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Chiral Lagrangian with confinement from the QCD Lagrangian

Yu. A. Simonov
Jefferson Laboratory, Newport News, Virginia 23606
and State Research Center, Institute of Theoretical and Experimental Physics, Moscow, Russia
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An effective Lagrangian for the light quark in the field of a static source is derived systematically using the exact field correlator expansion. The lowest Gaussian term is bosonized using nonlocal colorless bosonic fields and a general structure of an effective chiral Lagrangian is obtained containing all sets of fields. The new and crucial result is that the condensation of the scalar isoscalar field which is the usual onset of chiral symmetry breaking and is constant in space-time, assumes here the form of the confining string, and contributes to the confining potential, while the remaining bosonic fields describe mesons with the $q\bar{q}$ quark structure and pseudoscalars play the role of Nambu-Goldstone fields. Using a derivative expansion, the effective chiral Lagrangian is deduced containing both confinement and chiral effects for heavy-light mesons. The pseudovector quark coupling constant is computed to be exactly unity in the local limit, in agreement with earlier large N_c arguments.

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I. INTRODUCTION

It was understood long ago [1] that chiral symmetry breaking in QCD is responsible for the low mass of pions and therefore the low-energy limit of QCD can be adequately described by the effective chiral Lagrangians [2]. In this approach, the Nambu-Goldstone particles are described by local field variables and the resulting effective chiral Lagrangian (ECL) is local. The most general and practically useful form of the chiral Lagrangian was given in [3] and contains around ten phenomenological parameters (14 to the fourth order in p), to be found from experiment.

Being successful in describing low-energy processes with Nambu-Goldstone mesons, ECL has two major defects. First, it does not take into account the quark structure of mesons, and consequently, e.g., the form factor computed in ECL can display in the meson only mesonic degrees of freedom.

Second, ECL completely disregards the phenomenon of confinement, which is also important at small energies, and hence degrees of freedom of vacuum gluons, creating confining string, are not taken into account.

There have been attempts to cure the first defect; namely, the model Lagrangians have been suggested which take into account both quarks and mesons [4,5]. In particular, the instanton model of the QCD vacuum has been used to derive the ECL and the quark-meson Lagrangian (QML) [6]. As a result, interesting interconnections of quark and chiral degrees of freedom have been demonstrated in the example of the nucleon [7].

However, instantons are suppressed in the realistic vacuum of QCD [8,9], and moreover the internal consistency of the instanton vacuum without confinement is seriously questioned [10,11]. Moreover, instantonic vacuum lacks confinement and therefore cannot cure the second defect of ECL and QML, therefore model ECL and QML obtained in [6,7] disregard confinement fully.

Finally, the effective quark Lagrangian (EQL) obtained after averaging over gluon degrees of freedom contains in principle an infinite number of terms with a growing number of quark fields. It was shown in [12] that specifically for instanton vacuum all higher terms are of the same order, while only the lowest term (the so-called 't Hooft Lagrangian) is taken in the standard instanton lore.

It is the purpose of the present paper to start a new and systematic approach to the derivation of QML and ECL from the first principles—the QCD Lagrangian. In so doing, the most important is to keep gauge invariance at every stage, therefore we shall consider the simplest gauge-invariant system of a light guark in the field of a static antiquark (generalization to only light quark systems will be done later).

After averaging over gluon fields, one obtains EQL with an infinite number of terms, containing as kernels irreducible gluon field correlators (FC). Recent measurements for the realistic QCD vacuum have shown that FC create a hierarchy, where the lowest (Gaussian) correlator is dominating [8,9] while the next (fourth order) correlator contributes around 1% to the static $Q\bar{Q}$ interaction (note that this situation is drastically different from that of instantonic vacuum, where higher correlators are equally important). We assume that a similar hierarchy should be present also in our problem of a light-heavy quark, which allows us to study the resulting EQL term by term, paying most attention to the lowest, $4q$ piece. The next step is the bosonization procedure, i.e., an identical transformation introducing nonlocal colorless bosonic fields having different Lorentz and flavor indices due to the use of Fierz transformation.

A special attention deserves the $6q$ term, where bosonization may be done introducing bosonic fields for $\bar{q}q$ combination or else introducing baryonic fields for the $3q$ combination, deserves special attention. As a result, one obtains QML or quark-baryon Lagrangian (QBL) in the most general and rigorous form. These questions will be considered in a separate publication [13].

As the next step, one can use the stationary point analysis to obtain nonlinear equations for effective bosonic fields. This is done in a rigorous way and the resulting equations contain for the scalar-isoscalar part the same equation as was derived previously [14] in a gauge-invariant Dyson-Schwinger approach not using bosonization. The careful study of that equation in [14–16] has shown that it describes in the light-heavy system both confinement and chiral symmetry breaking, which coexist. This fact means that confinement in the language of effective meson fields enters in the form of a condensate of the scalar field inside the string, while other fields describe subdominant features of the $q\bar{Q}$ dynamics.

The paper is organized as follows: In the next section, the averaging over vacuum gluonic fields is done and EQL is obtained. Special attention is devoted to the gauge invariance and parallel transporters necessary to ensure it for nonlocal $q\bar{q}$ combinations. In Sec. III, the bosonization procedure is done and Fierz transformation is introduced to obtain final QML with proper classification in Lorentz and flavor indices.

In Sec. IV, the resulting ECL is obtained and the stationary point equations are derived and compared to that previously obtained in the Dyson-Schwinger approach. In Sec. V, the derivative expansion of the nonlocal bosonized Lagrangian is done and nonlocal forms of the lowest second- and fourth-order terms in this expansion are obtained. Keeping only a pion field in addition to quarks, one derives in Sec. VI the effective Lagrangian which appears to have the expected form with the pseudovector quark-pion coupling constant g_A^q. The latter is equal exactly to 1, in agreement with an earlier large N_c argument in which a local approximation is made. The concluding section is devoted to a discussion of confinement and chiral properties of the resulting ECL in the heavy-light meson case.

II. THE EFFECTIVE QUARK LAGRANGIAN

Consider the QCD partition function in the Euclidean space-time,

$$Z=\int DAD\psi D\psi^+ e^{-S_0(A)+\int\ {}^f\psi^+(i\hat{\partial}+im+g\hat{A})^f\psi d^4x}, \quad (1)$$

where $S_0(A)=\frac{1}{4}\int[F^a_{\mu\nu}(x)]^2d^4x$, m is the current quark mass, and the quark operator ${}^f\psi_{a\alpha}(x)$ has flavor index f ($f=1,\ldots,n_f$), color index a ($a=1,\ldots,N_c$), and Lorenz bispinor index α ($\alpha=1,2,3,4$).

The next step is to integrate over DA_μ with the weight $S_0(A)$. This is the gluon vacuum averaging which is denoted by $\langle\rangle_A$. Before doing this, however, one should choose the gauge-invariant system and, using an appropriate gauge, express A_μ through the field-strength operator $F_{\mu\nu}$ which would finally appear in gauge-invariant combinations—field correlators (FC) (see [17] for review and more discussion), namely,

$$\begin{aligned} &g^n\langle F_{\mu_1\nu_1}(x_1)\Phi_{C_1}(x_1,x_2)F_{\mu_2\nu_2}(x_2)\\ &\quad\times\Phi_{C_2}(x_2,x_3),\ldots,F_{\mu_n\nu_n}(x_n)\Phi_{C_n}(x_n,x_1)\rangle_A\\ &\equiv\Delta^{(n)}, \end{aligned}\quad (2)$$

where parallel transporters are defined as

$$\Phi_C(x,y)=P\exp\left(ig\int_{C(x,y)}A_\mu dz_\mu\right) \quad (3)$$

and the open contour $C(x,y)$ connects points x and y and can be arbitrary otherwise.

To achieve this goal, one can use the so-called contour gauge [18,19] which is especially convenient in the case of a light quark moving in the field of a static antiquark. One has, for the contour $z_\mu(s,x)$ starting at point x and ending at $Y=z(0,x)$,

$$\begin{aligned} A_\mu(x)&=\int_0^1 ds\frac{\partial z_\nu(s,x)}{\partial s}\frac{\partial z_\rho(s,x)}{\partial x_\mu}F_{\nu\rho}[z(s)]\\ &\equiv\int_Y^x d\Gamma_{\mu\nu\rho}(z)F_{\nu\rho}(z). \end{aligned}\quad (4)$$

In the particular case in which the contour $z_\mu(s,x)$ goes along the shortest (straight) way to the x_4 axis and then along this axis to some point Y, which can be at $-\infty$, one has the so-called modified Fock-Schwinger gauge [20], which was extensivelly used in [14] to get EQL. Here one has

$$A_\mu(\mathbf{x},x_4)=\int_0^{x_i}\alpha_\mu(u)du_iF_{i\mu}(u,x_4) \quad (5)$$

and $\alpha_4(u)\equiv1$, while for $\mu=1,2,3$, α_μ is equal to

$$\alpha(u)=\frac{u_i}{x_i},\quad i=1,2,3.$$

It is convenient to write all expressions in a gauge-invariant way, using the property [19] that Φ_C given by Eq. (3) is identically equal to unity when the contour $C(x,y)$ lies on $z_\mu(s,x)$ or $z_\mu(s,y)$. Therefore, one can define gauge-covariant operators referred to the point Y,

$$\begin{aligned} \psi^{(Y)}(x)&=\Phi_{C(x,Y)}\psi(x)\equiv\Phi(Y,x)\psi(x),\\ \psi^{+(Y)}(x)&=\psi^+(x)\Phi(x,Y),\\ F^{(Y)}_{\mu\nu}(x)&=\Phi(Y,x)F_{\mu\nu}(x)\Phi(x,Y). \end{aligned}\quad (6)$$

Here the contour $C(x,Y)$ in $\phi(Y,x)$ goes along $z(s,X)$ from Y to x, and in the opposite direction in $\Phi(x,Y)$.

For the field correlators referred to the same point Y, one can write (ab,cd are fundamental color indices; Lorentz indices and $1/N_c$ terms are suppressed for simplicity reasons)

$$\begin{aligned} &g^2\langle[F^{(Y)}(x)]_{ab}[F^{(Y)}(y)]_{cd}\rangle\\ &\quad=\frac{\delta_{ad}\delta_{bc}}{N_c^2}g^2\langle\mathrm{tr}[F^{(Y)}(x)F^{(Y)}(y)]\rangle, \end{aligned}\quad (7)$$

$$g^3\langle [F^{(Y)}(x)]_{ab}[F^{Y}(y)]_{cd}[F^{(Y)}(z)]_{ef}\rangle$$

$$=\delta_{bc}\delta_{de}\delta_{af}\left[-\frac{g^3\,\mathrm{tr}\langle xzy\rangle}{N_c(N_c^4-1)}+\frac{N_c}{N_c^4-1}g^3\,\mathrm{tr}\langle xyz\rangle\right]$$

$$+\delta_{be}\delta_{fc}\delta_{ad}\left[-\frac{g^3\,\mathrm{tr}\langle xyz\rangle}{N_c(N_c^4-1)}+\frac{N_c g^3\,\mathrm{tr}\langle xzy\rangle}{N_c^4-1}\right], \qquad (8)$$

where notation is introduced, e.g., $\mathrm{tr}\langle xyz\rangle \equiv\langle \mathrm{tr}[F^{(Y)}(x)F^{(Y)}(y)F^{(Y)}(z)]\rangle$.

Derivation of Eqs. (7) and (8) is given in Appendix A.

Let us first concentrate on the bilocal correlator (7). From Eqs. (4) and (5) it is clear that in the average value of $\langle A_\mu(x)A_\nu(y)\rangle$ the arguments of $F[z(s,x)]$ and $F[z(s,y)]$ are separated by the distance $r\sim T_g$, where T_g is the gluonic correlation length [21], which was measured on the lattice [22] and estimated analytically [23] to be $T_g\sim 0.2$ fm (or even smaller if data on glue lump masses [24] are used). For such distances $r\sim T_g$, which satisfy $r\ll|x-Y|$, $|y-Y|$, or for the gauge (5), $r\ll|\mathbf{x}|$, $|\mathbf{y}|$, one has an estimate

$$\langle \mathrm{tr}\,F^{(Y)}(x)F^{(Y)}(y)\rangle=\langle \mathrm{tr}[F(x)\Phi(x,y)F(y)\Phi(y,x)]\rangle + O\left(\frac{r^2}{\mathbf{x}^2},\frac{r^2}{\mathbf{y}^2}\right), \qquad (9)$$

where the correlator on the right-hand side of Eq. (9) is connected by straight lines from x to y. A similar estimate holds for the triple FC (8), and in what follows we shall use the straight-line form (9) which is independent of the position of the reference point Y.

For that correlator, one can use the general representation found in [25],

$$\frac{g^2}{N_c}\langle \mathrm{tr}\,F_{\mu\nu}(x)\Phi(x,y)F_{\rho\sigma}(y)\Phi(y,x)\rangle$$

$$=D(x-y)(\delta_{\mu\rho}\delta_{\nu\sigma}-\delta_{\mu\sigma}\delta_{\nu\rho})+\Delta^{(1)}_{\mu\nu,\rho\sigma}, \qquad (10)$$

where $\Delta^{(1)}$ has the structure of a full derivative and therefore does not contribute to confinement; its nonperturbative (NP) part is much smaller than that of D and we shall omit it.

The function $D(u)$ has a NP part which was measured in [22] and has the exponential form

$$D(u)=D(0)\exp\left(-\frac{|u|}{T_g}\right). \qquad (11)$$

Finally, the string tension σ can be expressed through $D(u)$ (and higher correlators) and at least for static quarks $D(u)$ yields a dominant (up to a few percent [8,9]) contribution to σ,

$$\sigma=\tfrac{1}{2}\int D(u)d^2u. \qquad (12)$$

Having in mind all the above relations, one can now average over gluonic fields in Eq. (1) to obtain

$$Z=\int D\psi D\psi^+ e^{\int {}^f\psi^+(i\hat\partial+im)^f\psi d^4x}e^{L^{(2)}_{\mathrm{EQL}}+L^{(3)}_{\mathrm{EQL}}+\cdots}, \qquad (13)$$

where the EQL proportional to $\langle\langle A^n\rangle\rangle$ is denoted by $L^{(n)}_{\mathrm{EQL}}$,

$$L^{(2)}_{\mathrm{EQL}}=\frac{g^2}{2}\int d^4x\,d^4y\,{}^f\psi^+_{a\alpha}(x)^f\psi_{b\beta}(x)^g\psi^+_{c\gamma}(y)\times{}^g\psi_{d\varepsilon}(y)\langle A^{(\mu)}_{ab}(x)A^{(\nu)}_{cd}(y)\rangle\gamma^{(\mu)}_{\alpha\beta}\gamma^{(\nu)}_{\gamma\varepsilon}, \qquad (14)$$

$$L^{(3)}_{\mathrm{EQL}}=\frac{g^3}{3!}\int d^4x\,d^4y\,d^4z\,{}^f\psi^+_{a\alpha}(x)^f\psi_{b\beta}(x)\times{}^g\psi^+_{c\gamma}(y)^g\psi_{d\varepsilon}(y)^h\psi^+_{e\rho}(z)^h\psi_{f\sigma}(z)\times\langle A^{(\mu)}_{ab}(x)A^{(\nu)}_{cd}(y)A^{(\lambda)}_{ef}(z)\rangle\gamma^{(\mu)}_{\alpha\beta}\gamma^{(\nu)}_{\gamma\varepsilon}\gamma^{(\lambda)}_{\rho\sigma}. \qquad (15)$$

An average of gluonic fields can be computed using Eqs. (5), (7), and (8) as

$$g^2\langle A^{(\mu)}_{ab}(x)A^{(\nu)}_{cd}(y)\rangle=\frac{\delta_{bc}\delta_{ad}}{N_c}\int_0^x du_i\alpha_\mu(u)\int_0^y dv_k\alpha_\nu(v)\times D(u-v)(\delta_{\mu\nu}\delta_{ik}-\delta_{i\nu}\delta_{k\mu}). \qquad (16)$$

The corresponding expression for the triple average $\langle A^3\rangle$ is given in Appendix A.

As was argued in [26], the dominant contribution at large distances from the static antiquark is given by the color-electric fields, therefore we shall write down explicitly $L^{(2)}_{\mathrm{EQL}}(el)$ for this case, i.e., taking $\mu=\nu=4$ in Eqs. (14) and (16), while the general case, also for the generalized gauge (4), is given in Appendix B. As a result, one has

$$L^{(2)}_{\mathrm{EQL}}(el)=\frac{1}{2N_c}\int d^4x\int d^4y\,{}^f\psi^+_{a\alpha}(x)^f\psi_{b\beta}(x)\times{}^g\psi^+_{b\gamma}(y)^g\psi_{a\varepsilon}(y)\gamma^{(4)}_{\alpha\beta}\gamma^{(4)}_{\gamma\varepsilon}J(x,y), \qquad (17)$$

where $J(x,y)$ is

$$J(x,y)=\int_0^x du_i\int_0^y dv_i D(u-v), \quad i=1,2,3. \qquad (18)$$

Note that we have omitted everywhere for simplicity the upper index (Y) in $\psi^{(Y)}(x)$, but it is implied, since otherwise both Eqs. (14) and (15) are not gauge-invariant.

III. BOSONIZATION OF EFFECTIVE QUARK LAGRANGIANS

We shall separate out white bilinears in Eq. (17) following the standard procedure given for the general $L^{(n)}_{\mathrm{EQL}}$ in [14,27],

$$\Psi^{fg}_{\alpha\varepsilon}(x,y)\equiv{}^f\psi^+_{a\alpha}(x)^g\psi_{a\varepsilon}(y)={}^f\psi^+_{a'\alpha}(x)\Phi_{a'a}(x,Y)\Phi_{ac}(Y,y)^g\psi_{c\varepsilon}(y) \qquad (19)$$

and introduce the isospin generators $t_{fg}^{(n)}$:

$$\sum_{n=0}^{n_f^2-1} t_{fg}^{(n)} t_{ij}^{(n)} = \tfrac{1}{2}\delta_{fj}\delta_{gi}; \quad t^{(0)} = \frac{1}{\sqrt{2n_f}}\hat{1}. \tag{20}$$

Hence the bilinears in Eq. (17) can be written as

$$\Psi_{\alpha\varepsilon}^{fg}(x,y)\Psi_{\gamma\beta}^{gf}(y,x) = 2\sum_{n=0}^{n_f^2-1} \Psi_{\alpha\varepsilon}^{(n)}(x,y)\Psi_{\gamma\beta}^{(n)}(y,x), \tag{21}$$

where we have defined

$$\Psi_{\alpha\varepsilon}^{(n)}(x,y) \equiv {}^f\psi_{a\alpha}^{+}(x) t_{fg}^{(n)g}\psi_{a\varepsilon}(y). \tag{22}$$

Now one can use the Fierz transformation [28] (see Appendix C for more details),

$$\gamma_{\alpha\beta}^{(4)}\gamma_{\alpha'\beta'}^{(4)} = \tfrac{1}{4}\sum_{k=1}^{5} \bar{O}_{\alpha\beta'}^{(k)}\bar{O}_{\alpha'\beta}^{(k)} \tag{23}$$

with

$$\bar{O}_{\alpha\beta}^{(1)} = \delta_{\alpha\beta}, \quad \bar{O}^{(2)}\bar{O}^{(2)} = \gamma^{(4)}\gamma^{(4)} - \gamma^{(i)}\gamma^{(i)},$$

$$\bar{O}^{(3)}\bar{O}^{(3)} = (\gamma^5\gamma^{(4)})(\gamma^5\gamma^{(4)}) - (\gamma^5\gamma^{(i)})(\gamma^5\gamma^{(i)}),$$

$$\bar{O}^{(4)}\bar{O}^{(4)} = (\sigma_{ik})(\sigma_{ik}) - (\sigma_{4k})(\sigma_{4k}) - (\sigma_{k4})(\sigma_{k4}), \tag{24}$$

$$\bar{O}_{\alpha\beta}^{(5)} = i(\gamma^{(5)})_{\alpha\beta}; \quad \sigma_{\mu\nu} = \frac{\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu}{2i}.$$

Introducing Eqs. (21) and (23) into Eq. (17), one obtains

$$L_{\text{EQL}}^{(2)}(el) = -\int d^4x \int d^4y\, \Psi^{(n,k)}(x,y) \times \Psi^{(n.k)}(y,x)\tilde{J}(x,y), \tag{25}$$

where we have defined

$$\Psi^{(n,k)}(x,y) = \tfrac{1}{2}\Psi_{\alpha\varepsilon}^{(n)}(x,y)\bar{O}_{\varepsilon\alpha}^{(k)}, \quad \tilde{J} \equiv \frac{1}{N_c}J. \tag{26}$$

Bosonization is now done in a standard way using the identity (signs and indices of summation and integration are suppressed)

$$e^{-\Psi\tilde{J}\Psi} = \int (\det\tilde{J})^{1/2} D\chi \exp[-\chi\tilde{J}\chi + i\Psi\tilde{J}\chi + i\chi\tilde{J}\Psi] \tag{27}$$

and hence the partition function with the only Gaussian contribution $L^{(2)}$ assumes the form

$$Z = \int D\psi D\psi^{+} D\chi \exp L_{\text{QML}} \tag{28}$$

where the effective quark-meson Lagrangian is

$$L_{\text{QML}}^{(2)} = \int d^4x \int d^4y \{ {}^f\psi_{a\alpha}^{+}(x)[(i\hat{\partial} + im)_{\alpha\beta}\delta(x-y) + iM_{\alpha\beta}^{(fg)}(x,y)]^g\psi_{a\beta}(y) - \chi^{(n,k)}(x,y)\tilde{J}(x,y)\chi^{(n,k)}(y,x)\} \tag{29}$$

and the effective quark-mass operator is

$$M_{\alpha\beta}^{(fg)}(x,y) = \sum_{n,k} \chi^{(n,k)}(x,y)\bar{O}_{\alpha\beta}^{(k)} t_{fg}^{(n)}\tilde{J}(x,y). \tag{30}$$

In a similar way, one can bosonize the term $L_{\text{EQL}}^{(3)}$. However, the computations are more lengthy and we shall present the results in a separate publication [13].

IV. THE EFFECTIVE CHIRAL LAGRANGIAN L_{ECL} AND STATIONARY POINT ANALYSIS

We are now in a position to integrate over quark fields in Eq. (28) and obtain the effective chiral Lagrangian $L_{\text{ECL}}^{(2)}$. The result is

$$Z = \int D\chi e^{L_{\text{ECL}}^{(2)}(\chi)} \tag{31}$$

with

$$L_{\text{ECL}}^{(2)}(\chi) = -\int d^4x\, d^4y \sum_{n,k} \chi^{(n,k)}(x,y) \times \tilde{J}(x,y)\chi^{(n,k)}(y,x) + N_c \operatorname{tr}\ln[(i\hat{\partial} + im)\delta(x-y) + iM(x,y)]. \tag{32}$$

In Eq. (32) we have taken into account that M is colorless, and the sign tr refers to the summation (integration) over Lorentz, flavor indices, and space-time coordinates.

As the next standard step, one finds the stationary point equations to determine the ground-state values for the auxiliary bosonic fields χ. Taking the functional derivative of $L_{\text{ECL}}^{(2)}$ with respect to $\chi^{(n,k)}$, one obtains

$$2\chi^{(n,k)}(x,y)\tilde{J}(x,y) = -iN_c \operatorname{tr}[S(x,y)\bar{O}^{(k)}t^{(n)}]\tilde{J}(x,y), \tag{33}$$

where we have defined as in [14]

$$S(x,y) = -[(i\hat{\partial} + im)\hat{1} + i\hat{M}]_{x,y}^{-1}. \tag{34}$$

The set of (nonlinear) Eqs. (33) and (34) is the central result of the present paper. In what follows, we shall study the properties of the solutions and compare this result to the previously obtained equations in [14], where another method was used, namely a large N_c approximation in the Dyson-Schwinger equations for the heavy-light system with one fixed flavor. In this case, one should take $n_f = 1$ and $t^{(0)} = 1/\sqrt{2}$. Moreover, only the scalar part (since it is dominant at large distances from the heavy source [14,26]) was considered, hence one can write instead of Eq. (30)

$$M \to M_0(x,y) = \frac{1}{\sqrt{2}} \chi(x,y) \tilde{J}(x,y) \tag{35}$$

and Eq. (33) reduces to the equation

$$iM_0(x,y) = [\gamma_4 S(x,y) \gamma_4]_{k=1} J(x,y), \tag{36}$$

where we have used the relation

$$(\gamma_4 S \gamma_4)_{\alpha\gamma} = \gamma^{(4)}_{\alpha\beta'} S_{\beta'\alpha'} \gamma^{(4)}_{\alpha'\gamma} = \tfrac{1}{4} \sum_k O^{(k)}_{\alpha\gamma} \operatorname{tr}(S O^{(k)}). \tag{37}$$

Equation (36) is exactly the same as Eq. (15) in [14] where the color-electric field component is retained, namely in the full answer

$$iM_0(x,y) = J(x,y) \gamma_\mu S(x,y) \gamma_\mu - J_{ik} \gamma_k S(x,y) \gamma_i \tag{36$'$}$$

one keeps only $\mu=4$, $k=1$. As follows from the definition of S, Eq. (34), one has another equation to complete a full set,

$$\begin{aligned}(-i\hat{\partial} - im) S(x,y) - i \int M(x,z) S(z,y) d^4 z \\ = \delta^{(4)}(x-y).\end{aligned} \tag{38}$$

Let us look more closely at the scalar part of mass operator, $M_0(x,y)$, Eqs. (35) and (36). The properties of the kernel $J(x,y)$ (18) have been thoroughly investigated in [14–16], and it was shown there that when $\mathbf{x}$ is close to $\mathbf{y}$, then $J(x,y)$ is growing linearly at large $|\mathbf{x}|$, e.g., when for simplicity $D(u)$ is taken in the Gaussian form

$$D(u) = D(0) e^{-(\mathbf{u}^2 + \mathbf{u}_4^2)/4T_g^2}, \tag{39}$$

then

$$J(\mathbf{x} \sim \mathbf{y}, |\mathbf{x}| \to \infty) = |\mathbf{x}| 2 T_g \sqrt{\pi} D(0) e^{-(x_4 - y_4)^2/4T_g^2} \tag{40}$$

whereas

$$\sigma = \tfrac{1}{2} \int D(u) d^2 u = 2\pi T_g^2 D(0). \tag{41}$$

Moreover, $S(x,y)$ [Eq. (34)] (where only M_0 is retained) at large distances displays the properties of the smeared δ function (see [14–16] for discussion and numerical estimates),

$$\gamma_4 S(x,y) \gamma_4 \sim \tilde{\delta}^{(3)}(\mathbf{x} - \mathbf{y}), \tag{42}$$

and as a result the product $J(x,y)\gamma_4 S(x,y)\gamma_4$ behaves linearly in $|\mathbf{x}|$ at large $|\mathbf{x}|$, in such a way that in Eq. (38) one has

$$\int M_0(x,z) S(z,y) d^4 z \to \sigma |\mathbf{x}| S(x,y). \tag{43}$$

Thus for $|\mathbf{x}| \gg T_g$ one has

$$M_0(x,z) \approx \sigma |\mathbf{x}| \tilde{\delta}^{(4)}(x-z)[1 + O(T_g/|\mathbf{x}|)] \tag{44}$$

and $\tilde{\delta}^{(4)}(x-z)$ is smeared off at a distance of the order of T_g. To proceed, we disregard first in the sum (30) all terms except for the scalar and pseudoscalar fields,

$$M(x,y) = \chi_S(x,y) \frac{\tilde{J}(x,y)}{\sqrt{2 n_f}} + \hat{\chi}_\pi(x,y) i \gamma_5 \tilde{J}(x,y) \tag{45}$$

with

$$\hat{\chi}_\pi(x,y) = \chi_\pi^{(f)} t^f.$$

The form (45) can be equivalently parametrized in a nonlinear way as follows:

$$\begin{gathered}\hat{M}(x,y) = M_S(x,y) \hat{U}(x,y), \\ \hat{U} = \exp(i\gamma_5 \hat{\phi}), \\ \hat{\phi}(x,y) = \phi^f(x,y) t^f.\end{gathered} \tag{46}$$

Now using the normalization property

$$\operatorname{tr}(t^{(n)} t^{(m)}) = \tfrac{1}{2} \delta_{nm}, \quad n,m = 0,1,\dots,n_f - 1, \tag{47}$$

one easily obtains

$$\tfrac{1}{4} \operatorname{tr}(\hat{M}\hat{M}^+) = \tfrac{1}{2}(\chi_s^2 + \chi_\pi^2) \tilde{J}^2(x,y) = M_S^2 n_f \tag{48}$$

and hence the first term in Eq. (32) can be written as

$$(\chi_s^2 + \chi_\pi^2) \tilde{J}(x,y) = 2 n_f \tilde{J}^{-1}(x,y) M_S^2(x,y) \tag{49}$$

and the total Lagrangian (32) is

$$\begin{aligned}L^{(2)}_{\mathrm{ECL}}(M_S, \hat{\phi}) = -2 n_f [\tilde{J}(x,y)]^{-1} M_S^2(x,y) \\ + N_c \operatorname{tr} \log[(i\hat{\partial} + im)\hat{1} + iM_S \hat{U}].\end{aligned} \tag{50}$$

The stationary point equations assume the form

$$\begin{aligned}\frac{\delta L^{(2)}_{\mathrm{ECL}}}{\delta M_S(x,y)} &= -4 n_f [\tilde{J}(x,y)]^{-1} M_S(x,y) \\ &\quad - N_c \operatorname{tr}(S i e^{i\gamma_5 \hat{\phi}}) \\ &= 0\end{aligned} \tag{51}$$

with

$$S(x,y) = -[i\hat{\partial} + im + iM_S \hat{U}]^{-1}_{x,y}, \tag{52}$$

$$\frac{\delta L^{(2)}_{\mathrm{ECL}}}{\delta \hat{\phi}(x,y)} = N_c \operatorname{tr}(S M_S e^{i\gamma_5 \hat{\phi}} \gamma_5). \tag{53}$$

The solution with $\hat{\phi} = 0$, $M_S = M_S^{(0)}$ satisfies Eq. (53) while Eq. (51) may be rewritten in the form

$$iM_S^{(0)}(x,y) = \frac{N_c}{4} \operatorname{tr} S \tilde{J}(x,y) = N_c (\gamma_4 S \gamma_4)_{\mathrm{sc}} \tilde{J}(x,y), \tag{54}$$

where $(\Gamma)_{sc}$ means the scalar part of an operator, as defined in Eq. (37), and one can see that one recovers Eq. (36), derived before in [14] in a different formalism.

V. THE DERIVATIVE EXPANSION OF $L^{(2)}_{\rm ECL}$

In this section, the chiral Lagrangian will be written as a series in powers of derivatives of the field $\hat{U}(x,y)$. A similar procedure for the local case and in the absence of confinement was systematically done in [5,29,30], and more recently in [31]. For the case of instanton model it was done in [7]. In all cases at some moment the local limit of the resulting chiral Lagrangian is done, and confining properties of the kernel $M_S(x,y)$ are not taken into account. In what follows, we keep both $\hat{\phi}$ and M_S nonlocal and the confining property (44) is exploited. It should be noted that nonlocality of the field $\hat{\phi}$ is a necessary consequence of its quark-antiquark structure and this structure is lost when the localization approximation is done. Since the radius of the pion is around 0.6 fm, it is only for small momenta that the localization procedure is justified. We now turn to the second term on the right-hand side of Eq. (50) which contains a pionic field and we make an expansion of its real part in powers of derivatives of the field U. Note that the imaginary part of the effective chiral Lagrangian was studied in [5,7,30] and it starts with the terms of the fifth power in the pionic field; it will not be studied below. Defining the real part of the pionic effective action $\mathrm{Re}\,L_{\rm eff}[\pi]$, one has

$$\mathrm{Re}\,L_{\rm eff}[\pi]=-\frac{N_c}{2}\log\det\frac{D^+D}{D_0^+D_0}, \tag{55}$$

where the following notations are used:

$$D=i\hat{\partial}+im+iM_S\hat{U},\quad D_0=i\hat{\partial}+im+iM_S. \tag{56}$$

Moreover,

$$D^+D=-\partial^2-\partial_\mu(M_S\hat{U}^+)\gamma_\mu+(m+M_S)^2+mM_S(\hat{U}^++\hat{U}), \tag{57}$$

$$D_0^+D_0=-\partial^2+(m+M_S)^2. \tag{58}$$

To simplify, we set $m=0$ and rewrite Eq. (55) as

$$\mathrm{Re}\,L_{\rm eff}[\pi]=-\frac{N_c}{2}\mathrm{tr}\log(1-G_0\partial_\mu M_S\hat{U}^+\gamma_\mu), \tag{59}$$

where we have defined

$$G_0(x,y)=(-\partial^2+M_S^2)^{-1}_{x,y}. \tag{60}$$

In Eq. (59), the sign "tr" implies the sum over Lorentz and flavor indices and an integral over coordinates, having in mind that every factor appearing in Eq. (59) under the logarithm sign should be considered as a matrix in the space-time coordinates. Expanding the logarithm in Eq. (59) and keeping only second- and fourth-order terms in $\partial_\mu\hat{U}^+$, one obtains

$$\mathrm{Re}\,L_{\rm eff}[\pi]=N_c\,\overline{\mathrm{tr}}\,\mathrm{Re}[G_0\partial_\mu U^+G_0\partial_\mu U+\tfrac{1}{2}G_0\partial_\mu U^+G_0\partial_\nu UG_0\partial_\alpha U^+G_0\partial_\beta U(\delta_{\mu\nu}\delta_{\alpha\beta}+\delta_{\mu\beta}\delta_{\nu\alpha}-\delta_{\mu\alpha}\delta_{\nu\beta})]+\cdots. \tag{61}$$

Here $\overline{\mathrm{tr}}$ implies summing over flavor indices and integration over coordinates, and $U=M_S(x,y)e^{i\hat{\phi}(x,y)}$. We consider now the first term on the right-hand side of Eq. (61) and write explicitly the coordinate part of $\overline{\mathrm{tr}}$,

$$\mathrm{Re}\,L^{(2)}_{\rm eff}[\pi]=N_c\,\mathrm{tr}_f\,\mathrm{Re}\left(\int d^4x\,G_0(x,y)\partial_\mu U^+(y,z)d^4z\,G_0(z,u)d^4u\,\partial_\mu U(u,x)\right). \tag{62}$$

In the limit when $M_S(x,y)$ becomes local,

$$U(x,y)\approx\tilde{\delta}^4(x-y)M_S(x)e^{I\hat{\phi}(x)}=\tilde{\delta}^4(x-y)U(x),$$

one has

$$\mathrm{Re}\,L^2_{\rm eff}[\pi]=N_c\,\mathrm{tr}_f\,\mathrm{Re}\left(\int d^4xd^4yG_0(x,y)\partial_\mu U^+(y)G_0(y,x)\partial_\mu U(x)\right). \tag{63}$$

One can see that Eq. (63) yields a nonlocal chiral Lagrangian, the nonlocality being given by the range of the quark Green's function $G_0(x,y)$. In the case of a model without confinement, e.g., the instanton model or the Nambu–Jona-Lasinio (NJL) model, $G_0(x,y)$ would represent the free Green's function of a massive quark, with the constituent mass μ created by chiral symmetry breaking, $\mu\approx0.3$ GeV. Hence the range of nonlocality in this model case is large, and the derivative expansion in powers of ∂U, which is a standard systematic procedure in [5,7,30], is justified for small momenta $p \leqslant \mu$. At this point, one should remember that in Eqs. (61)–(63) both $M_S(x,y)$ and $U(x,y)$ [and hence $S(x,y)$, Eq. (34)] are defined gauge invariantly with respect to the contour Y, which for simplicity was taken to be the x_4 axis. Therefore, the resulting string in $\hat{M}, M_S$ goes from the points (x,y) (coinciding when T_g tends to zero) to the contour Y. Physically it means that the effective Lagrangian (62) describes the bosonic field (e.g., a pion) in the presence of the heavy quark. It will enable us to define in the following section matrix elements of heavy-light meson transitions with emission of a pion. The case of an effective boson Lagrangian for light quarks and antiquarks (i.e., without the heavy quark line) will be considered in the second paper of this series.

VI. PIONIC TRANSITIONS IN HEAVY-LIGHT MESONS

In this section, the main emphasis will be on the pionic part of the quark mass operator, which enters the quark-meson Lagrangian as

$$\Delta L = i \int d^4x\, d^4y\; \psi^+(x) \hat{M}(x,y) \psi(y), \tag{64}$$

where $\hat{M}(x,y)$ according to Eq. (46) can be written in the form

$$\hat{M}(x,y) = M_S(x,y) e^{i\gamma_5 \hat{\phi}(x,y)}. \tag{65}$$

In the limit of small T_g, one obtains for $M_S(x,y)$ a localized expression

$$M_S(x,y) \approx \sigma |\mathbf{x}| \delta^{(4)}(x-y), \quad |\mathbf{x}| \gg T_g \tag{66}$$

and the corresponding Lagrangian linearized in $\hat{\phi}$ is (in the Minkowskian space-time)

$$\Delta L^{(1)} = \int \bar{\psi}(x) \sigma |\mathbf{x}| \gamma_5 \frac{\pi^A \lambda^A}{F_\pi} \psi(x) dt\, d^3x, \tag{67}$$

where F_π is known [3] to be $F_\pi = 94$ MeV. For the pionic transition between heavy-light states, one should compute the matrix elements

$$\langle M_2(\mathbf{p}_2), \pi(\mathbf{k}) | \Delta L^{(1)} | M_1(\mathbf{p}_1) \rangle. \tag{68}$$

Neglecting the recoil momentum of a heavy-light meson, one effectively reduces the problem to the calculation of the matrix element,

$$W_{21} = \int \bar{\psi}_2(x) \frac{\sigma|\mathbf{x}|}{F_\pi} \gamma_5 \frac{\lambda^A e^{i\mathbf{kx}}}{\sqrt{2\omega_\pi(\mathbf{k}) V}} \psi_1(\mathbf{x}) d^3x, \tag{69}$$

and the decay probability is

$$w = 2\pi |W_{21}|^2 \delta(E_1 - E_2 - \omega) \frac{V d^3k}{(2\pi)^3}. \tag{70}$$

In Eq. (66), $\bar{\psi}_2(x), \psi_1(x)$ are quark wave functions of the heavy-light states 2 and 1, respectively, which can be taken from the solutions of the corresponding Dirac equations, found in [32,26]. At this point, one can rewrite the matrix element of $\Delta L^{(1)}$ between two stationary quark states ψ_m, ψ_n. Using the Dirac equations

$$[\boldsymbol{\alpha}\mathbf{p} + \beta(m + \sigma|\mathbf{x}|) + V_{\mathrm{Coul}}] \psi_n = \epsilon_n \psi_n, \tag{71}$$

$$\bar{\psi}_m[-\boldsymbol{\alpha}\mathbf{p} + \beta(m + \sigma|\mathbf{x}|) + V_{\mathrm{Coul}}] = \epsilon_m \bar{\psi}_m, \tag{72}$$

one obtains

$$\langle m|\Delta L^{(1)}|n\rangle = \frac{1}{2F_\pi} \langle m| -2m\gamma_5 \hat{\pi} + \beta\gamma_5(\epsilon_m - \epsilon_n)\hat{\pi} + \gamma_5 \beta \boldsymbol{\alpha}\mathbf{p} \hat{\pi} |n\rangle. \tag{73}$$

In the chiral limit, $m=0$, one can rewrite the last two terms inside the brackets in Eq. (73) as

$$\gamma_5 \left(\boldsymbol{\gamma} \frac{\partial \hat{\pi}}{\partial \mathbf{x}} + i\beta \frac{\partial \hat{\pi}}{\partial t} \right) = \gamma_5 \gamma_\mu \partial_\mu \hat{\pi}. \tag{74}$$

One obtains from Eq. (73) the form of the quark-pion interaction which one usually writes as (see [33,34] and references therein)

$$\Delta L^{\mathrm{ch}} = g_A^q \, \mathrm{tr}(\bar{\psi} \gamma_\mu \gamma_5 \omega_\mu \psi), \qquad \omega = i(u\partial_\mu u^+ - u^+ \partial_\mu u), \tag{75}$$

where $u = \sqrt{\hat{U}}$ and g_A^q is the axial vector coupling of the (constituent) quark. It is easy to see that Eq. (73) is the first term in the expansion of Eq. (75) in powers of the field $\hat{\pi}$, and it is gratifying that the parameter g_A^q is defined theoretically to be equal exactly to 1 in our local approximation, which is in agreement with the large N_c argument in [33].

VII. CONCLUDING REMARKS

We have performed a systematic bosonization procedure starting from the QCD Lagrangian and have derived the nonlocal chiral Lagrangian and its limiting local form for quarks interacting with a pionic field. By the method of construction, the resulting Lagrangian is applicable for quark-pion interaction when the quark is coupled by the string to the heavy antiquark. Therefore, our results can be immediately applied to the pionic transitions in the heavy-light mesons. The one-pion transitions have been studied using the form (75) in [34], and it was found from comparison to experiment that the values of g_A^q around 0.7 are preferred. It makes it reasonable to study the nonlocal version of the chiral Lagrangian (73), since nonlocality effectively decreases the resulting matrix elements, possibly explaining the mismatch between our theoretical value $g_A^q = 1$ and the observed value of 0.7. In a similar way, one can obtain matrix elements for double pion emission in heavy boson transitions, which are of special interest for $(B,D),(B,D^*)$ semileptonic decays. The case of purely light mesons can be treated in a similar way, but needs another string configuration to be taken into account. We plan to do in the subsequent paper of this series.

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APPENDIX A: VACUUM AVERAGES OF FIELD CORRELATORS

For the field operator transported to the point Y,

$$F^{(Y)}_{\mu\nu}(x)=\Phi(Y,x)F_{\mu\nu}(x)\Phi(x,Y) \tag{A1}$$

so that gauge transformation has the form

$$F^{(Y)}_{\mu\nu}(x)\to U^+(Y)F^{(Y)}_{\mu\nu}(x)U(Y), \tag{A2}$$

the vacuum average of any product can be expressed through Kronecker symbols, e.g.,

$$g^2\langle[F^{(Y)}(x)]_{ab}[F^{(Y)}(y)]_{cd}\rangle=\delta_{ad}\delta_{bc}\mathcal{P}(x,y), \tag{A3}$$

where ab,cd are fundamental color indices and $\mathcal{P}(x,y)$ can be easily connected to the color trace, i.e., to the FC,

$$\mathcal{P}(x,y)=\frac{\delta_{ad}\delta_{bc}}{N_c^2}g^2\langle\mathrm{tr}[F^{(Y)}(x)F^{(Y)}(y)]\rangle. \tag{A4}$$

Similar rules apply to products of any number of $F^{(Y)}$. They can be deduced from the global $SU(N_c)$ rules for the tensors averaged with the Haar measure, namely one can make a gauge transformation

$$[F^Y_{\mu\nu}(x)]_{ab}\to\Omega^+(Y)_{ac}[F^Y_{\mu\nu}(x)]_{cd}\Omega(Y)_{db} \tag{A5}$$

and average over $D\Omega$ with the usual Haar measure. In this way one obtains relation (8). At this point, it is interesting to note that the product of three F's may have another representation in the $SU(3)$ group, since there one can use the totally antisymmetric tensor e_{abc}.

For example, one can write the following equality:

$$e_{ace}e_{bdf}=\delta_{ab}(\delta_{cd}\delta_{ef}-\delta_{cf}\delta_{ed})+\delta_{ad}(\delta_{cf}\delta_{eb}-\delta_{cb}\delta_{ef})+\delta_{af}(\delta_{cb}\delta_{ed}-\delta_{cd}\delta_{eb}). \tag{A6}$$

Therefore, one can use two forms of writing for the product. In the first case, one writes

$$\langle[F^{(Y)}(x)]_{ab}[F^{(Y)}(y)]_{cd}[F^{(Y)}(z)]_{ef}\rangle=\mathcal{P}_1(x,y,z)\delta_{bc}\delta_{de}\delta_{af}+\mathcal{P}_2(x,y,z)\delta_{bc}\delta_{fc}\delta_{ad} \tag{A7}$$

and finds $\mathcal{P}_1,\mathcal{P}_2$ by multiplying both sides of Eq. (A7) with the corresponding contribution of δ symbols. In this way, one arrives at Eq. (8) in the main text. If instead one uses Eq. (A6) instead of one of the combinations, or separates Eq. (A6) out of Eq. (8), one arrives at the white $(3q),(3\bar{q})$ combinations.

APPENDIX B: VACUUM AVERAGES OF $\langle(A)^n\rangle$

One can use the generalized contour gauge expression for $A_\mu(x)$,

$$A_\mu(x)=\int_Y^x dz_\nu\frac{\partial z_\rho}{\partial x_\mu}F_{\nu\rho}(z)\equiv\int_Y^x d\Gamma_{\mu\nu\rho}(z)F_{\nu\rho}(z) \tag{B1}$$

to represent the average of the product of any number of operators A_μ as

$$\langle(A^{(x_1)}_{\mu_1})_{a_1b_1}\cdots[A_{\mu_n}(x_n)]_{a_nb_n}\rangle=\int_Y^{x_1}d\Gamma_{\mu_1\nu_1\rho_1}(z_1)\cdots\int_Y^{x_n}d\Gamma_{\mu_n\nu_n\rho_n}(z_n)\times\langle[F^{(Y)}_{\nu_1\rho_1}(z_1)]_{a_1b_1}\cdots[F^{(Y)}_{\nu_n\rho_n}(z_n)]_{a_nb_n}\rangle. \tag{B2}$$

In the particular case of the modified Fock-Schwinger gauge, one has $Y=0,\ x_i\to\mathbf{x}_i$,

$$d\Gamma_{\mu\nu\rho}(z)=\alpha_\mu(z)dz_\nu\delta_{\rho\mu}. \tag{B3}$$

APPENDIX C: FIERZ TRANSFORMATIONS

In this appendix, the derivation is given of Fierz tranformations for combinations of γ_μ matrices met in the text above. It is based on the clear presentation done in the book [29]. Note, however, that we are always working with the Euclidean γ matrices, therefore some details and coefficients obtained below are different from [28].

We start with the general expansion for any 4×4 matrix,

$$\gamma=\tfrac{1}{4}\sum_A C_A\gamma_A,\quad A=1,\ldots,16, \tag{C1}$$

$$\gamma_A=1,\quad A=1,$$

$$\gamma_A=\gamma_\mu,\quad \mu=1,2,3,4;\quad A=2,3,4,5,$$

$$\gamma_A=\sigma_{\mu\nu}=\frac{\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu}{2i}, \quad A=6,7,8,9,10,11, \tag{C2}$$

$$\gamma_A=\gamma_5\gamma_\mu, \quad A=12,13,14,15,$$

$$\gamma_A=\gamma_5, \quad A=16.$$

In Eq. (C1), one can derive the general representation for any matrices F_{mk},G_{il}, indeed from the relation

$$\tfrac{1}{4}\sum_A \Delta_A\gamma^A_{ml}\gamma^A_{ik}=\delta_{mk}\delta_{il},$$

changing $m\to m'$, $l\to l'$, and multiplying with $F_{mm'}G_{l'l}$ one obtains

$$F_{mk}G_{il}=\tfrac{1}{4}\Delta\sum_A \Delta_A(F\gamma_A G)_{ml}(\gamma_A)_{ik}. \tag{C3}$$

For Euclidean matrices γ_A one easily obtains $\Delta_a=-1$ for $A=12,13,14,15$ and $\Delta_A=1$ otherwise. Taking $F=G=\gamma^4$, one obtains

$$\gamma^4_{mk}\gamma^4_{il}=\tfrac{1}{4}\sum_A \Delta_A(\gamma^4\gamma_A\gamma^4)_{ml}(\gamma_A)_{ik}=\tfrac{1}{4}\{(1)(1)+(\gamma^4)(\gamma^4)-(\gamma^i)(\gamma^i)+(\sigma^{ik})(\sigma^{ik})-(\sigma^{4k})(\sigma^{4k})-(\sigma^{k4})(\sigma^{k4})+(\gamma^5\gamma^4)$$
$$\times(\gamma^5\gamma^4)-(\gamma^5\gamma^i)(\gamma^5\gamma^i)-(\gamma^5)(\gamma^5)\}=\tfrac{1}{4}\sum_{k=1}^{5}\bar{O}^{(k)}_{ml}\bar{O}^{(k)}_{ik}. \tag{C4}$$

The preceding relation coincides with Eq. (23) and operators $\bar{O}^{(k)}$ are given in Eq. (24). Note that in the curly brackets in Eq. (C4) each product of γ matrices, $(\gamma\gamma)(\gamma\gamma)$, has the same order of indices as in $\bar{O}^{(k)}_{ml}\bar{O}^{(k)}_{ik}$. In a similar way, one can represent the combination of spatial γ matrices, $n=1,2,3$, with no summation over n,

$$\gamma^n_{mk}\gamma^n_{il}=\tfrac{1}{4}\{(1)(1)+(\gamma^4)(\gamma^4)-(\gamma^m)(\gamma^m)+(\gamma^n)(\gamma^n)+(\sigma^{\mu\nu})(\sigma^{\mu\nu})_{n\neq\mu\nu}-(\sigma^{n\nu})(\sigma^{n\nu})-(\sigma^{\nu n})(\sigma^{\nu n})-(\gamma^5\gamma^\mu)(\gamma^5\gamma^\mu)_{\mu\neq n}$$
$$+(\gamma^5\gamma^n)(\gamma^5\gamma^n)-(\gamma^5)(\gamma^5)\}_{ml,ik}. \tag{C5}$$

Summing Eq. (C5) over n and adding Eq. (C4), one obtains

$$(\gamma^\mu)_{mk}(\gamma^\mu)_{il}=\{(1)(1)-\tfrac{1}{2}(\gamma^\mu)(\gamma^\mu)-\tfrac{1}{2}(\gamma^5\gamma^\mu)(\gamma^5\gamma^\mu)-(\gamma^5)(\gamma^5)\}_{ml,ik}. \tag{C6}$$

Now in the case of a generalized contour gauge as in Appendix B, one has to make Fierz transformation of the combination

$$(\gamma^\mu)_{mk}(\gamma^\nu)_{il}=\tfrac{1}{4}\sum_A \Delta_A(\gamma^\mu\gamma_A\gamma^\nu)_{ml}(\gamma_A)_{ik}. \tag{C7}$$

Note that scalar and pseudoscalar combinations occur on the left-hand side of Eq. (C7) only for $\mu=\nu$.

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