Fermionic signature of the lattice monopoles

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We consider fermions in the field of static monopolelike configurations in Euclidean space-time. In all the cases considered, there exists an infinite number of zero modes, labeled by a frequency $i\omega$. The existence of such modes is a manifestation of the instability of the vacuum in the presence of the monopoles and massless fermions. In Minkowski space, the corresponding phenomenon is well known and is a cornerstone of the theory of magnetic catalysis. Moreover, the well-known zero mode of Jackiw and Rebbi corresponds to the limiting case $\omega = 0$. We provide arguments as to why the chiral condensate could be linked to the density of the monopoles in the infrared cluster. A mechanism which can naturally explain the equivalence of the critical temperatures for the deconfinement and chiral transitions is proposed. We discuss the possible implications for the phenomenology of the lattice monopoles.

I. INTRODUCTION

The condensation of monopoles is widely believed to be the confinement mechanism. It is natural then to try to reduce chiral symmetry breaking to monopole physics as well. There are many numerical investigations of the possible connection between monopoles and chiral symmetry breaking, see, e.g., [1]. On the theoretical side, the analysis proceeds usually along the lines of the Banks-Casher criterion [2], which relates chiral symmetry breaking to the density of zero fermionic modes in a given bosonic background. In the case of the monopole-dominated vacuum, the elementary bosonic configuration is usually assumed to be a monopole-antimonopole pair. The reason is that in the field of a monopole-antimonopole pair, there exist normalizable zero modes studied first in Ref. [3].

So far, the properties of the fermions on the lattice have been studied mostly in the quenched approximation. However, detailed measurements with dynamical fermions are imminent, see, e.g., [4]. In view of this, it is worth reexamining, the problem of fermions in a monopole-dominated vacuum. In particular, we feel that it is important to consider in more detail fermions in the field of a single monopole, not of a monopole-antimonopole pair. Indeed, if the QCD vacuum were dominated by magnetic dipoles, the standard explanation of confinement would not work. We consider three monopolelike configurations which were introduced earlier. We demonstrate that in all three cases there exists an infinite number of solutions to the $d=4$ Dirac equation $D_\mu \gamma_\mu \psi(i\omega) = 0$ in the Euclidean space-time labeled with an imaginary frequency $i\omega$. The solutions signal the instability of the fermionic vacuum in the presence of magnetic monopoles. The instability is well known in the language of Minkowski space and is the starting point of the theory of monopole catalysis [5–7].

The instability of the fermionic vacuum in the presence of monopoles implies, in fact, the inconsistency of the quenched approximation to study chiral symmetry breaking

in the monopole-dominated vacuum. We will comment on this in the final section.

II. FERMIONIC MODES

A. Equations

In this section, we rewrite the formalism of Jackiw and Rebbi [8] in the Euclidean space-time. We will study solutions of the Dirac equation,

$$\gamma_\mu \left(i\partial_\mu - \tfrac{1}{2} \tau^a A_\mu^a \right) \Psi = 0, \qquad (1)$$

where τ^a are the isospinor Pauli matrices and γ_μ are the four-dimensional (4D) Euclidean Dirac matrices:

$$\gamma_0 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \quad \vec{\gamma} = \begin{pmatrix} 0 & i\vec{\sigma} \\ -i\vec{\sigma} & 0 \end{pmatrix},$$

σ_α are the Pauli matrices, and 1 is the 2×2 unit matrix.

Moreover, we will consider the static monopole gauge fields defined as

$$A_0^a = n^a \Phi(r), \quad A_i^a = \varepsilon^{aik} n^k A(r), \qquad (2)$$

where $n^i = x^i/r$ and $r^2 = x_i^2$. At large distances,

$$\Phi(\infty) = \eta \geqslant 0, \quad \lim_{r \to \infty} rA(r) = -1. \qquad (3)$$

The case $\Phi \equiv 0$ and $A = -1/r$ at all the distances corresponds to the pointlike Wu-Yang monopole [9]. Note that all the exact monopole solutions of the pure Yang-Mills equations are gauge copies of this monopole. We will consider also fields regular at origin, $A(0) = 0$.

We look for solutions of the Dirac equation (1) of the following form:

$$\Psi_k(t,\vec{x};\omega)=\frac{1}{\sqrt{2\pi}}e^{-i\omega t}\varphi_k(\vec{x};\omega),$$

where ω and k are real, continuous, and discrete parameters, respectively. The solutions obey the normalization condition

$$\int dt\int d^3x\Psi_k^\dagger(t,\vec{x};\omega)\Psi_{k'}(t,\vec{x};\omega')=\delta(\omega-\omega')\delta_{k,k'},$$

$$(4)$$

and the three-dimensional component of the zero mode is normalized as follows:

$$\int d^3x\varphi_k^\dagger(t,\vec{x};\omega)\varphi_{k'}(t,\vec{x};\omega)=\delta_{k,k'}.\qquad(5)$$

The upper and lower components of the three-dimensional spinor field are denoted as

$$\varphi(\vec{x})=\begin{pmatrix}\chi^+(\vec{x})\\\chi^-(\vec{x})\end{pmatrix}.\qquad(6)$$

Following Ref. [8], we regard the fields $\chi_{i\alpha}^\pm$ as matrices. The spinor Pauli matrices act on this field as the matrix multiplication, $(\vec{\sigma}\chi^\pm)_{i\alpha}=\vec{\sigma}_{ij}\chi_{j\alpha}^\pm$, while the isospinor Pauli matrices act as follows: $(\vec{\tau}\chi^\pm)_{i\alpha}=\vec{\tau}_{\alpha\beta}\chi_{j\beta}^\pm\equiv(\chi^\pm\vec{\tau}^{tr})_{i\alpha}$, where $\vec{\tau}^{tr}$ is the transposed matrix, and $\vec{\tau}^{tr}=\varepsilon\vec{\tau}\varepsilon$ with $\varepsilon=i\tau^2$ being the totally antisymmetric tensor in two dimensions. In these notations, the Dirac equation (1) becomes

$$(-\omega M^\pm-\tfrac{1}{2}A_0^aM^\pm\sigma^a)\pm\left[(\vec{\sigma},\vec{\partial})M^\pm-\frac{i}{2}A_i^a\sigma_iM^\pm\sigma^a\right]=0,$$

$$(7)$$

$$M^\pm=\chi^\pm\varepsilon=g^\pm\cdot\mathbf{1}+g_a^\pm\cdot\sigma^a,\qquad(8)$$

where $g^\pm$ and $g_a^\pm$ are unknown functions used to parametrize the matrix $M^\pm$. We are looking for spherically symmetric s-wave solutions of the equations, $g^\pm=g^\pm(r)$ and $g_a^\pm=f^\pm(r)n_a$, where $f^\pm$ are scalar functions. Substituting this ansatz into Eq. (7), we get two sets of differential equations:

$$K^\pm f^\pm\mp\omega g^\pm=0,\quad D^\pm g^\pm\mp\omega f^\pm=0,\qquad(9)$$

where

$$K^\pm=\frac{\partial}{\partial r}+\frac{2}{r}+A(r)\mp\frac{1}{2}\Phi(r),$$

$$D^\pm=\frac{\partial}{\partial r}-A(r)\mp\frac{1}{2}\Phi(r).\qquad(10)$$

We are mainly interested in the nonzero frequency case, $\omega\neq0$, so that the functions $f^\pm$ and $g^\pm$ are related to each other:

$$f^\pm=\pm\frac{1}{\omega}D^\pm g^\pm.\qquad(11)$$

Four first-order equations (9) are reduced then to two differential equations of the second order:

$$K^\pm D^\pm g^\pm-\omega^2 g^\pm=0.\qquad(12)$$

The solutions of these equations are discussed in the next subsection.

B. Zero-mode solutions

The normalizable fermionic modes can be readily found in case of the pointlike $\mathbb{Z}_2$ Wu-Yang monopole:

$$\Psi_L(r;\omega)=\frac{|\omega|^{1/2}}{4\pi r}\begin{pmatrix}[1-\mathrm{sgn}(\omega)(\vec{\sigma},\vec{n})]\varepsilon\\0\end{pmatrix}e^{-i\omega t-|\omega|r},$$

$$\Psi_R(r;\omega)=\frac{|\omega|^{1/2}}{4\pi r}\begin{pmatrix}0\\[1+\mathrm{sgn}(\omega)(\vec{\sigma},\vec{n})]\varepsilon\end{pmatrix}e^{-i\omega t-|\omega|r},$$

$$(13)$$

where $\omega\in(-\infty,+\infty)$.

Next, let us consider a nonvanishing A_0 component of the gauge field [19] $\Phi\equiv2\mu$. The zero modes can again be found explicitly and the functional form of the solutions depends on the value of ω:

$$\omega\leq-\mu\quad\begin{aligned}\Psi_L(r;\omega)&=\frac{\sqrt{-\mu-\omega}}{4\pi r}\begin{pmatrix}[1+(\vec{\sigma},\vec{n})]\varepsilon\\0\end{pmatrix}e^{-i\omega t-(\omega+\mu)r},\\[2mm]\Psi_R(r;\omega)&=\frac{\sqrt{\mu-\omega}}{4\pi r}\begin{pmatrix}0\\[1-(\vec{\sigma},\vec{n})]\varepsilon\end{pmatrix}e^{-i\omega t-(\omega-\mu)r},\end{aligned}\qquad(14)$$

$$-\mu<\omega<\mu\quad\begin{aligned}\Psi_R^{(1)}(r;\omega)&=\frac{\sqrt{\mu+\omega}}{4\pi r}\begin{pmatrix}0\\[1+(\vec{\sigma},\vec{n})]\varepsilon\end{pmatrix}e^{-i\omega t-(\omega+\mu)r},\\[2mm]\Psi_R^{(2)}(r;\omega)&=\frac{\sqrt{\mu-\omega}}{4\pi r}\begin{pmatrix}0\\[1-(\vec{\sigma},\vec{n})]\varepsilon\end{pmatrix}e^{-i\omega t-(\mu-\omega)r},\end{aligned}\qquad(15)$$

$$\Psi_L(r;\omega) = \frac{\sqrt{\omega-\mu}}{4\pi r}\begin{pmatrix} [1-(\vec{\sigma},\vec{n})]\varepsilon \\ 0 \end{pmatrix} e^{-i\omega t-(\omega-\mu)r},$$

$$\omega > \mu \tag{16}$$

$$\Psi_R(r;\omega) = \frac{\sqrt{\omega+\mu}}{4\pi r}\begin{pmatrix} 0 \\ [1+(\vec{\sigma},\vec{n})]\varepsilon \end{pmatrix} e^{-i\omega t-(\omega+\mu)r}.$$

Both cases considered so far correspond to monopoles of zero size. A famous example of a monopole with a core of finite size is provided by the 't Hooft–Polyakov monopole [10], which involves also a Higgs field. In the case of QCD, the role of the Higgs field is rather commonly ascribed to the A_0 component of the gauge field. In particular, such field configurations were considered in Ref. [3] in connection with the monopole physics and chiral symmetry breaking.

For analytical studies, it is convenient to consider the so-called Bogomol'nyi limit [11] where the field configuration is known explicitly:

$$A(r) = \frac{2\mu}{\sinh(2\mu r)} - \frac{1}{r}, \quad \Phi = \frac{2\mu}{\tanh(2\mu r)} - \frac{1}{r}. \tag{17}$$

The gauge field is regular at the origin, $A(0)=\Phi(0)=0$. The mass parameter 2μ defines the monopole size and, simultaneously, the value of the Higgs condensate at spatial infinity. This field configuration is known as the Bogomol'nyi-Prasad-Sommerfield (BPS) dyon [12].

The solutions of Eqs. (9), (10), and (17) are

$$g_i^{\pm}(r) = \frac{N}{r}\left[\frac{\sqrt{4\mu^2+\omega^2\sinh^2(2\mu r)}+|\omega|\cosh 2\mu r}{\sqrt{2}\mu}\right]^{(-1)^i|\omega|/(2\mu)}\left[\frac{\cosh(2\mu r)-\sqrt{4\mu^2+\omega^2\sinh^2(2\mu r)}}{\cosh(2\mu r)+\sqrt{4\mu^2+\omega^2\sinh^2(2\mu r)}}\right]^{(-1)^i/2}$$

$$\times\left[\frac{\sinh(2\mu r)}{2\mu r}\right]^{\pm 1/2}, \tag{18}$$

where the subscript $i=1,2$ corresponds to two independent solutions of Eq. (9). The constant N is defined by the normalization condition (5).

The asymptotics of our solutions (18),

$$g_i^{\pm}(r) \propto (\mu r)^{(-1)^i-1}+O(\mu^2 r^2), \quad r\to 0,$$

$$g_i^{\pm}(r) \propto \exp\left\{\frac{1}{2}(-1)^i(\pm 1+|\omega|/\mu)r\right\}+O(e^{-2\mu r}), \quad r\to\infty, \tag{19}$$

indicate that the $i=1$ solution is not normalizable at the small-r region while the nonsingular $i=2$ solution is normalizable. At large distances, the g_2^+ solution is always growing exponentially and thus is not normalizable. However, the solution g_2^- can be normalized provided $|\omega|\leq 1/2$.

Thus we get the following normalizable solutions:

$$g^+(r)=0, \quad g^-(r)$$

$$=\frac{N}{r}\left[\frac{\sqrt{4\mu^2+\omega^2\sinh^2(2\mu r)}+|\omega|\cosh 2\mu r}{\sqrt{2}\mu}\right]^{|\omega|/2\mu}\left[\frac{2\mu r}{\sinh(2\mu r)}\frac{2\mu\cosh(2\mu r)-\sqrt{4\mu^2+\omega^2\sinh^2(2\mu r)}}{2\mu\cosh(2\mu r)+\sqrt{4\mu^2+\omega^2\sinh^2(2\mu r)}}\right]^{1/2}. \tag{20}$$

The functions $f^{\pm}$ are defined by Eqs. (11) and (20):

$$f^+(r)=0, \quad f^-(r)=\frac{1}{\omega\sinh(2\mu r)}[\sqrt{4\mu^2+\omega^2\sinh^2(2\mu r)}-2\mu]g^-(r). \tag{21}$$

Note that at large distances we recover the relation $f^-(r)=\text{sgn}(\omega)g^-(r)$, cf. Eq. (13).

Finally, combining Eqs. (20) and (21), we get the right-handed fermion zero mode:

$$\Psi_R = \frac{N}{r} \left[\frac{2\mu r}{\sinh(2\mu r)} \frac{2\mu \cosh(2\mu r) - \sqrt{4\mu^2 + \omega^2 \sinh^2(2\mu r)}}{2\mu \cosh(2\mu r) + \sqrt{4\mu^2 + \omega^2 \sinh^2(2\mu r)}} \right]^{1/2} \left[\frac{\sqrt{4\mu^2 + \omega^2 \sinh^2(2\mu r)} + |\omega| \cosh 2\mu r}{\sqrt{2}\mu} \right]^{|\omega|/2\mu}$$

$$\times \left(\left[\mathbb{1} + \frac{\sqrt{4\mu^2 + \omega^2 \sinh^2(2\mu r)} - 2\mu}{\omega \sinh(2\mu r)} (\vec{\sigma}, \vec{n}) \right] \begin{pmatrix} 0 \\ \varepsilon \end{pmatrix} \right), \tag{22}$$

where the frequency ω is restricted by the condition

$$\omega \lesssim \mu. \tag{23}$$

This solution coincides (up to a gauge transformation) with the fermion zero mode solution found in Ref. [13].

The solutions (14)–(16), and (22) can be linked to the fermion zero modes in the Georgi-Glashow model coupled to the fermions [20]. This model has been considered by Jackiw and Rebbi [8], who found the static fermion zero mode in the background of the 't Hooft–Polyakov monopole:

$$\Psi = N \exp \left\{ \int_0^r dr' [A(r') - \tfrac{1}{2}\Phi(r')] \right\} \cdot \begin{pmatrix} 0 \\ \varepsilon \end{pmatrix}, \tag{24}$$

where the constant N is determined from the three-dimensional normalizability condition, Eqs. (5) and (6).

The link between our zero modes and the Jackiw-Rebbi solution (24) can easily be established. First, let us consider the Wu-Yang monopole case, Eqs. (14)–(16). Setting $\omega=0$, we restrict ourselves to two solutions (15). The linear combination $\Psi_R^{(1)} + \Psi_R^{(2)}$ coincides with the Jackiw-Rebbi mode (24), while $\Psi_R^{(1)} - \Psi_R^{(2)}$ becomes its gauge copy.

The identification in the case of the 't Hooft–Polyakov monopole, Eq. (22), is even more straightforward. Setting $\omega=0$, we get

$$\Psi = \frac{1}{\mu} \sqrt{\frac{2\pi}{r \sinh(2\mu r)}} \tanh(\mu r) \cdot \begin{pmatrix} 0 \\ \varepsilon \end{pmatrix}.$$

This expression coincides with the zero mode (24) where the fields $A(r)$ and $\Phi(r)$ are given in Eq. (17).

C. Perturbations on the potential

The monopole field configurations which allow for exact zero-mode solutions assume fixation of the gauge field A_μ at all distances. In reality of course, one can hope to imitate the lattice monopole fields only to some extent. In particular, there arise cutoffs both at large and small distances, and the next question is, what are the corresponding changes in the structure of the zero modes? We address this question, on the qualitative level, in this subsection. First, let us notice that although in all the cases considered we found an infinite number of fermionic zero modes, the status of these modes is somewhat different. Namely, in the first case [see Eqs. (13)] there is symmetry between the left- and right-handed modes. In the third case, there are right-handed modes alone [see Eq.

(22)]. Finally, the modes in the second case considered [see Eqs. (14)–(16), are of mixed nature.

The difference in the number of the right- and left-handed modes is controlled in fact by the chiral anomaly:

$$N_R - N_L = \int dt \int d^3 r \frac{\mathbf{H}^a \cdot \mathbf{E}^a}{32\pi^2}, \tag{25}$$

where $\mathbf{H}^a$ and $\mathbf{E}^a$ are color magnetic and electric fields, respectively. The crucial point is that the product $(\mathbf{H}^a \cdot \mathbf{E}^a)$ in the second and third examples considered in the previous subsection does not disappear already on the classical level. Note that to make use of Eq. (25) in our case, one should introduce a finite range of integration over the time coordinate, $-T < t < T$, where T is large.

Now, if we modify the gauge field configurations, the difference $N_R - N_L$ changes smoothly as far as the change in the right-hand side of Eq. (25) is smooth. The corresponding analysis is trivial enough.

The situation is much more nontrivial in the case of the Wu-Yang pointlike monopole. Namely, let us introduce a cutoff at small distances so that $A_i \sim -1/r$ only as far as $r > r_0$ while at short distances the potential vanishes, $A_i(0) = 0$. Then the zero modes found in the previous subsection become nonrenormalizable. In other words, the zero modes disappear altogether. To see this, it is convenient to use the following relation:

$$\int d^3 x \{ (\tilde{D} g^\pm)^2 + [\omega^2 - \tfrac{1}{4}\Phi^2(r)] g^{\pm,2} \}$$

$$= 4\pi (r^2 g^{\pm,2}) \big|_0^\infty. \tag{26}$$

where $\tilde{D} = \partial/\partial r - A(r)$, and independence of the functions $g^\pm$ on the angular variables is assumed. Equation (26) can be readily obtained by multiplying Eq. (12) by the functions $g^\pm$, integrating over the whole space, and integrating by parts. Equation (26) implies that $g \sim 1/r$ even if $A_i(0) = 0$. It follows then from Eq. (11) that the function $f \sim 1/r^2$ at small r and the zero-mode solution is not renormalizable.

We will comment on the physical meaning of this discontinuity in the next section. Here we would like to notice only that the introduction of the lattice spacing $a \neq 0$ allows us to introduce renormalizable solutions in any case. Moreover, to be sensitive to the monopole field in the infrared, we should restrict ourselves to $\omega \ll 1/r_0$. Then the normalization integral over the functions (13) is dominated by $r \sim 1/\omega$. Upon the modification of A_i at small distances, there emerges a new

contribution from the distances of order a. The new contribution does not dominate provided that the product $(\omega \cdot r_0)(r_0/a)$ is small.

III. PHENOMENOLOGICAL APPLICATIONS: CONCLUSIONS

The existence of an infinite number of the zero fermionic modes indicates the instability of the fermionic vacuum in the presence of the monopolelike field configurations. And, indeed, the instability of the fermionic vacuum in the presence of the monopoles or dyons is well known and is the starting point of the theory of monopole catalysis [5–7]. In particular, it is well known that the S-wave interaction of massless fermions with Abelian monopoles is anomalous in the sense that for some chiralities there exist only incoming waves while for other chiralities there exist only outgoing waves; see the discussion in [14].

In terms of this analogy, one can also easily understand the drastic effect on the zero modes of the modification of the Wu-Yang monopole field on short distances; see Sec. II C. Indeed, if one applies the Dirac equation to study the motion of a massless fermion in the field of the 't Hooft–Polyakov monopole, then the result is that the fermion changes its charge due to the W-boson exchange on the core of the monopole; see, e.g., [14]. As a result, the sign of the magnetic moment is changed as well, and the fermion of opposite chirality is emitted as a particle of the same chirality with energy of order $1/r_0$. In our language, the modification of the potential at arbitrarily small distances leads to the concentration of the wave function at these distances. As a result, any weak interaction, like the interaction with W bosons, becomes crucial. Moreover, the energy of the fermion is not conserved. Thus, the wave function becomes sensitive only to the modifications of the monopole configuration at short distances, which are very difficult to describe realistically for the lattice monopoles. Instead, we suggested (see Sec. II C) to use the lattice regularization and choice of ω to remain sensitive to the monopole configuration at large distances.

The instability of the fermionic vacuum revealed through the existence of an infinite number of zero modes implies that the results of the numerical simulations in the quenched approximation and those with dynamical fermions differ substantially. While studying the solutions of the Dirac equation is sufficient to establish the instability of the perturbative vacuum, it is much more difficult to find the true fermionic vacuum in the presence of monopoles. To this end, one has to consider the full field theory. This allows us to include the effect of the anomaly which arises at the quantum level. The simplification is that because of the S-wave nature of the fermions, the corresponding theory is effectively two-dimensional [5,6].

The outcome of the calculations [5,6] is that a fermionic condensate is formed around the monopoles:

$$\langle \bar{\psi}(r)\psi(r) \rangle = \frac{1}{4\pi r^2} f(r,t),$$

where r is the distance to the monopole center. The function f was estimated in the leading order in Ref. [5]:

$$|f(r,t)| = \frac{1}{2\pi r}. \tag{27}$$

At least naively, one can think of this result as a manifestation of the Pauli principle: the decay of the vacuum is stopped once the fermionic states which correspond to the fermions falling on the center are occupied. Since the final field configuration (27) is static, it can be thought of as Euclidean as well.

So far we have not discussed the question of how much the monopolelike configurations considered above resemble the lattice monopoles. In particular, an obvious reservation is that we used an approximation of a pointlike Abelian monopole. Such a bosonic field configuration would have an infinite action and could not be important in lattice simulations (for review and further references, see, e.g., [15]). However, as the latest measurements strongly indicate [16], the monopole size is much smaller than the distance between the monopoles. As a result, the pointlike monopoles might in fact be a valid approximation to interpret the results of the lattice simulations.

Reversing the question, we can say that by studying the properties of the fermionic zero modes in the quenched approximation, one can independently judge how close the lattice monopoles are to one or another theoretical description. An advantage of this approach is that it is gauge-invariant.

The mechanism of the chiral symmetry breaking discussed in the paper may also allow us to predict the properties of the chiral condensate in the confinement phase as well as to provide a link between the chiral and deconfinement phase transitions. An essential point in the Callan-Rubakov analysis of the appearance of the chiral condensate near the static monopole lies in the fact that the Abelian monopole is static. Obviously, the fermions need a finite time to be attracted to the monopole core by the interaction between the fermion spin and magnetic charge of the monopole. As a result, short-lived monopoles (i.e., the monopole-antimonopole pairs collapsing in a short time) cannot be responsible for the chiral symmetry generation. The analogue of the short-living monopoles in the Euclidean space-time is small-sized monopole loops which are indeed observed in the lattice simulations [18]. However, besides these short monopole trajectories, a large monopole cluster is present in the confinement phase of the theory. The Minkowskian counterparts of these lattice monopoles should serve as agents of the chiral symmetry breaking according to our considerations above.

This picture naturally links the confinement and chiral phase transitions. Namely, chiral condensates must be nonzero in the presence of the large monopole cluster, i.e., in the confinement phase. In the high-temperature, deconfinement phase, the large monopole cluster is absent [18], and, as a result, the chiral symmetry breaking ceases to exist.

Naively, one may suggest that the chiral condensate is proportional to the density of the largest monopole cluster, ρ_{IR}, at least at zero temperatures,

$$\langle \bar{\psi}\psi \rangle = C_{\rho}\rho_{\mathrm{IR}}, \qquad (28)$$

since both quantities have the same dimension (mass^3). However, at nonzero temperatures the dependence should be more general:

$$\langle \bar{\psi}\psi \rangle = F(\rho_{\mathrm{IR}},T), \qquad (29)$$

where the function F obeys the property $F(0,T)=0$. Suggestion (28) may be checked in future lattice simulations at various lattice couplings.

In conclusion, let us summarize the findings of the present paper.

In the quenched approximation, one usually relies on the Banks-Casher relation [2] to study the relevance of various bosonic field configurations to the chiral symmetry breaking. However, we see that in the quenched approximation, the evaluation of the chiral condensate is inconsistent in the presence of the monopolelike configurations because of the instability of the fermionic vacuum. This inconsistency is another manifestation of the monopole catalysis phenomenon [5–7].

It would be very interesting to search numerically for solutions of the Dirac equations (in the given gluon field background) corresponding to the fermionic zero modes with imaginary frequencies, as discussed above. If such solutions exist, it would demonstrate independently the relevance of the monopolelike configurations—usually defined in an Abelian gauge (see Ref. [17] for a review)—in a non-Abelian gauge theory. The advantage of such a search for the monopoles is that it does not depend on the Abelian gauge explicitly.

Qualitatively, since the quenched approximation is in fact not adequate to describe the fermionic vacuum, the backreaction of the fermions on the gluon fields can be unexpectedly strong. In particular, extra monopole-antimonopole attraction is generated. Further analytical studies of the effect are now in progress.

The chiral condensate seems to be linked with the density of the monopoles in the largest monopole cluster. The proposed mechanism is very attractive since it can naturally explain the equivalence of the critical temperatures for the deconfinement and chiral transitions.

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[20] We do not describe this model and refer the reader to Ref. [8] for details. The essential point is that the Dirac equation for *static* zero modes has the same form both in the 4D QCD and 3D in the Georgi-Glashow model coupled to fermions (with identification of the zero component of the gauge field in the former case with the adjoint scalar field in the latter).