Thermodynamics of clan production

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Scenarios for particle production in the GeV and TeV regions are reviewed. The expected increase with the c.m. energy of the average number of clans for the soft component and the decrease for the semihard one indicate possible classical and quantum behavior of gluons, respectively. Clan thermodynamics, discussed in the paper, appears as the natural framework to deal with such phenomena.

I. INTRODUCTION

The phenomenological analysis of multiparticle production in hadron-hadron collisions in the GeV region [1,2] revealed interesting substructures, i.e., soft and semihard events, without and with minijets, respectively, each class of events being described by a negative binomial (NB) (Pascal) multiplicity distribution (MD) with different values of the parameters, the average charged multiplicity $\bar{n}$, and $k = \bar{n}^2/(D^2 - \bar{n})$. Here D is the dispersion. The weighted composition of the two MD's leads to the observed final charged particle MD. A clan structure analysis in terms of the average number of clans, $\bar{N}$, and the average number of particles per clan, $\bar{n}_c$, allows us to interpret nicely the onset of the above-mentioned substructures in the GeV region.

By extrapolating $\bar{n}$ and k behavior to the TeV region, three possible scenarios [1,2] for the semihard component have been investigated. The first one assumes that Koba-Nielsen-Olesen (KNO) scaling is satisfied both in the soft and semi-hard component. This situation should be compared with what is assumed in the other two scenarios for the semihard component where KNO scaling is strongly violated or has a QCD-inspired behavior through the center-of-mass (c.m.) energy dependence of the corresponding NB MD parameters. Since, at the present stage of QCD, calculations of MD's and correlations in the GeV and TeV regions cannot be performed in a sound way, we can only rely on QCD-inspired extrapolations of the parameters. The last two scenarios for the semihard component (the soft component is taken to be the same in all three scenarios), although more realistic than the first one, lead to a decreasing average number of clans and to the corresponding increase of the average number of particles per clan as the c.m. energy increases. Since clans are independently produced by assumption, it would be important to understand the real meaning of their decrease for c.m. energies in the TeV region, a fact which seems to widen the motivations at the origin of the first introduction of clan concept in high-energy phenomenology [3,4].

In addition, it should be pointed out that clan structure analysis can be generalized to the huge class of discrete infinitely divisible distributions (IDD), to which NB(Pascal)MD belongs, and therefore any result obtained in the framework of clan structure analysis can be easily extended to the full class of IDD.

In the present paper, after a short introduction on the most appealing statistical theories of multiparticle production, a new interpretation of n charged particles multiplicity distribution, $p(n)$, for the class of IDD is proposed in terms of the canonical and grand-canonical partition functions. Then the connection of this new interpretation for the description of the soft and semihard substructures in hh collisions in terms of the NB (Pascal) MD is examined.

II. AN ALTERNATIVE APPROACH TO MULTIPARTICLE PRODUCTION

One of the best known statistical approaches to multiparticle dynamics is Feynman's fluid analogy [5,6], where the cross section for the production of n particles plays the role of the partition function in the canonical ensemble, as it is an integral over phase space of the square of a matrix element which plays the role of the Gibbs distribution, $e^{-H/k_B T}$. In this approach, the volume is identified with the extension of phase space and the fugacity z with the dummy variable u appearing in the definition of the generating function G:

$$G(u) \equiv \sum_n u^n p(n). \tag{1}$$

This identification is unsatisfactory because one has to satisfy at the same time the definitions of the average number of particles, from the grand-canonical ensemble,

$$\langle n \rangle = z \frac{\partial \ln G}{\partial z}, \tag{2}$$

and from the definition of the generating function,

$$\langle n \rangle = \frac{dG}{du}\bigg|_{u=1} = \frac{d \ln G}{du}\bigg|_{u=1}. \tag{3}$$

The above formulas can be satisfied at the same time only in the limit of zero chemical potential.

Another approach was proposed by Scalapino and Sugar [7]: they defined the probability amplitude to produce a particle at rapidity y, denoted by $\Pi(y)$, as a random field variable, then introduced a functional $F[\Pi]$ which played a role analogous to the free energy for a system in thermal equilibrium. One can then obtain the n-particle inclusive distribution by averaging the product of the squares of the amplitudes, $\Pi^2(y_1) \cdots \Pi^2(y_n)$, with a weight given by $e^{-F[\Pi]}$.

Lacking the knowledge necessary to calculate $F[\Pi]$ from the underlying dynamics, the authors parametrized it (following Ginzburg and Landau) in retaining the first three terms in a series expansion, then solved the model in a few particular cases. Remarkably, to leading order in the size of the allowed full rapidity range, they obtain a generating function which has the form of an IDD.

More recent results obtained in the above-mentioned frameworks, concerning KNO scaling and phase transitions, can be found in [8–14].

Stimulated by these results, we propose a new simplified approach to the statistical theory of multiparticle production, heavily based on IDD properties and valid for any chemical potential.

We denote by $Q_n(V,T)$ the partition function in the canonical ensemble for a system with a fixed number n of particles, volume V, and temperature T, and by $\mathcal{Q}(z,V,T)$ the grand-canonical partition function for a system with fugacity z, volume V, and temperature T; the chemical potential μ is defined by $z = \exp(\mu/k_B T)$, where k_B is the Boltzmann constant.

We recall the relation between the partition functions,

$$\mathcal{Q}(z,V,T) = \sum_{n=0}^{\infty} z^n Q_n(V,T). \tag{4}$$

Accordingly, in a statistical mechanics framework, the probability $p(n)$ of finding n particles in the system is the following:

$$p(n) = \frac{z^n Q_n(V,T)}{\mathcal{Q}(z,V,T)} = \frac{1}{n!} \frac{z^n}{\mathcal{Q}} \frac{\partial^n \mathcal{Q}}{\partial z^n}\bigg|_{z=0}. \tag{5}$$

Noticing that $Q_0(V,T) = 1$, we find immediately that the grand-canonical partition function is the inverse of the void probability $p(0)$, i.e., of the probability to find no particles in the system:

$$p(0) = [\mathcal{Q}(z,V,T)]^{-1}. \tag{6}$$

This result is very general: the void probability is the inverse of the grand-canonical partition function and all properties of the system can be obtained from it [15].

Consider now the wide class of power series distributions (PSD), usually defined as follows:

$$p(n) = \frac{a_n b^n}{\gamma(b)}, \tag{7}$$

with a_n and b free parameters, while proper normalization requires that

$$\gamma(b) = \sum_{n=0}^{\infty} a_n b^n. \tag{8}$$

Notice that a_0 can always be chosen to be 1 [by redefining $\gamma(b)$ as $\gamma(b)/a_0$]. Then one has for the void probability,

$$p(0) = \frac{1}{\gamma(b)}. \tag{9}$$

Comparing Eqs. (5)–(7), our new approach is characterized by the following correspondence:

$$z \leftrightarrow b,$$

$$Q_n \leftrightarrow a_n, \tag{10}$$

$$\mathcal{Q} \leftrightarrow \gamma(b) = p(0)^{-1}.$$

A very interesting property of this novel identification is that a_n is the canonical partition function for a system with a fixed number of particles n, and in particular a_1 is the canonical partition function for a system with one particle. This means that if we know the multiplicity distribution of a thermodynamical system, and cast it into a PSD form, we can not only deduce the grand-partition function but also identify the fugacity of the system and the canonical partition function. As an intriguing example of this correspondence, motivated by the phenomenological analysis of multiparticle production in the GeV region, we will in the next section examine the NB(Pascal)MD.

III. THE NEGATIVE BINOMIAL DISTRIBUTION

Any discrete IDD can be written as a compound Poisson distribution (CPD) [16], i.e., the number of clans $\bar{N}$ can be defined in such a way that the void probability is

$$p(0) = \exp(-\bar{N}). \tag{11}$$

Comparing with Eq. (6), we notice that for any IDD, the average number of clans is the logarithm of the grand partition function,

$$\bar{N} = \ln \mathcal{Q}. \tag{12}$$

All thermodynamical properties can then be obtained by differentiating the average number of clans. In particular, being for the grand canonical ensemble $PV = k_B T \ln \mathcal{Q}$, we obtain the following equation of state:

$$PV = \bar{N} k_B T, \tag{13}$$

this tells us that our system behaves as an ideal gas of clans, an interpretation which fits very nicely with the idea that clans are independent objects, as implied by the definition of CPDs.

The NB(Pascal)MD, with the parameters $\bar{n}$ and k,

$$p(n) = \frac{k(k+1)\cdots(k+n-1)}{n!}\left(\frac{\bar{n}}{\bar{n}+k}\right)^n\left(\frac{k}{\bar{n}+k}\right)^k, \tag{14}$$

is an example of a PSD, with the following identification:

$$a_n = \frac{k(k+1)\cdots(k+n-1)}{n!},$$

$$b = \frac{\bar{n}}{\bar{n}+k}, \tag{15}$$

$$\gamma(b) = (1-b)^{-k}.$$

Furthermore, the NBMD also belongs to the class of discrete IDD, the multiplicity distribution inside each clan being of logarithmic type.

We obtain, therefore, the following value for $\bar{N}$:

$$\bar{N} = -k \ln(1-b), \tag{16}$$

which also gives the grand-canonical partition function, applying Eq. (12):

$$\mathcal{Q} = (1-b)^{-k}. \tag{17}$$

Comparing now with our proposed correspondence, Eq. (10), we find that k is a_1, i.e., the canonical partition function for a system with one particle, and must therefore be a function of V and T: $k = k(V,T)$; we also find that b is the fugacity of the system, i.e., $b = \exp(\mu/k_B T)$, and it is a scaling function of $\bar{n}/k$, see Eq. (15).

We calculate now, using the standard thermodynamical relations, the average number of particles in the system, $\langle n \rangle$, which turns out to be equal to the $\bar{n}$ parameter of the NBMD:

$$\langle n \rangle = k_B T \left(\frac{\partial \bar{N}}{\partial \mu} \right)_{T,V} = -k_B T k \left(\frac{\partial \ln(1-b)}{\partial \mu} \right)_{T,V}$$

$$= k_B T \frac{k}{1-b} \frac{b}{k_B T} = \frac{kb}{1-b} = \bar{n}, \tag{18}$$

consistently with the above-mentioned relation $b = \bar{n}/(\bar{n}+k)$. We also obtain that the average number of particles per clan, $\bar{n}_c$, is a function only of the fugacity of the system:

$$\bar{n}_c = \frac{b}{(b-1)\ln(1-b)}. \tag{19}$$

This result is very interesting because formally, as already pointed out, b is a scaling function of $\bar{n}/k$, and experimentally $\bar{n}_c$ is seen to vary with the width of the rapidity interval at fixed c.m. energy: if one were to identify (pseudo)rapidity with volume, one would expect that changing $\Delta\eta$ at fixed c.m. energy would imply changing the volume at constant temperature and fugacity (intensive variables), thus keeping $\bar{n}_c$ constant, contrary to observations. We must conclude that in the present approach we cannot identify rapidity with volume as a simple thought would suggest, but we should allow volume to vary also with other physical quantities.

We now turn our attention to the generating function (GF) for the multiplicity distribution, defined in Eq. (1). In the general case illustrated by Eq. (5), we easily find

$$G(u) = \sum_n \frac{u^n z^n Q_n}{\mathcal{Q}} = \frac{\mathcal{Q}(uz,V,T)}{\mathcal{Q}(z,V,T)} = \frac{p(0)|_z}{p(0)|_{uz}}, \tag{20}$$

which is the ratio of the grand-canonical partition function (or of the void probability) at two different fugacities. This is a very general expression valid for any system in the grand-canonical ensemble.

The GF for a CPD can always be written as

$$G_{\text{CPD}}(u) = \exp[\bar{N} g(u) - \bar{N}], \tag{21}$$

where $g(u)$ is the GF for the multiplicity distribution within each clan [it satisfies $g(0) = 0$].

Because of Eq. (12), we can write for the class of CPDs

$$G(u) = e^{-\bar{N}} \mathcal{Q}(uz,V,T). \tag{22}$$

However, $\ln \mathcal{Q}(uz,V,T) = \bar{N}(uz,V,T)$, hence we obtain

$$G(u) = \exp[\bar{N}(uz) - \bar{N}(z)], \tag{23}$$

which can be interpreted as a function of the *difference* in the average numbers of clans for a system with fugacity uz to that for a system at the actual fugacity z, keeping the same volume and temperature. For the GF within one clan, we find further

$$g(u) = \frac{\bar{N}(uz)}{\bar{N}(z)} = \frac{\Omega(uz,V,T)}{\Omega(z,V,T)} = \frac{P(uz,V,T)}{P(z,V,T)}. \tag{24}$$

Interestingly, this is the *ratio* of the average number of clans for two systems with unequal fugacities.

In addition, it is interesting to remark that, remembering that parameter k depends on V and T, a complete thermodynamics can be built in the just-mentioned framework. Its main quantities are listed in the following and explicitly calculated in the Appendix.

The equation of state is

$$\frac{PV}{k_B T} = k \ln\left(1 + \frac{\langle n \rangle}{k}\right), \tag{25}$$

thus the average number of clans can be expressed in terms of the thermodynamic potential Ω:

$$\bar{N} = -\Omega/k_B T. \tag{26}$$

The Helmholtz free energy can be rewritten in a form symmetric in $\bar{n}$ and k:

$$-\frac{A}{k_B T} = \langle n \rangle \ln\left(1 + \frac{k}{\langle n \rangle}\right) + k \ln\left(1 + \frac{\langle n \rangle}{k}\right). \tag{27}$$

The average internal energy is

$$\frac{U}{k_B T} = \bar{N}\left(\frac{\partial \ln k}{\partial \ln T}\right)_V. \tag{28}$$

The entropy is

$$S = k_B\left\{-\frac{A}{k_B T} + T\left(\frac{\partial k}{\partial T}\right)_V \ln\left(1 + \frac{\langle n \rangle}{k}\right)\right\}. \tag{29}$$

which coincides with $-A/T$ in the limit of $(\partial k/\partial T)_V \rightarrow 0$, which gives also $U \rightarrow 0$.

In the next section, we will focus our attention on clan thermodynamics of final charged particle MD.

IV. CLAN THERMODYNAMICS AND THE NB(PASCAL)MD

In this section, an attempt is made in order to interpret in the present approach a surprising finding in some of the possible scenarios for hadron-hadron collisions in the TeV region discussed in Refs. [1,2], i.e., the unexpected decrease with c.m. energy of the average number of clans for semihard events in scenarios 2 and 3.

We are guided by two considerations: (i) The occurrence of the NB(Pascal)MD—as is the case in the scenarios mentioned above both for semihard and soft events—is usually interpreted as the result of a two-step process: to the independent production of clans during the first step, it follows their decay according to a logarithmic distribution, which can be obtained by a weighted average of geometric (Bose-Einstein) distributions during the second step. (ii) The validity of the generalized local parton hadron duality (GLPHD).

It should be pointed out that clan ancestors are independently produced and Poissonianly distributed, by assumption, and a clan is, by definition, a group of partons of common ancestor; a clan consists of at least one parton, its ancestor. Each ancestor can be considered as an independent intermediate gluon source. All correlations among generated partons are exhausted within each clan.

Clan ancestors can be produced either very early in the production process at higher virtualities or later at lower virtualities.

In the first case, the ancestor's "temperature" (an unknown function in this approach of the average p_T and the rapidity) is expected to be higher: this expectation, together with the lack of mutual correlations among ancestors, emphasizes their overall quasiclassical behavior: ancestor production in this case is competitive with the increase of gluon population within each clan. This situation is qualitatively closer to that expected at the hadron level for soft events and semihard events in scenario 1.

In the second case, ancestors are produced later, at lower virtualities: their "temperature" should also be lower, with even "colder" generated gluons. Their virtuality is lower. Accordingly, quantum effects should be expected to be enhanced in events sharing these properties: new produced gluons prefer to stay together with other relatives within each clan rather than to become ancestors and initiate a new clan. The k parameter is in this case lower and closer to that of a Bose-Einstein distribution, which occurs for $k=1$. These remarks are consistent with the interpretation of $1/k$ (see [4]) as a measure of aggregation of partons into clans: it corresponds to the ratio of the probability to have two gluons (particles at the hadron level) in the same clan over the probability to have two gluons in two separate clans, i.e., the smaller k parameter corresponds to a higher aggregation among produced gluons into clans. In addition, being $1/k$ linked to the integral of two parton rapidity correlations via

second-order factorial cumulants, the decrease of k implies stronger two-parton correlations. In conclusion, generated gluons prefer to stay together rather than to stay far apart, higher parton density regions are generated, the probability to create a new gluon is enhanced (a typical quantum effect), clans become more populated, and their average number is reduced. This situation is closer qualitatively to that expected at the hadron level for semihard events in scenarios 2 and 3 of Refs. [1,2].

The just-mentioned considerations and Eqs. (16), (17), and (19) of Sec. III fully outline in the present approach the importance of the behavior of the fugacity variable b.

The remaining question is how clan thermodynamics results at the parton level can be extended to final particles through the hadronization mechanism. A possible answer to this question comes from GLPHD [17], which says that all inclusive distributions are proportional at the two levels of investigation:

$$Q_{n,\text{hadrons}}(y_1,\ldots,y_n) = \rho^n Q_{n,\text{partons}}(y_1,\ldots,y_n), \quad (30)$$

which corresponds for NBMD parameters to

$$k_{\text{hadron}} = k_{\text{parton}}, \qquad \bar{n}_{\text{hadron}} = \rho \bar{n}_{\text{parton}}. \quad (31)$$

GLPHD can be applied separately to soft and semihard components thus solving our problem. In particular, in this framework minijets production is related to the existence of regions of high gluon densities and final particle production should be sensitive to the mentioned quantum effects, by increasing two-particle correlations and BE effects.

Motivated by these considerations, the behavior of parameter b as a function of c.m. energy, as well as the b dependence of $\bar{N}$, and $\bar{n}_c$, have been explored as a suggestive example in the different above-mentioned scenarios phenomenologically described in terms of NBMD's. In addition, in view of their simple connections with $\bar{N}$ and $\bar{n}_c$, the probability of having no particles in the event, $p(0)$, and the void scaling function, $\mathcal{V}(\bar{n}/k)$, have been studied as a function of fugacity b. It turns out that the analysis in terms of $p(0)$ and $\mathcal{V}(\bar{n}/k)$ variables confirms the main result of the new approach, i.e., that the reduction of the average number of clans with the increase of c.m. energy is a quantum effect.

We proceed now to discuss the thermodynamical behavior of multiparticle production according to the scenarios described in Sec. I and fully characterized by the c.m. energy dependence shown in Fig. 1. We start with the soft component thermodynamical behavior since it is assumed to be the same in the all above-mentioned scenarios (see Fig. 2).

We observe that fugacity b is growing very fast from 0.25 to 0.75 with c.m. energies below 100 GeV and then smoothly varying from 0.75 to 0.9 at 14 TeV. The explanation of this behavior will be given in terms of the following three known possible interpretations of the occurrence on NB(Pascal)MD in high-energy physics phenomenology, which in terms of parameters $\bar{n}$ and k are:
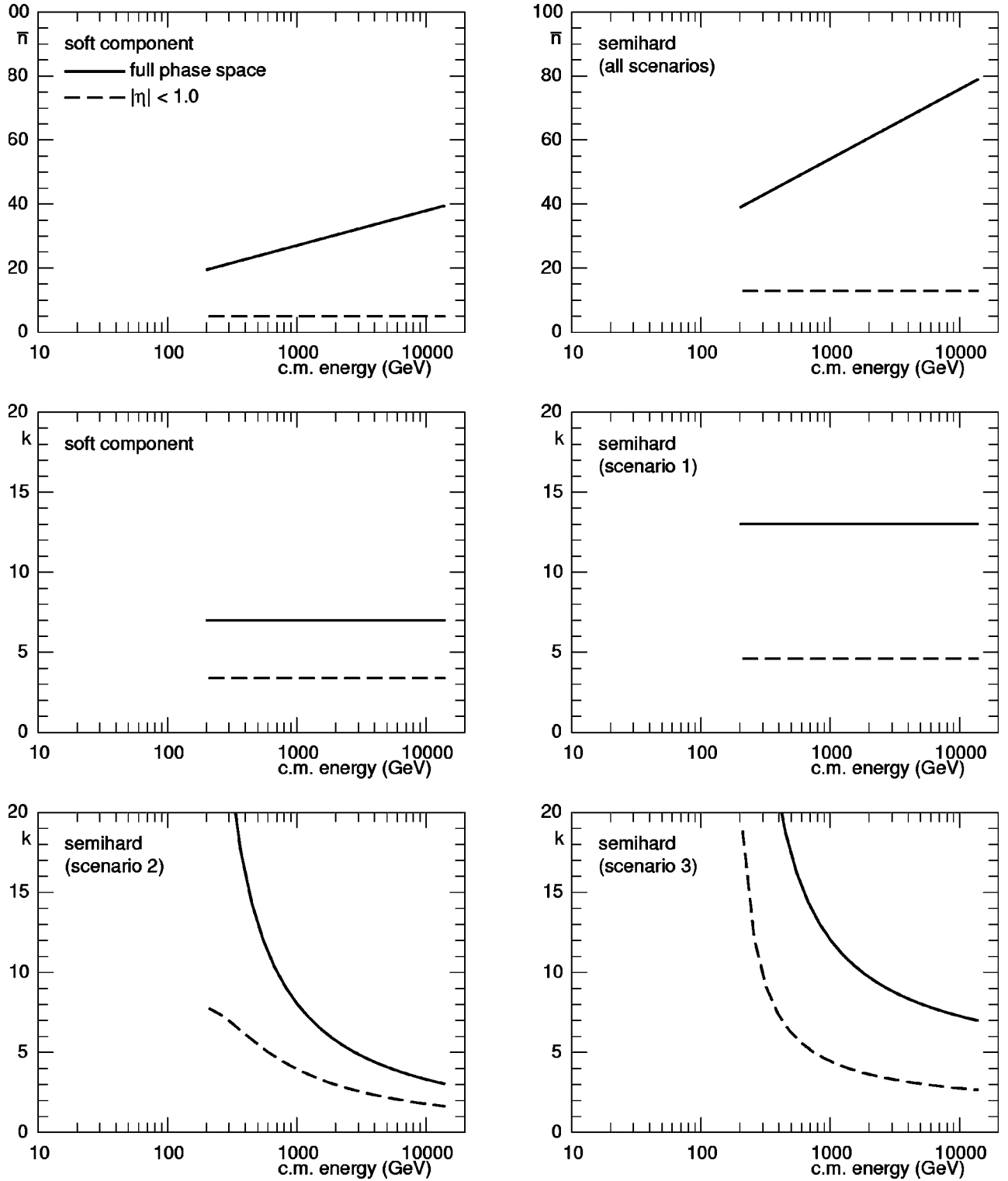
FIG. 1. Center-of-mass energy dependence of standard NBMD parameters $\bar{n}$ and k in the three scenarios described in the Sec. I. The top two panes show the behavior of $\bar{n}$ (it is the same in all scenarios for both the semihard and the soft component). The lower four panes show the k parameter for the soft component (equal in all scenarios) and the semihard one in the three scenarios.

$$(a) \quad \frac{(n+1)p(n+1)}{p(n)} = \alpha + \beta n, \quad (32a)$$

where $\alpha = k\bar{n}/(\bar{n}+k)$ and $\beta = \bar{n}/(\bar{n}+k)$; notice that for α

$= \beta$, i.e., for $k=1$, the MD $p(n)$ becomes a Bose-Einstein distribution, for $\beta = 0$, i.e., $k \to \infty$, a Poissonian distribution (the Poissonian limit), and for $\alpha = 0$, a logarithmic distribution, which can be expressed as the superposition of Bose-Einstein distributions;
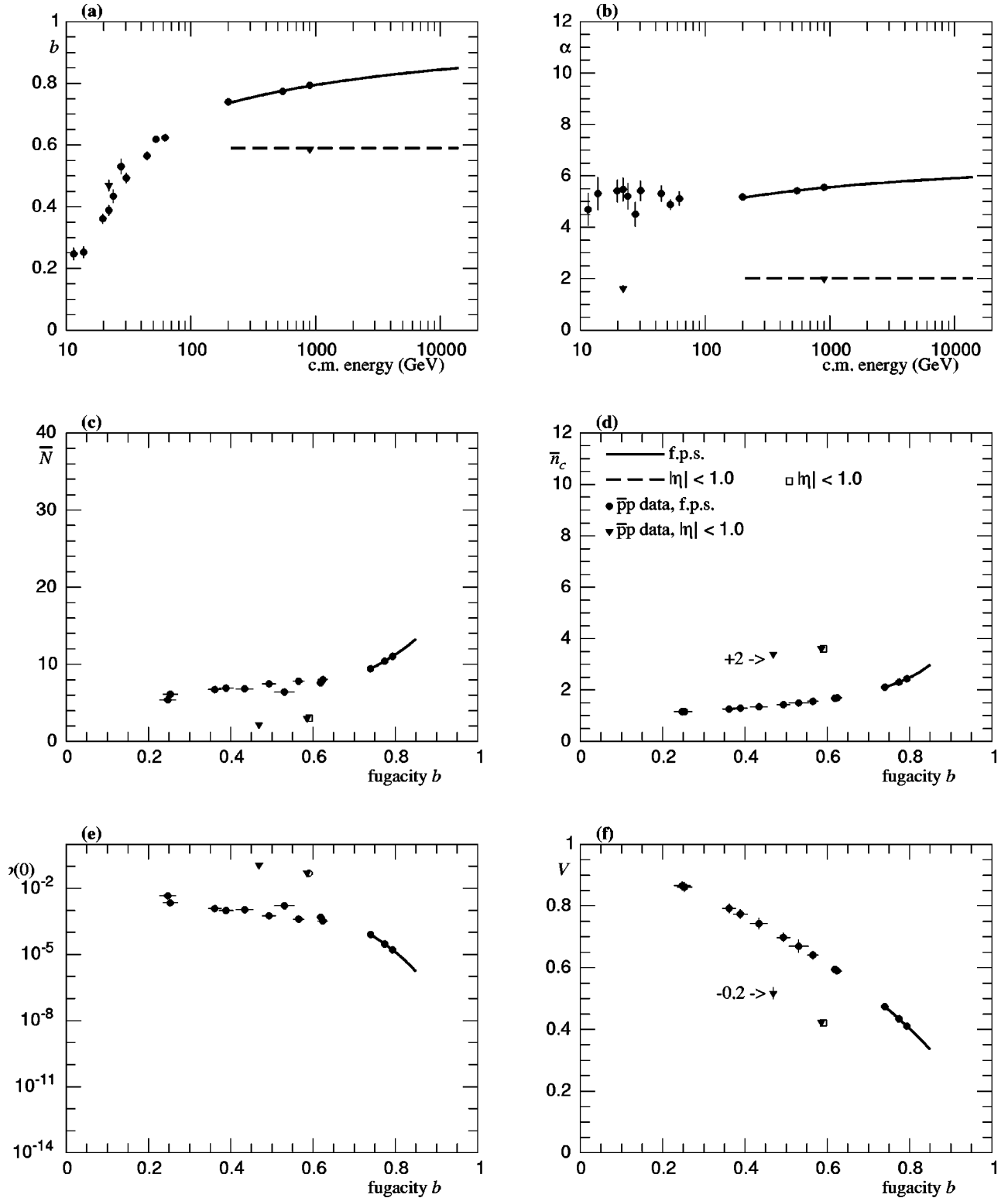
FIG. 2. Results for the soft component (equal in all scenarios). In this figure, the lines and the open points show the results from our extrapolations: the solid line refers to full phase space, the dashed lines and open square to the interval $|\eta|<1$. The last point on each line corresponds to a c.m. energy of 14 TeV. The solid circles show full phase-space data from ISR and UA5, the solid triangles refer to UA5 data in the interval $|\eta|<1$. (a) Fugacity b as a function of c.m. energy; (b) α parameter as a function of c.m. energy; (c) average number of clans vs fugacity; notice that this is also a plot of the grand partition function in logarithmic scale, since $\bar{N}=\log\mathcal{Q}$, Eq. (12); (d) average number of particles per clans vs fugacity; here for clarity the curves and the data for $|\eta|<1$ are shifted up by two units; (e) void probability vs fugacity; (f) void scaling function $\mathcal{V}$ vs fugacity; also here curves and the data for $|\eta|<1$ are shifted down by 0.2 units.
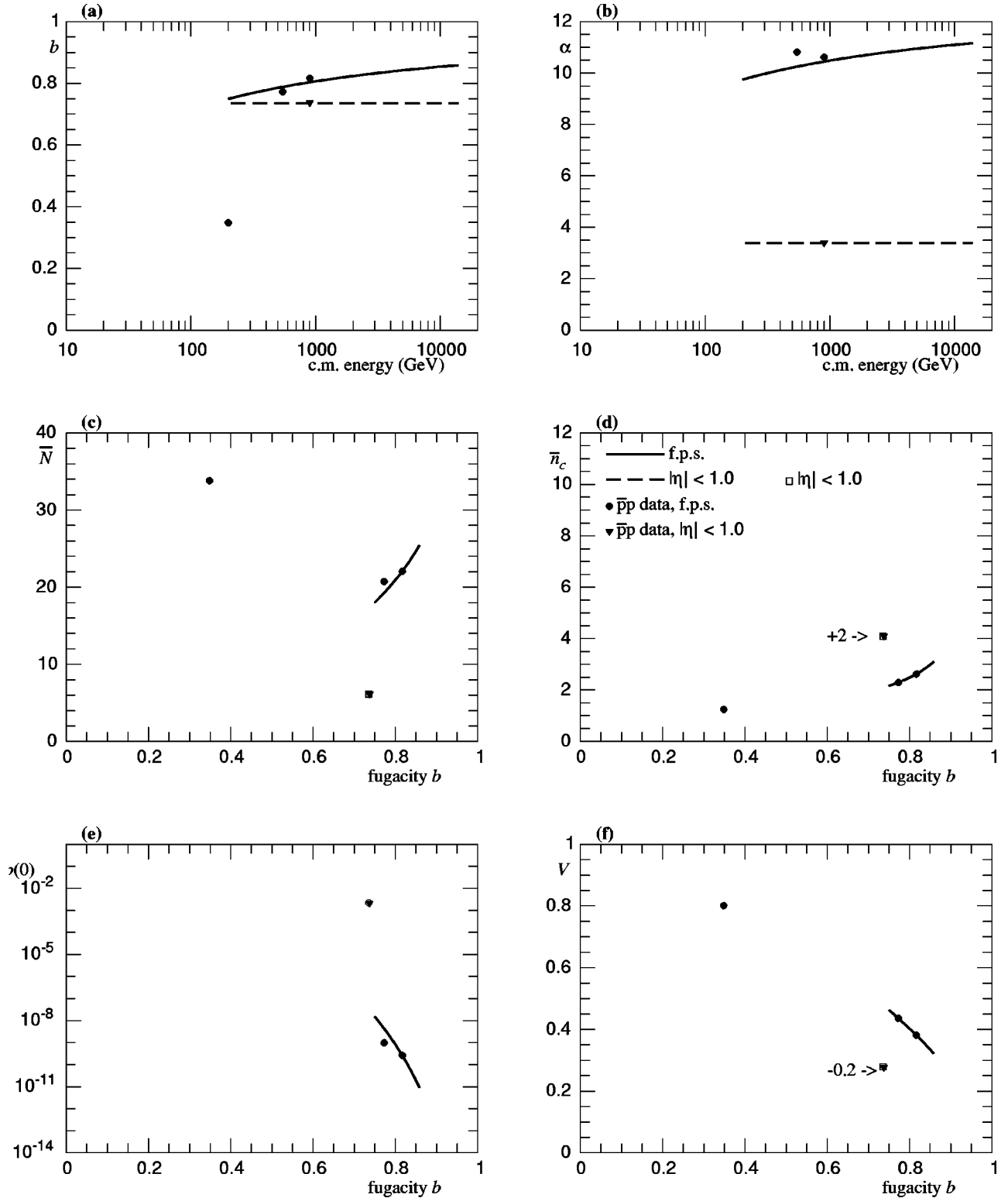
FIG. 3. Same as Fig. 2, but the semihard component in scenario 1 is shown.

$$(b) \qquad \bar{N} = k \ln\left(1 + \frac{\bar{n}}{k}\right), \qquad \bar{n}_c = \frac{\bar{n}}{k \ln(1 + \bar{n}/k)},$$

$$(32b)$$

where $\bar{N}$ is the average number of clans and $\bar{n}_c$ is the average number of particles per clan;

$$(c) \qquad p(0) = \left(\frac{k}{\bar{n} + k}\right)^k, \qquad \mathcal{V} = \frac{k}{\bar{n}} \ln\left(1 + \frac{\bar{n}}{k}\right), \quad (32c)$$

where $p(0)$ is the probability of generating zero charged particles and $\mathcal{V}(\bar{n}/k)$ is the void scaling function; the occur-
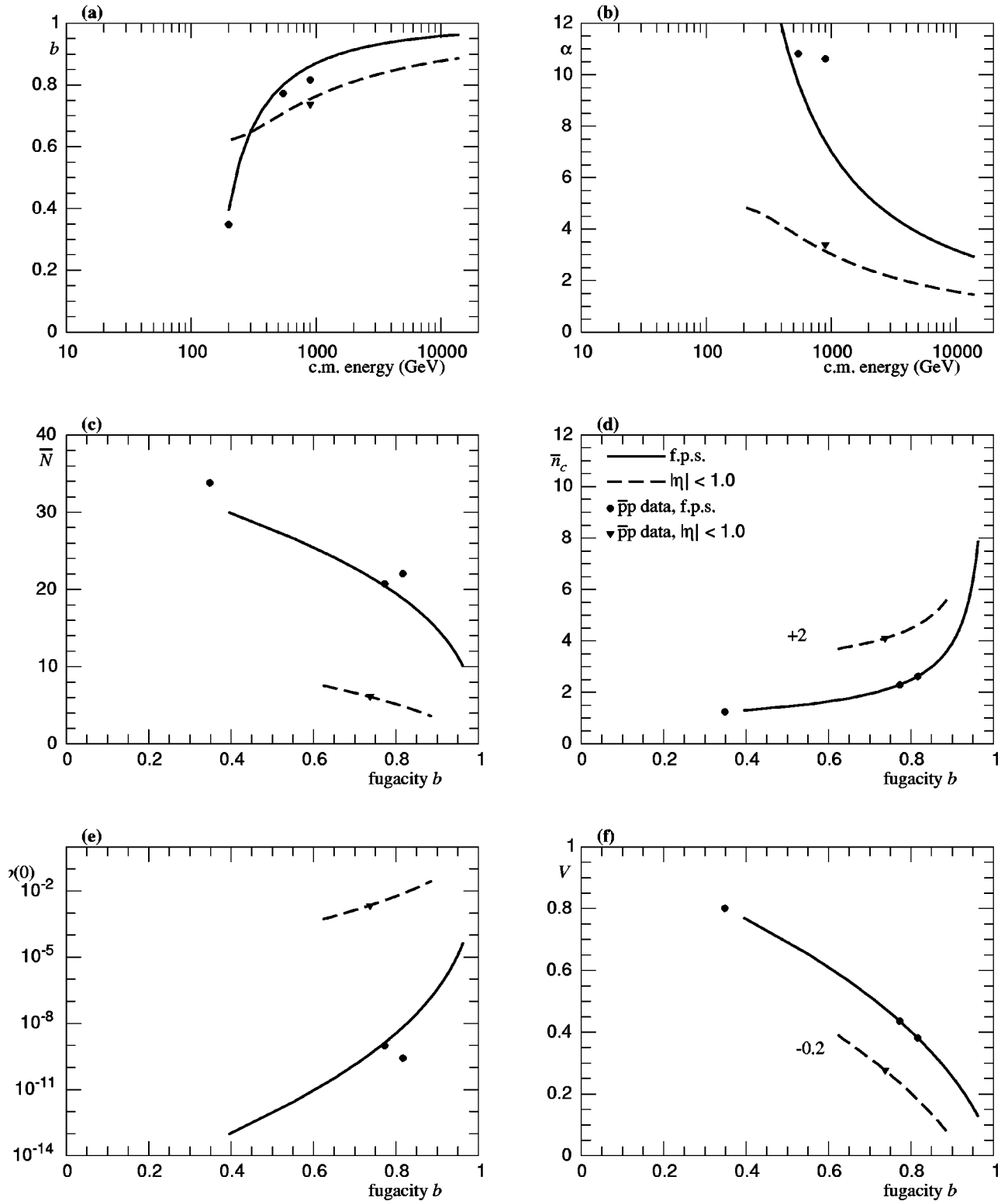
FIG. 4. Same as Fig. 2, but the semihard component in scenario 2 is shown.

rence of scaling in the product of the first two moments $\bar{n}$ and $1/k$ indicates two-particle correlation dominance for hierarchical systems, and the distance from point one [Poisson limit $\mathcal{V}(0)=1$] on the scaling function is larger, and the particles are more numerous and correlated.

It should be pointed out that the β parameter in Eq. (32a) coincides with the fugacity b discussed above in our thermo-dynamical approach and therefore b, like $\mathcal{V}(\bar{n}/k)$, is a scaling
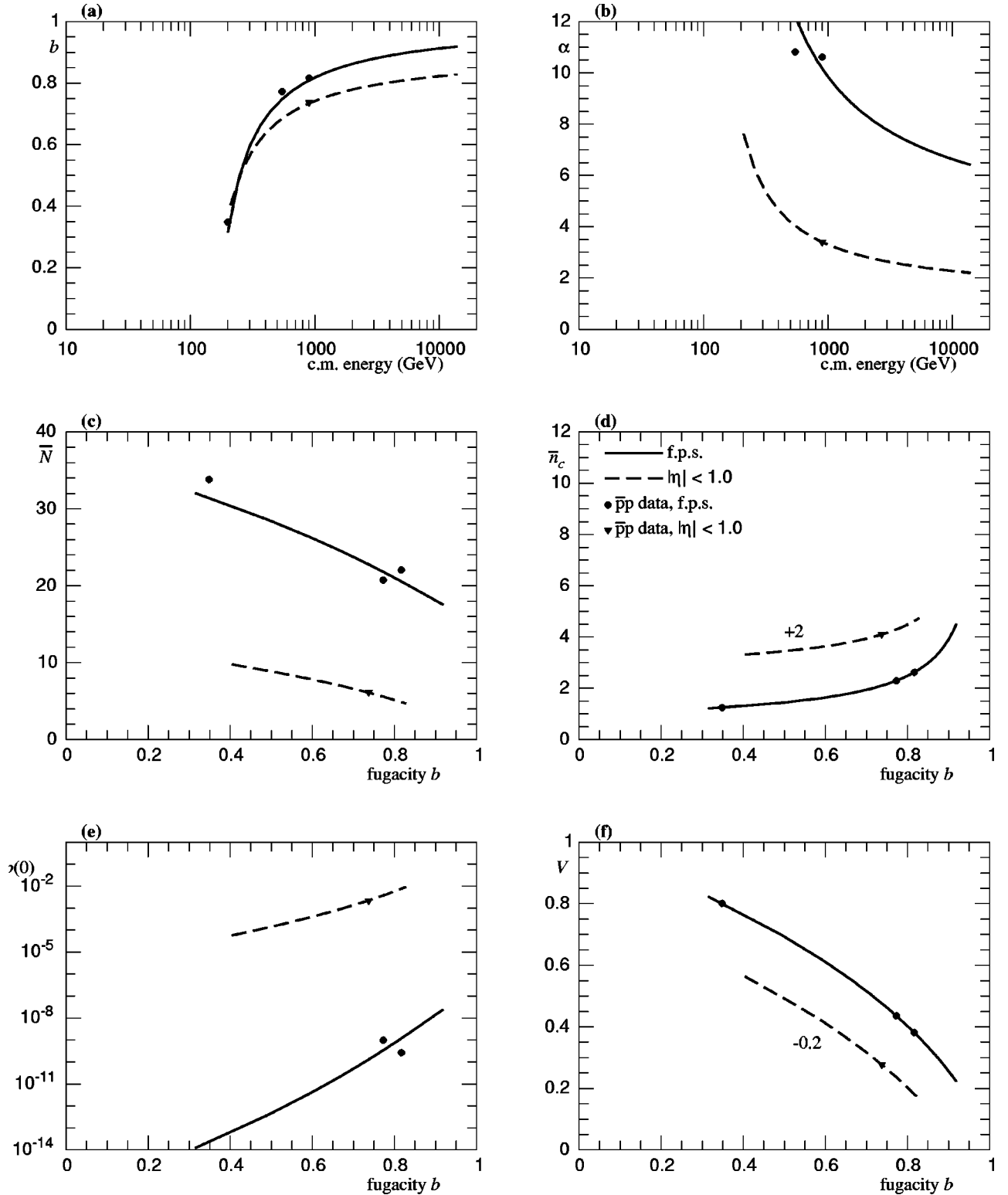
FIG. 5. Same as Fig. 2, but the semihard component in scenario 3 is shown.

function of $\bar{n}/k$, and the α parameter corresponds to the average charged multiplicity, $\bar{n}$, for a classical system ($k \rightarrow \infty$).

In this sense, the relative behavior of $\beta = b$ and $\alpha = kb$ as the c.m. energy increases in view of Eq. (32a) and the discussion at the beginning of this section can be considered an indication of the relative importance of a behavior closer to a quantum one, i.e., harder, with respect to a behavior closer to a quasiclassical, i.e., softer, for a class of events. A very slow increase of b with c.m. energy and an almost constant behavior of $\alpha = kb$ is the main characteristic of the class of soft events as shown in Figs. 2(a) and 2(b).

This fact is confirmed by inspection of Figs. 2(c) and 2(d), where it is shown that $\bar{N}$ is a very slow growing function of the fugacity of the system throughout the ISR region and below that region (≈ 7), and then a quickly growing function of the same variable in the GeV region up to 14 in the TeV region (14 TeV); $\bar{n}_c$ as a function of the fugacity has a similar behavior, from ≈ 1.5 to ≈ 3.

Accordingly, the probability of creating zero charged particles, $p(0)$, is decreasing throughout the same regions from 10^{-2} to 10^{-5} (for $b=0.9$ at 14 TeV c.m. energy); in addition [Fig. 2(f)], the void scaling function $\mathcal{V}(\bar{n}/k)$ turns out to populate for larger values of fugacity variable sections of the curve far from the Poissonian limit [$\mathcal{V}(0)=1$], showing a clear increase of two-particle correlations in this region as expected for a hierarchical system.

It should be noticed that in the soft scenario (no minijets), constant k parameter behavior in rapidity intervals as requested by KNO scaling in the GeV and TeV regions implies that also the other variables remain constant in the same regions.

In scenario 1, as already pointed out, the semihard component is assumed to have a very similar behavior to the soft one: KNO scaling is satisfied and minor changes in the general trend of the variables both in full phase space and in (pseudo)rapidity intervals are straightforward consequences of the smaller constant k parameter value suggested by NB fits for the semihard component in the GeV and extrapolated to the TeV region (Fig. 3).

Coming to the second scenario, the assumption of strong KNO scaling violation for the semihard component (an extreme point of view with respect to that of scenario 1) implies a completely new panorama with dramatic changes. Fugacity b [Fig. 4(a)] is growing very fast from 0.4 at 200 GeV c.m. energy up to 0.96 at 14 TeV almost saturating the maximum allowed value, which is 1, and the α parameter [Fig. 4(b)] is decreasing very rapidly from ≈ 16 at 200 GeV to ≈ 3 at 14 TeV.

The combined information contained in the two figures leads to the same conclusion, i.e., the proposed semihard scenario behavior is much closer to a quantum one than the soft scenario favoring the production of regions of higher particle density. This interpretation is confirmed by studying general trends of the other variables as a function of fugacity b. The average number of clans $\bar{N}$ is decreasing in full phase space from ≈ 30 ($b\approx 0.35$), to ≈ 10 at 14 TeV ($b\approx 0.96$) and the average number per clan, $\bar{n}_c$, is increasing from ≈ 1 to ≈ 8 in the same interval. Accordingly, the probability of zero particle production is increasing from $\approx 10^{-13}$ to $\approx 10^{-4}$, i.e., gap probability is increasing with fugacity and c.m. energy; in parallel, the void scaling function $\mathcal{V}(\bar{n}/k)$ is populating sections with much higher b values than in scenario 1 corresponding to regions much farther from the Poissonian limit. One interesting point concerns smaller (pseudo)rapidity intervals (say $|\eta|<1$): the general trend is that $\bar{N}$ is lower than in full phase space as are corresponding $\bar{n}_c$ values, thus suggesting the onset of regions with higher particle densities and lower temperatures. The probability of generating zero

particles, in view of the higher densities, is therefore much higher and the void probability is far from the Poisson limit.

Scenario 3 is a QCD-inspired scenario for the semihard component: it assumes for parameter k a QCD behavior (see Fig. 5). This scenario gives a panorama for our variables which is intermediate between the two extremes, 1 and 2. Fugacity b is increasing very fast with c.m. energy as in scenario 2, but the α parameter has a sweeter trend (it is ≈ 6 at 14 TeV) indicating stronger independent production; this fact is clearly shown in Figs. 5(c) and 5(d), where $\bar{N}$ is larger and $\bar{n}_c$ is smaller than in scenario 2. Differences in $p(0)$ and $\mathcal{V}(\bar{n}/k)$ behaviors in full phase space as well as in (pseudo) rapidity intervals with respect to scenario 2 are all consequences of the just-mentioned remarks.

V. CONCLUSIONS

Clan thermodynamics has been investigated in order to explain the decrease with c.m. energy of $\bar{N}$ for the semihard component and its increase for the soft one in the most realistic scenarios of multiparticle production in the GeV and TeV regions in hh collisions. It turns out that these two behaviors for clans point out structures closer to classical or quantum properties of gluons. A thermodynamical approach to multiparticle production was constructed on this basis. Results were determined in the framework of NB(Pascal)MD applied separately in the two components and can be extended to any infinitely divisible distribution.

APPENDIX

The main quantities of clan thermodynamics are explicitly calculated in the following.

The Helmholtz free energy is

$$A = \langle n \rangle \mu - PV = \frac{kb}{1-b}\mu - k_B T \bar{N}. \quad (A1)$$

The average internal energy is

$$U = k_B T^2 \left(\frac{\partial \bar{N}}{\partial T} \right)_{b,V}$$

$$= -k_B T^2 \left(\frac{\partial k}{\partial T} \right)_V \ln(1-b)$$

$$= k_B T^2 \left(\frac{\partial k}{\partial T} \right)_V \ln\left(1 + \frac{\langle n \rangle}{k} \right)$$

$$= k_B T^2 \frac{\bar{N}}{k} \left(\frac{\partial k}{\partial T} \right)_V. \quad (A2)$$

The entropy is

$$S = \frac{U-A}{T} = k_B \bar{N} - \langle n \rangle \frac{\mu}{T} + k_B T \frac{\bar{N}}{k} \left(\frac{\partial k}{\partial T} \right)_V$$

$$= k_B \bar{N} + k_B \langle n \rangle \ln \left(1 + \frac{k}{\langle n \rangle} \right) + k_B T \left(\frac{\partial k}{\partial T} \right)_V \ln \left(1 + \frac{\langle n \rangle}{k} \right)$$

$$= k_B \left\{ \left[k + T \left(\frac{\partial k}{\partial T} \right)_V \right] \ln \left(1 + \frac{\langle n \rangle}{k} \right) + \langle n \rangle \ln \left(1 + \frac{k}{\langle n \rangle} \right) \right\}. \tag{A3}$$

The specific heat at constant volume is

$$C_v = 2 k_B T \left(\frac{\partial k}{\partial T} \right)_V \ln \left(1 + \frac{\langle n \rangle}{k} \right) + k_B T^2 \left(\frac{\partial^2 k}{\partial T^2} \right)_V \ln \left(1 + \frac{\langle n \rangle}{k} \right)$$

$$+ k_B T^2 \left(\frac{\partial k}{\partial T} \right)_V \frac{1}{1 + \langle n \rangle / k} \left(- \frac{\langle n \rangle}{k^2} \right) \left(\frac{\partial k}{\partial T} \right)_V$$

$$= 2 \frac{U}{T} + k_B T^2 \left(\frac{\partial^2 k}{\partial T^2} \right)_V \ln \left(1 + \frac{\langle n \rangle}{k} \right)$$

$$- k_B T^2 \left(\frac{\partial k}{\partial T} \right)_V^2 \frac{\langle n \rangle}{k(k + \langle n \rangle)}. \tag{A4}$$

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