{B},\ B^n,\ \hat{B}^n$.

The $\tilde{W}_{5\mu}$ can be decomposed by sin modes and we have

$$n\ \text{mode:}\quad D_\mu\hat{W}^n_5+gf\hat{W}_\mu\hat{W}^n_5+nM_cW^n_\mu+nM_c\hat{W}^n_\mu$$

$$+\frac{1}{2}gf\sum_{i=1}^{n-1}(W^i_\mu+\hat{W}^i_\mu)\hat{W}^{n-i}_5$$

$$+\frac{1}{2}gf\sum_{i=1}^{\infty}[(W^i_\mu+\hat{W}^i_\mu)\hat{W}^{n+i}_5$$

$$+(W^{n+i}_\mu+\hat{W}^{n+i}_\mu)\hat{W}^i_5], \tag{26}$$

while for $\tilde{B}_{5\mu}$, we have

$$n \text{ mode:} \quad \partial_\mu \hat{B}_5^n + nM_c B_\mu^n + nM_c \hat{B}_\mu^n. \tag{27}$$

It is remarkable that there is no zero mode for the sin KK modes for $V_{5\mu}$ and it is related to the assumption of the compactified space-time.

The gauge fixing term of SU(2) is decomposed by cos modes and we have

$$0 \text{ mode:} \quad D^\mu \hat{W}_\mu, \tag{28}$$

$$n \text{ mode:} \quad D^\mu \hat{W}_\mu^n - n\xi_W M_c \hat{W}_5^n + \frac{1}{2} gf \sum_{i=1}^{n-1} (W^{i\mu} + \hat{W}^{i\mu}) \hat{W}_\mu^{n-i} + \frac{1}{2} gf \sum_{i=1}^{\infty} [(W^{i\mu} + \hat{W}^{i\mu}) \hat{W}_\mu^{n+i} + (W^{(n+i)\mu} + \hat{W}^{(n+i)\mu}) \hat{W}_\mu^i], \tag{29}$$

and that of U(1) is decomposed as

$$0 \text{ mode:} \quad \partial^\mu \hat{B}_\mu, \tag{30}$$

$$n \text{ mode:} \quad \partial^\mu \hat{B}_\mu^n - n\xi_B M_c \hat{B}_5^n. \tag{31}$$

Since we are only interested in low-energy physics where zero modes play the main part, we will only keep those terms containing zero modes and neglect those pure interactions of KK excitations. Then after integrating out the fifth dimension, we get the reduced YM Lagrangian, which reads

$$\begin{aligned}
L_{\rm YM}^{\rm eff} = &-\frac{1}{4}(2\pi R_c)\Big[(W_{\mu\nu} + D_\mu \hat{W}_\nu - D_\nu \hat{W}_\mu + gf\hat{W}_\mu \hat{W}_\nu)^2 + gf(W_{\mu\nu} + D_\mu W_\nu - D_\nu W_\mu + gf\hat{W}_\mu \hat{W}_\nu) \sum_{n=1}^{\infty} (W_\mu^n W_\nu^n + \hat{W}_\mu^n W_\nu^n \\
&+ W_\mu^n \hat{W}_\nu^n + \hat{W}_\mu^n \hat{W}_\nu^n)\Big] - \frac{1}{4}(\pi R_c) \sum_{n=1}^{\infty} [D_\mu W_\nu^n - D_\nu W_\mu^n + D_\mu \hat{W}_\nu^n - D_\nu \hat{W}^n{}_\mu + gf(\hat{W}_\mu W_\nu^n - \hat{W}_\nu W_\mu^n + \hat{W}_\mu \hat{W}_\nu^n - \hat{W}_\nu \hat{W}_\mu^n)]^2 \\
&+ \frac{1}{2}(\pi R_c) \sum_{n=1}^{\infty} [D_\mu \hat{W}_5^n + gf\hat{W}_\mu \hat{W}_5^n + nM_c W_\mu^n + nM_c \hat{W}_\mu^n]^2 - \frac{1}{2\xi_W}(2\pi R_c)(D^\mu \hat{W}_\mu)^2 - \frac{1}{2\xi_W}(\pi R_c) \sum_{n=1}^{\infty} (D^\mu \hat{W}_\mu^n \\
&- \xi_W n M_c \hat{A}_5^n)^2 + (2\pi R_c)\bar{c}[-D^\mu(D_\mu + gf\hat{W}_\mu)]c + (\pi R_c) \sum_{n=1}^{\infty} \overline{c^n}[-D^\mu(D_\mu + gf\hat{W}_\mu) - n^2 \xi_W M_c^2] c^n \\
&+ (\pi R_c) gf \sum_{n=1}^{\infty} (D^\mu \overline{c^n} W_\mu^n c + D^\mu \bar{c} W_\mu^n c^n) + \cdots - \frac{1}{4}(2\pi R_c)[B_{\mu\nu} + \hat{B}_{\mu\nu}]^2 - \frac{1}{4}(\pi R_c) \sum_{n=1}^{\infty} [B_{\mu\nu}^n + \hat{B}_{\mu\nu}^n]^2 \\
&+ \frac{1}{2}(\pi R_c) \sum_{n=1}^{\infty} [nM_c B_\mu^n + nM_c \hat{B}_\mu^n + \partial_\mu \hat{B}_5^n]^2 - \frac{1}{2\xi_B}(2\pi R_c)(\partial^\mu \hat{B}_\mu)^2 - \frac{1}{2\xi_B}(\pi R_c) \sum_{n=1}^{\infty} (\partial^\mu \hat{B}_\mu^n)^2 \\
&+ (2\pi R_c)\overline{c_B}(-\partial^\mu \partial_\mu) c_B + (\pi R_c)\overline{c_B^n}(-\partial^\mu \partial_\mu - n^2 \xi_B M_c^2) c_B^n,
\end{aligned} \tag{32}$$

where the omitted terms are only related to the non-Abelian SU(2) gauge symmetry and the U(1) part is exact.

By utilizing the rescaling relations

$$W(B, c_W, c_B) \to \sqrt{2\pi R_c} W(B, c_W, c_B), g(g') \to \frac{1}{\sqrt{2\pi R_c}} g(g'), \tag{33}$$

$$W^n(B^n, c_W^n, c_B^n, W_5^n, B_5^n) \to \sqrt{\pi R_c} W^n(B^n, c_W^n, c_B^n, W_5^n, B_5^n), \tag{34}$$

the final effective Lagrangian of 4D reads

$$L^{\rm eff}_{\rm YM,4D}=-\frac{1}{4}\bigg[(W_{\mu\nu}+D_\mu\hat{W}_\nu-D_\nu\hat{W}_\mu+gf\hat{W}_\mu\hat{W}_\nu)^2+\mathbf{R}gf(W_{\mu\nu}+D_\mu\hat{W}_\nu-D_\nu\hat{W}_\mu+gf\hat{W}_\mu\hat{W}_\nu)\sum_{n=1}^{\infty}(W^n_\mu W^n_\nu+\hat{W}^n_\mu W^n_\nu+W^n_\mu\hat{W}^n_\nu$$
$$+\hat{W}^n_\mu\hat{W}^n_\nu)\bigg]-\frac{1}{4}\sum_{n=1}^{\infty}[D_\mu W^n_\nu-D_\nu W^n_\mu+D_\mu\hat{W}^n_\nu-D_\nu\hat{W}^n_\mu+gf(\hat{W}_\mu W^n_\nu-\hat{W}_\nu W^n_\mu+\hat{W}_\mu\hat{W}^n_\nu-\hat{W}_\nu\hat{W}^n_\mu)]^2+\frac{1}{2}\sum_{n=1}^{\infty}[D_\mu\hat{W}^n_5$$
$$+gf\hat{W}_\mu\hat{W}^n_5+nM_cW^n_\mu+nM_c\hat{W}^n_\mu]^2-\frac{1}{2\xi_W}(D^\mu\hat{W}_\mu)^2-\frac{1}{2\xi_W}\sum_{n=1}^{\infty}(D^\mu\hat{W}^n_\mu-\xi_WnM_c\hat{A}^n_5)^2+\overline{c}_W[-D^\mu(D_\mu+gf\hat{W}_\mu)]c_W$$
$$+\sum_{n=1}^{\infty}\overline{c^n_W}[-D^\mu(D_\mu+gf\hat{W}_\mu)-n^2\xi_WM_c^2]c^n_W+gf\sum_{n=1}^{\infty}(D^\mu\overline{c^n}W^n_\mu c+D^\mu\overline{c}W^n_\mu c^n)+\cdots-\frac{1}{4}[B_{\mu\nu}+\hat{B}_{\mu\nu}]^2$$
$$-\frac{1}{4}\sum_{n=1}^{\infty}[B^n_{\mu\nu}+\hat{B}^n_{\mu\nu}]^2+\frac{1}{2}\sum_{n=1}^{\infty}[nM_cB^n_\mu+nM_c\hat{B}^n_\mu+\partial_\mu\hat{B}^n_5]^2-\frac{1}{2\xi_B}(\partial^\mu\hat{B}_\mu)^2$$
$$-\frac{1}{2\xi_B}\sum_{n=1}^{\infty}(\partial^\mu\hat{B}^n_\mu)^2+\overline{c}_B(-\partial^\mu\partial_\mu)c_B+\overline{c^n_B}(-\partial^\mu\partial_\mu-n^2\xi_BM_c^2)c^n_B, \tag{35}$$

where $R=2$, which arises from the different normalization factor of the zero mode and KK excitations.

There are several remarkable features of the reduced effective Lagrangian in 4D given in Eq. (35).

(i) In the dimension reduction procedure, the zero modes are still massless, and the corresponding gauge symmetry is unbroken and is explicit in the background field gauge. KK excitations are the adjoint representations of the SU(2) symmetry in 4D, as pointed out in [19]. To break the symmetries of the zero mode, other assumptions should be introduced.

(ii) There are infinite KK excitations. For each massive KK mode, the spectrum consists of a massive quantum field, its corresponding Goldstone field, its corresponding ghost field, and a massive background field.

(iii) There are infinite interaction terms among KK excitations which are controlled by only two gauge coupling constants, g and g'. This structure cannot sustain the quantum corrections even if we truncate the infinite KK tower to finite. The intrinsic reason is that the underlying theory defined in 5D is nonrenormalizable, as already pointed out in [2].

(iv) For the vector boson field of U(1) symmetry, there is no interaction among KK modes, while for the vector boson field of SU(2) symmetry, there exist gauge interactions between different KK modes. This fact will bring out some interesting phenomenologies, as we will show below.

(v) Because of the momentum conservation of the extra dimensions, all of the interaction terms between the zero mode A^0 and a KK excitation A^n contain at least two A^n's, as shown in Eq. (35).

IV. INTEGRATING OUT THE KK EXCITATIONS AT THE ONE-LOOP LEVEL

In this section, we will extract the effective Lagrangian up to the one-loop level by integrating out KK excitations. The method we will use is the functional integral. The functional method to integrate out a heavy DOF is quite standard, and Refs. [20,21] provide a detailed procedure. Normally, the background field method and Stuckeberg transformation are used to integrate out the quantum DOF. After that, the equation of motion of the heavy fields is used to eliminate the classic heavy DOF from the Lagrangian. In [20,21], the authors use this method to investigate the effect of heavy Higgs bosons, and in [22] the authors use this method to study that of the heavy fermion. To integrate out KK excitations, we assume that KK excitations are massive and heavier than all particles of the SM.

A. Tree-level relations

First we provide the classic equation of motion (EOM) of those background fields (BF). The EOM of the BF of the zero mode is given as

$$D^\mu W_{\mu\nu}-M_W^2W_\mu$$
$$=gf\sum_{n=1}^{\infty}W^\mu(D_\nu W^n_\mu-D_\mu W^n_\nu)+J_\mu(\text{light}), \tag{36}$$

where $J_\mu(\text{light})$ means the currents of light DOFs of the SM which are light compared with massive KK excitations. The EOM of the BF of the nth KK excitation is given as

$$D^\mu(D_\mu W^n_\nu-D_\nu W^n_\mu)-n^2M_c^2W^n_\nu$$
$$=W_{\mu\nu}W^{n\mu}+\frac{g_n}{g}J_\mu(\text{light})+\cdots. \tag{37}$$

The omitted terms are terms of KK excitations which can be safely neglected. For a vector gauge boson field of U(1), the EOM is simple. Considering that there is no interaction among KK excitations of U(1) symmetry and the brane fluctuation greatly suppresses the interactions between KK excitations and light DOFs, below we will omit the KK excitations of the U(1) part.

The equation of motion of a classic KK excitation can be formulated in momentum presentation as

$$[(p^2-n^2M_c^2)+f(W^0)]W_\mu^n=g_nJ_\mu, \tag{38}$$

where p^2 is the momentum of the W^n, nM_c is its mass, and $f(W^0)$ includes the terms of interactions between the zero-mode W^0 and the KK mode W^n. The J_μ is the current of matter of the SM and g_n is the brane fluctuation suppression factor. In the low-energy region, the terms with momentum p will be set to zero, and the W_μ^n can be represented by the light degree of freedom as given below,

$$W_\mu^n \approx -\frac{g_n}{(n^2M_c^2)}J_\mu[1+f(W^0)/(n^2M_c^2)+\cdots]. \tag{39}$$

Therefore, at tree level, after integrating out the massive KK excitations, we will get terms like

$$\frac{(g_n)^2}{(n^2M_c^2)}J_\mu J^\mu[1+f(W^0)/(n^2M_c^2)+\cdots]. \tag{40}$$

By invoking the heavy exponential suppression argument, we regard these terms as being over order $O(1/M_c^4)$ and neglect them in our consideration.

At tree level up to $O(1)$, to integrate out KK excitations means to set the field of KK excitations (both classic and quantum field) to zero. We get the tree level effective Lagrangian

$$L_{\rm YM}^{\rm eff,\,tree}=-\frac{1}{4}[(W_{\mu\nu}+D_\mu\hat{W}_\nu-D_\nu\hat{W}_\mu+gf\hat{W}_\mu\hat{W}_\nu)^2]-\frac{1}{2\xi_W}(D^\mu\hat{W}_\mu)^2+\overline{c}_W[-D^\mu(D_\mu+gf\hat{W}_\mu)]c_W. \tag{41}$$

This Yang-Mills Lagrangian is the standard one in the background gauge.

Up to the order $O(1/M_c^2)$, after integrating out massive KK excitations, we will get terms like

$$\sum_{n=1}^{\infty}\frac{g_n^2}{(nM_c)^2}J_\mu J^\mu+\cdots. \tag{42}$$

Under the assumption of brane fluctuation suppression, we regard these terms as being higher than $O(1/M_c^4)$ and will omit them in the analysis below.

B. Integrating out KK excitations

To extract the effective Lagrangian at the one-loop level, we reformulate the effective Lagrangian given in Eq. (35) and only keep those bilinear terms,

$$\mathcal{L}=\hat{W}_\mu\Delta_{WW}^{\mu\nu}\hat{W}_\nu+\overline{c}_W\Delta_{c_Wc_W}c_W+\sum_{n=1}^{\infty}\hat{W}_\mu^n\Delta_{W^nW^n}^{\mu\nu}\hat{W}_\nu^n+\sum_{n=1}^{\infty}\hat{W}_\mu\Delta_{WW^n}^{\mu\nu}\hat{W}_\nu^n+\sum_{n=1}^{\infty}\hat{W}_\mu^n\Delta_{W^nW}^{\mu\nu}\hat{W}_\nu+\sum_{n=1}^{\infty}\hat{W}_5^n\Delta_{W_5^nW_5^n}\hat{W}_5^n$$
$$+\sum_{n=1}^{\infty}\overline{c}_W^n\Delta_{c_W^nc_W^n}c_W^n+\sum_{n=1}^{\infty}\overline{c}_W\Delta_{c_Wc_W^n}c_W^n+\sum_{n=1}^{\infty}\overline{c}_W^n\Delta_{c_W^nc_W}c_W+\cdots, \tag{43}$$

$$\Delta_{WW}^{\mu\nu}=\frac{1}{2}\left[D^2g^{\mu\nu}-\left(1-\frac{1}{\xi_w}\right)D^\mu D^\nu-gW^{\rho\sigma}\mathcal{J}_{\rho\sigma}^{\mu\nu}\right], \tag{44}$$

$$\Delta_{W^nW^n}^{\mu\nu}=\frac{1}{2}\left[(D^2+n^2M_c^2)g^{\mu\nu}-\left(1-\frac{1}{\xi_w}\right)D^\mu D^\nu-gW^{\rho\sigma}\mathcal{J}_{\rho\sigma}^{\mu\nu}\right], \tag{45}$$

$$\Delta_{WW^n}^{\mu\nu}=\frac{1}{2}gf[W^{n\mu}D^\nu-g^{\mu\nu}W^{n\alpha}D_\alpha+(D^\mu W^{n\nu})-(D^\nu W^{n\mu})], \tag{46}$$

$$\Delta_{WW^n}^{\mu\nu}=\Delta_{W^nW}^{\mu\nu}, \tag{47}$$

$$\Delta_{W_5^nW_5^n}=\frac{1}{2}(-D^2-\xi_wn^2M_c^2), \tag{48}$$

$$\Delta_{c_W c_W} = -D^2, \tag{49}$$

$$\Delta_{c_W^n c_W^n} = -D^2 - \xi_w n^2 M_c^2, \tag{50}$$

$$\Delta_{c_W c_W^n} = -gfD^\mu W_\mu^n, \tag{51}$$

$$\Delta_{c_W^n c_W} = -gfD^\mu W_\mu^n, \tag{52}$$

where $W_{\mu\nu} = W_{\mu\nu}^a t_G^a$, $(t_G^a)_{bc} = if^{bac}$ are structure constants and the generator adjoint representations of the non-Abelian group, and $\mathcal{J}_{\rho\sigma}^{\mu\nu}$ is the generator of Lorentz transformations on 4-vectors and is defined as

$$\mathcal{J}_{\rho\sigma}^{\mu\nu} = i(\delta_\rho^\mu \delta_\sigma^\nu - \delta_\sigma^\mu \delta_\rho^\nu). \tag{53}$$

Linear terms can be eliminated by using the classic EOMs. From the result listed above, it is apparent that the quadratic operators of KK excitations are very similar to that of the zero mode.

We have omitted those terms which contribute at the two-loop level. One feature is worthy of mention: KK excitations always appear at least in pair due to the momentum conservation of the fifth dimension. This fact is very important for us to understand the decoupling behavior of KK excitations. It is also remarkable that there exist mixings among the quantum fields of KK modes, and in order to integrate out the quantum part of KK excitations, we must diagonalize the bilinear terms. (There are also mixings among different quantum fields of KK excitations which have been omitted, which is reasonable according to the auxiliary power counting rule that will be introduced below.) Then we get the one-loop effective Lagrangian by integrating out the massive KK excitations:

$$\begin{aligned} L_{\text{YM, KK}}^{\text{eff,1-loop}} &= \frac{1}{2}\sum_{n=1}^{\infty} \ln\det[\tilde{\Delta}_{W^n W^n}\delta^{(4)}(x-y)] \\ &+ \frac{1}{2}\sum_{n=1}^{\infty} \ln\det[\Delta_{W_5^n W_5^n}\delta^{(4)}(x-y)] \\ &- \sum_{n=1}^{\infty} \ln\det[\tilde{\Delta}_{c_W^n c_W^n}\delta^{(4)}(x-y)], \end{aligned} \tag{54}$$

$$\begin{aligned} &= \frac{1}{2}\sum_{n=1}^{\infty} \operatorname{tr}\ln[\tilde{\Delta}_{W^n W^n}\delta^{(4)}(x-y)] \\ &+ \frac{1}{2}\sum_{n=1}^{\infty} \operatorname{tr}\ln[\Delta_{W_5^n W_5^n}\delta^{(4)}(x-y)] \\ &- \sum_{n=1}^{\infty} \operatorname{tr}\ln[\tilde{\Delta}_{c_W^n c_W^n}\delta^{(4)}(x-y)], \end{aligned} \tag{55}$$

where

$$\tilde{\Delta}_{W^n W^n} = \Delta_{W^n W^n} - \Delta_{W W^n}^\dagger \Delta_{WW}^{-1} \Delta_{W W^n}, \tag{56}$$

$$\tilde{\Delta}_{c_W^n c_W^n} = \Delta_{c_W^n c_W^n} - \Delta_{c_W c_W^n}^\dagger \Delta_{c_W c_W}^{-1} \Delta_{c_W c_W^n}. \tag{57}$$

The signs of the contributions of ghost scalars and normal scalars are different due to the fact that ghost fields satisfy anticommutation relations.

To this step, the quantum fields of KK excitations have been integrated out and the functional trace and logarithm have to be evaluated. There are several methods to deal with this evaluation [24–27]. Below we will first use the method [26] to analyze those relevant terms. After doing this, we will use the heat kernel [27] to evaluate the trace and logarithm.

C. The auxiliary counting rule

We study the $\operatorname{tr}\ln\tilde{\Delta}_{W^n W^n}$ first. We have

$$\begin{aligned} &\tilde{\Delta}_{W^n W^n}(x,\partial_x)\delta^{(4)}(x-y) \\ &= \int \frac{d^4p}{(2\pi)^4}\tilde{\Delta}_{W^n W^n}(x,\partial_x)\exp[ip(x-y)] \\ &= \int \frac{d^4p}{(2\pi)^4}\exp[ip(x-y)]\tilde{\Delta}_{W^n W^n}(x,\partial_x+ip). \end{aligned} \tag{58}$$

Then, the trace can be determined,

$$\begin{aligned} &\operatorname{tr}\ln[\tilde{\Delta}_{W^n W^n}(x,\partial_x)\delta^{(4)}(x-y)] \\ &= \int d^4x \int \frac{d^4p}{(2\pi)^4}\operatorname{tr}\ln[\tilde{\Delta}_{W^n W^n}(x,\partial_x+ip)]. \end{aligned} \tag{59}$$

Here "tr" means the sum over group and spin indices. $\tilde{\Delta}_{W^n W^n}(x,\partial_x+ip)$ can be expanded in terms of derivatives,

$$\tilde{\Delta}_{W^n W^n}(x,\partial_x+ip) = \sum_{m=0}^{\infty}\frac{(-i)^m}{m!}\left[\frac{\partial^m}{\partial p_{\mu_1}\cdots\partial p_{\mu_m}}\tilde{\Delta}_{W^n W^n}(x,ip)\right]\partial_{\mu_1}\cdots\partial_{\mu_m}. \tag{60}$$

In the 't Hooft–Feynman gauge, it yields an expression like

$$\tilde{\Delta}_{W^n W^n}(x,\partial_x+ip) = (p^2 - n^2 M_c^2)\delta^{ab} + \Pi^{ab}(x,p,\partial_x). \tag{61}$$

Dropping an irrelevant constant, we get

$$\operatorname{tr}\tilde{\Delta}_{W^n W^n}(x,\partial_x+ip) = \sum_{m=1}^{\infty}\frac{(-1)^{m+1}}{n}\operatorname{tr}\left(\frac{\Pi}{p^2 - n^2 M_c^2}\right)^m. \tag{62}$$

We are interested in those terms caused by the mixing among KK modes. According to the standard procedure given in [21], when expanding $\ln\tilde{\Delta}_{W^nW^n}(x,\partial_x+ip)$ we determine the leading powers of p, W^n, and M_c for each term generated and introduce an auxiliary parameter ζ, which counts these powers,

$$p_\mu\to\zeta,\quad M_c\to\zeta,\quad W^n\to\zeta^{-2}\frac{g_n}{n^2g}. \tag{63}$$

We would like to mention that the W^n is not only suppressed by its mass, but also by the brane fluctuation factor g_n/g. This counting rule tells us that the contribution of $\Delta^\dagger_{WW^n}\Delta^{-1}_{WW}\Delta_{WW^n}$ is suppressed at least by $1/M_c^4(g_n/g)^2$.[2] So we can neglect this term and extract terms reliably up to $1/M_c^2$. Then the procedure to evaluate the trace and logarithm is greatly simplified. For the operator $\tilde{\Delta}_{c_W^nc_W^n}(x,\partial_x+ip)$, we have the same conclusion. So we have

$$S^{\text{eff,1-loop}}_{\text{YM, KK}}=\frac{i}{2}\int_x[\operatorname{tr}\ln\Delta_{W^nW^n}-\operatorname{tr}\ln\Delta_{c_W^nc_W^n}]+O\left(\frac{1}{M_c^4}\right). \tag{64}$$

To get the above equation, we have used the relation $\Delta_{c_W^nc_W^n}=\Delta_{W_5^nW_5^n}$.

D. Evaluating the trace and logarithm by using the method of a heat kernel

Now, it becomes easy to evaluate the trace and logarithm by utilizing the method of a heat kernel [27], up to $O(p^6)$, which reads

$$S_{\text{loop}}=-\frac{1}{2(4\pi)^{d/2}}\int_x\left\{m^d\Gamma\left(-\frac{d}{2}\right)(\operatorname{tr}a_0^W-\operatorname{tr}a_0^{c_W})+m^{d-2}\Gamma\left(1-\frac{d}{2}\right)(\operatorname{tr}a_1^W-\operatorname{tr}a_1^{c_W})+m^{d-4}\Gamma\left(2-\frac{d}{2}\right)(\operatorname{tr}a_2^W-\operatorname{tr}a_2^{c_W})+m^{d-6}\Gamma\left(3-\frac{d}{2}\right)(\operatorname{tr}a_3^W-\operatorname{tr}a_3^{c_W})+\cdots\right\}, \tag{65}$$

where a_i^a are the Seeley-DeWitt coefficients of the corresponding quadratic operators. For the generic operator of the form $\Delta=D^2+M^2+\sigma$, the Seeley-DeWitt coefficients in the coincidence limit read

$$a_0|=1, \tag{66}$$

$$a_1|=-\sigma, \tag{67}$$

$$a_2|=\frac{1}{2}\sigma^2-\frac{g^2}{12}F^{\mu\nu}F_{\mu\nu}+\frac{1}{6}[D_\mu,[D^\mu,\sigma]], \tag{68}$$

$$a_3|=-\frac{1}{6}\sigma^3+\frac{1}{12}(\{\sigma,D^2\sigma\}+D^\mu\sigma D_\mu\sigma)-\frac{1}{60}D^2D^2\sigma+i\frac{g}{60}[D_\alpha F^{\alpha\mu},D_\mu\sigma]+\frac{g^2}{60}(2\{F_{\mu\nu}F^{\mu\nu},\sigma\}+F_{\mu\nu}\sigma F^{\mu\nu})-\frac{g^2}{45}D_\alpha F^{\alpha\mu}D^\beta F_{\beta\mu}-\frac{g^2}{180}D_\alpha F_{\beta\gamma}D^\alpha F^{\beta\gamma}-\frac{g^2}{60}\{F_{\mu\nu},D^2F^{\mu\nu}\}-i\frac{g^3}{30}F_{\mu\nu}F^{\mu\alpha}F^\nu_{\ \alpha}. \tag{69}$$

The a_0 terms will contribute divergently but can be removed by redefining the vacuum. The a_1 term simply vanishes for $\Delta_{W^nW^n}$ and $\Delta_{c_W^nc_W^n}$. The a_2 term is nonzero and contributes to the hidden operators [24] in $O(p^4)$, which read

$$L^{\text{1-loop}}_{\text{eff}}(p^4)=-\frac{1}{2}\frac{1}{(4\pi)^{d/2}}\sum_{n=1}^\infty(n^2M_c^2)^{d-4}\Gamma\left(2-\frac{d}{2}\right)\times(EC_W^4-EC_{c_W}^4)\frac{g^2}{4}W^{\mu\nu}W_{\mu\nu}, \tag{70}$$

where

$$EC_i^4=\left[\frac{1}{3}d_i(j)-4c_i(j)\right]C_i(G),\quad i=W,c_W, \tag{71}$$

where the $C_i(G)$ is the quadratic Casmir operator of the adjoint representation of the group, the $d(j)$ is the number of spin components [23], and

[2]Even though the term $\operatorname{tr}X\equiv\operatorname{tr}\Delta^{-1}_{W^nW^n}\Delta^\dagger_{WW^n}\Delta^{-1}_{WW}\Delta_{WW^n}$ can provide contributions of order M_c^2 and $\ln M_c$, these contributions are proportional to $1/M_c^2(g_n/g)^2$ and $\ln M_c^2/M_c^4(g_n/g)^2$. Under the assumption of brane fluctuation suppression, we will omit them in the analysis below.

$$d(j)=1, \quad \text{for scalar (ghost)},$$
$$=4, \quad \text{for vector boson}, \tag{72}$$

while $c(j)$ is the trace over spin indices and is defined as

$$\mathrm{tr}[\mathcal{J}^{\rho\sigma}\mathcal{J}^{\alpha\beta}]=(g^{\rho\alpha}g^{\sigma\beta}-g^{\rho\beta}g^{\sigma\alpha})c(j), \tag{73}$$

and $c(j)$ has values as given below,

$$c(j)=0 \quad \text{for scalar (ghost)},$$
$$=2 \quad \text{for vector boson}. \tag{74}$$

The hidden operators can be eliminated by redefining the wave function and gauge coupling of the zero mode. So we see that up to $O(p^4)$, the KK excitations completely decouple from the low-energy observables. The underlying reasons for this decoupling behavior of KK excitations can be traced back to the momentum conservation of the fifth dimension and the gauge structure of the Lagrangian given in Eq. (35).

However, up to $O(p^6)$, the contribution of KK excitations is nonzero, and we have

$$L_{\mathrm{eff}}^{\text{1-loop}}(p^6)=-\frac{1}{2(4\pi)^{d/2}}\sum_{n=1}^{\infty}(nM_c)^{d-6}\Gamma\left(3-\frac{d}{2}\right)\times[(EC_W^6-EC_{c_W}^6)O_1^6+(FC_W^6-FC_{c_W}^6)O_2^6]=c_1^6O_1^6+c_2^6O_2^6, \tag{75}$$

where

$$O_1^6=g^2(D_\mu W^{\mu\nu})^a(D^\alpha W_{\alpha\nu})^a, \tag{76}$$

$$O_2^6=g^3W^{a\mu\nu}W_\mu^{b\ \alpha}W_{\nu\alpha}^c f^{abc}, \tag{77}$$

$$EC_i^6=\frac{1}{30}[-d_i(j)+10c_i(j)]C_i(G), \quad i=W,c_W, \tag{78}$$

$$FC_i^6=\frac{1}{180}\{2d_i(j)-15[c_i'(j)+2c_i(j)]\}C_i(G), \quad i=W,c_W, \tag{79}$$

with

$$c'(j)=0 \quad \text{for scalar (ghost)}, \tag{80}$$

$$=8 \quad \text{for vector boson}, \tag{81}$$

which is defined from

$$\mathrm{tr}(\mathcal{J}^{\mu\nu}\mathcal{J}^{\rho\sigma}\mathcal{J}^{\alpha\beta})W_{\mu\nu}^aW_{\rho\sigma}^bW_{\alpha\beta}^c f^{abc}=-ic'(j)W^{a\mu\nu}W_\mu^{b\rho}W_{\nu\rho}^c f^{abc}. \tag{82}$$

To get Eqs. (70) and (75), we have used the partial integration, the Bianchi identity of which reads

$$D_\mu W_{\nu\rho}+D_\nu W_{\rho\mu}+D_\rho W_{\mu\nu}=0, \tag{83}$$

and the relations of adjoint representations,

$$\mathrm{tr}[t_G^a t_G^b]=C_2(G)\delta^{ab}, \quad \mathrm{tr}[t_G^a t_G^b t_G^c]=i\frac{C_2(G)}{2}f^{abc},$$
$$[D_\mu,D_\nu]=D_\mu^{ae}D_\nu^{eb}-D_\nu^{ae}D_\mu^{eb}=-igW_{\mu\nu}^c(t_G^c)_{ab}. \tag{84}$$

It is remarkable that the contribution of a vector boson is much larger than that of a scalar (ghost), since a vector boson has four components and has a spin coupling with the background field, the contribution of which is represented by $c(j)$ and $c'(j)$.

V. DISCUSSIONS AND CONCLUSIONS

We know that the low-energy oblique parameters U, S, and T [28] always put a very stringent constraint on the possible new physics [29]. According to the standard electroweak chiral Lagrangian up to $O(p^4)$ [30], U, S, and T are related with the coefficients of operators up to $O(p^4)$, while the result given in Eq. (70) tells us that at the $O(p^4)$ order, these low-energy precision tests will not put any constraint on the brane fluctuation and deconstructing–extra-dimension models.

The operators O_1^6 and O_2^6 belong to the contact operators in the complete set of operators of order $O(p^6)$ [14]. The EOM of the zero mode given in Eq. (36) can change the operator O_1^6 to the following form:

$$O_1^6=g^2[m_W^2W_\mu+J_\mu(\text{light})][m_W^2W^\mu+J^\mu(\text{light})]. \tag{85}$$

From Eq. (85) we know that KK excitations can contribute to the low-energy fermion scattering processes.

The operator O_2^6 will contribute to the anomalous trilinear vector couplings (say WWZ and $WW\gamma$), the anomalous quartic photonic vector couplings ($WW\gamma\gamma$ and $WWZ\gamma$), and higher-order gauge couplings.

In 5D, the operators O_i^6 will contribute convergently even when the KK excitations are infinite, since the sum

$$\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6} \tag{86}$$

is finite. But in higher dimension, i.e., $(4+\delta)$D and $\delta\geqslant 2$, the sum is given by

$$\text{sum KK}\equiv\frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{\vec{n}^2}\approx\frac{\pi^{\delta/2}}{\Gamma\left(1+\frac{\delta}{2}\right)}\int_{n=1}^{N_{UV}}n^{\delta-3}dn,$$
$$\approx\frac{\pi}{2}\ln N_{UV} \quad \text{for} \quad \delta=2,$$
$$\approx\frac{\pi^{\delta/2}}{2\Gamma\left(1+\frac{\delta}{2}\right)}\frac{1}{\delta-2}(N_{\mathrm{UV}}^{\delta-2}-1) \quad \text{for} \quad \delta\geqslant 3. \tag{87}$$

The operator O_i^6 will contribute divergently and the meaningful theoretical prediction can only be made when the explicit ultraviolet cutoff $M_{\rm UV}$ is chosen (the relation between $N_{\rm UV}$ and $M_{\rm UV}$ is given as $N_{\rm UV}=M_{\rm UV}/M_c$). This fact reflects that the brane fluctuation suppression mechanism can work well at tree level. But at loop level and in the bosonic part, a more radical mechanism is needed in order to regularize the divergences caused by the infinite KK towers.

For the general $(4+\delta)$D extra dimensions model with brane fluctuation, the coefficients of the operators O_1^6 and O_2^6 will depend upon the number of extra dimensions δ, the size of the compactification scale M_c, and the explicit ultraviolet cutoff $M_{\rm UV}$ of the effective theory.

The magnitude of c_i^6 is determined by the loop factor $1/(16\pi^2)$, the M_c^2, the symmetric factor 6, the EC_i^6, and the sum over KK excitations. The loop factor is about 10^{-2}, the M_c^2 is assumed to be in the range of 0.5–1 TeV and can provide a factor about 10^{-5}–10^{-6} GeV^{-2}, and the EC_W^6 are about 3. If we take $M_c=500$ GeV, $M_{\rm UV}=10$ TeV, and $\delta=2$, the $c_i^6\approx10^{-6}$–10^{-7} GeV^2; if we take $M_c=500$ GeV, $M_{UV}=10$ TeV, and $\delta=4$, the c_2^6 can reach 10^{-3}–10^{-4} GeV^{-2}.

The present experimental accuracy on the anomalous triple vector coupling λ_V [31] is of order -6.2×10^{-2} to 1.47×10^{-1} [32]. The relation between λ_V and c_2^6 is given as

$$g^2c_2^6=\frac{\lambda_V}{M_W^2}, \tag{88}$$

and the λ_V can be expressed as

$$\lambda_V=6\times\alpha_W\left(\frac{M_W}{M_c}\right)^2(FC_W^6-FC_{c_W}^6)\text{sumKK},$$
$$\approx1.0\times10^{-3}\left(\frac{\Lambda}{M_c}\right)^2\text{sumKK}, \tag{89}$$

where $\Lambda=1$ TeV. If we take $M_c=0.5$ TeV and $\delta=1$, the value of λ_V is 1.0×10^{-3}. The typical value of λ_V is of order 10^{-3}–10^{-4}, which is within the reach of a 500 GeV linear collider (LC) [33] and the CERN Large Hadron Collider (LHC).

About the anomalous quartic coupling, according to the analysis of [15,34], operator O_2^6 can in principle be detected via the process $e^+e^-\rightarrow W^+W^-\gamma$. The present experimental accuracy from LEP2 is order of 10^{-2} GeV^{-2} and will increase to 10^{-5} GeV^{-2} at the LC and LHC.

We would like to mention that if the extra dimension(s) are compactified on a T^δ torus, the contributions of KK excitations will double. This is because there is not only the contribution of cosine modes, but also sine modes for each field in the bulk.

In conclusion, we study the bosonic part in the brane fluctuation model where the couplings of the fermionic and bosonic currents on the brane and KK excitations are exponentially suppressed. Since the couplings among vector bosons do not suffer this suppression substantially, they could help us to probe extra dimensions in the future at the LC and LHC. But due to the momentum conservation and the gauge structure of zero mode and KK excitations, up to $O(p^4)$, KK excitations decouple from the low-energy physics. However, up to $O(p^6)$, it is still possible to detect the effects of KK excitations through precision measurement of the bosonic sector of the SM in the LHC and LC.

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