Baryon magnetic moments in the effective quark Lagrangian approach

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An effective quark Lagrangian is derived from first principles through bilocal gluon field correlators. It is used to write down equations for baryons, containing both perturbative and nonperturbative fields. As a result one obtains magnetic moments of octet and decuplet baryons without the introduction of constituent quark masses and using only string tension as input. Magnetic moments come out on average in reasonable agreement with experiment, except for nucleons and Σ^-. The predictions for the proton and neutron are shown to be in close agreement with the empirical values once we choose the string tension to yield the proper nucleon mass. Pionic corrections to the nucleon magnetic moments have been estimated. In particular, the total result of the two-body current contributions is found to be small. Inclusion of the anomalous magnetic moment contributions from pion and kaon loops leads to an improvement of the predictions.

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I. INTRODUCTION

The QCD dynamics of $q\bar{q}$ and $3q$ systems is governed by two basic phenomena: confinement and chiral symmetry breaking (CSB), which should be treated in a fully relativistically covariant way. Confinement is usually introduced for static quarks via the area law of the Wilson loop [1] or equivalently through the field correlators in the field correlator method (FCM) [2,3].

For spinless quarks or neglecting spin-dependent mass corrections, one can envisage a self-consistent method which treats confinement as the area law also for light quarks in a relativistically covariant way. Such a method was introduced originally in [4] for mesons, in [5] for baryons, and in [6] for heavy-light mesons, and later on in [7] the method was generalized taking into account the dynamical degrees of freedom of the QCD string, which naturally appears due to the area law.

As a result Regge trajectories have been found in [7] with the correct string slope $(2\pi\sigma)^{-1}$. It was realized later on, that the method used in [4–7] can be more generally developed in the framework of the so-called einbein formalism; see [8–10]. Spin corrections have been considered in [11] for heavy mesons and in [6] for heavy-light ones. In the general case of light quarks spin-dependent correlations have been introduced in [12] and for gluons in [13]. For a general review with explicit formulas see [14]. Baryon Regge trajectories have been found in [5]. In all cases the basic formalism is the FCM and the Feynman-Schwinger (or world-line) path integral representation [3,15,16] which is well suited for relativistic quarks when spin is considered as a perturbation.

The main difficulty which was always present in this method has been the perturbative treatment of spin degrees of freedom (which is incorrect, e.g., for the pion) and the absence of spontaneous CSB effects in general [17]. Recently a new type of formalism was suggested to treat simultaneously confinement and CSB and a nonlinear equation was derived for a light quark in the field of a heavy antiquark [18]. This equation, derived directly from the QCD Lagrangian, was found to produce linear confinement and CSB for the light quark and the explicit form of the effective quark mass operator $M(x,y)$ was defined obeying both these properties.

The eigenvalues and eigenfunctions of the nonlocal and nonlinear equations have been determined and a nonzero condensate was computed in [19], confirming that CSB is really present in the equations. In an additional study [20] it was demonstrated that magnetic field correlators do not contribute to the large distance confinement; however, they strongly modify the confinement for lowest levels and heavy-light masses corrected in this way are favorably compared in [20] to the experiment and results of other calculations.

Moreover, it was shown in [21] that lattice data strongly support the dominance of the Gaussian (bilocal) correlator, estimating the correction due to higher correlators to 1%–2%. Since the method of [18] is quite general and allows one to treat also multiquark systems, it can be applied to the $q\bar{q}$ and $3q$ systems, to find dynamical equations for them, which contain confinement and CSB [22]. To make these equations tractable, one systematically exploits the large-N_c limit and mostly confine ourselves to the simplest field correlators—the so-called Gaussian approximation; it was in particular shown in [18] that the sum over all correlators does not change the qualitative results. However, the kernel of equations becomes much more complicated.

In the present paper we study the baryon magnetic moments based on the derived effective Lagrangian without constituent quark masses. The paper is organized as follows. In Sec. II the general effective quark Lagrangian from the standard QCD Lagrangian is obtained by integrating out gluonic degrees of freedom, and the nonlinear equation for the single quark propagator S (attached to the string in a gauge-invariant way) is derived, following the procedure in [22]. Section III is devoted to the baryon Green's function, which can be expressed in the lowest order of our approximation scheme (neglecting gluon and pion exchanges) in terms of three independent quark Green's function, resulting in a Hamiltonian as a sum of three quark terms. In Sec. IV the next order approximation is written down when perturbative gluon exchanges are taken into account, including the nonperturbative interaction between quarks violating the factorized form of the zeroth-order approximation. The next section is devoted to the calculation of magnetic moments of baryons both in octet and decuplet representations of the SU(3) flavor group. In Sec. VI we discuss the corrections to magnetic moments due to pion exchange contributions.

II. EFFECTIVE QUARK LAGRANGIAN

As was discussed in the previous section, one can obtain an effective quark Lagrangian by averaging over background gluonic fields. We shall repeat this procedure following [18], now paying special attention to the dependence on the contour in the definition of contour gauge and introducing the operation of averaging over contour manifold. The QCD partition function for quarks and gluons can be written as

$$Z=\int DAD\psi D\psi^{\dagger}\exp[L_0+L_1+L_{\rm int}], \tag{1}$$

where we are using Euclidean metric and define

$$L_0=-\frac{1}{4}\int d^4x(F^a_{\mu\nu})^2, \tag{2}$$

$$L_1=-i\int {}^f\psi^{\dagger}(x)(\hat{\partial}+m_f)\,{}^f\psi(x)d^4x, \tag{3}$$

$$L_{\rm int}=\int {}^f\psi^{\dagger}(x)g\hat{A}(x)\,{}^f\psi(x)d^4x. \tag{4}$$

Here m_f is the (current) mass of the quark field ${}^f\psi_{a\alpha}$ with flavor f, color a, and bispinor index α.

To express $A_\mu(x)$ through $F_{\mu\nu}$ one can use the generalized Fock-Schwinger gauge [23] with the contour $C(x)$ from the point x to x_0, which can also lie at infinity:

$$A_\mu(x)=\int_c F_{\lambda\beta}(z)\frac{\partial z_\beta(s,x)}{\partial x_\mu}\frac{\partial z_\lambda}{\partial s}ds. \tag{5}$$

Now one can integrate out the gluonic field $A_\mu(x)$ and introduce an arbitrary integration over the set of contours $C(x)$ with weight $D_\kappa(C)$, since Z is gauge invariant it does not depend on the contour $C(x)$. One obtains

$$Z=\int D\kappa(C)D\psi D\psi^{\dagger}\exp\{L_1+L_{\rm eff}\}, \tag{6}$$

where the effective quark Lagrangian $L_{\rm eff}$ is defined as

$$\exp(L_{\rm eff})=\left\langle \exp\int {}^f\psi^{\dagger}\hat{A}\,{}^f\psi d^4x\right\rangle_A. \tag{7}$$

When the quark fields are treated statically the right-hand side (RHS) of Eq. (7) reduces to the well-known Wilson loop. To study the averaging of the gluonic field configurations we adopt the correlator method, based on the series expansion of the exponent operator. Using the cluster expansion, $L_{\rm eff}$ can be written as an infinite sum containing averages $\langle(\hat{A})^k\rangle_A$. At this point one can exploit the Gaussian approximation, neglecting all correlators $\langle(\hat{A})^k\rangle$ of degree higher than $k=2$. Numerical accuracy of this approximation was discussed and tested in [21]. One expects that for static quarks corrections to Gaussian approximation amount to less than 2%–3%.

The resulting effective Lagrangian is quartic in ψ,

$$L^{(4)}_{\rm eff}=\frac{1}{2N_c}\int d^4xd^4y\,{}^f\psi^{\dagger}_{a\alpha}(x)\,{}^f\psi_{b\beta}(x)\,{}^g\psi^{\dagger}_{b\gamma}(y)\,{}^g\psi_{a\delta}(y)$$
$$\times J_{\alpha\beta;\gamma\delta}(x,y)+O(\psi^6), \tag{8}$$

$$J_{\alpha\beta,\gamma\delta}(x,y)=(\gamma_\mu)_{\alpha\beta}(\gamma_\nu)_{\gamma\delta}J_{\mu\nu}(x,y), \tag{9}$$

and $J_{\mu\nu}$ is expressed as

$$J_{\mu\nu}(x,y)=g^2\int_C^x\frac{\partial u_\omega}{\partial x_\mu}du_\varepsilon\int_C^y\frac{\partial v_{\omega'}}{\partial y_\nu}dv_{\varepsilon'}\frac{\rm tr}{N_c}$$
$$\times\langle F_{\varepsilon\omega}(u)F_{\varepsilon'\omega'}(v)\rangle. \tag{10}$$

$L_{\rm eff}$, Eq. (8), is written in the contour gauge [23]. It can be identically rewritten in the gauge-invariant form if one substitutes parallel transporters $\Phi(x,x_0),\Phi(y,x_0)$ (identically equal to unity in this gauge) into Eqs. (8) and (10), multiplying each $\psi(x)$ and $\psi(y)$, respectively, and replacing $F(u)$ in Eq. (10) by $\Phi(x,u)F(u)\Phi(u,x_0)$ and similarly for $F(v)$.

After that $L_{\rm eff}$ becomes gauge invariant, but in general contour dependent, if one keeps only the quartic term (8), and neglects all higher terms. A similar problem occurs in the cluster expansion of Wilson loop, when one keeps only lowest correlators, leading to the (erroneous) surface dependence of the result. The situation here is the same as with a sum of QCD perturbation series, which depends on the normalization mass μ for any finite number of terms in the series. This unphysical dependence is usually treated by fixing μ at some physically reasonable value μ_0 (of the order of the inverse size of the system).

The integration over contours $D\kappa(C)$ in Eq. (6) resolves this difficulty in a similar way. Namely, the partition function Z formally does not depend on contours (since it is integrated over a set of contours) but depends on the weight $D\kappa(C)$. We choose this weight in such a way that the contours would generate a string of minimal length between q and $\bar{q}$. Thus the physical choice of the contour corresponds to the minimization of the meson (baryon) mass over the class of strings, in the same way as the choice of $\mu=\mu_0$ corresponds to the minimization of the dropped higher perturbative terms. As a practical outcome, we shall keep the integral $D\kappa(C)$ till the end and finally use it to minimize the string between the quarks.

Until this point we have made only one approximation—neglected all field correlators except the Gaussian one. Recent lattice calculations (see Refs. [24,25]) estimate the accuracy of this approximation at the level of a few percent. Now one must use another approximation, i.e., assume a large-N_c expansion and keep the lowest term. As was shown in [18] this enables one to replace in Eq. (8) the colorless product ${}^f\psi_b(x)\,{}^g\psi_b^\dagger(y)=\mathrm{tr}[\,{}^f\psi(x)\Phi(x,x_0)\Phi(x_0,y)\,{}^g\psi^+(y)]$ b the quark Green's function

$$ {}^f\psi_{b\beta}(x)\,{}^g\psi^\dagger_{b\gamma}(y)\rightarrow\delta_{fg}N_cS_{\beta\gamma}(x,y). \tag{11}$$

$L_{\rm eff}^{(4)}$ assumes the form

$$L_{\rm eff}^{(4)}=-i\int d^4xd^4y\,{}^f\psi^\dagger_{a\alpha}(x)\,{}^fM_{\alpha\delta}(x,y)\,{}^f\psi_{a\delta}(y), \tag{12}$$

where the quark mass operator is

$$ {}^fM_{\alpha\delta}(x,y)=-J_{\mu\nu}(x,y)(\gamma_\mu\,{}^fS(x,y)\gamma_\nu)_{\alpha\delta}. \tag{13}$$

From Eq. (12) it is evident that ${}^fS$ satisfies

$$(-i\hat{\partial}_x-im_f)\,{}^fS(x,y)-i\int {}^fM(x,z)d^4z\,{}^fS(z,y)$$
$$=\delta^{(4)}(x-y). \tag{14}$$

Equations (13) and (14) were first derived in [18]. From Eqs. (6) and (12) one should expect that at large N_c the $q\bar{q}$ and $3q$ dynamics is expressed through the quark mass operator (13), which should contain both confinement and CSB. Indeed, the analysis performed in Refs. [18–20] reveals that confinement is present in the long-distance form of $M(x,y)$, when both distances $|\mathbf{x}|,|\mathbf{y}|$ of the light quark from the heavy antiquark (placed at the origin) are large.

We shall now make several simplifying assumptions to clarify the structure of $M(x,y)$. First of all we take the class of contours C going from any point $x=(x_4,\mathbf{x})$ to the chosen point $x_0=(x_4,\vec{r}^{(0)})$ and then to $(-\infty,\vec{r}^{(0)})$ along the x_4 axis. For this class the corresponding gauge was studied in [26]. Second, we take the dominant part of $J_{\mu\nu}$ in Eq. (13), namely, J_{44}, which is proportional to the correlator of color-electric fields. This yields a linear confining interaction, while the other components J_{ik},J_{i4},J_{4i}, $i=1,2,3$, have been neglected, containing magnetic fields and yielding momentum-dependent corrections. (It is easy to take into account these contributions in a more detailed analysis.)

The correlator $\langle FF\rangle$ in Eq. (10) can be expressed through the scalar correlator $D(x)$, defined as [2]

$$\frac{\mathrm{tr}\,g^2}{N_c}\langle F_{\alpha\beta}(u)\Phi(u,v)F_{\gamma\delta}(v)\Phi(v,u)\rangle$$
$$=D(u-v)(\delta_{\alpha\gamma}\delta_{\beta\delta}-\delta_{\alpha\delta}\delta_{\beta\gamma})+O(D_1), \tag{15}$$

where the correlator D_1, not contributing to confinement, is neglected. As a result one has, for M [19,20],

$$ {}^fM_{C_{x_4}}(x,y)={}^fM^{(0)}I+{}^fM^{(i)}\hat{\sigma}_i+{}^fM^{(4)}\gamma_4+{}^fM^{(i)}_\gamma\gamma_i. \tag{16}$$

Here we have defined

$$\hat{\sigma}_i=\begin{pmatrix}\sigma_i & 0\\ 0 & \sigma_i\end{pmatrix}. \tag{17}$$

The dominant part of M, ${}^fM^{(0)}$ is linearly growing at large $|\mathbf{x}|,|\mathbf{y}|$ and in the most simple case of Gaussian form of $D(x)$ can be written as

$$ {}^fM^{(0)}(x,y)=\frac{1}{2T_g\sqrt{\pi}}e^{-(x_4-y_4)^2/4T_g^2}\sigma\left|\frac{\mathbf{x}+\mathbf{y}}{2}\right|\tilde{\delta}^{(3)}(\mathbf{x}-\mathbf{y}), \tag{18}$$

where T_g is the correlation time characterizing the time scale of correlations in the fluctuations of the gluon background field. It has been studied in lattice gauge simulations [24] and found to be of the order of $\frac{1}{4}$ fm. Following Ref. [19] we have adopted a value $T_g=0.24$ fm.

Similarly, in the space dimension we assume for $\tilde{\delta}$ in Eq. (18) a smeared δ function, which can be represented as [19,20]

$$\tilde{\delta}^{(3)}(\mathbf{x}-\mathbf{y})\approx\exp\left(-\frac{|\mathbf{x}-\mathbf{y}|^2}{b^2}\right)\left(\frac{1}{b\sqrt{\pi}}\right)^3,\quad b\sim 2T_g. \tag{19}$$

Here again T_g is the gluon correlation length, which enters $D(u)$ as $D(u)=D(0)\exp(-u^2/4T_g^2)$. In Eq. (18) the parameter σ has been introduced. It corresponds to the string tension, as can be seen from Eq. (14). For asymptotic large $|x|$ we find that the kernel (18) leads to a linear confining interaction $\sigma|x|$ [19,20]. We are now in the position to derive the $q\bar{q}$, $3q$ Green's function, which will be done in the next section.

III. EQUATIONS FOR THE BARYON GREEN'S FUNCTION

Equations for the $3q$ system can be written in the same way as for the $q\bar{q}$ system. We again shall assume the large-N_c limit in the sense that $1/N_c$ corrections from $q\bar{q}$ pairs to the quark Green's function and the effective mass can be neglected. We now write down the explicit expressions for $N_c=3$.

The initial and final field operators are

$$\Psi_{in}(x,y,z)=e_{abc}\Gamma^{\alpha\beta\gamma}\psi_{a\alpha}(x,C(x))\psi_{b\beta}(y,C(y))$$
$$\times\psi_{c\gamma}(z,C(z)), \tag{20}$$

where a,b,c, are color indices, α,β,γ are Lorentz bispinor indices, and transported quark operators are

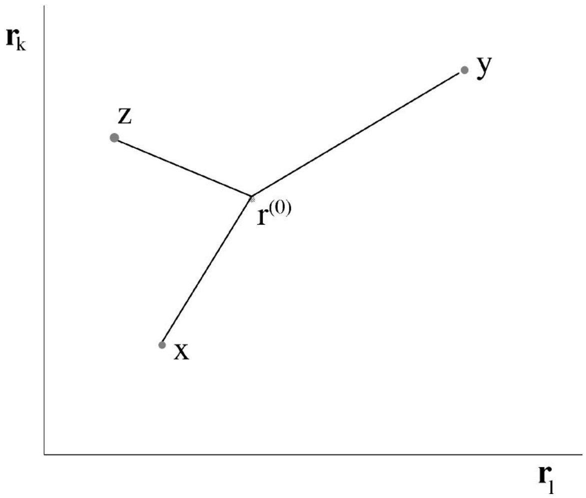

FIG. 1. The set of contours C for the instantaneous configuration of three quarks at $x=(x_4,\vec{x})$, $y=(x_4,\vec{y})$, and $z=(x_4,\vec{z})$, passing through the common point $x_0=(x_4,\vec{r}^{(0)})$.

$$\psi_{a\alpha}(x,C(x))=(\Phi_C(x,\overline{x})\psi_\alpha(\overline{x}))_a. \tag{21}$$

The contour $C(x)$ in Φ_C can be arbitrary, but it is convenient to choose it in the same class of contours that is used in $D\kappa(C)$ and in the generalized Fock–Schwinger gauge [23]. $\Gamma^{\alpha\beta\gamma}$ is the Lorentz spinor tensor securing proper baryon quantum numbers. One can also choose other operators, but it does not influence the resulting equations. In Eq. (20) we have omitted flavor indices in Γ and $\psi(x,C)$, to be easily restored in the final expressions.

Using now the effective Lagrangian (12) valid at large N_c, we obtain, for the $3q$ Green's function,

$$G^{(3q)}(x,y,z|x',y',z')$$
$$=\frac{1}{N}\int D\kappa(C)D\psi D\psi^\dagger\Psi_{\rm fin}(x',y',z')\Psi^\dagger_{\rm in}(x,y,z)$$
$$\times\exp(L_1+L_{\rm eff}). \tag{22}$$

Integrating out quark degrees of freedom and neglecting the determinant at large N_c, one has

$$G^{(3q)}=\int D\kappa(C)(e\Gamma)(e'\Gamma')$$
$$\times\{S(x,x')S(y,y')S(z,z')+\text{perm}\}, \tag{23}$$

where for simplicity color and bispinor indices are suppressed together with parallel transporters in initial and final states. One can also define unprojected (without Γ,Γ') $3q$ Green's function $G^{(3q)}_{\rm un}$ with three initial and three final bispinor indices instead of projected by Γ,Γ' quantum numbers of the baryon.

The set of contours $C(x)$ in Eq. (23) should be chosen to yield a stationary point of the action (6). For the particular case of small correlation time T_g, i.e., taking $x_4=y_4=z_4$, we may assume that this can be achieved by a single choice of contours passing through the point $x_0=(x_4,\mathbf{r}^{(0)})$ (see Fig. 1). This can readily be generalized for the noninstantaneous case.

One can write, for $G^{(3q)}_{\rm un}$,

$$(-i\hat{\partial}_x-im_1-i\hat{M}_1)(-i\hat{\partial}_y-im_2-i\hat{M}_2)$$
$$\times(-i\hat{\partial}_z-im_3-i\hat{M}_3)G^{(3q)}_{\rm un}$$
$$=\delta^{(4)}(x-x')\delta^{(4)}(y-y')\delta^{(4)}(z-z'), \tag{24}$$

with, e.g., $\hat{M}_1G\equiv\int M(x,u)G(u,x')d^4u$. One can simplify the form (24) for $G^{(3q)}$ taking into account that $M(x,x')$ actually does not depend on $(x_4+x_4')/2$. Hence the interaction kernel of $G^{(3q)}$ does not depend on relative energies, as in [27]. Similarly to [27,28] one can introduce the Fourier transform of $G^{(3q)}$ in time components and take into account energy conservation $E=E_1+E_2+E_3$. One obtains

$$G^{(3q)}(E,E_2,E_3)\simeq\int D\kappa(C)(e\Gamma)(e'\Gamma')$$
$$\times\frac{1}{(E-E_2-E_3-H_1)(E_2-H_2)(E_3-H_3)}, \tag{25}$$

where we have used the notation

$$H_i=m_i\beta^{(i)}+\mathbf{p}^{(i)}\boldsymbol{\alpha}^{(i)}+\beta^{(i)}M(\mathbf{r}^{(i)}-\mathbf{r}^{(0)}). \tag{26}$$

Moreover, we have taken in $M(x,x')$ the limit of small T_g. As in [27] one can now integrate over E_2,E_3 to obtain, finally,

$$G^{(3q)}(E,\mathbf{r}_i,\mathbf{r}_i')\simeq\int D\kappa(C)(e\Gamma)(e'\Gamma')\frac{1}{(E-H_1-H_2-H_3)}. \tag{27}$$

From Eq. (27) one obtains an equation for the $3q$ wave function similar to that of the $q\overline{q}$ system:

$$(H_1+H_2+H_3-E)\psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)=0. \tag{28}$$

For a (arbitrary) given point $\mathbf{r}^{(0)}$ the solution of Eq. (28) is of a factorizable form. Hence, when we treat $\mathbf{r}^{(0)}$ as a constant parameter, the three-quark wave function is simply expressed in terms of single-quark orbitals. However, in general the point $\mathbf{r}^{(0)}$ should be found by minimizing the interaction between the three quarks, yielding for $\mathbf{r}^{(0)}$ the so-called Torricelli point. As a result we obtain for the effective string not an additive two-body confining interaction, but a single-string Y junction, which is of a genuine three-body nature.

Here we do not consider, in Eq. (28), $\mathbf{r}^{(0)}$ to be expressed through three-quark positions, as required by the Torricelli point, but in a first approximation take it as a constant parameter. This allows us to have the three-quark solutions in factorized form, leaving calculation of the dynamical correlations induced by nonfactorizability to a further study.

In the nonrelativistic approximation $m_i \gg \sqrt{\sigma}$ one has

$$\sum_{i=1}^{3}\left[\frac{(\mathbf{p}^{(i)})^2}{2m_i}+\sigma|\mathbf{r}^{(i)}-\mathbf{r}^{(0)}|\right]\Psi=\varepsilon\Psi, \quad \varepsilon=E-\sum m_i. \tag{29}$$

IV. PERTURBATIVE CORRECTIONS TO FACTORIZED SOLUTIONS

The effective Lagrangian (8) and the effective mass operator $M(x,y)$, Eq. (13), do not take into account the perturbative interaction between the quarks in the baryon. To this end we separate the gluonic field A_μ into a background B_μ and perturbative parts $A_\mu = B_\mu + a_\mu$ and use the 't Hooft identity to integrate in the partition function independently over both parts of A_μ as was done in [30].

We shall use the following representation of gauge transformations,

$$B_\mu \to U^\dagger\left(B_\mu+\frac{i}{g}\partial_\mu\right)U, \quad a_\mu \to U^\dagger a_\mu U, \tag{30}$$

and keep for a_μ the background gauge condition [29,30]

$$D_\mu(B)a_\mu=0, \quad D_\mu(B)=\partial_\mu - igB_\mu. \tag{31}$$

As a result of the perturbative gluon exchange between different quarks in the baryon there will appear an additional vertex in the effective Lagrangian [19]

$$\Delta L = g^2\int^f \psi^\dagger(x)\gamma_\mu^f\psi(x)\int^g \psi^\dagger(y)\gamma_\nu^g\psi(y) \times\langle a_\mu(x)a_\nu(y)\rangle dx\, dy. \tag{32}$$

In what follows we shall be interested only in the color Coulomb interaction which results from Eq. (32) assuming the simplest form of gluon propagator and neglecting at first for simplicity the influence of the background field on it, namely,

$$\int \langle a_\mu(x)a_\nu(y)\rangle d(x_4-y_4) = \frac{\delta_{\mu\nu}C_2}{4\pi^2}\int\frac{d(x_4-y_4)}{(\overline{x}-\overline{y})^2+(x_4-y_4)^2}=\frac{\delta_{\mu\nu}C_2}{4\pi|\mathbf{x}-\mathbf{y}|}. \tag{33}$$

Now taking the background into account, one arrives at the picture of the gluon a_μ propagating inside the film—the world sheet of the string, created by the background between three-quark world lines and the string junction, as is shown in Fig. 2. Depending on the choice of $\mathbf{r}^{(0)}$ we will get in general an effective interaction of a two-body or three-body nature. Because of the presence of the QCD background, the strength of the resulting Coulomb interaction is expected to be different from the perturbative OGE contribution and as a result different from the interaction used for example in the Breit equation [31].

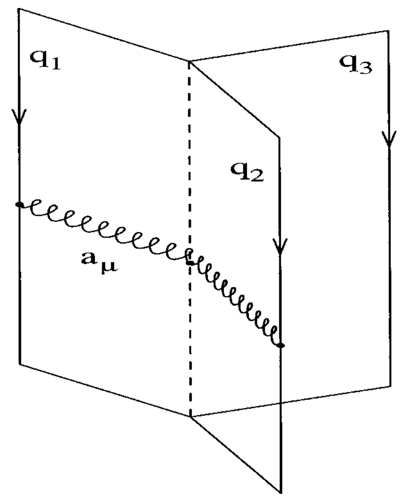

FIG. 2. A schematic view of the gluon propagating inside the world sheet of the string.

Because of its attractive nature, the color Coulomb contribution leads to smaller baryon masses and gives rise to composite systems with a smaller radius. As a result the magnetic moments become smaller. In the remaining part of the paper we neglect the effect from the Coulomb interaction. To study this a more involved analysis is needed, where also the hyperfine interaction has to be included.

V. BARYON MAGNETIC MOMENTS WITHOUT QUARK CONSTITUENT MASSES

Since the calculation of magnetic moments as well as baryon masses does not involve large momentum transfer, one can use for that purpose the Hamiltonian equation (28). According to the results of Sec. IV, H_i can be represented as

$$H_i = m_i\beta^{(i)}+\mathbf{p}^{(i)}\boldsymbol{\alpha}^{(i)}+\beta^{(i)}M^{(i)}(\mathbf{r}^{(i)}-\mathbf{r}^{(0)}). \tag{34}$$

The baryon solution of Eq. (34) can be represented as

$$\Psi_{JM}=\Gamma_{JM}^{\alpha\beta\gamma}(f_1f_2f_3)e_{abc}\psi_{a\alpha}^{f_1}(\mathbf{r}^{(1)}-\mathbf{r}^{(0)})\psi_{b\beta}^{f_2}(\mathbf{r}^{(2)}-\mathbf{r}^{(0)}) \times\psi_{c\gamma}^{f_3}(\mathbf{r}^{(3)}-\mathbf{r}^{(0)}), \tag{35}$$

where a,b,c and α,β,γ refer to color and Lorentz indices, respectively, and f_i is the flavor index. In what follows we shall use only the lowest orbitals (lowest eigenvalues solutions) for quarks and therefore the orbital excitation indices are everywhere omitted. The orbital wave function can be decomposed in the standard way,

$$\psi_\alpha^f(\boldsymbol{\rho})=\frac{1}{\rho}\begin{pmatrix}G(\rho)\Omega_{jlM}\\ iF(\rho)\Omega_{jl'M}\end{pmatrix}=\begin{pmatrix}g(\rho)\Omega_{jlM}\\ if(\rho)\Omega_{jl'M}\end{pmatrix}, \quad \boldsymbol{\rho}=\mathbf{r}-\mathbf{r}^{(0)}, \tag{36}$$

and the color index is omitted, since the orbital satisfies a "white" (vacuum averaged) equation

$$H_i\psi_{\alpha_i}^{f_i}=\varepsilon_{n_i}^{(i)}\psi_{\alpha_i}^{f_i}. \tag{37}$$

Therefore the only remnant of color is the requirement that Ψ_{JM} be symmetric in all coordinates besides color. From Eq. (28) we see that the mass of the baryon, corresponding to Eq. (35), is given by

$$M_B=\sum_{i=1}^{3} \epsilon_{n_i}^{(i)}. \tag{38}$$

To define the magnetic moment one may introduce an external e.m. field A, $\mathbf{p}^{(i)}\to\mathbf{p}^{(i)}-e_q^{(i)}\mathbf{A}$, $\mathbf{A}=\frac{1}{2}(\mathbf{H}\times\mathbf{r})$, and calculate perturbatively the energy shift

$$\Delta E=-\boldsymbol{\mu}\mathbf{H}. \tag{39}$$

Because of the symmetry of the problem, it is enough to consider only the perturbation of one orbital, say, for the first quark,

$$H_1\to H_1+\Delta H_1, \quad \Delta H_1=-e_q^{(1)}\boldsymbol{\alpha}^{(1)}\mathbf{A}. \tag{40}$$

Hence, denoting

$$\Psi^{(1)}=\begin{pmatrix}\varphi^{(1)}\\ \chi^{(1)}\end{pmatrix},$$

$$\begin{aligned}\langle\Delta H_1\rangle&=-e_q^{(1)}(\varphi^{(1)*},\chi^{(1)*})\\ &\quad\times\begin{pmatrix}0 & \boldsymbol{\sigma}^{(1)}\mathbf{A}\\ \boldsymbol{\sigma}^{(1)}\mathbf{A} & 0\end{pmatrix}\begin{pmatrix}\varphi^{(1)}\\ \chi^{(1)}\end{pmatrix}\\ &=-e_q^{(1)}(\varphi^{(1)*}\boldsymbol{\sigma}^{(1)}\mathbf{A}\chi^{(1)}\\ &\quad+\chi^{*(1)}\boldsymbol{\sigma}^{(1)}\mathbf{A}\varphi^{(1)}).\end{aligned} \tag{41}$$

Using Eq. (36) and a simple derivation given in Appendix A one obtains, for the contribution of the first quark to the magnetic moment operator in spin space,

$$\boldsymbol{\mu}^{(1)}=-\frac{2e_q^{(1)}}{3}\int g^*(r)f(r)rd^3r\Omega^*_{jlM}\boldsymbol{\sigma}^{(1)}\Omega_{jlM}. \tag{42}$$

For the lowest orbital $j=\frac{1}{2}$, $l=0$, $M=\frac{1}{2}$, $\boldsymbol{\sigma}\to\sigma_z$, one obtains

$$\mu_z\equiv 3\mu_z^{(1)}=-2e_q^{(1)}\sigma_z^{(1)}\int g^*(r)f(r)rr^2dr, \tag{43}$$

where the superscript 1 denotes the contribution of the first quark to the magnetic moment. The normalization condition is

$$\int (|g|^2+|f|^2)r^2dr=1. \tag{44}$$

Note that everywhere we put $\mathbf{r}^{(1)}-\mathbf{r}^{(0)}=\mathbf{r}$. In the case of a local linear confining interaction using the Dirac equation one can express $\mu^{(i)}$ through $g(r)$ only (see Appendix A for details):

$$\mu_z^{(i)}=\frac{e_q^{(i)}\sigma_z^{(i)}}{3}\int_0^\infty\frac{|g|^2r^2(2\sigma r+3\varepsilon)}{(\varepsilon+\sigma r)^2}dr. \tag{45}$$

Constructing the fully symmetrical $3q$ wave function for the nucleon with total spin up one has, for the proton,

$$\begin{aligned}\Psi^P_{symm}=N'\Big\{&\frac{2}{3}[u_+(1)d_-(2)+d_-(1)u_+(2)]u_+(3)\\ &-\frac{1}{3}[d_+(1)u_-(2)+u_-(1)d_+(2)]u_+(3)\\ &-\frac{1}{3}[u_+(1)u_-(2)+u_-(1)u_+(2)]d_+(3)\\ &-\frac{1}{3}[u_+(1)d_+(2)+d_+(1)u_+(2)]u_-(3)\\ &+\frac{2}{3}u_+(1)u_+(2)d_-(3)\Big\},\end{aligned} \tag{46}$$

where $N'=1/\sqrt{2}$, and subscripts $(\pm)$ refer to the spin projection. In a similar way for the neutron one replaces $u\leftrightarrow d$ and obtains

$$\begin{aligned}\Psi^n_{symm}=N'\Big\{&\frac{2}{3}[d_+(1)u_-(2)+u_-(1)d_+(2)]d_+(3)\\ &-\frac{1}{3}[u_+(1)d_-(2)+d_-(1)u_+(2)]d_+(3)\\ &-\frac{1}{3}[d_+(1)d_-(2)+d_-(1)d_+(2)]u_+(3)\\ &-\frac{1}{3}[d_+(1)u_+(2)+u_+(1)d_+(2)]d_-(3)\\ &+\frac{2}{3}d_+(1)d_+(2)u_-(3)\Big\}.\end{aligned} \tag{47}$$

The matrix elements are computed easily:

$$\langle\Psi^p_{symm}|e_q^{(1)}\sigma_z^{(1)}|\Psi^p_{symm}\rangle=\frac{1}{3}e, \tag{48}$$

$$\langle\Psi^n_{symm}|e_q^{(1)}\sigma_z^{(1)}|\Psi^n_{symm}\rangle=-\frac{2}{9}e, \tag{49}$$

where e is the charge of the proton. From Eqs. (48) and (49) one immediately gets the famous relation

$$\frac{\mu^{(n)}}{\mu^{(p)}}=-\frac{2}{3}. \tag{50}$$

Writing for identical orbitals the magnetic moment as a product,

$$\mu_B=3\langle\Psi_{symm}|e_q^{(1)}\sigma_z^{(1)}|\Psi_{symm}\rangle\lambda, \tag{51}$$

where

$$\lambda\equiv-\frac{2}{3}\int g^*(r)f(r)r^3dr. \tag{52}$$

TABLE I. Ground state energy ϵ_0 of the orbitals and the predicted magnetic moments of the nucleons in units of nuclear magneton for various values of σ. The experimental values are also listed.

σ (GeV^2)	$\epsilon_0(u,d)$ (MeV)	$\epsilon_0(s)$ (MeV)	μ_{proton}	$\mu_{neutron}$
0.09	297	439	2.81	−1.87
0.12	342	482	2.44	−1.63
0.15	380	519	2.20	−1.46
		Expt.	2.79	−1.91

It is clear that inclusion of higher orbitals will change the magnetic moment of proton and neutron, similarly to the case of tritium and ^{3}He, where the admixture of the orbital momentum $L=2$ changes the magnetic moment by 7%–8%. In our case the orbital momentum is brought by all three quarks symmetrically, and these components appear in the wave function due to mixing through the tensor and spin-orbit forces between quarks.

Equations (51) and (52) can readily be generalized when the quarks have different orbital wave functions. For the single-quark orbitals we have taken the solution of the Dyson-Schwinger-Dirac equation with nonlocal kernel from Refs. [19,20]. Assuming for the field correlator a Gaussian form

$$D(u)=D(0)\exp(-u^2/4T_g^2), \quad D(0)=\frac{\sigma}{2\pi T_g^2}, \tag{53}$$

with $T_g=0.24$ fm the ground state orbital solution is determined. In Table I are shown the calculated ground state energy of the orbitals for various flavor states. For the current masses we have used $m_u=m_d=5$ MeV and $m_s=200$ MeV.

Using these orbitals we calculate the nucleon magnetic moment for various values of the string tension σ. The results are also shown in Table I. From the table we see that the predictions depends sensitively on the string tension σ. Increasing the value of σ leads to a larger ground state energy of the orbitals and smaller size of the magnetic moment. This in accordance with an analysis where the small component of the orbital is treated perturbatively. Similarly the presence of a Coulomb interaction yields a lower ground state energy of the orbital, resulting in a larger value in magnitude of the magnetic moment. Close agreement with the experimental values of the magnetic moment is found when $\sigma=0.09$ GeV^2. In this case the mass of the nucleon is predicted to be 891 MeV. It is gratifying to see that the magnetic moments are reasonable in the regime where also the predicted mass of the nucleon is close to the empirical value.

The explicit forms of Ψ_{symm} for other baryons are given in Appendix B. Note that due to the strange quark mass their orbitals are different from those of u,d quarks and therefore the decomposition (51) has to be modified. Some useful formulas can be found in Appendix B.

The resulting values for baryon magnetic moments are given in Table II, where they are compared with experimental values. Considering the case of $\sigma=0.12$ GeV^2 we see that there is a rather close agreement with the experimental magnetic moments, with the largest deviations found for the nucleon and Σ^-. As discussed for the case of the nucleon improvement of the predicted mass of the composite system also leads to magnetic moments closer to the experimental values. This applies also for the case of the Δ isobar. Hence we may hope that inclusion of the Coulomb and hyperfine splitting interactions will improve the predictions. Moreover, pionic effects are expected to be present. As a result significant mesonic current contributions to the magnetic moments may occur. In the next section we study the dominant corrections from the pion to the one- and two-body current.

TABLE II. The magnetic moment of the baryons in units of nuclear magneton for various values of σ. Calculations and experimental results.

B	μ_B $\sigma=0.09$ GeV^2	μ_B $\sigma=0.12$ GeV^2	μ_B $\sigma=0.15$ GeV^2	Expt.
p	2.81	2.44	2.20	2.79
n	−1.87	−1.63	−1.46	−1.91
Σ^-	−1.03	−0.89	−0.79	−1.16
Σ^0	0.85	0.74	0.67	
Σ^+	2.72	2.37	2.14	2.46
Λ	−0.66	−0.60	−0.56	−0.61
Ξ^-	−0.57	−0.53	−0.50	−0.65
Ξ^0	−1.51	−1.34	−1.23	−1.25
Δ^{++}	5.62	4.89	4.39	4.52
Δ^+	2.81	2.44	2.20	
Δ^0	0.00	0.00	0.00	
Δ^-	−2.81	−2.44	−2.20	
Σ^{+*}	3.09	2.66	2.37	
Σ^{0*}	0.27	0.21	0.18	
Σ^{-*}	−2.54	−2.23	−2.02	
Ξ^{0*}	0.55	0.43	0.35	
Ξ^{-*}	−2.26	−2.02	−1.84	
Ω^-	−1.99	−1.80	−1.67	−2.02

VI. MESONIC CONTRIBUTIONS

In this section we carry out in our single-orbital model an estimate of the magnitude of the pionic-current corrections to the magnetic moment of the nucleon. Because of the quark coupling to effective mesonic degrees of freedom, one- and two-body current contributions to the magnetic moments of the baryons exist from the virtual excitations of mesons. Assuming as in Ref. [32] that there exists an effective one-meson exchange between quarks in the three-quark system this leads to meson exchange current contributions to the magnetic moment. The leading correction is due to the pion-in-flight and pair term; see Ref. [32]. Effects from the heavier mesons like the ρ are in general less important.

Our starting point is the e.m. current matrix element

$$M_\mu=\langle\Psi|J_\mu(Q)|\Psi\rangle, \tag{54}$$

where Ψ is the three-quark wave function and Q is the photon momentum.

We first consider the single-quark current contribution. For the single-quark current operator we use

$$J_\mu^{\gamma qq} \equiv 3 J_\mu^{\gamma qq}(1) = 3 e_q^{(1)} \gamma_\mu^{(1)} \prod_{n=2}^{3} \gamma_0^{(n)}, \qquad (55)$$

and for the wave function normalization, Eq. (44) for the single-particle orbitals is taken. This choice has the nice property that the zeroth component of the current at $Q=0$ is found to give the proper charge of the three-quark system, i.e.,

$$M_0 = \langle \Psi | J_0(Q=0) | \Psi \rangle = \sum_{n=1}^{3} e_q^{(n)}. \qquad (56)$$

The result for the magnetic moment, obtained in the previous section, can readily be recovered from our single-quark current matrix element. Following Ref. [33], the magnetic moment can be calculated by taking the curl of the space component of the current matrix element in the Breit system. In doing so, the magnetic moment can be deduced from the e.m. current as

$$\mu_z = \frac{e}{2M_p} G_{mag}(Q=0) = -\frac{i}{2}[\nabla_Q \times \mathbf{M}]_z \quad (Q=0), \qquad (57)$$

where M_p is the proton mass, e the proton charge, and G_{mag} the Sachs e.m. magnetic form factor. The matrix element (57) can easily be evaluated in momentum space. Introducing the Fourier transform of the wave function of the single-quark orbital,

$$\tilde{\psi}_\alpha^f(\mathbf{k}) = \begin{pmatrix} \tilde{g}(k)\Omega_{jlM} \\ \tilde{f}(k)\Omega_{jl'M} \end{pmatrix} = 4\pi \int \begin{pmatrix} (-i)^l j_l(k\rho) g(\rho) \Omega_{jlM} \\ i(-i)^{l'} j_{l'}(k\rho) f(\rho) \Omega_{jl'M} \end{pmatrix} \rho^2 d\rho, \qquad (58)$$

with j_l the spherical Bessel functions, we may after some algebra reduce Eq. (57) in momentum space to

$$\mu_z = 3\mu_z^{(1)} = 3\langle \psi_{symm} | e_q^{(1)} \sigma_z^{(1)} | \psi_{symm} \rangle \tilde{\lambda}. \qquad (59)$$

We thus find

$$\tilde{\lambda} = \frac{-1}{2N} \int \int d^3p\, d^3q \prod_{n=2}^{3} [|\tilde{g}(k_n)|^2 + |\tilde{f}(k_n)|^2] \times \left(\tilde{g}(k_1) \frac{4}{3k_1} \tilde{f}(k_1) - \frac{\partial \tilde{g}(k_1)}{\partial k_1} \frac{2}{3} \tilde{f}(k_1) + \tilde{g}(k_1) \frac{2}{3} \frac{\partial \tilde{f}(k_1)}{\partial k_1} \right)_{Q^2=0}, \qquad (60)$$

where N is the normalization factor:

$$N = \int \int d^3p\, d^3q \prod_{n=1}^{3} [|\tilde{g}(k_n)|^2 + |\tilde{f}(k_n)|^2]. \qquad (61)$$

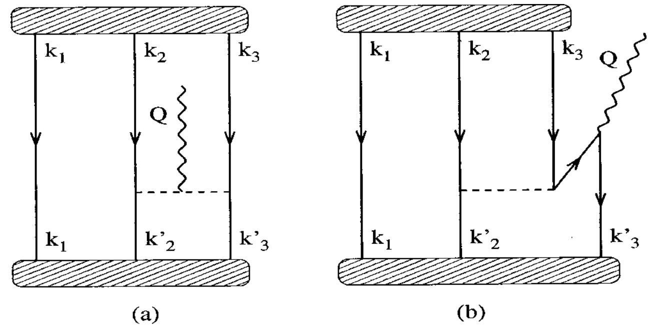

FIG. 3. The diagrams corresponding to the pionic contributions to the current: (a) the pion-in-flight diagram, (b) the pair term. The bound state of the quarks is represented by the blobs at the beginning and the end of the diagrams.

The momenta are expressed in terms of the Jacobi coordinates as

$$\mathbf{k}_1 = -\frac{2}{\sqrt{3}}\mathbf{q} + \frac{1}{3}\mathbf{P}, \quad \mathbf{k}_1' = -\frac{2}{\sqrt{3}}\mathbf{q}' + \frac{1}{3}\mathbf{P}',$$
$$\mathbf{k}_2 = \mathbf{p} + \frac{1}{\sqrt{3}}\mathbf{q} + \frac{1}{3}\mathbf{P}, \quad \mathbf{k}_2' = \mathbf{p}' + \frac{1}{\sqrt{3}}\mathbf{q}' + \frac{1}{3}\mathbf{P}', \qquad (62)$$
$$\mathbf{k}_3 = -\mathbf{p} + \frac{1}{\sqrt{3}}\mathbf{q} + \frac{1}{3}\mathbf{P}, \quad \mathbf{k}_3' = -\mathbf{p}' + \frac{1}{\sqrt{3}}\mathbf{q}' + \frac{1}{3}\mathbf{P}'.$$

Imposing the Breit system, $\mathbf{P}+\mathbf{P}'=0$, and momentum conservation gives $\mathbf{P}'=-\mathbf{P}=\mathbf{Q}/2$, $\mathbf{p}'=\mathbf{p}$ and $\sqrt{3}(\mathbf{q}-\mathbf{q}')=\mathbf{Q}$.

Use has been made of the identity

$$\langle \Omega_{jlM}(\hat{k}_1) | (\hat{\mathbf{k}}_1)_i (\hat{\mathbf{k}}_1)_j | \Omega_{jlM}(\hat{k}_1) \rangle = \frac{1}{3}\delta_{ij}, \qquad (63)$$

with $l=0$ and Eqs. (48) and (49). The magnetic moment expression (51) from the previous section is readily recovered when we replace the integration over the Jacobi momenta in Eqs. (60) and (61) by $\Pi_{n=1}^{3} dk_n$.

We now turn to the pionic two-body current contributions, assuming a γ_5 theory. The resulting pion-in-flight and pair current operators, shown in Fig. 3, are given, respectively, by

$$\mathbf{J}_{\gamma\pi\pi}^{(23)} = -2ieg_{\pi qq}^2 \gamma_5^{(2)} \gamma_5^{(3)} (\boldsymbol{\tau}^{(2)} \times \boldsymbol{\tau}^{(3)})_z \times \frac{\boldsymbol{\Delta}}{\left[\left(\boldsymbol{\Delta} - \frac{1}{2}\mathbf{Q}\right)^2 + m_\pi^2\right]\left[\left(\boldsymbol{\Delta} + \frac{1}{2}\mathbf{Q}\right)^2 + m_\pi^2\right]} \times \frac{\Lambda_\pi^4}{\left[\left(\boldsymbol{\Delta} - \frac{1}{2}\mathbf{Q}\right)^2 + \Lambda_\pi^2\right]\left[\left(\boldsymbol{\Delta} + \frac{1}{2}\mathbf{Q}\right)^2 + \Lambda_\pi^2\right]} \times \left(1 + \frac{\left(\boldsymbol{\Delta} - \frac{1}{2}\mathbf{Q}\right)^2 + m_\pi^2}{\left(\boldsymbol{\Delta} + \frac{1}{2}\mathbf{Q}\right)^2 + \Lambda_\pi^2} + \frac{\left(\boldsymbol{\Delta} + \frac{1}{2}\mathbf{Q}\right)^2 + m_\pi^2}{\left(\boldsymbol{\Delta} - \frac{1}{2}\mathbf{Q}\right)^2 + \Lambda_\pi^2} \right) \qquad (64)$$

TABLE III. The single-quark current contribution $\mu_N^{(1)}$ to the magnetic moment in units of nuclear magneton, together with the two-body corrections and the anomalous correction $\delta\mu_N^{(1)}$ arising from the pion one-loop diagrams. Also are shown the total combined prediction of our calculations and the experimental results.

N	$\mu_N^{(1)}$	$\mu_N^{(\pi\pi\gamma)}$	$\mu_N^{(N\bar{N}\gamma)}$	$\delta\mu_N^{(1)}$	μ_N^{tot}	Expt.
			$\sigma=0.09$ GeV^2			
p	2.81	0.20	−0.21	0.12	2.92	2.79
n	−1.87	−0.20	0.21	−0.16	−2.02	−1.91
			$\sigma=0.12$ GeV^2			
p	2.44	0.19	−0.18	0.11	2.56	2.79
n	−1.63	−0.19	0.18	−0.14	−1.78	−1.91
			$\sigma=0.15$ GeV^2			
p	2.20	0.18	−0.16	0.10	2.32	2.79
n	−1.46	−0.18	0.16	−0.13	−1.61	−1.91

and

$$\mathbf{J}^{(23)}_{\gamma N\bar{N}}=-ieg^2_{\pi qq}\gamma_5^{(2)}\gamma_5^{(3)}(\boldsymbol{\tau}^{(2)}\times\boldsymbol{\tau}^{(3)})_z \times\left[\frac{(\gamma^0-1)^{(3)}}{4m_q}\boldsymbol{\gamma}^{(3)}\frac{1}{\left[\left(\boldsymbol{\Delta}-\frac{1}{2}\mathbf{Q}\right)^2+m_\pi^2\right]}\right. \times\frac{\Lambda_\pi^4}{\left[\left(\boldsymbol{\Delta}-\frac{1}{2}\mathbf{Q}\right)^2+\Lambda_\pi^2\right]^2}-\frac{(\gamma^0-1)^{(2)}}{4m_q}\boldsymbol{\gamma}^{(2)} \left.\times\frac{1}{\left[\left(\boldsymbol{\Delta}+\frac{1}{2}\mathbf{Q}\right)^2+m_\pi^2\right]}\frac{\Lambda_\pi^4}{\left[\left(\boldsymbol{\Delta}+\frac{1}{2}\mathbf{Q}\right)^2+\Lambda_\pi^2\right]^2}\right]. \tag{65}$$

In Eqs. (64) and (65), Q is the photon momentum, $\Delta=\mathbf{p}-\mathbf{p}'$. A monopole form factor with cutoff mass $\Lambda_\pi=675$ MeV has been used. The last two terms in the last factor in Eq. (64) correspond to contact terms, which are needed to satisfy current conservation. The quark propagator in Eq. (65) has been replaced by its negative-energy part,

$$\frac{i}{\not{p}-m}\Rightarrow\frac{i}{2\sqrt{\mathbf{p}^2+m^2}}\frac{\mathbf{p}\boldsymbol{\gamma}-m+\sqrt{\mathbf{p}^2+m^2}\gamma^0}{p^0+\sqrt{\mathbf{p}^2+m^2}} \approx\frac{i}{4m}(\gamma^0-1), \tag{66}$$

as the positive-energy part has already been included in the single-quark current matrixelement [35]. Moreover, the pair contribution (65) consists of four terms where the photon can interact with quarks 2 and 3 prior to and after the pion-quark interaction.

The photopion vertex is described by an effective interaction Lagrangian

$$\mathcal{L}_{\pi\pi\gamma}=-\frac{1}{2}eA_\mu(\vec{\pi}\times\partial^\mu\vec{\pi})_z+\frac{1}{2}eA_\mu(\partial^\mu\vec{\pi}\times\vec{\pi})_z. \tag{67}$$

From the two-body operators $\mathbf{J}_{2b}$, Eqs. (64)–(65), we may write down the current matrix element between the three-quark state:

$$\mathbf{M}_{2b}=3\mathbf{M}_{2b}^{(1)}=3\frac{1}{N}\int\int d^3p\,d^3q\,\bar{\Psi}\gamma_0^{(1)}\mathbf{J}_{2b}^{(23)}\Psi. \tag{68}$$

Taking the curl of Eq. (68) the magnetic moment can be determined. The resulting expressions are given in Appendix C. As a check using the obtained magnetic moment operators we have determined the exchange magnetic moment contribution to the trinucleon system. Our results agree with those obtained by Kloet and Tjon [33].

To get an estimate of the exchange current contributions in the three-quark case we have used for the couplings and cutoff mass the values from Ref. [32]. They are taken to be $g^2_{qq\pi}/4\pi=0.67$. The results for the magnetic moments are shown in Table III. Our estimates are in strong disagreement with those obtained in Ref. [32]. The pion-in-flight contribution is substantially smaller than found in Ref. [32] using the chiral constituent model [34]. This may be partially due to the three-quark wave function used, which has a matter radius smaller than in our case. Moreover, it contains only nonrelativistic components. The pair contribution is found to be comparable to the pion-in-flight term, leading to an almost cancellation of the mesonic current pionic contributions.

The presence of mesonic degrees of freedom will modify the single-quark current. The resulting e.m. current operator can in general be characterized by a large number of off-shell form factors [36–38], which reduces to 2 when we assume that the initial and final quarks are on mass shell. Using this

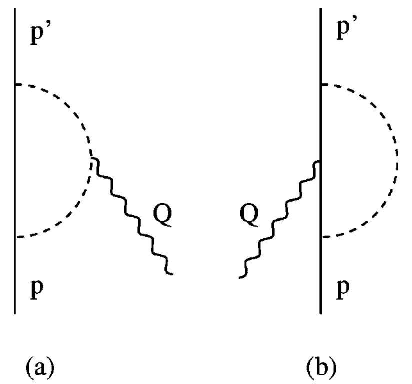

FIG. 4. The diagrams contributing to the anomalous magnetic moment of the single quark.

approximation we may estimate the resulting anomalous magnetic κ term due to the the pionic contributions. Near $Q^2=0$ we have

$$J_\mu^{\gamma qq}=e_q\gamma_\mu+\kappa_q\frac{ie}{2M_p}\sigma_{\mu\nu}q_\nu, \quad (69)$$

where $\kappa_q=\kappa_s+\kappa_v\tau_z$ for the u,d quarks. The κ coefficients can be determined in a simple model, assuming that the loop corrections are given by only the one-loop pionic contributions to the e.m. vertex. Similarly as in the two-body current case we approximate the single-quark orbital by a free quark propagation with a constituent mass given by the ground state orbital energy. With the above simplifying assumptions the calculation amounts to calculating the magnetic moment contributions of the diagrams shown in Fig. 4. Using the same cutoff mass regularization as for the two-body currents we find for the anomalous magnetic moment in units of the nuclear magneton:

$$\begin{aligned}\kappa^{(a)}&=\kappa_v^{(a)}\tau_z\\ &=ig_{\pi qq}^2\tau_z\frac{4M_p}{3m_q^3}\int\frac{d^4k}{(2\pi)^4}\frac{4(p\cdot k)^2-p^2k^2}{[k^2-2pk+i\epsilon][k^2-m_\pi^2+i\epsilon]^2}\\ &\quad\times\left(\frac{\Lambda_\pi^2}{k^2-\Lambda_\pi^2}\right)^2\left(1+2\frac{k^2-m_\pi^2}{k^2-\Lambda_\pi^2}\right) \quad (70)\end{aligned}$$

and

$$\begin{aligned}\kappa^{(b)}=\kappa_s^{(b)}+\kappa_v^{(b)}\tau_z=-ig_{\pi qq}^2\frac{1-\tau_z}{2}\frac{2M_p}{3m_q^3}\int\frac{d^4k}{(2\pi)^4}\\ \times\frac{4(p\cdot k)^2-p^2k^2}{[k^2-2pk+i\epsilon]^2[k^2-m_\pi^2+i\epsilon]}\left(\frac{\Lambda_\pi^2}{k^2-\Lambda_\pi^2}\right)^2, \quad (71)\end{aligned}$$

TABLE IV. The quark anomalous magnetic moments in units of the nucleon magneton in the one-loop approximation for various string tensions σ. The first set is the prediction for only the pion loops, while the second set is with both pion and kaon loops included.

σ (GeV^2)	κ_u	κ_d	κ_s
	Pion loops		
0.09	0.101	−0.160	0.0
0.12	0.092	−0.140	0.0
0.15	0.085	−0.126	0.0
	Pion and kaon loops		
0.09	0.132	−0.151	−0.034
0.12	0.121	−0.133	−0.032
0.15	0.112	−0.120	−0.031

where p is the momentum of the quark. For details we refer to Appendix D. Equation (70) corresponds to the coupling of the photon to the pion, Eq. (71) to the coupling of the photon to the quark.

In Table IV are shown the calculated anomalous magnetic moments of the u, d, and s quarks for $\Lambda=675$ MeV for various choices of σ. Clearly, the results depend on the constituent quark masses. These are given in Table I for the considered string tensions.

Using Eq. (57) the κ term in Eq. (69) yields a nucleon magnetic moment correction

$$\delta\mu_z=3\,\delta\mu_z^{(1)}=3\langle\psi_{symm}|\kappa_q(1)\sigma_z(1)|\psi_{symm}\rangle\lambda_0, \quad (72)$$

with

$$\lambda_0=\frac{\int r^2dr(|g|^2-|f|^2)}{\int r^2dr(|g|^2+|f|^2)}. \quad (73)$$

In Table III the predictions for the nucleon are shown including also the one-pion loop contributions (72) and two-body currents. Our results obtained for the one-loop corrections are smaller than reported by Glozman and Riska [39]. This is due to the inclusion of the lower component in the single-quark orbitals. Neglecting these we recover the results of Ref. [39]. From Table III we see that the proton and neutron magnetic moment is in reasonable agreement with experiment for a string tension of $\sigma=0.1$ GeV^2. For this value of the string tension the model predicts a nucleon mass of 940 MeV, remarkably close to the empirical value. The anomalous magnetic moment contributions are found to be of the order of 10%.

As a result of the one-loop contributions, the magnetic moments of the other baryons are modified. Corrections from kaon loops have also been considered. Because of the larger kaon mass, the contributions are expected in general to be

TABLE V. The magnetic moment μ_B of the baryon octet and decuplet in units of nuclear magneton, including the anomalous contribution $\delta\mu_B$ arising from the pion and kaon one-loop diagrams and the pion exchange corrections for different string tension σ. Also are shown the experimental results.

B	$\delta\mu_B$ $\sigma=0.09$ GeV^2	μ_B	$\delta\mu_B$ $\sigma=0.12$ GeV^2	μ_B	$\delta\mu_B$ $\sigma=0.15$ GeV^2	μ_B	Expt.
p	0.15	2.95	0.14	2.59	0.12	2.34	2.79
n	−0.16	−2.02	−0.14	−1.78	−0.13	−1.61	−1.91
Σ^-	−0.13	−1.16	−0.11	−1.00	−0.10	−0.89	−1.16
Σ^0	0.00	0.85	0.00	0.74	0.00	0.67	
Σ^+	0.12	2.84	0.11	2.48	0.11	2.25	2.46
Λ	−0.02	−0.68	−0.02	−0.62	−0.02	−0.58	−0.61
Ξ^-	0.00	−0.57	0.00	−0.53	0.00	−0.50	−0.65
Ξ^0	−0.06	−1.57	−0.05	−1.39	−0.05	−1.28	−1.25
Δ^{++}	0.26	5.88	0.24	5.13	0.22	4.61	4.52
Δ^+	0.07	2.88	0.07	2.51	0.07	2.27	
Δ^0	−0.11	−0.11	−0.10	−0.10	−0.08	−0.08	
Δ^-	−0.30	−3.11	−0.26	−2.70	−0.24	−2.44	
Σ^{+*}	0.15	3.24	0.14	2.80	0.13	2.50	
Σ^{0*}	−0.04	0.23	−0.03	0.18	−0.03	0.15	
Σ^{-*}	−0.22	−2.76	−0.20	−2.43	−0.18	−2.20	
Ξ^{0*}	0.04	0.59	0.04	0.47	0.03	0.38	
Ξ^{-*}	−0.14	−2.40	−0.13	−2.15	−0.12	−1.96	
Ω^-	−0.07	−2.06	−0.06	−1.86	−0.06	−1.73	−2.02

smaller in size than those of the pion loops. In Table IV the calculated anomalous moments of the strange quark due to the kaon one-loop corrections are given. In the calculations a cutoff mass of $\Lambda=675$ MeV has been used. The isoscalar and isovector anomalous magnetic moment pieces are also changed by the kaon loop contributions. From Table IV we see that the kaon loop contributions are indeed smaller in magnitude as compared to the pion loop ones. The full results for the magnetic moments of the baryon octet and decuplet, including the pionic exchange currents and the pion and kaon one-loop contributions, are summarized in Table V. For the value of the string tension $\sigma=0.1$ the overall agreement with the experimental data is reasonable. From the table we see that the anomalous magnetic moment contribution leads to an improvement of the predictions.

VII. CONCLUSION

We have written down the general effective quark Lagrangian as obtained from the standard QCD Lagrangian by integrating out the gluonic degrees of freedom. Considering the baryon Green's function, neglecting gluon and meson exchanges, we find in lowest order of the approximation scheme that it is given by a product of three independent single-quark Green's functions. As a result the Hamiltonian can be written as a sum of three quark terms, where the single-quark solutions satisfy the Dyson-Schwinger equation with a nonlocal kernel.

The nonlinear equation for the single-quark propagator S (attached to the string in a gauge-invariant way) has been solved in the Gaussian correlator approximation. The resulting three-quark wave function has been used to determine the magnetic moments of the baryons. This has been done for both the octet and decuplet of the SU(3) flavor group.

Comparing the predictions we find that the magnetic moments are mostly in close overall agreement with the experiment for a string tension of $\sigma=0.1$ GeV^2. We find that the predicted magnetic moment of the nucleon is improved substantially once we choose a string tension to give a reasonable nucleon mass. The same applies for the Δ isobar. Effects due to the presence of virtual mesons are in general expected to be important. We have estimated the pionic one-loop and one-pion exchange contributions to the magnetic moment. The single-quark corrections from pionic loops are found to be of the order of 10%, whereas the total effect of two-body current contributions are predicted to be small, to be contrasted to the results of Ref. [32]. This is due to the cancellation of the pion-in-flight and pair terms in the present model. Because of the anomalous magnetic contributions, there seems to be somewhat an improvement of the predictions.

We have assumed here that the baryon wave function can be described as a product of single-quark orbitals, i.e., neglecting correlation effects. Our results in this approximation for the magnetic moments of baryons are encouraging, but are in need of including higher-order corrections. In particular, the mass spectrum obtained from our lowest-order approximation does not contain the N-Δ mass splitting. This is due to neglecting contributions like the hyperfine interaction arising from the one-gluon interaction. This induces correlations in the three-quark wave function and its magnitude may give us insight into whether our basic description in this paper in terms of simply single-quark orbitals is a reasonable one. Moreover, it is clearly of interest to investigate how the magnetic moments are changed when effects from color Coulomb and hyperfine interactions are accounted for.

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APPENDIX A: MAGNETIC MOMENT CALCULATION IN COORDINATE SPACE

The one-quark contribution to the magnetic moment can be written as in Eq. (45):

$$\langle \Delta H_1 \rangle = -e_q^{(1)} \int (\varphi^{(1)*} \boldsymbol{\sigma}^{(1)} \mathbf{A} \chi^{(1)} + \chi^{(1)*} \boldsymbol{\sigma}^{(1)} \mathbf{A} \varphi^{(1)}) d^3 r, \tag{A1}$$

where $\mathbf{A} = \frac{1}{2}(\mathbf{H} \times \mathbf{r})$ is the vector potential of external constant magnetic field.

Inserting in Eq. (A1) $\varphi^{(1)} = g(r)\Omega_{jlM}$ and $\chi^{(1)} = i f(r) \Omega_{jl'M}$, and taking into account that $\Omega_{jl'M} = -(\boldsymbol{\sigma}\mathbf{n})\Omega_{jlM}$, one easily obtains

$$\langle \Delta H_1 \rangle = -\frac{1}{2} e_q^{(1)} \int d^3 r (g^* f + f^* g) r \Omega_{jlM}^* \times \{(\boldsymbol{\sigma}\mathbf{n})(\mathbf{n}\,\mathbf{H}) - \boldsymbol{\sigma}\mathbf{H}\} \Omega_{jlM}. \tag{A2}$$

Equation (A2) contains the matrix element $\int d\mathbf{n} \Omega_{jlM}^* n_i n_k \Omega_{jlM}$, which simplifies when $l=0$, so that $\langle n_i n_k \rangle = \frac{1}{3}\delta_{ik}$.

In this case one obtains, taking into account relation $\langle \Delta H_1 \rangle = \Delta E = -\boldsymbol{\mu}^{(1)}\mathbf{H}$,

$$\begin{aligned}\boldsymbol{\mu}^{(1)} &= -\frac{1}{3} e_q^{(1)} \int [g^*(r) f(r) + f^*(r) g(r)] r d^3 r \\ &\quad \times \Omega_{jlM}^* \boldsymbol{\sigma}^{(1)} \Omega_{jlM} \\ &= -\boldsymbol{\sigma}^{(1)} \frac{2}{3} e_q^{(1)} \int \mathrm{Re}[g^*(r) f(r)] r^3 dr. \end{aligned} \tag{A3}$$

In the case of a local scalar potential $U(r)$ one can further express $f(r)$ through $g(r)$ using the Dirac equation for the one-quark state:

$$r f(r) = \frac{1}{\varepsilon + m + U(r)} \left(\frac{d}{dr}(gr) + \frac{\kappa}{r} gr \right). \tag{A4}$$

Introducing Eq. (A4) into Eq. (A3) and integrating by parts one obtains

$$\mu_z^{(i)} = \frac{e_q^{(i)} \sigma_z^{(i)}}{3} \int \frac{|g|^2 r^2 dr}{(\varepsilon + m + U)^2} [3(\varepsilon + m + U) - r U'(r)]. \tag{A5}$$

For $U(r) = \sigma r$ one obtains Eq. (49).

APPENDIX B: MAGNETIC MOMENT OF THE MULTIPLET

In this appendix the calculation of the nucleon magnetic moment is generalized to the baryon octet and decuplet. By analogy with the fully symmetrical $3q$ wave function for the nucleon, Eqs. (46) and (47), wave functions for the baryon multiplets can be formulated. The flavor octet with total spin 1/2 up becomes

$$\begin{aligned}\Psi_{symm}^{p} = \frac{\sqrt{2}}{6}\{&2d_- u_+ u_+ - u_- d_+ u_+ - d_+ u_- u_+ + 2u_+ d_- u_+ \\ &- u_+ u_- d_+ - u_- u_+ d_+ - u_+ d_+ u_- - d_+ u_+ u_- \\ &+ 2u_+ u_+ d_-\}, \end{aligned} \tag{B1}$$

$$\begin{aligned}\Psi_{symm}^{n} = \frac{\sqrt{2}}{6}\{&2u_- d_+ d_+ - d_- u_+ d_+ - u_+ d_- d_+ + 2d_+ u_- d_+ \\ &- d_+ d_- u_+ - d_- d_+ u_+ - d_+ u_+ d_- - u_+ d_+ d_- \\ &+ 2d_+ d_+ u_-\}, \end{aligned} \tag{B2}$$

$$\begin{aligned}\Psi_{symm}^{\Sigma^+} = \frac{\sqrt{2}}{6}\{&2s_- u_+ u_+ - u_- s_+ u_+ - s_+ u_- u_+ + 2u_+ s_- u_+ \\ &- u_+ u_- s_+ - u_- u_+ s_+ - u_+ s_+ u_- - s_+ u_+ u_- \\ &+ 2u_+ u_+ s_-\}, \end{aligned} \tag{B3}$$

$$\begin{aligned}\Psi_{symm}^{\Sigma^0} = \frac{-1}{6}\{&u_+ d_- s_+ + d_+ u_- s_+ + s_+ d_- u_+ + s_+ u_- d_+ \\ &- 2u_+ s_- d_+ - 2d_+ s_- u_+ + u_- d_+ s_+ + d_- u_+ s_+ \\ &- 2s_- d_+ u_+ - 2s_- u_+ d_+ + u_- s_+ d_+ + d_- s_+ u_+ \\ &- 2u_+ d_+ s_- - 2d_+ u_+ s_- + s_+ d_+ u_- + s_+ u_+ d_- \\ &+ u_+ s_+ d_- + d_+ s_+ u_-\}, \end{aligned} \tag{B4}$$

$$\begin{aligned}\Psi_{symm}^{\Sigma^-} = \frac{\sqrt{2}}{6}\{&2s_- d_+ d_+ - d_- s_+ d_+ - s_+ d_- d_+ + 2d_+ s_- d_+ \\ &- d_+ d_- s_+ - d_- d_+ s_+ - d_+ s_+ d_- - s_+ d_+ d_- \\ &+ 2d_+ d_+ s_-\}, \end{aligned} \tag{B5}$$

$$\begin{aligned}\Psi_{symm}^{\Lambda} = \frac{\sqrt{3}}{6}\{&u_- d_+ s_+ - d_- u_+ s_+ + u_- s_+ d_+ - d_- s_+ u_+ \\ &- u_+ d_- s_+ + d_+ u_- s_+ - s_+ d_- u_+ + s_+ u_- d_+ \\ &+ s_+ d_+ u_- - s_+ u_+ d_- - u_+ s_+ d_- + d_+ s_+ u_-\}, \end{aligned} \tag{B6}$$

$$\Psi^{\Xi^0}_{symm}=\frac{\sqrt{2}}{6}\{2u_-s_+s_+-s_-u_+s_+-u_+s_-s_++2s_+u_-s_+ -s_+s_-u_+-s_-s_+u_+-s_+u_+s_--u_+s_+s_- +2s_+s_+u_-\}, \quad (B7)$$

$$\Psi^{\Xi^-}_{symm}=\frac{\sqrt{2}}{6}\{2d_-s_+s_+-s_-d_+s_+-d_+s_-s_++2s_+d_-s_+ -s_+s_-d_+-s_-s_+d_+-s_+d_+s_--d_+s_+s_- +2s_+s_+d_-\}, \quad (B8)$$

where the subscripts ($\pm$) refer to the spin projection. For the flavor decuplet with total spin 3/2 up we have

$$\Psi^{\Delta^{++}}_{symm}=u_+u_+u_+ , \quad (B9)$$

$$\Psi^{\Delta^+}_{symm}=\frac{1}{\sqrt{3}}\{u_+u_+d_++u_+d_+u_++d_+u_+u_+\}, \quad (B10)$$

$$\Psi^{\Delta^0}_{symm}=\frac{1}{\sqrt{3}}\{d_+d_+u_++d_+u_+d_++u_+d_+d_+\}, \quad (B11)$$

$$\Psi^{\Delta^-}_{symm}=d_+d_+d_+ , \quad (B12)$$

$$\Psi^{\Sigma^+}_{symm}=\frac{1}{\sqrt{3}}\{u_+u_+s_++u_+s_+u_++s_+u_+u_+\}, \quad (B13)$$

$$\Psi^{\Sigma^0}_{symm}=\frac{1}{\sqrt{6}}\{u_+d_+s_++d_+u_+s_++u_+s_+d_++s_+u_+d_+ +d_+s_+u_++s_+d_+u_+\}, \quad (B14)$$

$$\Psi^{\Sigma^-}_{symm}=\frac{1}{\sqrt{3}}\{d_+d_+s_++d_+s_+d_++s_+d_+d_+\}, \quad (B15)$$

$$\Psi^{\Xi^0}{}_{symm}=\frac{1}{\sqrt{3}}\{s_+s_+u_++s_+u_+s_++u_+s_+s_+\}, \quad (B16)$$

$$\Psi^{\Xi^-}_{symm}=\frac{1}{\sqrt{3}}\{s_+s_+d_++s_+d_+s_++d_+s_+s_+\}, \quad (B17)$$

$$\Psi^{\Omega^-}_{symm}=s_+s_+s_+ . \quad (B18)$$

These fully symmetrical wave functions, Eqs. (B1)–(B18), can be written symbolically as

$$\psi^N_{JM,symm}=\Gamma^{\alpha\beta\gamma}_{JM}(f_1f_2f_3)\psi^{f_1}_\alpha\psi^{f_2}_\beta\psi^{f_3}_\gamma . \quad (B19)$$

As the orbital of the s quark is heavier than the u- and d-quark orbitals, Eq. (51) has to be split up in contributions from the u,d quarks and from the s quark. Using the symmetrical wave function, Eq. (B19), this is realized by writing

$$\mu_z=3\mu_z^{(1)}=3\sum_{f_1f_2f_3}\langle\Gamma^{\alpha\beta\gamma}_{JM}(f_1f_2f_3)\psi^{f_1}_\alpha\psi^{f_2}_\beta\psi^{f_3}_\gamma|e_q(1)\sigma_z(1) \times|\Gamma^{\alpha\beta\gamma}_{JM}(f_1f_2f_3)\psi^{f_1}_\alpha\psi^{f_2}_\beta\psi^{f_3}_\gamma\rangle\lambda_{f_1}, \quad (B20)$$

TABLE VI. The matrix elements of the e.m. current for the baryons.

N	$\mu_N/3$
p	$\frac{1}{3}\lambda_u$
n	$-\frac{2}{9}\lambda_u$
Σ^+	$\frac{8}{27}\lambda_u+\frac{1}{27}\lambda_s$
Σ^0	$\frac{2}{27}\lambda_u+\frac{1}{27}\lambda_s$
Σ^-	$-\frac{4}{27}\lambda_u+\frac{1}{27}\lambda_s$
Λ	$-\frac{1}{9}\lambda_s$
Ξ^0	$-\frac{4}{27}\lambda_s-\frac{2}{27}\lambda_u$
Ξ^-	$-\frac{4}{27}\lambda_s+\frac{1}{27}\lambda_u$
Δ^{++}	$\frac{2}{3}\lambda_u$
Δ^+	$\frac{1}{3}\lambda_u$
Δ^0	0
Δ^-	$-\frac{1}{3}\lambda_u$
Σ^{+*}	$\frac{4}{9}\lambda_u-\frac{1}{9}\lambda_s$
Σ^{0*}	$\frac{1}{9}\lambda_u-\frac{1}{9}\lambda_s$
Σ^{-*}	$-\frac{2}{9}\lambda_u-\frac{1}{9}\lambda_s$
Ξ^{0*}	$-\frac{2}{9}\lambda_s+\frac{2}{9}\lambda_u$
Ξ^{-*}	$-\frac{2}{9}\lambda_s-\frac{1}{9}\lambda_u$
Ω^-	$-\frac{1}{3}\lambda_s$

with

$$\lambda_{f_i}=-\frac{2}{3}\int g^*_{f_i}(r)f_{f_i}(r)r^3dr. \quad (B21)$$

The flavor index f_i can take the values u, d, or s. Note that $\lambda_u=\lambda_d$ as the same orbital is taken for the u and d quarks. Evaluating Eq. (B20) for the different baryon wave functions Eqs. (B1)–(B18) result in the expressions in Table VI.

APPENDIX C: PIONIC TWO-BODY CONTRIBUTION TO THE MAGNETIC MOMENT

In this appendix the pion-in-flight and pair contributions to the magnetic moment of the nucleons are given. Following Ref. [33] these contributions are determined by taking the curl of the pionic two-body currents, Eq. (68). The three-quark state Ψ is given by the product of three single-quark orbitals, Eq. (35). Because of symmetry considerations, it suffices to calculate the magnetic moment contribution of pion exchange between, say, the second and third quarks only and multiply the result by a factor of 3 to include the contribution of the other possible permutations of quark pairs.

Considering the pion-in-flight contribution first [Eq. (64)], taking the curl gives rather long expressions which can be divided into two parts:

$$\delta\mu_z^{proton}=-\delta\mu_z^{neutron}=3(\delta\mu_z^A+3\delta\mu_z^B). \quad (C1)$$

The first part gives the larger contribution and can be written as

$$\delta\mu_z^A = \lim_{Q^2\to 0}\frac{2eg_{\pi qq}^2}{3(2\pi)^3N}\int d^3q d^3p d^3p' \frac{1}{(\boldsymbol{\Delta}^2+m_\pi^2)^2}[|\tilde g(k_1)|^2+|\tilde f(k_1)|^2]\Big\{\tilde g(k_2')\tilde f(k_2)\tilde g(k_3')\tilde f(k_3)\frac{1}{3k_2k_3}(\mathbf{p}\,\boldsymbol{\Delta}-p_z\Delta_z)$$
$$+\tilde g(k_2')\tilde f(k_2)\tilde f(k_3')\tilde g(k_3)\frac{1}{6k_2k_3'}[(2\mathbf{p}-\mathbf{p}'+\sqrt{3}\mathbf{q})\boldsymbol{\Delta}-(2p_z-p_z'+\sqrt{3}q_z)\Delta_z]+\tilde f(k_2')\tilde g(k_2)\tilde g(k_3')\tilde f(k_3)\frac{1}{6k_2'k_3}$$
$$\times[(2\mathbf{p}-\mathbf{p}'-\sqrt{3}\mathbf{q})\boldsymbol{\Delta}-(2p_z-p_z'-\sqrt{3}q_z)\Delta_z]+\tilde f(k_2')\tilde g(k_2)\tilde f(k_3')\tilde g(k_3)\frac{-2}{3k_2'k_3'}(\mathbf{p}'\boldsymbol{\Delta}-p_z'\Delta_z)\Big\}$$
$$\times\left(1+2\frac{\boldsymbol{\Delta}^2+m_\pi^2}{\boldsymbol{\Delta}^2+\Lambda_\pi^2}\right)\left(\frac{\Lambda_\pi^2}{\boldsymbol{\Delta}^2+\Lambda_\pi^2}\right)^2. \tag{C2}$$

The second part comes from the curl applied to the wave functions

$$\delta\mu_z^B = \lim_{Q^2\to 0}\frac{2eg_{\pi qq}^2}{3(2\pi)^3N}\int d^3q d^3p d^3p' \frac{1}{(\boldsymbol{\Delta}^2+\mu^2)^2}[|\tilde g(k_1)|^2+|\tilde f(k_1)|^2]\frac{1}{3}(\hat{\mathbf{k}}_2'\times\boldsymbol{\Delta})_z\Big\{\frac{\partial\tilde g(k_2')}{\partial k_2'}\tilde f(k_2)\tilde g(k_3')\tilde f(k_3)(\hat{\mathbf{k}}_2\times\hat{\mathbf{k}}_3)_z$$
$$-\frac{\partial\tilde g(k_2')}{\partial k_2'}\tilde f(k_2)\tilde f(k_3')\tilde g(k_3)(\hat{\mathbf{k}}_2\times\hat{\mathbf{k}}_3')_z-\frac{\partial\tilde f(k_2')}{\partial k_2'}\tilde g(k_2)\tilde g(k_3')\tilde f(k_3)(\hat{\mathbf{k}}_2'\times\hat{\mathbf{k}}_3)_z+\frac{\partial\tilde f(k_2')}{\partial k_2'}\tilde g(k_2)\tilde f(k_3')\tilde g(k_3)(\hat{\mathbf{k}}_2'\times\hat{\mathbf{k}}_3')_z$$
$$+\tilde f(k_2')\tilde g(k_2)\tilde g(k_3')\tilde f(k_3)\frac{1}{k_2'}(\hat{\mathbf{k}}_2'\times\hat{\mathbf{k}}_3)_z-\tilde f(k_2')\tilde g(k_2)\tilde f(k_3')\tilde g(k_3)\frac{1}{k_2'}(\hat{\mathbf{k}}_2'\times\hat{\mathbf{k}}_3')_z\Big\}\left(1+2\frac{\boldsymbol{\Delta}^2+m_\pi^2}{\boldsymbol{\Delta}^2+\Lambda_\pi^2}\right)\left(\frac{\Lambda_\pi^2}{\boldsymbol{\Delta}^2+\Lambda_\pi^2}\right)^2. \tag{C3}$$

The normalization factor N is the same as used before in the single-quark current contribution [Eq. (61)]. The momenta are expressed in terms of the Jacobi coordinates, Eqs. (62), again, but from imposing the Breit system and momentum conservation we now get $\mathbf{P}'=-\mathbf{P}=\mathbf{Q}/2$ and $2\sqrt{3}(\mathbf{q}'-\mathbf{q})=\mathbf{Q}$. In writing down these expressions use has been made of the spin-isospin operator sandwiched between the fully symmetric wave functions in the spin-isospin and orbital space of the three quarks:

$$\langle\psi_{symm}^p|(\boldsymbol{\tau}^{(1)}\times\boldsymbol{\tau}^{(2)})_z\sigma_i^{(1)}\sigma_j^{(2)}|\psi_{symm}^p\rangle=-\langle\psi_{symm}^n|(\boldsymbol{\tau}^{(1)}\times\boldsymbol{\tau}^{(2)})_z\sigma_i^{(1)}\sigma_j^{(2)}|\psi_{symm}^n\rangle=-\frac{2}{3}\epsilon_{ij3}. \tag{C4}$$

It should be noted that the spin-isospin factor (C4) is identical to that found for the trinucleon case. For all the other baryon wave functions given in Appendix B the matrix element of the considered two-body e.m. operators vanish, because of the isospin structure of the e.m. operator. Hence the considered two-body currents contribute only to the magnetic moment of the proton and neutron.

The second part $\delta\mu_z^B$ is a relativistic effect which enlarges the values by about 10% and which vanishes in the static limit as is shown at the end of this section.

In the same way the pair term can be analyzed. We find

$$\delta\mu_z^{proton}=-\delta\mu_z^{neutron}=3(\delta\mu_z^C+3\delta\mu_z^D), \tag{C5}$$

with

$$\delta\mu_z^C=\lim_{Q^2\to 0}\frac{eg_{\pi qq}^2}{2m_q(2\pi)^3N}\int d^3q d^3p d^3p'\frac{1}{\mathbf{\Delta}^2+m_\pi^2}\frac{1}{3}[|\tilde g(k_1)|^2+|\tilde f(k_1)|^2]\Bigg\{\frac{1}{k_2}\left(\frac{1}{3}-\frac{\mathbf{\Delta}\,\mathbf{k}_2-\Delta_z(k_2)_z}{\Delta^2+m_\pi^2}\right)\tilde g(k_2')\tilde f(k_2)\tilde g(k_3')\tilde g(k_3)$$
$$+\frac{1}{k_2'}\left(\frac{2}{3}+\frac{\mathbf{\Delta}\,\mathbf{k}_2'-\Delta_z(k_2')_z}{\Delta^2+m_\pi^2}\right)\tilde f(k_2')\tilde g(k_2)\tilde g(k_3')\tilde g(k_3)+\frac{1}{k_3}\left(\frac{1}{3}+\frac{\mathbf{\Delta}\,\mathbf{k}_3-\Delta_z(k_3)_z}{\Delta^2+m_\pi^2}\right)\tilde g(k_2')\tilde g(k_2)\tilde g(k_3')\tilde f(k_3)$$
$$+\frac{1}{k_3'}\left(\frac{2}{3}-\frac{\mathbf{\Delta}\,\mathbf{k}_3'-\Delta_z(k_3')_z}{\mathbf{\Delta}^2+m_\pi^2}\right)\tilde g(k_2')\tilde g(k_2)\tilde f(k_3')\tilde g(k_3)\Bigg\}\left(\frac{\Lambda_\pi^2}{\mathbf{\Delta}^2+\Lambda_\pi^2}\right)^2 \tag{C6}$$

and

$$\delta\mu_z^D=\lim_{Q^2\to 0}\frac{eg_{\pi qq}^2}{6m_q(2\pi)^3N}\int\int\int d^3q d^3p d^3p'\frac{1}{\mathbf{\Delta}^2+m_\pi^2}\frac{1}{3}[|\tilde g(k_1)|^2+|\tilde f(k_1)|^2]\Bigg\{\frac{\partial\tilde g(k_2')}{\partial k_2'}\tilde g(k_2)\{\tilde g(k_3')\tilde f(k_3)[\hat{\mathbf{k}}_2'\hat{\mathbf{k}}_3-(\hat k_2')_z(\hat k_3)_z]$$
$$-\tilde f(k_3')\tilde g(k_3)[\hat{\mathbf{k}}_2'\hat{\mathbf{k}}_3'-(\hat k_2')_z(\hat k_3')_z]\}+\tilde g(k_3')\tilde g(k_3)\Bigg(-\frac{\partial\tilde g(k_2')}{\partial k_2'}\tilde f(k_2)[\hat{\mathbf{k}}_2'\hat{\mathbf{k}}_2-(\hat k_2')_z(\hat k_2)_z]$$
$$+\frac{\partial\tilde f(k_2')}{\partial k_2'}\tilde g(k_2)[\hat{\mathbf{k}}_2'\hat{\mathbf{k}}_2'-(\hat k_2')_z(\hat k_2')_z]\Bigg)-\tilde f(k_2')\tilde g(k_2)\tilde g(k_3')\tilde g(k_3)\frac{1}{k_2'}[\hat{\mathbf{k}}_2'\hat{\mathbf{k}}_2'-(\hat k_2')_z(\hat k_2')_z]\Bigg\}\left(\frac{\Lambda_\pi^2}{\mathbf{\Delta}^2+\Lambda_\pi^2}\right)^2. \tag{C7}$$

In the nonrelativistic limit the lower component of the wave function can be expressed in the upper component as

$$\tilde f(k)=-\frac{|k|}{2m_q}\tilde g(k), \tag{C8}$$

where m_q is the constituent mass of the quark. As a result, the pionic two-body current contributions, Eqs. (C1)–(C3) and Eqs. (C5)–(C7), can be simplified considerably. We obtain, for the pion-in-flight contribution,

$$\delta\mu_z^A=\frac{eg_{\pi qq}^2}{6m_q^2(2\pi)^3N}\int d^3q d^3p d^3p'[|\tilde g(k_1)|^2+|\tilde f(k_1)|^2]\tilde g(k_2')\tilde g(k_2)\tilde g(k_3')\tilde g(k_3)\frac{\mathbf{\Delta}^2-\Delta_z\Delta_z}{(\mathbf{\Delta}^2+m_\pi^2)^2}\left(1+2\frac{\mathbf{\Delta}^2+m_\pi^2}{\mathbf{\Delta}^2+\Lambda_\pi^2}\right)\cdot\left(\frac{\Lambda_\pi^2}{\mathbf{\Delta}^2+\Lambda_\pi^2}\right)^2. \tag{C9}$$

For the pair term we get

$$\delta\mu_z^C=\frac{eg_{\pi qq}^2}{6m_q^2(2\pi)^3N}\int d^3q d^3p d^3p'[|\tilde g(k_1)|^2+|\tilde f(k_1)|^2]\tilde g(k_2')\tilde g(k_2)\tilde g(k_3')\tilde g(k_3)\left(\frac{\mathbf{\Delta}^2-\Delta_z^2}{(\mathbf{\Delta}^2+m_\pi^2)^2}-\frac{1}{\mathbf{\Delta}^2+m_\pi^2}\right)\left(\frac{\Lambda_\pi^2}{\mathbf{\Delta}^2+\Lambda_\pi^2}\right)^2, \tag{C10}$$

while $\delta\mu_z^B$ and $\delta\mu_z^D$ vanish. These expressions agree with the results of Refs. [33] and [35].

APPENDIX D: ANOMALOUS MAGNETIC MOMENT CONTRIBUTIONS FROM PION LOOPS

Our starting point is the e.m. currents, corresponding to the one-loop diagrams shown in Fig. 4:

$$J_\mu^{(a)}=-2ig_{\pi qq}^2e\tau_z\int\frac{d^4k}{(2\pi)^4}\frac{\gamma_5(\not p-\not k+m_q)\gamma_5(2k_\mu+Q_\mu)}{[(p-k)^2-m_q^2+i\epsilon][k^2-m_\pi^2+i\epsilon][(k+Q)^2-m_\pi^2+i\epsilon]}\frac{\Lambda_\pi^2}{k^2-\Lambda_\pi^2}\frac{\Lambda_\pi^2}{(k+Q)^2-\Lambda_\pi^2}$$
$$\times\left(1+\frac{k^2-m_\pi^2}{(k+Q)^2-\Lambda_\pi^2}+\frac{(k+Q)^2-m_\pi^2}{k^2-\Lambda_\pi^2}\right) \tag{D1}$$

and

$$J_\mu^{(b)}=-ig_{\pi qq}^2e\frac{1-\tau_z}{2}\int\frac{d^4k}{(2\pi)^4}\frac{\gamma_5(\not p'-\not k+m_q)\gamma_\mu(\not p-\not k+m_q)\gamma_5}{[(p'-k)^2-m_q^2+i\epsilon][(p-k)^2-m_q^2+i\epsilon][k^2-m_\pi^2+i\epsilon]}\left(\frac{\Lambda_\pi^2}{k^2-\Lambda_\pi^2}\right)^2. \tag{D2}$$

Since we have assumed a finite form factor at the πqq vertex, similar as in the two-body current case, the two additional terms are needed in the last factor of Eq. (D1) to satisfy current conservation. From these currents the anomalous magnetic moment has to be extracted. By applying the Gordon decomposition to the current, Eq. (69), near $Q^2=0$ it can be seen that the anomalous magnetic moment κ is the term proportional to $(-e/2M)K_\mu$ with $K_\mu=p_\mu+p'_\mu$. To isolate this term the currents are rewritten by explicit evaluation of the γ-matrix algebra and taking the limit $Q^2\to 0$. Using the approximation that the initial and final quarks are on mass shell we obtain

$$J_\mu^{(a)}=-2ig_{\pi qq}^2 e\tau_z\gamma^\nu\int\frac{d^4k}{(2\pi)^4}\frac{2k_\mu k_\nu}{[k^2-2pk+i\epsilon][k^2-m_\pi^2+i\epsilon]^2}\left(\frac{\Lambda_\pi^2}{k^2-\Lambda_\pi^2}\right)^2\left(1+2\frac{k^2-m_\pi^2}{k^2-\Lambda_\pi^2}\right)\equiv-2ig_{\pi qq}^2 e\tau_z\gamma^\nu C_{\mu\nu}^{(a)} \quad \text{(D3)}$$

and

$$J_\mu^{(b)}=ig_{\pi qq}^2 e\frac{1-\tau_z}{2}\gamma^\nu\int\frac{d^4k}{(2\pi)^4}\frac{2k_\mu k_\nu-k^2g_{\mu\nu}}{[k^2-2pk+i\epsilon]^2[k^2-m_\pi^2+i\epsilon]}\left(\frac{\Lambda_\pi^2}{k^2-\Lambda_\pi^2}\right)^2\equiv ig_{\pi qq}^2 e\frac{1-\tau_z}{2}\gamma^\nu C_{\mu\nu}^{(b)}. \quad \text{(D4)}$$

As the tensors $C^{\mu\nu}$ depend only on the initial and final momenta, they can be written as

$$C_{\mu\nu}^{(i)}=A_1^{(i)}K_\mu K_\nu+A_2^{(i)}K_\mu Q_\nu+A_3^{(i)}Q_\mu K_\nu+A_4^{(i)}Q_\mu Q_\nu+A_5^{(i)}g_{\mu\nu}, \quad \text{(D5)}$$

where $A_n^{(i)}$ are Lorentz invariants. It can readily be seen that only the first term $A_1^{(i)}$ contributes to the magnetic moment. Substituting Eq. (D5) into Eqs. (D3),(D4) and taking the initial and final quarks on mass shell we find, for the anomalous magnetic moment corrections,

$$\kappa^{(a)}=8iM_p m_q g_{\pi qq}^2\tau_z A_1^{(a)}, \quad \text{(D6)}$$

$$\kappa^{(b)}=-4iM_p m_q g_{\pi qq}^2\frac{1-\tau_z}{2}A_1^{(b)}. \quad \text{(D7)}$$

The Lorentz-invariant expression $A_1^{(i)}$ can immediately be determined from the tensor $C_{\mu\nu}^{(i)}$. We get

$$A_1^{(i)}=\frac{1}{3K^4}(4K^\mu K^\nu-K^2g^{\mu\nu})C_{\mu\nu}^{(i)}. \quad \text{(D8)}$$

Inserting Eq. (D8) into Eqs. (D6) and (D7) the expressions (70) and (71) are obtained.

The kaon one-loop diagrams can be calculated in similar way. The starting point is the expressions (D1) and (D2) again, where the mass of the pion is replaced by the mass of the kaon and the isospin structure is changed to $(\tau_z+3Y)/2$ and $-(\frac{2}{9}+\frac{4}{3}Y)$, respectively, in Eqs. (D1) and (D2) with Y the hypercharge. The expressions for the anomalous magnetic moment due to the kaon loop become

$$\kappa^{(a)}=ig_{Kqq}^2(\tau_z+3Y)\frac{2M_p}{3m_q^3}\int\frac{d^4k}{(2\pi)^4}\frac{4(p\cdot k)^2-p^2k^2+3m_q(M_q-m_q)p\cdot k}{[k^2-2pk+m_q^2-M_q^2+i\epsilon][k^2-m_K^2+i\epsilon]^2}\left(\frac{\Lambda_K^2}{k^2-\Lambda_K^2}\right)^2\left(1+2\frac{k^2-m_K^2}{k^2-\Lambda_K^2}\right), \quad \text{(D9)}$$

and

$$\kappa^{(b)}=ig_{Kqq}^2\left(\frac{2}{9}+\frac{4}{3}Y\right)\frac{2M_p}{3m_q^3}\int\frac{d^4k}{(2\pi)^4}\frac{4(p\cdot k)^2-p^2k^2+3m_q(M_q-m_q)p\cdot k}{[k^2-2pk+m_q^2-M_q^2+i\epsilon]^2[k^2-m_K^2+i\epsilon]}\left(\frac{\Lambda_K^2}{k^2-\Lambda_K^2}\right)^2, \quad \text{(D10)}$$

with M_q the mass of the intermediate quark, and m_q the mass of the initial and final quarks. The coupling constant g_{Kqq} and the cutoff Λ_K are taken the same as for the pion loop.

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