$B \to J/\psi K^*$ decays in QCD factorization

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(Received 8 November 2001; published 9 May 2002)

The hadronic decay $B \to J/\psi K^*$ is analyzed within the framework of QCD factorization. The spin amplitudes A_0, $A_\parallel$, and $A_\perp$ in the transversity basis and their relative phases are studied using various different form-factor models for the B-K^* transition. The effective parameters a_2^h for helicity $h=0,+,-$ states receive different nonfactorizable contributions and hence they are helicity-dependent, contrary to naive factorization where a_2^h are universal and polarization-independent. QCD factorization breaks down even at the twist-2 level for transverse hard spectator interactions. Although a nontrivial strong phase for the $A_\parallel$ amplitude can be achieved by adjusting the phase of an infrared-divergent contribution, the present QCD factorization calculation cannot say anything definite about the phase $\phi_\parallel$. Unlike $B \to J/\psi K$ decays, the longitudinal parameter a_2^0 for $B \to J/\psi K^*$ does not receive twist-3 corrections and is not large enough to account for the observed branching ratio and the fraction of the longitudinal polarization. Possible enhancement mechanisms for a_2^0 are discussed.

DOI: 10.1103/PhysRevD.65.094023 PACS number(s): 13.25.Hw, 12.38.Bx

I. INTRODUCTION

It has been well known that the factorization approach (naive or generalized) fails to explain the production ratio $R = \mathcal{B}(B \to J/\psi K^*)/\mathcal{B}(B \to J/\psi K)$ and the fraction of the longitudinal polarization Γ_L/Γ in $B \to J/\psi K^*$ decay. We consider two representative form-factor models for $B - K(K^*)$ transitions: the Ball-Braun (BB) model based on the light-cone sum rule (LCSR) analysis [1] and the Melikhov-Stech (MS) model [2] based on the constituent quark picture. Both are consistent with the lattice calculation at large q^2, the constraint from $B \to \phi K^*$ at lower q^2, and the constraint from heavy quark symmetry on the q^2 dependence of the heavy-light transition (see Sec. IV for more details). We see from Table I that in general the predicted longitudinal polarization is too small, whereas the production ratio is too large.

This is understandable because the parameter a_2, which governs $B \to J/\psi K(K^*)$ decays, is assumed to be universal according to the factorization hypothesis, namely $a_2^h(J/\psi K^*) = a_2(J/\psi K)$, where $h=0,+,-$ refer to the helicity states 00, $++$, and $--$, respectively. In the above-mentioned form-factor models, one has $h_0 = 5.98$, $h_+ = 6.23$, and $h_- = 043$ (in units of GeV^3) in the BB model and $h_0 = 5.47$, $h_+ = 5.92$, and $h_- = 0.73$ in the MS model, where h_i are the helicity amplitudes given by

$$h_0 = \frac{f_{J/\psi}}{2m_{K^*}}\left[(m_B^2 - m_{J/\psi}^2 - m_{K^*}^2)(m_B + m_{K^*}) \times A_1^{BK^*}(m_{J/\psi}^2) - \frac{4m_B^2 p_c^2}{m_B + m_{K^*}} A_2^{BK^*}(m_{J/\psi}^2)\right],$$

$$h_\pm = m_{J/\psi} f_{J/\psi}\left[(m_B + m_{K^*})A_1^{BK^*}(m_{J/\psi}^2) \pm \frac{2m_B p_c}{m_B + m_{K^*}} V^{BK^*}(m_{J/\psi}^2)\right]. \tag{1.1}$$

It is obvious that $h_+ > h_0 \gg h_-$. Therefore, under naive factorization $\Gamma_L/\Gamma \approx (a_2^0 h_0)^2/[(a_2^0 h_0)^2 + (a_2^+ h_+)^2] = h_0^2/(h_0^2 + h_+^2) \lesssim 1/2$ and R is expected to be greater than unity due to three polarization states for $J/\psi K^*$. These two problems will be circumvented if nonfactorized terms contribute differently to each helicity amplitude and to different decay modes so that $a_2^0(J/\psi K^*) > a_2^+(J/\psi K^*) \neq a_2^-(J/\psi K^*)$ and $a_2(J/\psi K) > a_2^h(J/\psi K^*)$. In other words, the present data imply that the effective parameter a_2^h should be non-universal and polarization-dependent. Recently two of us have analyzed charmless $B \to VV$ decays within the framework of QCD factorization [7]. We show that, contrary to phenomenological generalized factorization, nonfactorizable corrections to each partial-wave or helicity amplitude are not the same; the ef-

TABLE I. The ratio of vector meson to pseudoscalar production R and the longitudinal polarization fraction Γ_L/Γ in $B\to J/\psi K^{(*)}$ decays calculated in two representative form-factor models using the factorization hypothesis.

			Experiments			
	BB	MS	CDF [3]	CLEO [4]	BaBar [5]	Belle [6]
R	3.40	3.11	1.53 ± 0.32	1.45 ± 0.26	1.38 ± 0.11	1.43 ± 0.13
Γ_L/Γ	0.47	0.46	0.61 ± 0.14	0.52 ± 0.08	0.60 ± 0.04	0.60 ± 0.05

fective parameters a_i vary for different helicity amplitudes. The purpose of the present paper is to study the nonfactorizable effects in $B\to J/\psi K^*$ decay within the same framework of QCD factorization.

The decays $B\to J/\psi K(K^*)$ are of great interest as experimentally only a few color-suppressed modes in hadronic B decays have been measured so far. The recent measurement by BaBar [5] has confirmed the earlier CDF observation [3] that there is a nontrivial strong phase difference between polarized amplitudes, indicating final-state interactions. However, no such evidence is seen by CLEO [4] and more recently by Belle [6]. It is interesting to check if the current approach for B hadronic decays predicts a departure from factorization. Therefore, the measurements of various helicity amplitudes in $B\to J/\psi K^*$ decays will provide a nice foundation for testing factorization and differentiating various theory approaches in which the calculated nonfactorizable terms have real and imaginary parts.

It is known that in the QCD factorization approach, the coefficient a_2 is severely suppressed in the absence of hard spectator interactions. It has been shown in [8] that $|a_2|$ in $B\to J/\psi K$ is of order 0.11 to the leading twist order, to be compared with the experimental value of order 0.25. The twist-3 effect in hard spectator interactions will enhance a_2 to the value of $0.19^{+0.14}_{-0.12}$. We shall see later that, contrary to the $J/\psi K$ case, a_2^0 in $B\to J/\psi K^*$ does not receive twist-3 contributions and it is dominated by twist-2 hard spectator interactions.

The layout of the present paper is as follows. In Sec. II we first outline the necessary ingredients of the QCD factorization approach for describing $B\to J/\psi K^*$ and then we proceed to compute vertex and hard spectator interactions. The ambiguity of the experimental determination of spin amplitude phases is addressed in Sec. III. Numerical calculations and results are presented in Sec. IV. Discussions and conclusions are shown in Sec. V.

II. $B\to J/\psi K^*$ IN QCD FACTORIZATION

A. Factorization formula

The general $B\to J/\psi K^*$ amplitude consists of three independent Lorentz scalars:

$$A[B(p)\to J/\psi(\varepsilon_{J/\psi},p_{J/\psi})K^*(\varepsilon_{K^*},p_{K^*})] \propto \varepsilon^{*\mu}_{J/\psi}\varepsilon^{*\nu}_{K^*}(a g_{\mu\nu}+b p_\mu p_\nu + i c\,\epsilon_{\mu\nu\alpha\beta}p^\alpha_{J/\psi}p^\beta_{K^*}), \tag{2.1}$$

where $\epsilon^{0123}=+1$ in our convention, the coefficient c corresponds to the P-wave amplitude, and a,b to the mixture of S- and D-wave amplitudes. Three helicity amplitudes can be constructed as[1]

$$H_0=-\frac{1}{2m_{J/\psi}m_{K^*}}[(m_B^2-m_{J/\psi}^2-m_{K^*}^2)a+2m_B^2p_c^2 b],$$
$$H_\pm=a\pm m_B p_c\,c, \tag{2.2}$$

where p_c is the c.m. momentum of the vector meson in the B rest frame. If the final-state two vector mesons are both light as in charmless $B\to V_1V_2$ decays with V_1 being a recoiled meson and V_2 an ejected one, it is expected that $|H_0|^2 > |H_+|^2 > |H_-|^2$ owing to the argument that the amplitude H_+ is suppressed by a factor of $\sqrt{2}m_2/m_B$ as one of the quark helicities in V_2 has to be flipped, while the H_- amplitude is subject to further chirality suppression of order m_1/m_B [9]. However, for $B\to J/\psi K^*$ decay, $\sqrt{2}m_{J/\psi}/m_B$ is of order unity and hence in practice H_+ and H_0 can be comparable.

Note that the polarized decay amplitudes can be expressed in several different but equivalent bases. For example, the helicity amplitudes can be related to the spin amplitudes in the transversity basis $(A_0,A_\parallel,A_\perp)$ defined in terms of the linear polarization of the vector mesons, or to the partial-wave amplitudes (S,P,D) via

$$A_0=H_0=-\frac{1}{\sqrt{3}}S+\sqrt{\frac{2}{3}}D,$$
$$A_\parallel=\frac{1}{\sqrt{2}}(H_++H_-)=\sqrt{\frac{2}{3}}S+\frac{1}{\sqrt{3}}D, \tag{2.3}$$
$$A_\perp=\frac{1}{\sqrt{2}}(H_+-H_-)=P,$$

where we have followed the sign convention of [10]. The decay rate reads

[1]For $\overline{B}\to J/\psi\overline{K}^*$ decay the transverse amplitudes are given by $H_\pm=-a\pm m_Bp_c\,c$.

$$
\begin{aligned}
\Gamma(B\to J/\psi K^*) &= \frac{p_c}{8\pi m_B^2}\left|\frac{G_F}{\sqrt{2}}V_{cb}V_{cs}^*\right|^2(|H_0|^2+|H_+|^2+|H_-|^2),\\
&= \frac{p_c}{8\pi m_B^2}\left|\frac{G_F}{\sqrt{2}}V_{cb}V_{cs}^*\right|^2(|A_0|^2+|A_\perp|^2+|A_\parallel|^2),\\
&= \frac{p_c}{8\pi m_B^2}\left|\frac{G_F}{\sqrt{2}}V_{cb}V_{cs}^*\right|^2(|S|^2+|P|^2+|D|^2).
\end{aligned} \tag{2.4}
$$

The effective Hamiltonian relevant for $B\to J/\psi K^*$ has the form

$$
\mathcal{H}_{\rm eff}=\frac{G_F}{\sqrt{2}}\left\{V_{cb}V_{cs}^*[c_1(\mu)O_1(\mu)+c_2(\mu)O_2(\mu)] - V_{tb}V_{ts}^*\sum_{i=3}^{10}c_i(\mu)O_i(\mu)\right\}+\text{H.c.}, \tag{2.5}
$$

where

$$
\begin{aligned}
O_1&=(\bar{c}b)_{V-A}(\bar{s}c)_{V-A},\\
O_2&=(\bar{s}b)_{V-A}(\bar{c}c)_{V-A},\\
O_{3(5)}&=(\bar{s}b)_{V-A}\sum_{q'}(\bar{q}'q')_{V-A(V+A)},\\
O_{4(6)}&=(\bar{s}_\alpha b_\beta)_{V-A}\sum_{q'}(\bar{q}'_\beta q'_\alpha)_{V-A(V+A)},\\
O_{7(9)}&=\frac{3}{2}(\bar{s}b)_{V-A}\sum_{q'}e_{q'}(\bar{q}'q')_{V+A(V-A)},\\
O_{8(10)}&=\frac{3}{2}(\bar{s}_\alpha b_\beta)_{V-A}\sum_{q'}e_{q'}(\bar{q}'_\beta q'_\alpha)_{V+A(V-A)},
\end{aligned} \tag{2.6}
$$

with O_3–O_6 being the QCD penguin operators, O_7–O_{10} the electroweak penguin operators, and $(\bar{q}_1q_2)_{V\pm A}\equiv\bar{q}_1\gamma_\mu(1\pm\gamma_5)q_2$. Under factorization, the decay amplitude of $B\to J/\psi K^*$ reads

$$
A(B\to J/\psi K^*) = \frac{G_F}{\sqrt{2}}V_{cb}V_{cs}^*(a_2+a_3+a_5+a_7+a_9)X^{(BK^*,J/\psi)}, \tag{2.7}
$$

where

$$
\begin{aligned}
X^{(BK^*,J/\psi)} &\equiv\langle J/\psi|(\bar{c}c)_{V-A}|0\rangle\langle K^*|(\bar{b}s)_{V-A}|B\rangle\\
&=-if_{J/\psi}m_{J/\psi}\Bigg[(\varepsilon^*_{K^*}\cdot\varepsilon^*_{J/\psi})(m_B+m_{K^*})A_1^{BK^*}(m^2_{J/\psi})\\
&\quad-(\varepsilon^*_{K^*}\cdot p_B)(\varepsilon^*_{J/\psi}\cdot p_B)\frac{2A_2^{BK^*}(m^2_{J/\psi})}{m_B+m_{K^*}}\\
&\quad-i\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\mu}_{J/\psi}\varepsilon^{*\nu}_{K^*}p_B^\alpha p_{K^*}^\beta\frac{2V^{BK^*}(m^2_{J/\psi})}{m_B+m_{K^*}}\Bigg].
\end{aligned} \tag{2.8}
$$

Note that for $\bar{B}\to J/\psi\bar{K}^*$ decay, the factorizable amplitude $X^{(\bar{B}\bar{K}^*,J/\psi)}\equiv\langle J/\psi|(\bar{c}c)_{V-A}|0\rangle\langle\bar{K}^*|(\bar{s}b)_{V-A}|\bar{B}\rangle$ is the same as Eq. (2.8) except that the last term proportional to $i\epsilon_{\mu\nu\alpha\beta}$ has a positive sign. Comparing Eq. (2.8) with Eq. (2.2) leads to the helicity amplitudes

$$
H_0=-\tilde{a}(J/\psi K^*)h_0,\quad H_\pm=\tilde{a}(J/\psi K^*)h_\pm, \tag{2.9}
$$

where $\tilde{a}(J/\psi K^*)=a_2+a_3+a_5+a_7+a_9$. Note that the helicity amplitudes $H_\pm$ in $\bar{B}\to J/\psi\bar{K}^*$ are precisely the ones $H_\mp$ in $B\to J/\psi K^*$ decays. Hence, in the factorization approach one has $|H_-|>|H_+|$ for the former and $|H_+|>|H_-|$ for the latter. This is consistent with the picture that the s quark produced in the weak process $b\to c\bar{c}s$ in $\bar{B}\to J/\psi\bar{K}^*$ has helicity $-1/2$ in the zero quark mass limit. Therefore, the helicity of $\bar{K}^*$ in $\bar{B}\to J/\psi\bar{K}^*$ cannot be $+1$ and the corresponding helicity amplitude H_+ vanishes in the chiral limit [11].

B. QCD factorization

Under naive factorization, the coefficients a_i are given by $a_{2i}=c_{2i}+(1/N_c)c_{2i-1}$, $a_{2i-1}=c_{2i-1}+(1/N_c)c_{2i}$. Hence, $a_2^h(J/\psi K^*)=a_2(J/\psi K)$ for $h=0,+,-$. In the present paper, we will compute nonfactorizable corrections to $a_2^h(J/\psi K^*)$. The effective parameters a_i^h entering into the helicity amplitudes H_0 and $H_\pm$ are not the same.

The QCD-improved factorization approach advocated recently in [12] allows us to compute the nonfactorizable corrections in the heavy quark limit since only hard interactions between the (BV_1) system and V_2 survive in the $m_b\to\infty$ limit. Naive factorization is recovered in the heavy quark limit and to the zeroth order of QCD corrections. In this approach, the light-cone distribution amplitudes (LCDAs) play an essential role. The LCDAs of the vector meson are given by [13,12]

$$\langle V(P,\varepsilon)|\overline{q}(x)\gamma_\mu q'(0)|0\rangle = f_V m_V\int_0^1 d\xi\, e^{i\xi P\cdot x}\left[\frac{\varepsilon^*\cdot x}{P\cdot x}P_\mu \Phi_\parallel^V(\xi)+\left(\varepsilon_\mu^* - \frac{\varepsilon^*\cdot x}{P\cdot x}P_\mu\right)g_\perp^{(v)}(\xi)\right],$$

$$\langle V(P,\varepsilon)|\overline{q}(x)\gamma_\mu\gamma_5 q'(0)|0\rangle = \frac{1}{4}m_V\left(f_V - f_V^T\frac{m_q+m_{q'}}{m_V}\right)\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}P^\alpha x^\beta\int_0^1 d\xi\, e^{i\xi P\cdot x}g_\perp^{(a)}(\xi),$$

$$\langle V(P,\varepsilon)|\overline{q}(x)\sigma_{\mu\nu}q'(0)|0\rangle = -if_V^T(\varepsilon_\mu^* P_\nu - \varepsilon_\nu^* P_\mu)\int_0^1 d\xi\, e^{i\xi P\cdot x}\Phi_\perp^V(\xi) - if_V^T(P_\mu x_\nu - P_\nu x_\mu)\frac{\varepsilon^*\cdot x}{(P\cdot x)^2}m_V^2$$

$$\times\int_0^1 d\xi\, e^{i\xi P\cdot x}h_\parallel^{(t)}(\xi),$$

$$\langle V(P,\varepsilon)|\overline{q}(x)q'(0)|0\rangle = i\frac{1}{2}\left(f_V - f_V^T\frac{m_q+m_{q'}}{m_V}\right)(\varepsilon^*\cdot x)m_V^2\int_0^1 d\xi\, e^{i\xi P\cdot x}h_\parallel^{(s)}(\xi), \tag{2.10}$$

where $x^2=0$, ξ is the light-cone momentum fraction of the quark q in the vector meson, and f_V and f_V^T are vector and tensor decay constants, respectively, but the latter is scale-dependent. In Eq. (2.10), $\Phi_\parallel(\xi)$ and $\Phi_\perp(\xi)$ are twist-2 DAs, while $h_\parallel^{(s,t)}$, $g_\perp^{(v)}$, and $g_\perp^{(a)}$ are twist-3 ones. Since

$$\frac{\varepsilon\cdot x}{P\cdot x}P^\mu = \varepsilon_\parallel^\mu + \frac{\varepsilon\cdot x}{P\cdot x}\frac{m_V^2}{2P\cdot x}x^\mu, \tag{2.11}$$

it is clear that to order $\mathcal{O}(m_V^2/m_B^2)$ the approximated relation $(\varepsilon\cdot x)/(P\cdot x)\,P^\mu=\varepsilon_\parallel^\mu$ holds for a light vector meson, where $\varepsilon_\parallel^\mu$ $(\varepsilon_\perp^\mu)$ is the polarization vector of a longitudinally (transversely) polarized vector meson. Also, to a good approximation one has $\varepsilon_\parallel^\mu = P_V^\mu/m_V$ for a light vector meson such as K^*. Hence, $P\cdot\varepsilon_\perp=0$ and Eq. (2.10) can be simplified for K^* as

$$\langle K^*(P,\varepsilon)|\overline{q}(x)\gamma_\mu s(0)|0\rangle = f_{K^*}m_{K^*}\int_0^1 d\xi\, e^{i\xi P\cdot x}[\varepsilon_{\mu\parallel}^*\Phi_\parallel^{K^*}(\xi)+\varepsilon_{\mu\perp}^* g_\perp^{K^*(v)}(\xi)],$$

$$\langle K^*(P,\varepsilon)|\overline{q}(x)\gamma_\mu\gamma_5 s(0)|0\rangle = \frac{1}{4}m_{K^*}f_{K^*}\epsilon_{\mu\nu\alpha\beta}\varepsilon_\perp^{*\nu}P^\alpha x^\beta\int_0^1 d\xi\, e^{i\xi P\cdot x}g_\perp^{K^*(a)}(\xi),$$

$$\langle K^*(P,\varepsilon)|\overline{q}(x)\sigma_{\mu\nu}s(0)|0\rangle = -if_{K^*}^T(\varepsilon_{\mu\perp}^* P_\nu - \varepsilon_{\nu\perp}^* P_\mu)\int_0^1 d\xi\, e^{i\xi P\cdot x}\Phi_\perp^{K^*}(\xi),$$

$$\langle K^*(P,\varepsilon)|\overline{q}(x)s(0)|0\rangle = -\frac{1}{2}f_{K^*}m_{K^*}\int_0^1 d\xi\, e^{i\xi P\cdot x}h_\parallel'^{(s)}(\xi), \tag{2.12}$$

where $h'(\xi)=dh(\xi)/d\xi$ and we have neglected light quark masses and applied the relation

$$(P_\mu x_\nu - P_\nu x_\mu)\frac{\varepsilon\cdot x}{(P\cdot x)^2}m_V^2 = \frac{\varepsilon\cdot x}{P\cdot x}(P_\mu P_\nu - P_\nu P_\mu)+(\varepsilon_{\mu\parallel}P_\nu - \varepsilon_{\nu\parallel}P_\mu), \tag{2.13}$$

which vanishes for a light vector meson. From Eq. (2.12) we see that the twist-3 DA $h_\parallel^{(t)}$ of K^* does not make a contribution.

In the heavy quark limit, the B meson wave function is given by

$$\langle 0|\overline{b}_\alpha(x)q_\beta(0)|B(p)\rangle|_{x_+=x_\perp=0} = -\frac{if_B}{4}[(\not p+m_B)\gamma_5]_{\beta\gamma}\int_0^1 d\overline{\rho}\, e^{-i\overline{\rho}p_+x_-}[\Phi_1^B(\overline{\rho})+\not n_-\Phi_2^B(\overline{\rho})]_{\gamma\alpha}, \tag{2.14}$$

with $n_- = (1,0,0,-1)$ and the normalization conditions

$$\int_0^1 d\bar{\rho}\,\Phi_1^B(\bar{\rho}) = 1, \quad \int_0^1 d\bar{\rho}\,\Phi_2^B(\bar{\rho}) = 0. \tag{2.15}$$

Likewise, to the leading order in $1/m_c$, the J/ψ wave function has a similar expression

$$\langle J/\psi(p,\varepsilon)|\bar{c}_\alpha(x)c_\beta(0)|0\rangle|_{x_+=x_\perp=0} = \frac{f_{J/\psi}}{4}[\not{\varepsilon}^*(\not{p}+m_{J/\psi})]_{\beta\gamma}\int_0^1 d\xi\, e^{-i\xi p_+ x_-}[\Phi_1^{J/\psi}(\xi) + \not{n}_-\Phi_2^{J/\psi}(\xi)]_{\gamma\alpha}. \tag{2.16}$$

Since the J/ψ meson is heavy, the use of the light-cone wave function for J/ψ is problematic. The effects of higher twist wave functions have to be included and may not converge fast enough. Because the charmed quark in J/ψ carries a momentum fraction of order $\sim m_c/m_{J/\psi}$, the distribution amplitudes of J/ψ vanish in the end-point region. In the following study, we adopt $\Phi_{||}$ as the DA of the nonlocal vector current of J/ψ rather than $g_\perp^{(v)}$ as the DA of the $\epsilon_\perp$ component since the latter does not vanish at the end point. Hence, we will treat the J/ψ wave function on the same footing as the B meson. Comparing Eq. (2.16) with Eq. (2.10), we see that at the leading order in $1/m_c$ one has

$$\Phi_1^{J/\psi}(\xi) = \Phi_\parallel^{J/\psi}(\xi) = \Phi_\perp^{J/\psi}(\xi), \quad f_{J/\psi}^T = f_{J/\psi}. \tag{2.17}$$

The inclusion of vertex-type corrections and hard spectator interaction in QCD factorization leads to

$$a_2^h = c_2 + \frac{c_1}{N_c} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_c}c_1 F^h,$$

$$a_3^h = c_3 + \frac{c_4}{N_c} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_c}c_4 F^h,$$

$$a_5^h = c_5 + \frac{c_6}{N_c} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_c}c_6(-F^h - 12),$$

$$a_7^h = c_7 + \frac{c_8}{N_c} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_c}c_8(-F^h - 12),$$

$$a_9^h = c_9 + \frac{c_{10}}{N_c} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_c}c_{10} F^h, \tag{2.18}$$

where $C_F = (N_c^2 - 1)/(2N_c)$ and the superscript h denotes the polarization of the vector mesons: $h=0$ for the helicity 0 state and $h=\pm$ for the helicity $\pm$ ones. In the naive dimensional regularization (NDR) scheme for γ_5, F^h in Eq. (2.18) has the form

$$F^h = -12\ln\frac{\mu}{m_b} - 18 + f_I^h + f_{II}^h, \tag{2.19}$$

where the hard scattering function f_I^h arises from vertex corrections [see Figs. 1(a)–1(d)] and f_{II}^h from the hard spectator interactions with a hard gluon exchange between the emitted vector meson and the spectator quark of the B meson, as depicted in Figs. 1(e) and 1(f).

C. Vertex corrections

The calculation of vertex corrections in Fig. 1 is very similar to that in $B \to J/\psi K$ decay and the detail can be found in [8]. In terms of the two hard kernels f_I and g_I given by

$$f_I = \int_0^1 d\xi\,\Phi_\parallel^{J/\psi}(\xi)\left\{\frac{2z\xi}{1-z(1-\xi)} + (3-2\xi)\frac{\ln\xi}{1-\xi} + \left(-\frac{3}{1-z\xi} + \frac{1}{1-z(1-\xi)} - \frac{2z\xi}{[(1-z(1-\xi)]^2}\right)z\xi\ln z\xi + \left(3(1-z) + 2z\xi + \frac{2z^2\xi^2}{1-z(1-\xi)}\right)\frac{\ln(1-z)-i\pi}{1-z(1-\xi)}\right\} + \int_0^1 d\xi\,\Phi_\perp^{J/\psi}(\xi)\left\{-4r\frac{\ln\xi}{1-\xi} + \frac{4zr\ln z\xi}{1-z(1-\xi)} - 4zr\frac{\ln(1-z)-i\pi}{1-z(1-\xi)}\right\} \tag{2.20}$$

and

$$g_I = \int_0^1 d\xi\,\Phi_\parallel^{J/\psi}(\xi)\left\{\frac{-4\xi}{(1-z)(1-\xi)}\ln\xi + \frac{z\xi}{[1-z(1-\xi)]^2}\ln(1-z) + \left(\frac{1}{(1-z\xi)^2} - \frac{1}{[1-z(1-\xi)]^2} + \frac{2(1+z-2z\xi)}{(1-z)(1-z\xi)^2}\right)z\xi\ln z\xi - i\pi\frac{z\xi}{[1-z(1-\xi)]^2}\right\} + \int_0^1 d\xi\,\Phi_\perp^{J/\psi}(\xi)\left\{\frac{4r}{(1-z)(1-\xi)}\ln\xi - \frac{4rz}{(1-z)(1-z\xi)}\ln z\xi\right\}, \tag{2.21}$$

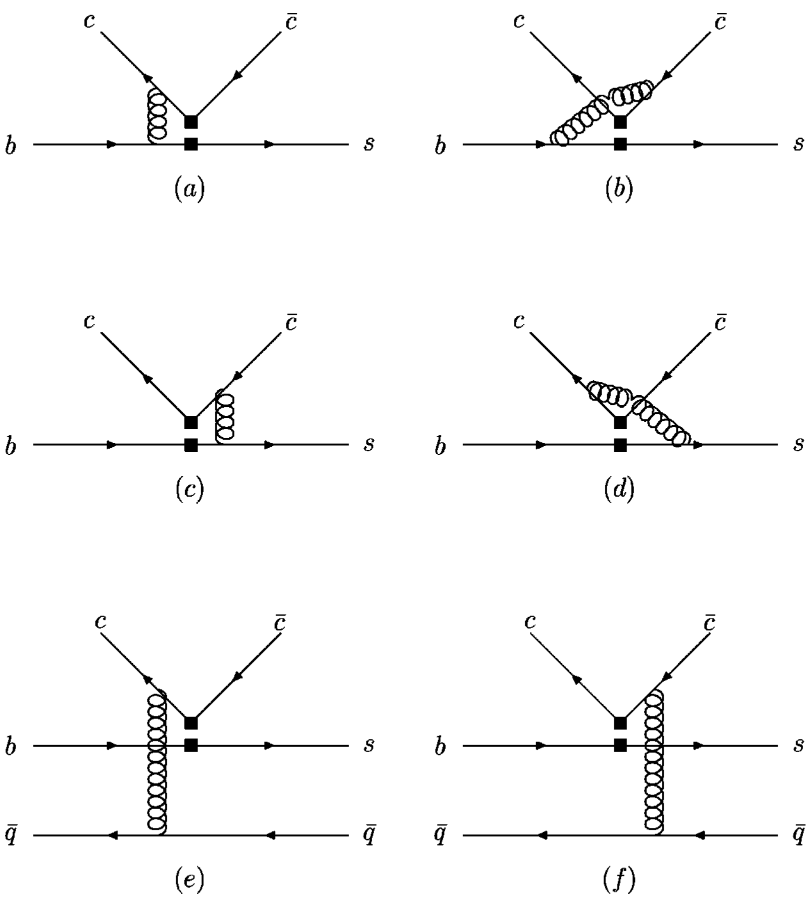

FIG. 1. Vertex and spectator corrections to $B \to J/\psi K^*$.

where $r = f^T_{J/\psi} m_c / (f_{J/\psi} m_{J/\psi})$ and $z \equiv m^2_{J/\psi}/m^2_B$, the first scattering function f^h_I induced from vertex corrections has the form

$$f_I^0 = f_I + g_I(1-z)\frac{A_0^{BK^*}(m^2_{J/\psi})}{\tilde{A}_3^{BK^*}(m^2_{J/\psi})},$$

$$f_I^{\pm} = f_I, \qquad (2.22)$$

where

$$\tilde{A}_3(q^2) = \frac{m_B + m_{K^*}}{2m_{K^*}} A_1(q^2) - \frac{m_B^2 - m_{J/\psi}^2 + m_{K^*}^2}{2m_{K^*}(m_B + m_{K^*})} A_2(q^2). \qquad (2.23)$$

In writing Eqs. (2.20) and (2.21), we have distinguished the contributions from $\Phi^{J/\psi}_{\parallel}$ and $\Phi^{J/\psi}_{\perp}$ for the reader's convenience, though later we will apply Eq. (2.17). Also notice that we have applied the relation [8][2]

$$r \equiv \frac{f^T_{J/\psi} m_c}{f_{J/\psi} m_{J/\psi}} = 2\left(\frac{m_c}{m_{J/\psi}}\right)^2 = 2\xi^2. \qquad (2.24)$$

Three remarks are in order. (i) As shown in [8], the transverse DA $\Phi^{J/\psi}_{\perp}$ contributes not only to the transverse amplitudes $H_{\pm}$ but also to the longitudinal amplitude H_0, and vice versa for the longitudinal DA $\Phi^{J/\psi}_{\parallel}$. This occurs because J/ψ is heavy: the coefficient in front of $\Phi_{\parallel}$ in Eq. (2.11) consists of not only the longitudinal polarization but also the transverse one. (ii) It is easily seen that in the zero J/ψ mass limit,

$$f_I^0 \to \int_0^1 d\xi\, \phi^{J/\psi}(\xi)\left(3\frac{1-2\xi}{1-\xi}\ln\xi - 3i\pi\right), \qquad (2.25)$$

[2]It is known from heavy quark effective theory (HQET) that below the $\overline{m}_c$ scale, where $\overline{m}_c$ is the pole mass of the charmed quark, the vector and tensor currents receive the same anomalous dimensions; that is, $f^T_{J/\psi}$ and $f_{J/\psi} m_c$ scale as the same power. Up to the m_b scale, f^T rescales with a factor $[\alpha_s(m_b)/\alpha_s(\overline{m}_c)]^{4/(3b)}$, m_c with $[\alpha_s(m_b)/\alpha_s(\overline{m}_c)]^{4/b}$, and the ratio of $f^T_{J/\psi}/f_{J/\psi}$ becomes $[\alpha_s(\overline{m}_c)/\alpha_s(m_b)]^{8/(3b)}$ $2m_c(m_b)/m_{J/\psi} = (1.1-1.2)\times 2m_c(m_b)/m_{J/\psi}$, where $b = (11N_c - 2n_f)/3$ and $m_c(m_b)$ is the running charmed quark mass at the m_b scale. However, the scale factor $[\alpha_s(\overline{m}_c)/\alpha_s(m_b)]^{8/(3b)} = 1.1-1.2$ is relatively small and can be neglected for our purposes.

in agreement with [12] for $B\to\pi\pi$, as it should be. (iii) The expression of $A_0/\tilde{A}_3$ in Eq. (2.22) can be further simplified by applying equations of motion. Neglecting the mass of light quarks, applying the equation $\bar{s}\not{p}_{K*}(1-\gamma_5)b=0$, and sandwiching it between the K^* and B states leads to the result

$$-\frac{m_{J/\psi}^2}{2m_Bm_{K*}}A_2(q^2)=A_3(q^2)-A_0(q^2) \qquad (2.26)$$

and hence $A_0^{BK*}(m_{J/\psi}^2)/\tilde{A}_3^{BK*}(m_{J/\psi}^2)=1$. Consequently, $f_I^0=f_I+g_I(1-z)$.

D. Hard spectator interactions

For hard spectator interactions, we write

$$f_{II}=f_{II(2)}+f_{II(3)}, \qquad (2.27)$$

where the subscript $(\cdots)$ denotes the twist dimension of the LCDA. To the leading-twist order, we obtain

$$f_{II(2)}^0=\frac{4\pi^2}{N_c}\frac{\alpha_s(\mu_h)}{\alpha_s(\mu)}\frac{f_Bf_{J/\psi}f_{K*}}{h_0}(1-z)\int_0^1 d\xi\, d\bar{\rho}\, d\bar{\eta}\,\Phi_1^B(\bar{\rho})\Phi^{J/\psi}(\xi)\Phi^{K*}(\bar{\eta})$$
$$\times\frac{\bar{\rho}-\bar{\eta}+(\bar{\rho}-2\xi+\bar{\eta})z+4\xi^2z}{\bar{\rho}(\bar{\rho}-\bar{\eta}+\bar{\eta}z)[(\bar{\rho}-\xi)(\bar{\rho}-\bar{\eta})+(\bar{\eta}\bar{\rho}-\bar{\eta}\xi-\bar{\rho}\xi)z]}. \qquad (2.28)$$

This can be further simplified by noting that $\bar{\rho}\sim O(\Lambda_{\rm QCD}/m_b)\to 0$ in the $m_b\to\infty$ limit. Hence,

$$f_{II(2)}^0=\frac{4\pi^2}{N_c}\frac{\alpha_s(\mu_h)}{\alpha_s(\mu)}\frac{f_Bf_{J/\psi}f_{K*}}{h_0}\int_0^1 d\bar{\rho}\frac{\Phi_1^B(\bar{\rho})}{\bar{\rho}}\int_0^1 d\xi\frac{\Phi^{J/\psi}(\xi)}{\xi}\int_0^1 d\bar{\eta}\frac{\Phi^{K*}(\bar{\eta})}{\bar{\eta}}, \qquad (2.29)$$

where the z terms in the numerator cancel after the integration over ξ via Eq. (2.24). Likewise, for transverse polarization states, we find

$$f_{II(2)}^{\pm}=-\frac{4\pi^2}{N_c}\frac{\alpha_s(\mu_h)}{\alpha_s(\mu)}\frac{2f_Bf_{J/\psi}f_{K*}^{\perp}m_{J/\psi}}{m_Bh_{\pm}}(1\pm1)\int_0^1 d\bar{\rho}\,d\xi\,d\bar{\eta}\,\Phi_1^B(\bar{\rho})\Phi^{J/\psi}(\xi)\Phi_{\perp}^{K*}(\bar{\eta})\frac{1-2\xi}{\bar{\rho}\bar{\eta}^2(1-z)}. \qquad (2.30)$$

Note that the hard gluon exchange in the spectator diagrams is not as hard as in the vertex diagrams. Since the virtual gluon's momentum squared there is $k^2=(-\bar{\rho}p_B+\bar{\eta}p_{K*})^2\approx-\bar{\rho}\bar{\eta}m_B^2\sim-\Lambda_hm_b$, where Λ_h is the hadronic scale $\sim$500 MeV, we will set $\alpha_s\approx\alpha_s(\sqrt{\Lambda_hm_b})$ in the spectator diagrams. The corresponding Wilson coefficients in the spectator diagrams are also evaluated at the $\mu_h=\sqrt{\Lambda_hm_b}$ scale. As for twist-3 contributions to hard spectator interactions, we find

$$f_{II(3)}^0=0 \qquad (2.31)$$

and

$$f_{II(3)}^{\pm}=\frac{4\pi^2}{N_c}\frac{\alpha_s(\mu_h)}{\alpha_s(\mu)}\frac{2f_Bf_{J/\psi}f_{K*}m_{J/\psi}m_{K*}}{m_B^2h_{\pm}}\int_0^1 d\bar{\rho}\frac{\Phi_1^B(\bar{\rho})}{\bar{\rho}}\int_0^1 d\xi\frac{\Phi^{J/\psi}(\xi)}{\xi}\int_0^1 d\bar{\eta}\left(\frac{g_{\perp}^{K*(v)}(\bar{\eta})}{\bar{\eta}(1-z)}\pm\frac{g_{\perp}^{K*(a)}(\bar{\eta})}{4\bar{\eta}^2(1-z)}\right). \qquad (2.32)$$

Since asymptotically $\Phi^{K*}(\bar{\eta})=6\bar{\eta}(1-\bar{\eta})$, the logarithmic divergence of the $\bar{\eta}$ integral in Eq. (2.29) implies that the spectator interaction is dominated by soft gluon exchanges between the spectator quark and the charmed or anticharmed quark of J/ψ. Hence, QCD factorization breaks down even at the twist-2 level for $f_{II(2)}^+$. Thus we will treat the divergent integral as an unknown "model" parameter and write

$$Y\equiv\int_0^1\frac{d\bar{\eta}}{\bar{\eta}}=\ln\left(\frac{m_B}{\mu_h}\right)(1+\rho_H), \qquad (2.33)$$

with ρ_H being a complex number whose phase may be caused by soft rescattering [12]. Note that linear divergences are canceled owing to the relation (2.24). Needless to say, how to treat the unknown parameter ρ_H is a major theoretical uncertainty in the QCD factorization approach.

E. Distribution amplitudes

If we apply the asymptotic form for the vector meson's LCDAs [13]

$$\Phi_\parallel^V(x)=\Phi_\perp^V(x)=g_\perp^{(a)}(x)=6x(1-x),$$
$$g_\perp^{(v)}(x)=\frac{3}{4}[1+(2x-1)^2], \tag{2.34}$$

it is easy to check that $f_{II(3)}^{-}=0$. Since the scale relevant to hard spectator interactions is of order $\mu_h=\sqrt{\Lambda_h m_b}\approx 1.5$ GeV, it is important to take into account the evolution of LCDAs from $\mu=\infty$ down to the lower scale. The leading-twist LCDA Φ_M can be expanded in terms of Gegenbauer polynomials $C_n^{3/2}$ [13]:

$$\Phi_M(x,\mu)=6x(1-x)\left(1+\sum_{n=1}^{\infty} a_{2n}^M(\mu)C_{2n}^{3/2}(2x-1)\right), \tag{2.35}$$

where the Gegenbauer moments a_n^M are multiplicatively renormalized. To $n=2$ we have

$$\Phi_\parallel^V(x,\mu)=6x(1-x)\left[1+3a_1^\parallel\xi+\frac{3}{2}a_2^\parallel(5\xi^2-1)\right], \tag{2.36}$$

$$\Phi_\perp^V(x,\mu)=6x(1-x)\left[1+3a_1^\perp\xi+\frac{3}{2}a_2^\perp(5\xi^2-1)\right],$$

where $\xi=2x-1$. For twist-3 DAs we follow [15] to use[3]

$$g_\perp^{(a)}(x,\mu)=6x(1-x)\left[1+a_1^\parallel\xi+\left\{\frac{1}{4}a_2^\parallel+\frac{5}{3}\zeta_3\left(1-\frac{3}{16}\omega_3^A+\frac{9}{16}\omega_3^V\right)\right\}(5\xi^2-1)\right]+6\delta_+[3x(1-x)+(1-x)\ln(1-x)+x\ln x]$$
$$+6\delta_-[(1-x)\ln(1-x)-x\ln x],$$
$$g_\perp^{(v)}(x,\mu)=\frac{3}{4}(1+\xi^2)+\frac{3}{2}a_1^\parallel\xi^3+\left(\frac{3}{7}a_2^\parallel+5\zeta_3\right)(3\xi^2-1)+\left[\frac{9}{112}a_2^\parallel+\frac{15}{64}\zeta_3(3\omega_3^V-\omega_3^A)\right](3-30\xi^2+35\xi^4)$$
$$+\frac{3}{2}\delta_+[2+\ln x+\ln(1-x)]+\frac{3}{2}\delta_-[2\xi+\ln(1-x)-\ln x], \tag{2.37}$$

where the Gegenbauer moments and couplings $\eta_3,\omega_3^{V,A},\delta_{+,-}$ for K^* at the scale $\mu^2=1$ GeV^2 and $\mu^2=5$ GeV^2 can be found in [15]. It turns out that the end-point behavior of $g_\perp^{(v)}$ for K^* is substantially modified and is very different from that of the asymptotic form (see Fig. 3 of [14]).

III. EXPERIMENTS

The angular analysis of $B^+\to J/\psi K^{*+}$ and $B^0\to J/\psi K^{*0}$ has been carried out by CDF [3], CLEO [4], and most recently by the B factories BaBar [5] and Belle [6]. The three polarized amplitudes are measured in the transversity basis with results summarized in Table IV below. Experimental results are conventionally expressed in terms of spin amplitudes $\hat{A}_{0,\perp,\parallel}$ normalized to unity, $|\hat{A}_0|^2+|\hat{A}_\perp|^2+|\hat{A}_\parallel|^2=1$. Since the measurement of interference terms in the angular distribution is limited to $\mathrm{Re}(A_\parallel A_0^*)$, $\mathrm{Im}(A_\perp A_0^*)$, and $\mathrm{Im}(A_\perp A_\parallel^*)$, there exists a phase ambiguity,

$$\phi_\parallel\to-\phi_\parallel,$$
$$\phi_\perp\to\pm\pi-\phi_\perp, \tag{3.1}$$
$$\phi_\perp-\phi_\parallel\to\pm\pi-(\phi_\perp-\phi_\parallel).$$

Take the BaBar measurement [5] as an example :

$$\phi_\perp=-0.17\pm0.17,\qquad \phi_\parallel=2.50\pm0.22,$$
$$\Rightarrow|H_+|<|H_-|, \tag{3.2}$$

where the phases are measured in radians. The other allowed solution is

$$\phi_\perp=-2.97\pm0.17,\qquad \phi_\parallel=-2.50\pm0.22,$$
$$\Rightarrow|H_+|>|H_-|. \tag{3.3}$$

TABLE II. Form factors $A_1^{BK^*}$, $A_2^{BK^*}$, and V^{BK^*} at $q^2=0$ and $q^2=m_{J/\psi}^2$ in various form-factor models.

	BSWI	BSWII	LF	NS	Yang	BB	MS	YYK
$A_1^{BK^*}(0)$	0.33	0.33	0.26	0.30	0.18	0.34	0.36	0.49
$A_1^{BK^*}(m_{J/\psi}^2)$	0.45	0.45	0.37	0.39	0.24	0.43	0.43	0.49
$A_2^{BK^*}(0)$	0.33	0.33	0.24	0.30	0.17	0.28	0.32	0.30
$A_2^{BK^*}(m_{J/\psi}^2)$	0.46	0.63	0.43	0.48	0.31	0.45	0.50	0.42
$V^{BK^*}(0)$	0.37	0.37	0.35	0.30	0.21	0.46	0.44	0.39
$V^{BK^*}(m_{J/\psi}^2)$	0.55	0.82	0.42	0.51	0.40	0.86	0.77	0.87

[3]Note that there is a slight difference for the expressions of $g_\perp^{(v,a)}$ in [15] and [14].

As pointed out in [11], the solution (3.2) indicates that $A_\parallel$ has a sign opposite to that of $A_\perp$ and hence $|H_+|<|H_-|$, in contradiction to what is expected from factorization. Therefore, we will compare solution (3.3) with the factorization approach. Obviously there is a 3-σ effect that $\phi_\parallel$ is different from π and this agrees with the CDF measurement. However, such an effect is not observed by Belle and CLEO (see Table IV). In Table IV, we will only list those amplitude phases from solution (3.3).

The measured branching ratios are

$$\mathcal{B}(B^+\to J/\psi K^{*+}) = \begin{cases} (13.7\pm 0.9\pm 1.1)\times 10^{-4} & \text{BaBar [5]} \\ (12.9\pm 0.8\pm 1.2)\times 10^{-4} & \text{Belle [6]} \\ (14.1\pm 2.3\pm 2.4)\times 10^{-4} & \text{CLEO [4]} \end{cases} \tag{3.4}$$

and

$$\mathcal{B}(B^0\to J/\psi K^{*0}) = \begin{cases} (12.4\pm 0.5\pm 0.9)\times 10^{-4} & \text{BaBar [5]} \\ (12.5\pm 0.6\pm 0.8)\times 10^{-4} & \text{Belle [6]} \\ (13.2\pm 1.7\pm 1.7)\times 10^{-4} & \text{CLEO [4].} \end{cases} \tag{3.5}$$

IV. NUMERICAL RESULTS

To proceed, we use the next-to-leading Wilson coefficients in the NDR scheme [16],

$$\begin{aligned} &c_1=1.082, \quad c_2=-0.185, \quad c_3=0.014, \\ &c_4=-0.035, \quad c_5=0.009, \quad c_6=-0.041, \\ &c_7/\alpha=-0.002, \quad c_8/\alpha=0.054, \quad c_9/\alpha=-1.292, \\ &c_{10}/\alpha=0.263, \quad c_g=-0.143, \end{aligned} \tag{4.1}$$

at $\mu=\overline{m}_b(m_b)=4.40$ GeV for $\Lambda^{(5)}_{\overline{\rm MS}}=225$ MeV taken from Table XXII of [16], with α being an electromagnetic fine-structure coupling constant. For the decay constants, we use

$$f_{K^*}=221\,\text{MeV}, \quad f_{J/\psi}=405\,\text{MeV}, \quad f_B=190\,\text{MeV}, \tag{4.2}$$

and we will assume $f_V^T=f_V$ for the tensor decay constant. For LCDAs we use those in Sec. II E and the B meson wave function,

$$\Phi_1^B(\overline{\rho})=N_B\overline{\rho}^2(1-\overline{\rho})^2\exp\left[-\frac{1}{2}\left(\frac{\overline{\rho}m_B}{\omega_B}\right)^2\right], \tag{4.3}$$

with $\omega_B=0.25$ GeV and N_B being a normalization constant.

In the following study, we will consider eight distinct form-factor models: the Bauer-Stech-Wirbel (BSWI) model [17,18], the modified BSW model (referred to as the BSWII model) [19], the relativistic light-front (LF) quark model [20], the Neubert-Stech (NS) model [21], the QCD sum-rule calculation by Yang [22], the Ball-Braun (BB) model based on the light-cone sum-rule analysis [1], the Melikhov-Stech (MS) model based on the constituent quark picture [2], and the Isgur-Wise scaling laws based on the SU(2) heavy quark symmetry (YYK) so that the form factor A_1 is mostly flat, A_2 is a monopole-type form factor, and V is a dipole-type one [23]. The values of the form factors $A_1^{BK^*}$, $A_2^{BK^*}$, and V^{BK^*} at $q^2=0$ and $q^2=m^2_{J/\psi}$ in various form-factor models are shown in Table II.

Among the eight form-factor models, only a few of them are consistent with the lattice calculations at large q^2, the constraint from $B\to\phi K^*$ at low q^2, and the constraint from heavy quark symmetry for the form-factor q^2 dependence. The BSWI model assumes a monopole behavior (i.e., $n=1$) for all the form factors. However, this is not consistent with heavy quark symmetry for the heavy-to-heavy transition. The BSWII model takes the BSW model results for the form factors at zero momentum transfer but makes a different ansatz for their q^2 dependence, namely a dipole behavior (i.e., $n=2$) is assumed for the form factors F_1, A_0, A_2, V, motivated by heavy quark symmetry, and a monopole dependence for F_0, A_1. However, the equality of the form factors $A_1^{BK^*}$ and $A_2^{BK^*}$ at $q^2=0$ is ruled out by recent measurements of $B\to\phi K^*$ decays [7]. Lattice calculations of V^{BK^*}, $A_0^{BK^*}$, and $A_1^{BK^*}$ at large q^2 [24] in conjunction with reasonable extrapolation to $q^2=m^2_{J/\psi}$ indicate that $V^{BK^*}(m^2_{J/\psi})$ is of order 0.70–0.80.

The parameters $\tilde{a}^h(J/\psi K^*)$ defined by

$$\tilde{a}^h(J/\psi K^*)=a_2^h+a_3^h+a_5^h+a_7^h+a_9^h \tag{4.4}$$

are calculated using Eq. (2.18) and their results are shown in Table III. Since the penguin parameters $a^h_{3,5,7,9}$ are small, in practice we have $\tilde{a}^h\approx a_2^h$. Note that $\tilde{a}_2^0$ and $\tilde{a}_2^-$ are independent of the parameter ρ_H introduced in Eq. (2.33); that is, they are infrared-safe. Since h_- is quite small due to the compensation between the $A_1^{BK^*}$ and $V_1^{BK^*}$ terms and $f^{\pm}_{II(3)}$ is inversely proportional to h_-, $\tilde{a}^-$ becomes more sensitive than $\tilde{a}^+$ to the form-factor model chosen.

From the experimental measurement of spin amplitudes, it is possible to extract the parameters $\tilde{a}^h$ in various form-factor models. We use the averaged decay rate $\Gamma(B\to J/\psi K^*)=(5.34\pm 0.23)\times 10^{-16}$ GeV obtained from Eqs. (3.4) and (3.5) and the central values of the spin amplitudes measured by BaBar [5] as an illustration,

$$\begin{aligned} |\hat{A}_0|^2&=0.597\pm 0.028\pm 0.024, \\ |\hat{A}_\perp|^2&=0.160\pm 0.032\pm 0.014, \\ |\hat{A}_\parallel|^2&=0.243\pm 0.034\pm 0.017. \end{aligned} \tag{4.5}$$

Then $\tilde{a}^0$ can be determined from $\Gamma_L(B\to J/\psi K^*)=\Gamma(B\to J/\psi K^*)\times|\hat{A}_0|^2$ and likewise for $\tilde{a}^\pm$. The results are shown in Table III. It is evident that the ‘‘experimental’’ val-

TABLE III. The calculated parameters $\tilde{a}^h(J/\psi K^*)$ $(h=0,+,-)$ for $B\to J/\psi K^*$ decay in QCD factorization using various form-factor models for the B-K^* transition. The experimental results for $\tilde{a}^h(J/\psi K^*)$ are obtained using the averaged branching ratio of $B\to J/\psi K^*$ measured by BaBar, Belle, and CLEO in conjunction with the central values of the BaBar measurement for the spin amplitudes $|\hat{A}_{0,\perp,\parallel}|^2$. Only the central values of $\tilde{a}^h_{\rm expt}$ are shown here.

	$\tilde{a}^0$	$\lvert\tilde{a}^0\rvert_{\rm expt}$	$\tilde{a}^+$	$\lvert\tilde{a}^+\rvert_{\rm expt}$	$\tilde{a}^-$	$\lvert\tilde{a}^-\rvert_{\rm expt}$
BSWI	$0.11-i0.06$	0.19	$0.16-i0.05$	0.18	$-0.01+i0.05$	0.06
BSWII	$0.15-i0.06$	0.25	$0.14-i0.05$	0.15	$-0.07+i0.05$	0.14
LF	$0.14-i0.06$	0.25	$0.19-i0.05$	0.23	$-0.02+i0.05$	0.07
NS	$0.14-i0.06$	0.25	$0.18-i0.05$	0.20	$-0.03+i0.05$	0.08
Yang	$0.23-i0.06$	0.43	$0.25-i0.05$	0.30	$-0.16+i0.05$	0.20
BB	$0.12-i0.06$	0.20	$0.14-i0.05$	0.16	$-0.15+i0.05$	0.23
MS	$0.13-i0.06$	0.22	$0.14-i0.05$	0.16	$-0.07+i0.05$	0.14
YYK	$0.09-i0.06$	0.16	$0.13-i0.05$	0.15	$-0.06+i0.05$	0.12

ues of $\tilde{a}^h$ are polarization-dependent, $|\tilde{a}^0|>|\tilde{a}^+|>|\tilde{a}^-|$, whereas the present QCD factorization calculation yields $|\tilde{a}^+|>|\tilde{a}^0|>|\tilde{a}^-|$.

Normalized spin amplitudes and their phases in $B\to J/\psi K^*$ decays calculated in various form-factor models using QCD factorization are exhibited in Table IV, where the unknown parameter ρ_H in Eq. (2.33) is taken to be real and unity. For comparison, we also carry out the analysis in the partial wave basis as the phases of S, P, and D partial wave amplitudes are the ones directly related to the long-range final-state interactions. We see from Table V that the predicted $|\hat{A}_0|^2$, $|D|^2$, and branching ratios are too small, whereas $|\hat{A}_\perp|^2=|P|^2$ is too large. It is also clear that a nontrivial phase $\phi_\parallel$ deviated from $-\pi$ is seen in some form-factor models, but it is still too small compared to the BaBar measurement. Nevertheless, a large phase $\phi_\parallel$ as implied by BaBar can be achieved by adjusting the phase of the complex parameter ρ_H, but admittedly it is rather arbitrary. In other words, the present QCD factorization calculation cannot say something definite for the phase $\phi_\parallel$. The partial wave decompositions S, P, and D corresponding to the relative orbital angular momentum $L=0,1,2$ between J/ψ and K^* uniquely determine the spin angular momentum. Our results are difficult to reconcile with the observation $|S|^2:|D|^2:|P|^2\simeq 3.5:1:1$ from recent Babar and Belle measurements.

There are several major theoretical uncertainties in the calculation: B-K^* form factors, the twist-3 LCDAs of K^* at the scale μ_h, and the infrared divergences occurring in twist-2 and twist-3 contributions. It has been advocated that Sudakov form-factor suppression may alleviate the soft divergence [25]. Hence, we have studied Sudakov effects explicitly and the detailed results will be presented in a future publication. When partons in the meson carry the transverse momentum through the exchange of gluons, the Sudakov suppression effect will be naturally generated due to large double logarithms $\exp[-(\alpha_s C_F/4\pi)\ln^2(Q^2/k_\perp^2)]$, which will suppress the long-distance contributions in the small $k_\perp$ region and give a sizable average $\langle k_\perp^2\rangle \sim \bar{\Lambda} m_B$, where $\bar{\Lambda}=m_B-m_b$. This can resolve the singularity problem occurring at the end point. Basically, there is no Sudakov suppression in the vertex correction since the end-point singularity

TABLE IV. Normalized spin amplitudes and their phases (in radians) in $B\to J/\psi K^*$ decays calculated in various form-factor models using QCD factorization. The branching ratios given in the table are for $B^+\to J/\psi K^{*+}$. For comparison, experimental results from CDF, CLEO, BaBar, and Belle are also exhibited.

	$\lvert\hat{A}_0\rvert^2$	$\lvert\hat{A}_\perp\rvert^2$	$\lvert\hat{A}_\parallel\rvert^2$	$\phi_\perp$	$\phi_\parallel$	$\mathcal{B}(10^{-3})$
BSWI	0.43	0.33	0.24	-3.05	-2.89	0.76
BSWII	0.38	0.36	0.26	3.13	-3.12	0.73
LF	0.41	0.34	0.25	-3.09	-2.95	0.69
NS	0.40	0.34	0.25	-3.10	-2.99	0.70
Yang	0.38	0.36	0.25	-3.12	-3.11	0.64
BB	0.41	0.34	0.25	-3.04	-3.05	0.77
MS	0.40	0.35	0.25	-3.08	-3.05	0.75
YYK	0.44	0.32	0.23	-2.99	-2.95	0.84
CLEO [4]	0.52 ± 0.08	0.16 ± 0.09	0.32 ± 0.12	-3.03 ± 0.46	-3.00 ± 0.37	1.41 ± 0.31
CDF [3]	0.59 ± 0.06	$0.13^{+0.13}_{-0.11}$	0.28 ± 0.12	-2.58 ± 0.54	-2.20 ± 0.47	
BaBar [5]	0.60 ± 0.04	0.16 ± 0.03	0.24 ± 0.04	-2.97 ± 0.17	-2.50 ± 0.22	1.37 ± 0.14
Belle [6]	0.60 ± 0.05	0.19 ± 0.06	0.21 ± 0.08	-3.15 ± 0.21	-2.86 ± 0.25	1.29 ± 0.14

TABLE V. Normalized partial wave amplitudes and their phases (in radians) in $B \to J/\psi K^*$ decays calculated in various form-factor models using QCD factorization and fitted from the data, where $\phi_P = \arg(PS^*), \phi_D = \arg(DS^*)$, and there exists a phase ambiguity: $\phi_D \to -\phi_D$ and $\phi_P \to \pm\pi - \phi_P$.

	$\|S\|^2$	$\|P\|^2$	$\|D\|^2$	ϕ_P	ϕ_D
BSWI	0.60	0.33	0.07	-0.04	2.75
BSWII	0.60	0.36	0.04	0.02	3.10
LF	0.60	0.34	0.06	-0.05	2.80
NS	0.60	0.34	0.06	-0.05	2.86
Yang	0.59	0.36	0.05	0.002	3.07
BB	0.60	0.34	0.06	0.05	2.99
MS	0.60	0.35	0.05	0.01	2.97
YYK	0.60	0.32	0.08	0.05	2.85
CLEO [4]	0.77 ± 0.19	0.16 ± 0.09	0.07 ± 0.03	0.04 ± 0.59	2.9 ± 0.59
CDF [3]	0.61 ± 0.34	$0.13^{+0.13}_{-0.11}$	0.26 ± 0.20	0.10 ± 0.34	2.17 ± 0.34
BaBar [5]	0.65 ± 0.13	0.16 ± 0.03	0.19 ± 0.10	-0.13 ± 0.21	2.44 ± 0.21
Belle [6]	0.66 ± 0.14	0.19 ± 0.06	0.15 ± 0.03	-0.14 ± 0.29	2.80 ± 0.29

in the hard kernel is canceled in the convolution. However, for the hard spectator interaction, we can have large Sudakov suppression effects at the end point since there are sizable $\langle k_\perp^2\rangle$ contributions in the propagators. Especially, the end-point singularities without $k_\perp$ do not compensate in the twist-3 contributions. We find that $\tilde{a}_2^0$ is suppressed whereas $\tilde{a}_2^-$ is enhanced by the Sudakov effect, and we conclude that Sudakov suppression cannot help to solve the discrepancy between theory and experiment.

V. DISCUSSIONS AND CONCLUSIONS

The hadronic decay $B \to J/\psi K^*$ is analyzed within the framework of QCD factorization. The spin amplitudes A_0, $A_\parallel$, and $A_\perp$ in the transversity basis and their relative phases are studied using various different form-factor models for the B-K^* transition. The effective parameters a_2^h for helicity $h = 0,+,-$ states receive different nonfactorizable contributions and hence they are helicity-dependent, contrary to naive factorization, where a_2^h are universal and polarization-independent. QCD factorization breaks down even at the twist-2 level for transverse hard spectator interactions. Although a nontrivial strong phase for the $A_\parallel$ amplitude can be achieved by adjusting the phase of an infrared divergent contribution, the present QCD factorization calculation cannot say anything definite about the phase $\phi_\parallel$. In QCD factorization we found that a_2^0 and a_2^- are infrared safe.

Unfortunately, our conclusion is somewhat negative. The longitudinal parameter a_2^0 calculated by QCD factorization, which is of order 0.15 in magnitude, is not large enough to account for the observed decay rates and the fraction of longitudinal polarization. In QCD factorization, the ratio R of vector meson to pseudoscalar production is close to unity with large uncertainties arising from the chirally enhanced and infrared-sensitive contributions to $B \to J/\psi K$ [8]. (In the naive factorization approach, R ranges from 1.3 to 4.2 [26], but it is difficult to account for R, Γ_L/Γ, and $|P|^2$ simultaneously.) This is mainly ascribed to the smallness of a_2^0. It is instructive to compare $a_2^0(J/\psi K^*)$ in $B \to J/\psi K^*$ decay with $a_2(J/\psi K)$ in $B \to J/\psi K$. It is found in [8] that $a_2(J/\psi K) = 0.19^{+0.14}_{-0.12}$ for $|\rho_H| \leq 1$ and that twist-2 as well as twist-3 hard spectator interactions are equally important. As for $a_2^0(J/\psi K^*)$, it is dominated by twist-2 hard spectator interactions. We have studied Sudakov form-factor suppression on end-point singularities and found that it does not help to solve the discrepancy between theory and experiment.

Since the predicted a_2^0 in QCD factorization is too small compared to experiment, one may explore other effects that have not been studied. One possibility is that soft final-state interactions (FSIs) may enhance a_2^0 substantially [27]. A recent observation of $\overline{B^0} \to D^{0(*)}\pi^0$ decay by Belle [28] and CLEO [29] indicates $a_2(D^{(*)}\pi) \sim 0.40-0.55$ much larger than the naive value of order 0.25. It is thus conceivable that some sort of inelastic FSIs could make substantial nonperturbative contributions to a_2^0. The other possibility arises from the gluon component in the K^* wave function. Consider the diagram in which one of outgoing charmed quarks emits a hard gluon before they form the J/ψ meson and the gluon fragments into a parton of the K^* meson. Neglecting the charmed quark mass, because the charmed quark's helicity is conserved in the strong interaction, this gluon has zero helicity, i.e., it is longitudinally polarized. Following the same argument right after Eq. (2.2), the hybrid K^* will make a contribution to H_0 and H_+. Although this amplitude is suppressed by order of $\Lambda_{\rm QCD}/m_b$ owing to the presence of an additional propagator compared to the leading diagram, it is enhanced by the large Wilson coefficient c_1 and hence cannot be ignored. A similar mechanism can also give a contribution to the $B \to J/\psi K$ mode but it is difficult to make a quantitative estimate since the chirally enhanced twist-3 contribution is still quite uncertain. Good candidates to search for evidence of this effect are $B \to \rho^0\rho^0, \rho^0\omega, \omega\omega$. Without taking into account the hard gluon emission, the branching ratios of these decays which are color-suppressed and dominated by $b \to d$ penguin contributions are of order 10^{-7} [30,31,7]. Nevertheless, they can receive large contributions, proportional to c_1 at the amplitude level, from the hard gluon emission mechanism so that the branching ratios become 10^{-6}–10^{-5}.

Note added. We learned of the paper by X.S. Nguyen and X.Y. Pham (NP) (Ref. [32]) in which a similar analysis in QCD factorization was carried out. However, their results differ from ours in some aspects: (i) There are some discrepancies between Eqs. (2.20)–(2.22) in the present paper and Eqs. (36) and (37) of NP. Also the expression of $F^{\pm}_{II}$ given by Eq. (39) of NP originally derived in [7] is valid only for two *light* vector mesons in the final state. It will undergo some modifications for heavy J/ψ. It should be stressed that Eq. (28) adopted by NP for describing LCDAs works only for a light vector meson, but not for a heavy meson such as J/ψ. (ii) For hard spectator interactions, we have considered contributions from leading wave functions of B and J/ψ and twist-3 DAs of K^* [see Eq. (2.32)], which are absent in NP. Also we have taken into account the relevant scale $\mu_h = \sqrt{\Lambda_h m_b}$ for hard spectator interactions. (iii) Unlike NP, we did not consider the higher twist expansion for the J/ψ wave function. The twist expansion of LCDAs is applicable for light mesons but it is problematic for heavy mesons such as J/ψ. Note that although twist-3 J/ψ contributions to hard spectator interactions were considered by NP, they did not consistently compute the twist-3 effects of J/ψ in vertex corrections.

ACKNOWLEDGMENTS

H.Y.C. wishes to thank the Physics Department, Brookhaven National Laboratory and C.N. Yang Institute for Theoretical Physics at SUNY Stony Brook for their hospitality. Y.Y.K. would like to thank the Physics Department, KIAS in Korea and C.W. Kim for their hospitality. This work was supported in part by the National Science Council of R.O.C. under Grant Nos. NSC90-2112-M-001-047, NSC90-2112-M-033-004, and NSC90-2811-M-001-019.

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