S-D mixing and searching for the $\psi(1^1P_1)$ state at the Beijing Electron-Positron Collider

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The $\psi(1\,^1P_1)$ state can be produced at the Beijing Electron-Positron Collider (BEPC) in the process $\psi' \rightarrow \psi(1\,^1P_1)+\pi^0$. We calculate the rate of this process taking account of the S-D mixing effect in ψ'. It is shown that the rate is about a factor of 3 smaller than the simple result without considering the S-D mixing effect. Possible detecting channels are suggested and it is shown that $\psi(1\,^1P_1)$ is able to be found with the accumulation of 3×10^7 events of ψ' at BEPC.

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Searching for the $^1P_1$ states of heavy quarkonia is of interest since the difference between the $^1P_1$ mass $M_{^1P_1}$ and the center-of-gravity (c.o.g.) of the ^3P_J mass $M_{\rm c.o.g.} = (5M_{^3P_2}+3M_{^3P_1}+M_{^3P_0})/9$ gives useful information about the spin-dependent interactions between the heavy quark and antiquark. Theoretical investigations of the general structure of the spin-dependent interactions and the formula for the spin-dependent potential up to $O(1/m^2)$ in the $1/m$ expansion have been carried out in various approaches [1,2]. Experimentally, neither the $\psi(1\,^1P_1)$ state nor the $\Upsilon(1\,^1P_1)$ state has been found yet.[1] The best way of searching for the $\psi(1\,^1P_1)$ state is to look for the process

$$\psi' \rightarrow \psi(1\,^1P_1)+\pi^0 \tag{1}$$

at an e^+e^- collider in the τ-charm energy range. Many years ago, the Crystal Ball Group searched for this process with a negative result, and the obtained upper limit for the branching ratio is [5]

$$B[\psi' \rightarrow \psi(1\,^1P_1)\pi^0] < 0.46\%, \quad 95\% \ \text{C.L.} \tag{2}$$

for $M_{\psi(1\,^1P_1)} = 3520$ MeV. Now the most promising collider for this kind of experiment is the Beijing Electron-Positron Collider (BEPC).

Theoretically, a simple single-channel calculation of the transition rate (1) based on the QCD multipole expansion [6,7] using the Cornell potential model [8] was given in our previous paper [9]. The obtained results are

$$\Gamma[\psi' \rightarrow \psi(1\,^1P_1)\pi^0] = 0.12\left(\frac{\alpha_M}{\alpha_E}\right) \ \text{keV},$$
$$B[\psi' \rightarrow \psi(1\,^1P_1)\pi^0] = (4.3\pm0.5)\left(\frac{\alpha_M}{\alpha_E}\right)\times10^{-4}, \tag{3}$$

where $\alpha_E = g_E^2/4\pi$ and $\alpha_M = g_M^2/4\pi$ are phenomenological coupling constants for color electric dipole and color magnetic dipole gluon radiations, respectively. The branching ratio here is obtained from the updated total width $\Gamma_{\rm tot}(\psi') = 227\pm31$ keV [10], and it is a little different from the value in Ref. [9]. The uncertainty in the branching ratio comes from the experimental error in $\Gamma_{\rm tot}(\psi')$. Phenomenological determination of the ratio α_M/α_E is not so certain [9]. From the theoretical point of view, the two coupling constants should not be so different, so we roughly take a possible range

$$\frac{\alpha_M}{\alpha_E} \approx 1\text{–}3 \tag{4}$$

in the calculation. We see that the theoretically predicted $B[\psi' \rightarrow \psi(1\,^1P_1)\pi^0]$ is consistent with the Crystal Ball limit (2), and is not much smaller than the limit, thus it is hopeful that we will find the $\psi(1\,^1P_1)$ state in the near future at BEPC.

Since the ψ' mass $M_{\psi'} = 3685.96\pm0.09$ MeV is quite close to the $D\bar{D}$ threshold $E_{\rm th} = 3729.0\pm1.0$ MeV [10], it is expected that the coupled-channel effect may affect the transition rate. Complete coupled-channel calculation of the transition rate is tedious. A coupled-channel formulation of the hadronic transitions between heavy quarkonium states and the transition rates for $\Upsilon' \rightarrow \Upsilon\pi\pi$, $\Upsilon'' \rightarrow \Upsilon\pi\pi$, and $\Upsilon'' \rightarrow \Upsilon'\pi\pi$ have been given in Ref. [11]. Further improved study of the coupled-channel theory of hadronic transitions in the heavy quarkonium systems will be presented in a future paper. We notice that an important aspect of the coupled-channel effect affecting the transition rate is state mixing. From a reasonable coupled-channel model, the unitarized quark model (UQM) [12], we see that ψ' is not a pure $2S$ state. Apart from negligibly small ingredients (an order of magnitude smaller), ψ' mainly contains ψ_{2S} and ψ_{1D} states. We denote this mixing as

$$\psi' = \psi_{2S}\cos\theta + \psi_{1D}\sin\theta,$$
$$\psi'' = -\psi_{2S}\sin\theta + \psi_{1D}\cos\theta. \tag{5}$$

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[1] In 1992, the E760 Collaboration claimed that they saw a significant enhancement in $\bar{p}+p \rightarrow J/\psi + \pi^0$ at $\sqrt{s} = 3526.2$ MeV which was supposed to be a candidate of $\psi(1\,^1P_1)$ [3]. However, such an enhancement has not been confirmed by the successive E835 experiment with significantly higher statistics [4].

The UQM gives $\theta\approx-8.5°$ [12]. It is the purpose of this paper to calculate the influence of such an S-D mixing on the transition rate (1).[2]

Instead of taking a specific model of the coupled-channel effect, we take a simple and phenomenological approach such as was done in studying hadronic transitions of $\psi(3770)$ in Ref. [13] and in studying radiative transitions of heavy quarkonia in Ref. [14], i.e., we determine the mixing angle θ by fitting the experimental value of the ratio of the leptonic decay rates $\Gamma(\psi'\to e^+e^-)/\Gamma(\psi''\to e^+e^-)$. In the simple Cornell potential model, the so-determined θ is [13]

$$\theta=-10°. \tag{6}$$

This is consistent with the mixing angle $\theta\approx-8.5°$ in the UQM mentioned above. In this paper, we take an improved potential model by Chen and Kuang, which reflects more about QCD and leads to more successful phenomenological results [15]. The potential reads

$$V(r)=kr-\frac{16\pi}{25}\frac{1}{rf(r)}\left[1+\frac{2\gamma_E+\frac{53}{75}}{f(r)}-\frac{462}{625}\frac{\ln f(r)}{f(r)}\right], \tag{7}$$

where $k=0.1491\ \mathrm{GeV}^2$ is the string tension related to the Regge slope [15], γ_E is the Euler constant, and $f(r)$ is

$$f(r)=\ln\left[\frac{1}{\Lambda_{\overline{\rm MS}}r}+4.62-\left(1-\frac{1}{4}\frac{\Lambda_{\overline{\rm MS}}}{\Lambda^I_{\overline{\rm MS}}}\right)\frac{1-\exp\left\{-\left[15\left(3\frac{\Lambda^I_{\overline{\rm MS}}}{\Lambda_{\overline{\rm MS}}}-1\right)\Lambda_{\overline{\rm MS}}r\right]^2\right\}}{\Lambda_{\overline{\rm MS}}r}\right]^2, \tag{8}$$

in which $\Lambda^I_{\overline{\rm MS}}=180\ \mathrm{MeV}$. Taking $\Lambda_{\overline{\rm MS}}=200\ \mathrm{MeV}$,[3] we obtain the following wave functions at the origin:

$$\psi_{2S}(0)=0.199\ \mathrm{GeV}^{3/2},\quad \frac{5}{\sqrt{2}}\frac{\psi''_{1D}(0)}{2m_c^2}=0.0262\ \mathrm{GeV}^{3/2}, \tag{9}$$

and then, as was done in Ref. [13], we determine

$$\theta=-12° \tag{10}$$

in this model. This is close to the value (6) in the Cornell model.

In general, the initial-state quarkonium Φ_i and the final-state quarkonium Φ_f in the hadronic transition $\Phi_i\to\Phi_f+h$ [h stands for light hadron(s)] can be described as

$$\Phi_i=\sum_{n_il_i}a^{(i)}_{n_il_i}\phi_{n_il_i},\qquad \Phi_f=\sum_{n_fl_f}a^{(f)}_{n_fl_f}\phi_{n_fl_f}, \tag{11}$$

where $\phi_{n_il_i}$ ($\phi_{n_fl_f}$) is the quarkonium state with principal quantum number n_i (n_f) and orbital angular momentum l_i (l_f), and the mixing coefficients satisfy $\Sigma_{n_il_i}a^{(i)}_{n_il_i}=1$ and $\Sigma_{n_fl_f}a^{(f)}_{n_fl_f}=1$. In the framework of the QCD multipole expansion [6,7], the hadronic transition (1) is mainly contributed by the E_1M_1 transition, and the transition amplitude is

$$\mathcal{M}_{E1M1}=i\frac{g_Eg_M}{6m_Q}\sum_{KL}\left(\frac{\langle\Phi_f|(s_Q-s_{\bar Q})_\alpha|KL\rangle\langle KL|x_\beta|\Phi_i\rangle}{E_i-E_{KL}}+\frac{\langle\Phi_f|x_\beta|KL\rangle\langle KL|(s_Q-s_{\bar Q})_\alpha|\Phi_i\rangle}{E_i-E_{KL}}\right)\langle\pi^0|E^a_\alpha B^a_\beta|0\rangle, \tag{12}$$

where m_Q is the heavy quark mass, s_Q ($s_{\bar Q}$) is the spin of the heavy quark (antiquark), K,L are the principal quantum number and the orbital angular momentum of the intermediate state, and E_i and E_{KL} are the energy eigenvalues of the initial state and the intermediate state, respectively.

For the process (1), the initial-state quarkonium $\Phi_i=\psi'$, i.e., $a^{(i)}_{20}=\cos\theta$, $a^{(i)}_{12}=\sin\theta$, and other $a^{(i)}_{n_il_i}$ vanish. Since the mass of the final-state quarkonium $\Phi_f=\psi(1\,^1P_1)$ is supposed to be close to $M_{\rm c.o.g.}=3525.3\ \mathrm{MeV}$ [10], which is not close to $E_{\rm th}$

[2]From Ref. [12], we see that state-mixings in Υ'' are an order of magnitude smaller, so we expect that the single-channel result of the rate of $\Upsilon''\to\Upsilon(1\,^1P_1)\pi\pi$ given in Ref. [9] will not be much affected by the state-mixing effect.

[3]Our calculation shows that the determined θ is not sensitive to the value of $\Lambda_{\overline{\rm MS}}$. For example, a 50 MeV variation of $\Lambda_{\overline{\rm MS}}$ causes only a $<1\%$ change of θ.

$=3729.0\pm1.0$ MeV, we expect that the state mixing effect in $\psi(1\,^1P_1)$ is small. So that we take $a_{11}^{(f)}=1$ and other $a_{n_f l_f}^{(f)}$ vanish. The matrix element $\langle\pi^0|E_\alpha^a B_\beta^a|0\rangle$ can be evaluated by using the Gross-Treiman-Wilczek formula [16]

$$\langle\pi^0|g_E^2 E_\alpha^a B_\beta^a|0\rangle=\frac{1}{3}\,\delta_{\alpha\beta}\langle\pi^0|g_E^2\mathbf{E}^a\cdot\mathbf{B}^a|0\rangle=\frac{1}{3}\,\delta_{\alpha\beta}\frac{4\pi^2}{\sqrt{2}}\left[\frac{m_d-m_u}{m_d+m_u}\right]f_\pi m_\pi^2, \tag{13}$$

where we have identified g_E as the QCD coupling constant g_s as in Ref. [9]. After evaluating the spin- and angular-momentum dependent parts of $\mathcal{M}_{E1M1}$, we obtain

$$\mathcal{M}_{E1M1}=i\,\frac{g_M}{g_E}\,\frac{4\pi^2}{18\sqrt{6}m_c}\{\cos\theta(f_{2011}^{110}+f_{2011}^{001})-\sqrt{2}\sin\theta(f_{1211}^{110}+f_{1211}^{201})\}\left[\frac{m_d-m_u}{m_d+m_u}\right]f_\pi m_\pi^2, \tag{14}$$

where the overlapping integral $f_{n_i l_i n_f l_f}^{LP_iP_f}$ is defined by

$$f_{n_i l_i n_f l_f}^{LP_iP_f}=\sum_K\frac{\langle R_{n_f l_f}|r^{P_f}|R_{KL}\rangle\langle R_{KL}|r^{P_i}|R_{n_i l_i}\rangle}{E_i-E_{KL}}, \tag{15}$$

in which $R_{n_i l_i}$, $R_{n_f l_f}$, and R_{KL} are radial wave functions of the initial-, final-, and intermediate-state quarkonium, respectively. The transition rate is then

$$\Gamma(\psi'\to\psi(1\,^1P_1)+\pi^0)=\frac{\pi^3}{143m_c^2}\,\frac{\alpha_M}{\alpha_E}|\cos\theta(f_{2011}^{110}+f_{2011}^{001})-\sqrt{2}\sin\theta(f_{1211}^{110}+f_{1211}^{201})|^2\frac{E_{\psi(1\,^1P_1)}}{M_{\psi'}}\left[\frac{m_d-m_u}{m_d+m_u}f_\pi m_\pi^2\right]^2|\mathbf{p}_\pi|, \tag{16}$$

where $E_{\psi(1\,^1P_1)}=(M_{\psi'}^2+M_{\psi(1\,^1P_1)}^2-m_\pi^2)/(2M_{\psi'})$ is the energy of $\psi(1\,^1P_1)$, and $|\mathbf{p}_\pi|$ is the absolute value of the pion momentum.

As in Ref. [7], when evaluating the transition amplitude, we describe the intermediate states by the string vibrational states proposed in Ref. [17]. Then for a given potential model, the radial wave functions and the overlapping integrals $f_{n_i l_i n_f l_f}^{LP_iP_f}$ can be calculated numerically. The results in the Chen-Kuang potential model with $\Lambda_{\overline{\rm MS}}=200$ MeV are

$$\theta=0°:\qquad \Gamma[\psi'\to\psi(1\,^1P_1)+\pi^0]=0.17\left(\frac{\alpha_M}{\alpha_E}\right)\ \text{keV},$$
$$B[\psi'\to\psi(1\,^1P_1)+\pi^0]=(6.1\pm0.7)\left(\frac{\alpha_M}{\alpha_E}\right)\times10^{-4}, \tag{17}$$

$$\theta=-12°:\qquad \Gamma[\psi'\to\psi(1\,^1P_1)+\pi^0]=0.06\left(\frac{\alpha_M}{\alpha_E}\right)\ \text{keV},$$
$$B[\psi'\to\psi(1\,^1P_1)+\pi^0]=(2.2\pm0.2)\left(\frac{\alpha_M}{\alpha_E}\right)\times10^{-4}. \tag{18}$$

Comparing the results in Eqs. (17) with those in Eqs. (3), we see that the results in the two models are close to each other. From Eqs. (17) and (18), we see that the state-mixing effect significantly reduces the rate due to the fact that the sign of $(f_{1211}^{110}+f_{1211}^{201})$ is opposite to that of $(f_{2011}^{110}+f_{2011}^{001})$.

The detection of the process (1) depends on the capability of the photon detector. In the two-body decay (1), the energies of $\psi(1\,^1P_1)$ and π^0 are fixed, i.e., $E_{\psi(1\,^1P_1)}=(M_{\psi'}^2+M_{\psi(1\,^1P_1)}^2-m_{\pi^0}^2)/(2M_{\psi'})$, $E_{\pi^0}=(M_{\psi'}^2-M_{\psi(1\,^1P_1)}^2+m_{\pi^0}^2)/(2M_{\psi'})$. π^0 decays 99% into two photons. The two photons in $\psi'\to\psi(1\,^1P_1)+\gamma+\gamma$ with invariant mass $M_{\gamma\gamma}^2=m_{\pi^0}^2$ come mainly from the π^0 decay in (1),[4] since the branching ratios of the cascade electromagnetic transitions $\psi'\to\eta_c'(3590)+\gamma\to\psi(1\,^1P_1)+\gamma+\gamma$ and $\psi'\to\psi(1\,^3P_2)$

[4]The branching ratio of the process $\psi'\to J/\psi+\pi^0$ is $(9.7\pm2.1)\times10^{-4}$ [10], so that this process can also be detected. However, it can be clearly distinguished from the signal process (1) by the measured value of $\omega_1+\omega_2=E_{\pi^0}$ since $M_{J/\psi}$ and $M_{\psi(1\,^1P_1)}$ are quite different.

$+\gamma \rightarrow \psi(1\,^1P_1)+\gamma+\gamma$ are of the order of 10^{-5} and 10^{-6}–10^{-5}, respectively [18]. If the momenta of the two photons can be measured with sufficient accuracy, one can look for the monotonic invariant mass $M_{\gamma\gamma}$ as the signal. Once the energies ω_1 and ω_2 of the two photons are measured, the mass $M_{\psi(1\,^1P_1)}$ can be determined from $\omega_1+\omega_2 = E_{\pi^0} = (M^2_{\psi'} - M^2_{\psi(1\,^1P_1)} + m^2_{\pi^0})/(2M_{\psi'})$. Now BES has already accumulated $3\times 10^6 \psi'$ events. According to the branching ratio in Eqs. (18) and taking into account a 10% detection efficiency, there can be 10^2–10^3 events of $\psi' \rightarrow \psi(1\,^1P_1)+\gamma+\gamma$. Unfortunately, the present BES photon detector is not efficient enough to do this kind of measurement. So one should take certain decay products of $\psi(1\,^1P_1)$ as the signal.

To calculate the branching ratios of various $\psi(1\,^1P_1)$ decay channels, the crucial thing is to estimate the radiative decay rate $\Gamma[\psi(1\,^1P_1)\rightarrow \eta_c\gamma]$ and the hadronic decay rate $\Gamma[\psi(1\,^1P_1)\rightarrow \text{light hadrons}]$. Since the state-mixing effect is supposed to be not important for $\psi(1\,^1P_1)$, we can simply do the single-channel calculation. In a recent paper [19], the rate $\Gamma[\psi(1\,^1P_1)\rightarrow \eta_c\gamma]$ was estimated by using the spin symmetry

$$\Gamma[\psi(1\,^1P_1)\rightarrow \eta_c\gamma] = \left(\frac{E_\gamma^{(1^1P1)}}{E_\gamma^{(1\,^3P_J)}}\right)^3 \Gamma[\psi(1\,^3P_J)\rightarrow \eta_c\gamma] \tag{19}$$

and the experimental data of $\Gamma[\psi(1\,^3P_J)\rightarrow \eta_c\gamma]$. The obtained value is [19]

$$\Gamma[\psi(1\,^1P_1)\rightarrow \eta_c\gamma] = 0.45 \ \text{MeV}. \tag{20}$$

For $\Gamma[\psi(1\,^1P_1)\rightarrow \text{light hadrons}]$, we take the following approximation as in Ref. [9]:

$$\frac{\Gamma[\psi(1\,^1P_1)\rightarrow \text{light hadrons}]}{\Gamma(J/\psi\rightarrow \text{light hadrons})} \approx \frac{\Gamma[\psi(1\,^1P_1)\rightarrow 3g]}{\Gamma(J/\psi\rightarrow 3g)}. \tag{21}$$

The rate $\Gamma(J/\psi\rightarrow 3g)$ can be obtained from [9]

$$\Gamma(J/\psi\rightarrow 3g) = [1-(2+R)B(J/\psi\rightarrow e^+e^-)]\Gamma_{\text{tot}}(J/\psi), \tag{22}$$

where $R = N_c(4/9+1/9+1/9) = 2$. Taking the updated data $\Gamma_{\text{tot}}(J/\psi) = 87\pm 5$ keV, $B(J/\psi\rightarrow e^+e^-) = (5.93\pm 0.10)\%$ [10], we obtain

$$\Gamma(J/\psi\rightarrow 3g) = (66\pm 4) \ \text{keV}. \tag{23}$$

We first take the conventional perturbative QCD (PQCD) approach to calculate the ratio in Eq. (21), in which it is assumed that the long-distance effect is factorized into the wave function of the quarkonium (naive factorization). The leading-order PQCD formula for the right-hand side of Eq. (21) has been given in Refs. [20,21]. In the Chen-Kuang potential model, the ratio on the right-hand side of Eq. (21) has been calculated in Ref. [22], which is 0.515. Thus we obtain

$$\Gamma[\psi(1\,^1P_1)\rightarrow 3g] = (45\pm 3) \ \text{keV}. \tag{24}$$

Then from Eqs. (20), (24), and some $\psi(1\,^1P_1)$ decay modes with small decay rates given in Ref. [22], we obtain, in PQCD,

$$\text{PQCD:} \quad \Gamma_{\text{tot}}(\psi(1\,^1P_1)) = (0.51\pm 0.01) \ \text{MeV},$$

$$B[\psi(1\,^1P_1)\rightarrow \eta_c\gamma] \approx (88\pm 2),$$

$$B[\psi(1\,^1P_1)\rightarrow \text{light hadrons}] \approx (8.8\pm 0.8)\%. \tag{25}$$

We see that both $\psi(1\,^1P_1)\rightarrow \eta_c\gamma$ and $\psi(1\,^1P_1) \rightarrow \text{light hadrons}$ are detectable, and the most promising mode is the radiative transition $\psi(1\,^1P_1)\rightarrow \eta_c\gamma$.

Next we take the nonrelativistic QCD (NRQCD) approach developed in recent years [23]. NRQCD is a more sophisticated approach in which the naive factorization assumption is avoided. It has been intensively studied in recent years and has been applied to studying the hadronic decays of heavy quarkonia [19,23,24,25]. The study is up to next-to-leading order (NLO) which contains unknown matrix elements of some operators.[5] Considering the large theoretical uncertainty and the large experimental errors in the input data in this study, taking the leading order (LO) NRQCD result concerning only one matrix element related to the wave function at the origin is already sufficient for the present purpose. The LO NRQCD result of $\Gamma[\psi(1\,^1P_1) \rightarrow \text{light hadrons}]$ is [23]

$$\text{NRQCD:} \quad \Gamma[\psi(1\,^1P_1)\rightarrow \text{light hadrons}] = (0.53\pm 0.08) \ \text{MeV}. \tag{26}$$

This is very different from the corresponding values in Eq. (24) in the conventional PQCD approach. With the updated value of $\psi(1\,^1P_1)\rightarrow \eta_c\gamma$ [cf. Eq. (20)], the NRQCD predicted total width of $\psi(1\,^1P_1)$ will be slightly larger than that given in Ref. [23], which is

$$\text{NRQCD:} \quad \Gamma_{\text{tot}}[\psi(1\,^1P_1)] = (1.1\pm 0.09) \ \text{MeV}. \tag{27}$$

Then we have

$$\text{NRQCD:} \quad B[\psi(1\,^1P_1)\rightarrow \eta_c\gamma] = (41\pm 3)\%,$$

$$B[\psi(1\,^1P_1)\rightarrow \text{light hadrons}] = (48\pm 7)\%. \tag{28}$$

[5]In a recent paper [19], the NLO value $\Gamma[\psi(1\,^1P_1)] \rightarrow \text{light hadrons} = (0.72\pm 0.32)$ MeV is obtained. It is larger than the leading-order value [cf. Eq. (26)] by 26%, which is within the uncertainty (44%) of the NLO result, and may be neglected relative to the large theoretical uncertainty and the large experimental errors in the input data in this study. Furthermore, the uncertainty in this NLO result is so large that, together with the experimental errors in the input data, it can hardly make clear predictions for the final results.

TABLE I. Numbers of events for the processes in (30) with 3×10^7 ψ' events in the Chen-Kuang potential model for $\theta=-12°$ and $\alpha_M/\alpha_E=1$–3 with the conventional PQCD approach to $\Gamma[\psi(1\,^1P_1)\to\text{light hadrons}]$. A 10% detection efficiency has been taken into account.

	$\psi'\to K\bar{K}\pi\gamma\gamma\gamma$	$\psi'\to\rho\rho\gamma\gamma\gamma$	$\psi'\to K^+K^{=}\pi^+\pi^{=}\gamma\gamma\gamma$
$\alpha_M=\alpha_E$	32 ± 13	15 ± 7	12 ± 5
$\alpha_M=2\alpha_E$	64 ± 27	30 ± 14	24 ± 10
$\alpha_M=3\alpha_E$	96 ± 40	45 ± 21	35 ± 15

We see that the predictions in [cf. Eq. (28)] are quite different from those of PQCD [cf. Eq. (25)]. Again, both $\psi(1\,^1P_1)\to\eta_c\gamma$ and $\psi(1\,^1P_1)\to$light hadrons modes are detectable, but here $\psi(1\,^1P_1)\to$light hadrons is slightly more promising.

Let us first consider the mode $\psi(1\,^1P_1)\to\eta_c\gamma$. We then need to take certain detectable decay channels of η_c as the signal. Possible modes are $\eta_c\to K\bar{K}\pi$, $\eta_c\to\rho\rho$, and $\eta_c\to\pi^+\pi^-K^+K^-$. The branching ratios are [10]

$$B(\eta_c\to K\bar{K}\pi)=(5.5\pm1.7),$$

$$B(\eta_c\to\rho\rho)=(2.6\pm0.9)\%, \tag{29}$$

$$B(\eta_c\to\pi^+\pi^-K^+K^-)=(2.0^{+0.7}_{-0.6})\%.$$

Thus we can look for the decay chains

$$\psi'\to\psi(1\,^1P_1)\pi^0\to\psi(1\,^1P_1)\gamma\gamma\to\eta_c\gamma\gamma\gamma\to K\bar{K}\pi\gamma\gamma\gamma,$$

$$\psi'\to\psi(1\,^1P_1)\pi^0\to\psi(1\,^1P_1)\gamma\gamma\to\eta_c\gamma\gamma\gamma\to\rho\rho\gamma\gamma\gamma,$$

$$\psi'\to\psi(1\,^1P_1)\pi^0\to\psi(1\,^1P_1)\gamma\gamma\to\eta_c\gamma\gamma\gamma \to K^+K^-\pi^+\pi^-\gamma\gamma\gamma. \tag{30}$$

If BES can accumulate 3×10^7 ψ' events in the near future, then from Eqs. (18), (25), and (29), and taking into account a 10% detection efficiency, we obtain the event numbers of the signals in (30) in the Chen-Kuang potential model for $\theta=-12°$. The results are listed in Table I. The corresponding event numbers in the NRQCD approach from Eqs. (18), (28), and (29) are listed in Table II. We see that the signals are detectable.

Next we consider the mode $\psi(1\,^1P_1)\to$light hadrons. We need to look at $\psi(1\,^1P_1)$ decaying into a certain exclusive hadronic channel h. As in Refs. [9,22], we assume that

$$\frac{\Gamma[\psi(1\,^1P_1)\to h]}{\Gamma(\eta_c\to h)}\approx\frac{\Gamma[\psi(1\,^1P_1)\to3g]}{\Gamma(\eta_c\to2g)}, \tag{31}$$

and we can estimate $\eta_c\to h$ from Eq. (23).

First, in the conventional PQCD, we have [20]

$$\frac{\Gamma(\eta_c\to2g)}{\Gamma(J/\psi\to3g)}=\frac{27\pi}{5(\pi^2-9)\alpha_s}\left(\frac{M^2_{J/\psi}}{M^2_{\eta_c}}\right), \tag{32}$$

which is model-independent. Then from Eq. (23) we have

$$\Gamma(\eta_c\to2g)=(6.4\pm0.4)\ \text{MeV}, \tag{33}$$

in which we have taken $\alpha_s(M_{\eta_c})\approx0.22$, $M_{J/\psi}=3096.87\pm0.04$ MeV, $M_{\eta_c}=2978.8\pm1.8$ MeV [10]. So, with Eq. (24), we have

$$\text{PQCD:}\quad \Gamma[\psi(1\,^1P_1)\to h]=(0.010\pm0.001)\Gamma(\eta_c\to h). \tag{34}$$

Next, for NRQCD, to LO the ratio $\Gamma(\eta_c\to2g)/\Gamma(J/\psi\to3g)$ is the same as Eq. (32) [23], so that we still have Eq. (33). Then from Eqs. (33) and (26), we have

$$\text{NRQCD:}\quad \Gamma[\psi(1\,^1P_1)\to h]=(0.083\pm0.018)\Gamma(\eta_c\to h). \tag{35}$$

With the experimental value $\Gamma_{\text{tot}}(\eta_c)=13.2^{+3.8}_{-3.2}$ MeV [10], we have

$$\text{PQCD:}\quad \Gamma[\psi(1\,^1P_1)\to K\bar{K}\pi]=(7.3\pm3.0)\ \text{keV},$$

$$\Gamma[\psi(1\,^1P_1)\to\rho\rho]=(3.4\pm1.5)\ \text{keV}, \tag{36}$$

$$\Gamma[\psi(1\,^1P_1)\to\pi^+\pi^-K^+K^-]=(2.6\pm1.1)\ \text{keV},$$

$$\text{NRQCD:}\quad \Gamma[\psi(1\,^1P_1)\to K\bar{K}\pi]=(60\pm32)\ \text{keV},$$

TABLE II. Numbers of events for the processes in (30) with 3×10^7 ψ' events in the Chen-Kuang potential model for $\theta=-12°$ and $\alpha_M/\alpha_E=1$–3 with the NRQCD approach to $\Gamma[\psi(1\,^1P_1)\to\text{light hadrons}]$. A 10% detection efficiency has been taken into account.

	$\psi'\to K\bar{K}\pi\gamma\gamma\gamma$	$\psi'\to\rho\rho\gamma\gamma\gamma$	$\psi'\to K^+K^{=}\pi^+\pi^{=}\gamma\gamma\gamma$
$\alpha_M=\alpha_E$	15 ± 7	7 ± 4	5 ± 3
$\alpha_M=2\alpha_E$	30 ± 22	14 ± 7	11 ± 5
$\alpha_M=3\alpha_E$	45 ± 21	21 ± 11	16 ± 8

TABLE III. Numbers of events for the processes in (40) with 3×10^7 ψ' events in the Chen-Kuang potential model for $\theta=-12°$ and $\alpha_M/\alpha_E=1$–3 with the conventional PQCD approach to $\Gamma[\psi(1\,^1P_1)\to\text{light hadrons}]$. A 10% detection efficiency has been taken into account.

	$\psi'\to K\bar{K}\pi\gamma\gamma$	$\psi'\to\rho\rho\gamma\gamma$	$\psi'\to K^+K^=\pi^+\pi^=\gamma\gamma$
$\alpha_M=\alpha_E$	9 ± 7	4 ± 3	3 ± 2
$\alpha_M=2\alpha_E$	18 ± 13	9 ± 6	7 ± 5
$\alpha_M=3\alpha_E$	28 ± 20	13 ± 10	10 ± 7

$$\Gamma[\psi(1\,^1P_1)\to\rho\rho]=(28\pm16)\ \text{keV}, \quad (37)$$

$$\Gamma[\psi(1\,^1P_1)\to\pi^+\pi^-K^+K^-]=(22\pm12)\ \text{keV}.$$

Then with Eqs. (25) and (26) we have

$$\text{PQCD:}\quad B[\psi(1\,^1P_1)\to K\bar{K}\pi]=(1.4\pm0.9)\%,$$

$$B[\psi(1\,^1P_1)\to\rho\rho]=(0.67\pm0.43)\%, \quad (38)$$

$$B[\psi(1\,^1P_1)\to\pi^+\pi^-K^+K^-]=(0.51\pm0.32)\%,$$

$$\text{NRQCD:}\quad B[\psi(1\,^1P_1)\to K\bar{K}\pi]=(5.5\pm3.3)\%,$$

$$B[\psi(1\,^1P_1)\to\rho\rho]=(2.6\pm1.7)\%, \quad (39)$$

$$B[\psi(1\,^1P_1)\to\pi^+\pi^-K^+K^-]=(2.0\pm1.2)\%.$$

Now, instead of looking for the chains in (30), one can look for

$$\psi'\to\psi(1\,^1P_1)\pi^0\to\psi(1\,^1P_1)\gamma\gamma\to K\bar{K}\pi\gamma\gamma,$$

$$\psi'\to\psi(1\,^1P_1)\pi^0\to\psi(1\,^1P_1)\gamma\gamma\to\rho\rho\gamma\gamma, \quad (40)$$

$$\psi'\to\psi(1\,^1P_1)\pi^0\to\psi(1\,^1P_1)\gamma\gamma \to K^+K^-\pi^+\pi^-\gamma\gamma.$$

Taking into account a 10% detection efficiency, the numbers of events for the chains in Eq. (40) with 3×10^7 ψ' events in the Chen-Kuang potential model for $\theta=-12°$ are listed in Table III (PQCD) and Table IV (NRQCD). We see that the two decay chains are all detectable, and the chains (30) are more promising in the PQCD approach, while the chains (40) are more promising in the NRQCD approach.

Comparing the numbers in Table I–IV, we see that the NRQCD approach reduces the numbers of events for the chains (30) by roughly a factor of 2, and increases the numbers of events for the chains (40) by roughly a factor of 4 relative to the conventional PQCD approach.

The above situation shows that the detection of the chains (30) and (40) is interesting not only for the $\psi(1\,^1P_1)$ searches at BEPC, but also for providing a test of the difference between the NRQCD predictions and the conventional PQCD predictions.

In conclusion, our calculation shows that the *S-D* mixing effect does affect the transition rate (1) significantly. It reduces the rate (branching ration) by about a factor of 3 relative to the single-channel result [cf. Eqs. (17) and (18)]. If the photon detector in the future BES III can be efficient enough to measure the momenta of the two photons from $\pi^0\to\gamma\gamma$, the search for the $\psi(1\,^1P_1)$ state via the process (1) will not be difficult at BEPC II. At the present BES, one should tag $\psi(1\,^1P_1)$ directly via its decay products. Our results in Tables I–IV show that the search for $\psi(1\,^1P_1)$ is possible with the present BES if an accumulation of 3×10^7 ψ' events can be achieved in the near future.

Finally, we would like to mention that we have also calculated the rate of $\psi(3770)\to J/\psi\pi\pi$ in the Chen-Kuang potential model for $\theta=-12°$ and $\Lambda_{\overline{\text{MS}}}=200$ MeV. The result is $\Gamma[\psi(3770)\to J/\psi\ \pi\pi]=170$ keV (for $c_2=3c_1$), or 37 keV (for $c_2=c_1$). This is close to the corresponding values $\Gamma[\psi(3770)\to J/\psi\ \ \pi\pi]=160$ keV (for $c_2=3c_1$), or 30 keV (for $c_2=c_1$) in the Cornell potential model [13]. Together with the results in Eqs. (17) and (3), we see that the results are insensitive to the potential models used in the calculation as compared with the uncertainties in the approach.

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TABLE IV. Numbers of events for the processes in (40) with 3×10^7 ψ' events in the Chen-Kuang potential model for $\theta=-12°$ and $\alpha_M/\alpha_E=1$–3 with the NRQCD approach to $\Gamma[\psi(1^1P_1)\to\text{light hadrons}]$. A 10% detection efficiency has been taken into account.

	$\psi'\to K\bar{K}\pi\gamma\gamma$	$\psi'\to\rho\rho\gamma\gamma$	$\psi'\to K^+K^=\pi^+\pi^=\gamma\gamma$
$\alpha_M=\alpha_E$	36 ± 25	17 ± 13	13 ± 9
$\alpha_M=2\alpha_E$	73 ± 51	34 ± 25	26 ± 19
$\alpha_M=3\alpha_E$	109 ± 76	52 ± 41	40 ± 28

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