

Implications of recent $\bar{B}^0 \rightarrow D^{(*)0} X^0$ measurements

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(Received 7 November 2001; published 2 May 2002)

The recent measurements of the color-suppressed modes $\bar{B}^0 \rightarrow D^{(*)0} \pi^0$ imply nonvanishing relative final-state interaction (FSI) phases among various $\bar{B} \rightarrow D \pi$ decay amplitudes. Depending on whether or not FSIs are implemented in the topological quark-diagram amplitudes, two solutions for the parameters a_1 and a_2 are extracted from data using various form-factor models. It is found that $|a_2(D\pi)| \sim 0.35 - 0.60$ and $|a_2(D^* \pi)| \sim 0.25 - 0.50$ with a relative phase of order 60° between a_1 and a_2 . If FSIs are not included in the quark-diagram amplitudes from the outset, $a_2^{\text{eff}}/a_1^{\text{eff}}$ and a_2^{eff} will become smaller. The large value of $|a_2(D\pi)|$ compared to $|a_2^{\text{eff}}(D\pi)|$ or naive expectation implies the importance of long-distance FSI contributions to color-suppressed internal W emission via final-state rescatterings of the color-allowed tree amplitude.

DOI: 10.1103/PhysRevD.65.094012

PACS number(s): 13.25.-k, 13.25.Hw

I. INTRODUCTION

For some time $B \rightarrow J/\psi K$ and $B \rightarrow J/\psi K^*$ have remained the only color-suppressed B meson two-body decay modes that have been measured experimentally. Recently, the long-awaited color-suppressed decay modes $\bar{B}^0 \rightarrow D^{(*)0} \pi^0$ were finally measured by both Belle [1] and CLEO [2] with the $D^0 \pi^0$ branching ratio larger than the upper limit previously reported [3]. The channels $\bar{B}^0 \rightarrow D^{(*)0} \eta$ and $\bar{B}^0 \rightarrow D^{(*)0} \omega$ were also observed by Belle [1]. We shall see below that the theoretical predictions based on the factorization approach in general are too small to account for the observed decay rates of color-suppressed modes $D^{(*)0} X^0$ with $X = \pi, \eta, \omega$. This has important implications for final-state interactions (FSIs).

Under the factorization hypothesis, the nonleptonic decay amplitudes are approximated by the factorized hadronic matrix elements multiplied by some universal, process-independent effective coefficients a_i . Based on the factorization assumption, one can catalog the decay processes into three classes. For class-I decays, the decay amplitudes, dominated by the color-allowed external W emission, are proportional to $a_1 \langle O_1 \rangle_{\text{fact}}$, where O_1 is a charged current–charged current 4-quark operator. For class-II decays, the decay amplitudes, governed by the color-suppressed internal W emission, are described by $a_2 \langle O_2 \rangle_{\text{fact}}$ with O_2 being a neutral current–neutral current 4-quark operator. The decay amplitudes of the class-III decays involve a linear combination of $a_1 \langle O_1 \rangle_{\text{fact}}$ and $a_2 \langle O_2 \rangle_{\text{fact}}$. If factorization works, the effective coefficients a_i in nonleptonic B or D decays should be channel-by-channel independent.

What is the relation between the coefficients a_i and the Wilson coefficients in the effective Hamiltonian approach? Under the naive factorization hypothesis, one has

$$\begin{aligned} a_1(\mu) &= c_1(\mu) + \frac{1}{N_c} c_2(\mu), \\ a_2(\mu) &= c_2(\mu) + \frac{1}{N_c} c_1(\mu), \end{aligned} \quad (1)$$

for decay amplitudes induced by current-current operators $O_{1,2}(\mu)$, where $c_{1,2}(\mu)$ are the corresponding Wilson coefficients and N_c is the number of colors. In the absence of QCD corrections, $c_1 = 1$ and $c_2 = 0$, and hence class-II modes governed by $a_2 = 1/N_c$ are obviously “color suppressed.” However, this naive factorization approach encounters two principal difficulties: (i) the coefficients a_i given by Eq. (1) are renormalization scale and γ_5 -scheme dependent, and (ii) it fails to describe the color-suppressed class-II decay modes. For example, the ratio $R = \Gamma(D^0 \rightarrow \bar{K}^0 \pi^0) / \Gamma(D^0 \rightarrow K^- \pi^+)$ is predicted to be only of order 3×10^{-4} due to the smallness of a_2 in the naive factorization approach, while experimentally it is measured to be 0.55 ± 0.06 [4]. It is known that the decay $D^0 \rightarrow \bar{K}^0 \pi^0$ is enhanced by two mechanisms. First, a_2 receives a large nonfactorizable correction. Second, the weak decay $D^0 \rightarrow K^- \pi^+$ followed by the inelastic rescattering $K^- \pi^+ \rightarrow \bar{K}^0 \pi^0$ can raise $\mathcal{B}(D^0 \rightarrow \bar{K}^0 \pi^0)$ dramatically by lowering $\mathcal{B}(D^0 \rightarrow K^- \pi^+)$.

Beyond naive factorization the parameters $a_{1,2}$ have the general expression

$$\begin{aligned} a_{1,2} &= c_{2,1}(\mu) + \frac{c_{1,2}(\mu)}{N_c} \\ &+ \text{nonfactorizable corrections}, \end{aligned} \quad (2)$$

where nonfactorizable corrections include vertex corrections, hard spectator interactions involving the spectator quark of the heavy meson, and FSI effects from inelastic rescattering, resonance effects, . . . , etc. In the generalized factorization approach of [5,6], one includes the vertex corrections which will compensate the renormalization scale and γ_5 -scheme dependence of the Wilson coefficients to render the $a_{1,2}$ scale and scheme independent. Contrary to the naive one, the improved generalized factorization scheme assumes that nonfactorizable effects are incorporated in a process-independent form. Since not all nonfactorizable effects are calculable by perturbative QCD, one will treat a_1 and a_2 as free parameters in the generalized factorization approach and extract them from experiment. The phenomenological analysis of

two-body decay data of D and B mesons will tell us if the generalized factorization hypothesis works reasonably well by studying the variation of the parameters $a_{1,2}$ from channel to channel.

The experimental measurement of $B \rightarrow J/\psi K$ leads to $|a_2(J/\psi K)| = 0.26 \pm 0.02$ [7]. This seems to be also supported by the study of $B \rightarrow D\pi$ decays: Assuming no relative phase between a_1 and a_2 , the result $a_2 \sim \mathcal{O}(0.20 - 0.30)$ [7,8] is inferred from the data of $\bar{B}^0 \rightarrow D^{(*)+} \pi^-$ and $B^- \rightarrow D^{(*)0} \pi^-$. However, as we shall show below, the above value of a_2 leads to too small decay rates for $\bar{B}^0 \rightarrow D^{(*)0} \pi^0$ when compared to recent measurements. In order to account for the observation, one needs a larger $a_2(D\pi)$ with a nontrivial phase relative to a_1 . The importance of FSIs has long been realized in charm decay since some resonances are known to exist at energies close to the mass of the charmed meson. We shall see in this work that, just as $D^0 \rightarrow \bar{K}^0 \pi^0$, both nonfactorizable effects and FSIs are also needed to explain the data of $\bar{B}^0 \rightarrow D^{(*)0} \pi^0$, though these two effects in B decays are naively expected to be not as dramatic as in the charm case.

The color-suppressed mode is a very suitable place for studying the effect of FSIs (especially the soft one) in weak decays. The ratio of the color-suppressed decay amplitudes with and without FSIs is $R_{\bar{K}\pi} \equiv |A(D^0 \rightarrow \bar{K}^0 \pi^0)/A(D^0 \rightarrow \bar{K}^0 \pi^0)_{\text{without FSIs}}| \approx 2.0$ and the relative phase between $D^0 \rightarrow \bar{K}^0 \pi^0$ and $D^0 \rightarrow K^- \pi^+$ is about 150° . It is expected that for the $\bar{B} \rightarrow D\pi$ decay, $R_{D\pi}$ and the relative phase among decay amplitudes will become smaller. The recent measurement of the $\bar{B}^0 \rightarrow D^0 \pi^0$ mode allows us to determine the above two quantities. We shall see that although the relative phase among $\bar{B} \rightarrow D\pi$ decay amplitudes becomes smaller, $R_{D^{(*)}\pi}$ does not decrease in a significant way from the charm to the bottom case. The implications and related physics will be discussed below in details.

II. FACTORIZATION

We begin by considering the branching ratios of the color-suppressed modes $\bar{B}^0 \rightarrow D^{(*)0} X^0$ ($X = \pi, \eta, \omega$) within the framework of the factorization approach. The $\bar{B}^0 \rightarrow D^0 \pi^0$ amplitude is given by

$$A(\bar{B}^0 \rightarrow D^0 \pi^0) = \frac{1}{\sqrt{2}}(-\mathcal{C} + \mathcal{E}), \quad (3)$$

where \mathcal{C} , \mathcal{E} are color-suppressed internal W -emission and W -exchange amplitudes, respectively. In terms of the factorized hadronic matrix elements, they read

$$\mathcal{C} = i \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* a_2(D\pi) (m_B^2 - m_\pi^2) f_D F_0^{B\pi}(m_D^2),$$

$$\mathcal{E} = i \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* a_2(D\pi) (m_D^2 - m_\pi^2) f_B F_0^{0 \rightarrow D\pi}(m_B^2), \quad (4)$$

where $a_2(D\pi)$ is a parameter to be determined from experiment. The annihilation form factor $F_0^{0 \rightarrow D\pi}(m_B^2)$ is expected to be suppressed at large momentum transfer, $q^2 = m_B^2$, corresponding to the conventional helicity suppression. Based on the argument of helicity and color suppression, one may therefore neglect short-distance (hard) W -exchange contributions. However, it is not clear if the long-distance contribution to the W exchange is also negligible. Likewise,

$$\begin{aligned} A(\bar{B}^0 \rightarrow D^0 \eta) &= i \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* a_2(D\eta) (m_B^2 - m_\eta^2) \\ &\quad \times f_D F_0^{B\eta}(m_D^2), \\ A(\bar{B}^0 \rightarrow D^{*0} \pi^0) &= - \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* a_2(D^* \pi) \sqrt{2} m_{D^*} \\ &\quad \times f_{D^*} F_1^{B\pi}(m_{D^*}^2), \end{aligned} \quad (5)$$

$$\begin{aligned} A(\bar{B}^0 \rightarrow D^0 \omega) &= \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* a_2(D\omega) 2m_\omega \\ &\quad \times f_D A_0^{B\omega}(m_D^2), \end{aligned}$$

and

$$\begin{aligned} A(\bar{B}^0 \rightarrow D^{*0} \omega) &= -i \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* a_2(D^* \omega) f_{D^*} m_{D^*} \\ &\quad \times \left[(\varepsilon_{D^*}^* \cdot \varepsilon_\omega^*) (m_B + m_\omega) A_1^{B\omega}(m_{D^*}^2) - (\varepsilon_{D^*}^* \cdot p_B) \right. \\ &\quad \times (\varepsilon_\omega^* \cdot p_B) \frac{2A_2^{B\omega}(m_{D^*}^2)}{m_B + m_\omega} \\ &\quad \left. + i \epsilon_{\mu\nu\alpha\beta} \varepsilon_\omega^{*\mu} \varepsilon_{D^*}^{*\nu} p_B^\alpha p_1^\beta \frac{2V^{B\omega}(m_{D^*}^2)}{m_B + m_\omega} \right]. \end{aligned} \quad (6)$$

Here factorization implies a universal a_2 , namely, $a_2(D^* \omega) = a_2(D\omega) = a_2(D\eta) = a_2(D^* \pi) = a_2(D\pi)$. In naive factorization, a_2 is not only small, of order 0.10, but also renormalization scale and scheme dependent. In the generalized factorization approach, the scale- and scheme-independent a_2 can be extracted from experiment and the factorization hypothesis is tested by studying a_2 to see if it is process independent or insensitive.

To proceed, we shall consider four distinct form-factor models: the Neubert-Rieckert-Stech-Xu (NRSX) model [9], the relativistic light-front (LF) quark model [10], the Neubert-Stech (NS) model [8], and the Melikhov-Stech

TABLE I. Form factors in various form-factor models. Except for the NRSX model, the relations $A_i^{B\omega}(q^2) = A_i^{B\rho}(q^2)$ ($i=0,1,2$) and $V^{B\omega}(q^2) = V^{B\rho}(q^2)$ are assumed in all the form-factor models. The pion in the B - π transition is referred to the charged one.

	$F_0^{B\pi}(m_D^2)$	$F_1^{B\pi}(m_{D^*}^2)$	$F_0^{B\eta}(m_D^2)$	$F_1^{B\eta}(m_{D^*}^2)$	$F_0^{BD}(m_\pi^2)$	$A_0^{BD^*}(m_\pi^2)$	$A_0^{B\omega}(m_D^2)$	$A_1^{B\omega}(m_{D^*}^2)$	$A_2^{B\omega}(m_{D^*}^2)$	$V^{B\omega}(m_{D^*}^2)$
NRSX	0.37	0.45	0.19	0.23	0.69	0.62	0.26	0.23	0.27	0.32
LF	0.34	0.39	0.18	0.22	0.70	0.73	0.25	0.16	0.17	0.28
MS	0.32	0.36	0.17	0.20	0.67	0.69	0.26	0.20	0.21	0.28
NS	0.27	0.32	0.15	0.18	0.63	0.64	0.22	0.20	0.22	0.22

(MS) model based on the constituent quark picture [11]. The NRSX model takes the Bauer-Stech-Wirbel (BSW) model [12] results for the form factors at zero momentum transfer but makes a different ansatz for their q^2 dependence, namely, a dipole behavior is assumed for the form factors F_1 , A_0 , A_2 , V , motivated by heavy quark symmetry, and a monopole dependence for F_0, A_1 , where we have followed the definition of form factors given in [12]. For the reader's convenience, the values of relevant form factors are listed in Table I (see [7] for some details about the NS model).

The form factors for $B \rightarrow \eta$ and $B \rightarrow \eta'$ transitions have been calculated by BSW [12] in a relativistic quark model. However, in their relativistic quark model calculation of $B \rightarrow \eta^{(\prime)}$ transitions, BSW considered only the $u\bar{u}$ component of the η and η' ; that is, the form factors calculated by BSW are actually $F_0^{B\eta_{u\bar{u}}}$ and $F_0^{B\eta'_{u\bar{u}}}$ induced from the $b \rightarrow u$ transition. It is thus more natural to consider the flavor basis of η_q and η_s defined by

$$\eta_q = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}), \quad \eta_s = s\bar{s}. \quad (7)$$

The wave functions of the η and η' are given by

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}, \quad (8)$$

where $\phi = \theta + \arctan \sqrt{2}$, and θ is the η - η' mixing angle in the octet-singlet basis. The physical form factors then have the simple expressions:

$$F_{0,1}^{B\eta} = \frac{1}{\sqrt{2}} \cos \phi F_{0,1}^{B\eta_{u\bar{u}}}, \quad F_{0,1}^{B\eta'} = \frac{1}{\sqrt{2}} \sin \phi F_{0,1}^{B\eta'_{u\bar{u}}}. \quad (9)$$

Using $F_0^{B\eta_{u\bar{u}}}(0) = 0.307$ and $F_0^{B\eta'_{u\bar{u}}}(0) = 0.254$ obtained from [12] and the mixing angle $\phi = 39.3^\circ$ (or $\theta = -15.4^\circ$) [13] we find $F_0^{B\eta}(0) = 0.168$ and $F_0^{B\eta'}(0) = 0.114$ in the BSW model and hence the NRSX model. For other form-factor models,¹ we shall apply the relation based on isospin-quartet symmetry

¹The form factors $F_0^{B\eta}(m_D^2) = 0.28$ and $F_1^{B\eta}(m_{D^*}^2) = 0.33$ for the NS model obtained in [14] are larger than ours by about a factor of 2.

$$F_{0,1}^{B\eta_{u\bar{u}}} = F_{0,1}^{B \rightarrow \eta'_{u\bar{u}}} = F_{0,1}^{B\pi} \quad (10)$$

and Eq. (9) to obtain the physical $B - \eta$ and $B - \eta'$ transition form factors.

As mentioned in the Introduction, in the absence of a relative phase between a_1 and a_2 , a value of a_2 in the range of 0.20 to 0.30 is inferred from the data of $\bar{B}^0 \rightarrow D^{(*)+} \pi^-$ and $B^- \rightarrow D^{(*)0} \pi^-$. For definiteness, we shall use the representative value $a_2 = 0.25$ for the purpose of illustration. The calculated branching ratios for $\bar{B}^0 \rightarrow D^{(*)0} X^0$ are shown in Table II for $f_D = 200$ MeV and $f_{D^*} = 230$ MeV. Evidently, the predicted rates for color suppressed modes are too small compared to recent measurements. It should be stressed that if there is no relative phase between a_1 and a_2 , then one cannot increase a_2 arbitrarily to fit the data as this will enhance the decay rate of the $\Delta I = 3/2$ mode $B^- \rightarrow D^{(*)0} \pi^-$ and destroy the agreement between theory and experiment for the charged mode. For example, fitting a_2 to the data of $D^0 \pi^0$ without FSIs will yield $a_2 = 0.45$ in the MS model, which in turn implies $\mathcal{B}(B^- \rightarrow D^0 \pi^-) = 7.9 \times 10^{-3}$ and this is obviously too large compared to the experimental value $(5.3 \pm 0.5) \times 10^{-3}$ [4]. In this case, one needs FSIs to convert $D^+ \pi^-$ into $D^0 \pi^0$. In contrast, if a_2 is of order 0.45, then a relative strong phase between a_1 and a_2 will be needed in order not to overestimate the $D^0 \pi^-$ rate. In either case, we conclude that FSIs are the necessary ingredients for understanding the data.

III. EXTRACTION OF a_1 AND a_2

In this section we will extract the parameters a_1 and a_2 in two different approaches. In the first approach, the topological amplitudes are assumed to incorporate all the information of strong interactions. Therefore, $a_{1,2}$ thus determined already include the effects of FSIs. In the second approach, one will assume that quark-diagram topologies in their original forms do not include FSIs from the outset.

A. Direct analysis

In terms of the quark-diagram topologies \mathcal{T} , \mathcal{C} , and \mathcal{E} , where \mathcal{T} is the color-allowed external W -emission amplitude, the other $\bar{B} \rightarrow D \pi$ amplitudes can be expressed as

$$\begin{aligned} A(\bar{B}^0 \rightarrow D^+ \pi^-) &= \mathcal{T} + \mathcal{E}, \\ A(B^- \rightarrow D^0 \pi^-) &= \mathcal{T} + \mathcal{C}, \end{aligned} \quad (11)$$

TABLE II. Predicted branching ratios (in units of 10^{-4}) of $\bar{B}^0 \rightarrow D^{(*)0} X^0$ ($X = \pi, \eta, \omega$) in the generalized approach with various form-factor models for $a_2 = 0.25$, $f_D = 200$ MeV, and $f_{D^*} = 230$ MeV.

Decay mode	NRSX	LF	MS	NS	Experiments	
					Belle [1]	CLEO [2]
$\bar{B}^0 \rightarrow D^0 \pi^0$	1.13	0.93	0.82	0.58	$3.1 \pm 0.4 \pm 0.5$	$2.74^{+0.36}_{-0.32} \pm 0.55$
$\bar{B}^0 \rightarrow D^{*0} \pi^0$	1.57	1.20	1.01	0.80	$2.7^{+0.8+0.5}_{-0.7-0.6}$	$2.20^{+0.59}_{-0.52} \pm 0.79$
$\bar{B}^0 \rightarrow D^0 \eta$	0.55	0.53	0.48	0.34	$1.4^{+0.5}_{-0.4} \pm 0.3$	
$\bar{B}^0 \rightarrow D^{*0} \eta$	0.76	0.68	0.58	0.46	$2.0^{+0.9}_{-0.8} \pm 0.4$	
$\bar{B}^0 \rightarrow D^0 \omega$	0.76	0.71	0.76	0.54	$1.8 \pm 0.5^{+0.4}_{-0.3}$	
$\bar{B}^0 \rightarrow D^{*0} \omega$	1.60	1.16	1.75	1.35	$3.1^{+1.3}_{-1.1} \pm 0.8$	

and they satisfy the isospin triangle relation

$$A(\bar{B}^0 \rightarrow D^+ \pi^-) = \sqrt{2}A(\bar{B}^0 \rightarrow D^0 \pi^0) + A(B^- \rightarrow D^0 \pi^-). \quad (12)$$

In writing Eqs. (3) and (11) it has been assumed that the topologies \mathcal{T} , \mathcal{C} , \mathcal{E} include the information of all strong interactions for physical $\bar{B} \rightarrow D\pi$ amplitudes (for an earlier discussion of quark-diagram amplitudes, see [15]). Now since all three sides of the $\bar{B} \rightarrow D\pi$ triangle are measured, we are able to determine the relative phases among the decay amplitudes. Using the data [4]

$$\begin{aligned} \mathcal{B}(\bar{B}^0 \rightarrow D^+ \pi^-) &= (3.0 \pm 0.4) \times 10^{-3}, \\ \mathcal{B}(B^- \rightarrow D^0 \pi^-) &= (5.3 \pm 0.5) \times 10^{-3}, \end{aligned} \quad (13)$$

$$\begin{aligned} \mathcal{B}(\bar{B}^0 \rightarrow D^{*+} \pi^-) &= (2.76 \pm 0.21) \times 10^{-3}, \\ \mathcal{B}(B^- \rightarrow D^{*0} \pi^-) &= (4.6 \pm 0.4) \times 10^{-3}, \end{aligned}$$

and the combined value of Belle and CLEO for the neutral modes (see Table II)

$$\begin{aligned} \mathcal{B}(\bar{B}^0 \rightarrow D^0 \pi^0) &= (2.92 \pm 0.46) \times 10^{-4}, \\ \mathcal{B}(\bar{B}^0 \rightarrow D^{*0} \pi^0) &= (2.47 \pm 0.67) \times 10^{-4}, \end{aligned} \quad (14)$$

we find (only the central values for phase angles are shown here)

$$\begin{aligned} \left. \frac{\mathcal{C} - \mathcal{E}}{\mathcal{T} + \mathcal{E}} \right|_{D\pi} &= (0.44 \pm 0.05) e^{i59^\circ}, \\ \left. \frac{\mathcal{C} - \mathcal{E}}{\mathcal{T} + \mathcal{C}} \right|_{D\pi} &= (0.34 \pm 0.03) e^{i37^\circ}, \\ \left. \frac{\mathcal{C} - \mathcal{E}}{\mathcal{T} + \mathcal{E}} \right|_{D^*\pi} &= (0.42 \pm 0.06) e^{i63^\circ}, \\ \left. \frac{\mathcal{C} - \mathcal{E}}{\mathcal{T} + \mathcal{C}} \right|_{D^*\pi} &= (0.34 \pm 0.05) e^{i44^\circ}, \end{aligned} \quad (15)$$

where we have employed the B meson lifetimes given in [4].

The same phases also can be obtained from the isospin analysis. Decomposing the physical amplitudes into their isospin amplitudes yields

$$\begin{aligned} A(\bar{B}^0 \rightarrow D^+ \pi^-) &= \sqrt{\frac{2}{3}} A_{1/2^+} + \sqrt{\frac{1}{3}} A_{3/2}, \\ A(\bar{B}^0 \rightarrow D^0 \pi^0) &= \sqrt{\frac{1}{3}} A_{1/2^-} - \sqrt{\frac{2}{3}} A_{3/2}, \\ A(B^- \rightarrow D^0 \pi^-) &= \sqrt{3} A_{3/2}. \end{aligned} \quad (16)$$

The isospin amplitudes are related to the topological quark-diagram amplitudes via

$$A_{1/2} = \frac{1}{\sqrt{6}}(2\mathcal{T} - \mathcal{C} + 3\mathcal{E}), \quad A_{3/2} = \frac{1}{\sqrt{3}}(\mathcal{T} + \mathcal{C}). \quad (17)$$

Intuitively, the phase shift difference between $A_{1/2}$ and $A_{3/2}$, which is of order 90° for $D \rightarrow \bar{K}\pi$ modes (see below), is expected to play a minor role in the energetic $B \rightarrow D\pi$ decay, the counterpart of $D \rightarrow \bar{K}\pi$ in the B system, as the decay particles are moving fast, not allowing adequate time for final-state interactions. Applying the relations (see, e.g., [8])

$$\begin{aligned} |A_{1/2}|^2 &= |A(\bar{B}^0 \rightarrow D^+ \pi^-)|^2 + |A(\bar{B}^0 \rightarrow D^0 \pi^0)|^2 \\ &\quad - \frac{1}{3} |A(B^- \rightarrow D^0 \pi^-)|^2, \\ |A_{3/2}|^2 &= \frac{1}{3} |A(B^- \rightarrow D^0 \pi^-)|^2, \\ \cos(\delta_{1/2} - \delta_{3/2}) &= \frac{3|A(\bar{B}^0 \rightarrow D^+ \pi^-)|^2 - 2|A_{1/2}|^2 - |A_{3/2}|^2}{2\sqrt{2}|A_{1/2}||A_{3/2}|}, \end{aligned} \quad (18)$$

we obtain

$$\left. \frac{A_{1/2}}{\sqrt{2}A_{3/2}} \right|_{D\pi} = (0.70 \pm 0.10) e^{i29^\circ},$$

TABLE III. Extraction of the parameters a_1 and a_2 from the measured $B \rightarrow D^{(*)} \pi$ rates by assuming a negligible W -exchange contribution. Note that $a_2(D\pi)$ and $a_2(D^* \pi)$ should be multiplied by a factor of (200 MeV/ f_D) and (230 MeV/ f_{D^*}), respectively.

Model	$ a_1(D\pi) $	$ a_2(D\pi) $	$a_2(D\pi)/a_1(D\pi)$	$ a_1(D^* \pi) $	$ a_2(D^* \pi) $	$a_2(D^* \pi)/a_1(D^* \pi)$
NRSX	0.85 ± 0.06	0.40 ± 0.05	$(0.47 \pm 0.05) \exp(i59^\circ)$	0.94 ± 0.04	0.31 ± 0.04	$(0.33 \pm 0.04) \exp(i63^\circ)$
LF	0.84 ± 0.06	0.44 ± 0.06	$(0.53 \pm 0.06) \exp(i59^\circ)$	0.80 ± 0.03	0.36 ± 0.05	$(0.45 \pm 0.06) \exp(i63^\circ)$
MS	0.88 ± 0.06	0.47 ± 0.06	$(0.53 \pm 0.06) \exp(i59^\circ)$	0.85 ± 0.03	0.389 ± 0.05	$(0.46 \pm 0.06) \exp(i63^\circ)$
NS	0.93 ± 0.06	0.56 ± 0.07	$(0.60 \pm 0.07) \exp(i59^\circ)$	0.91 ± 0.03	0.44 ± 0.06	$(0.48 \pm 0.06) \exp(i63^\circ)$

$$\left. \frac{A_{1/2}}{\sqrt{2}A_{3/2}} \right|_{D^* \pi} = (0.74 \pm 0.07) e^{i29^\circ}. \quad (19)$$

Similar results are also obtained in [16,17] using the preliminary Belle and CLEO measurements. It is easy to check that the ratio $(\mathcal{C} - \mathcal{E})/(\mathcal{C} + \mathcal{E})$ in Eq. (15) follows from Eqs. (17) and (19). It is also interesting to compare the above results with that for $D \rightarrow \bar{K}^{(*)} \pi$ decays [4]:

$$\left. \frac{A_{1/2}}{\sqrt{2}A_{3/2}} \right|_{\bar{K} \pi} = (2.70 \pm 0.14) e^{i90^\circ},$$

$$\left. \frac{A_{1/2}}{\sqrt{2}A_{3/2}} \right|_{\bar{K}^* \pi} = (3.97 \pm 0.25) e^{i104^\circ}. \quad (20)$$

The smaller isospin phase shift difference in B decays is in accord with expectation. Notice that while $\Delta I = 1/2$ and $3/2$ amplitudes in $\bar{B} \rightarrow D^{(*)} \pi$ are of the same size, the $D \rightarrow \bar{K} \pi$ decays are dominated by the isospin $\Delta I = 1/2$ amplitude. In the heavy quark limit, the ratio of $A_{1/2}/(\sqrt{2}A_{3/2})$ approaches to unity [17]. Evidently, the charm system exhibits a greater deviation than the B system from the heavy quark limit, as expected.

The ratio of a_2/a_1 can be extracted from Eq. (15) or Eq. (19). Noting that the factorized color-allowed tree amplitude reads

$$\mathcal{T} = i \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* a_1(D\pi) (m_B^2 - m_D^2) f_\pi F_0^{BD}(m_\pi^2), \quad (21)$$

and neglecting W -exchange contributions, we get

$$\left. \frac{a_2}{a_1} \right|_{D\pi} = (0.44 \pm 0.05) e^{i59^\circ}$$

$$\times \frac{f_\pi \frac{m_B^2 - m_D^2}{f_D \frac{m_B^2 - m_\pi^2}} F_0^{BD}(m_\pi^2)}{F_0^{B\pi}(m_D^2)}$$

$$= \frac{1 - \frac{A_{1/2}}{\sqrt{2}A_{3/2}} \Big|_{D\pi} f_\pi \frac{m_B^2 - m_D^2}{f_D \frac{m_B^2 - m_\pi^2}} F_0^{BD}(m_\pi^2)}{\frac{1}{2} + \frac{A_{1/2}}{\sqrt{2}A_{3/2}} \Big|_{D\pi} f_\pi \frac{m_B^2 - m_\pi^2}{f_D \frac{m_B^2 - m_\pi^2}} F_0^{B\pi}(m_D^2)}. \quad (22)$$

Likewise, for the $\bar{B} \rightarrow D^* \pi$ decays

$$\left. \frac{a_2}{a_1} \right|_{D^* \pi} = (0.42 \pm 0.06) e^{i63^\circ} \times \frac{f_\pi A_0^{BD^*}(m_\pi^2)}{f_{D^*} F_1^{B\pi}(m_{D^*}^2)}$$

$$= \frac{1 - \frac{A_{1/2}}{\sqrt{2}A_{3/2}} \Big|_{D^* \pi} f_\pi A_0^{BD^*}(m_\pi^2)}{\frac{1}{2} + \frac{A_{1/2}}{\sqrt{2}A_{3/2}} \Big|_{D^* \pi} f_{D^*} F_1^{B\pi}(m_{D^*}^2)}. \quad (23)$$

With the form factors given in various models, we are ready to extract a_1 and a_2 from the experimental data. The results are shown in Table III and the parameter a_2 falls into the range of $|a_2(D\pi)| \sim 0.35 - 0.60$ and $|a_2(D^* \pi)| \sim 0.25 - 0.50$. Note that the phases of a_2/a_1 , 59° for the $D\pi$ system and 63° for $D^* \pi$, are slightly different from that given in [17] based on the preliminary Belle and CLEO data. We see that although $|a_2(D\pi)|$ and $|a_2(D^* \pi)|$ agree to within one standard deviation, there is a tendency that the former is slightly larger than the latter. Hence, nonfactorizable effects could be process dependent, recalling that the experimental value for $B \rightarrow J/\psi K$ is $|a_2(J/\psi K)| = 0.26 \pm 0.02$ [7].

Ideally, the parameters a_1 and a_2 will be more precisely determined if the topologies \mathcal{T} , \mathcal{C} , and \mathcal{E} can be individually extracted from experiment. Indeed, this is the case for charm decays where \mathcal{T} , \mathcal{C} , and \mathcal{E} can be determined from $D \rightarrow \bar{K} \pi$, $D \rightarrow \bar{K} \eta$, and $D \rightarrow \bar{K} \eta'$ decays based on SU(3) flavor symmetry and it is found that $|\mathcal{T}|:|\mathcal{C}|:|\mathcal{E}| \sim 1.7:1.3:1.0$ [18]. Hence, the W -exchange amplitude that receives short-distance and long-distance contributions is not negligible at all in charm decay.² Unfortunately, one cannot extract those three quark-diagram amplitudes for B decays since the decay amplitudes of $\bar{B}^0 \rightarrow D^0(\eta, \eta')$ are proportional to $(\mathcal{C} + \mathcal{E})$, while $D^0 \pi^0$ is governed by $(-\mathcal{C} + \mathcal{E})$ [see Eq. (4)]. Therefore, the quark-diagram amplitudes \mathcal{C} and \mathcal{E} cannot be disentangled. Nevertheless, an accurate measurement of $D^0(\eta, \eta')$ will enable us to test the importance of the W exchange in $\bar{B} \rightarrow D\pi$ decays.

In principle, a_1 can be determined in a model-independent way from the measurement of the ratio of the decay rate of

²From [18] one can deduce that $x a_2/a_1 = \mathcal{C}/\mathcal{T} = (0.73 \pm 0.05) \exp(i152^\circ)$ for $D \rightarrow PP$ decays without making any assumption on the W exchange, to be compared with the value $(1.05 \pm 0.05) \exp(i149^\circ)$ obtained in [17] by neglecting the W exchange.

TABLE IV. Predicted branching ratios (in units of 10^{-4}) of $\bar{B}^0 \rightarrow D^{(*)0}(\eta, \eta')$ in various form-factor models by assuming $a_2(D^{(*)} \eta^{(\prime)}) = a_2(D^{(*)} \pi)$.

Decay mode	NRSX	LF	MS	NS	Experiment [1]
$\bar{B}^0 \rightarrow D^0 \eta$	1.43 ± 0.24	1.69 ± 0.28	1.69 ± 0.28	1.69 ± 0.28	$1.4^{+0.5}_{-0.4} \pm 0.3$
$\bar{B}^0 \rightarrow D^{*0} \eta$	1.20 ± 0.29	1.41 ± 0.35	1.41 ± 0.35	1.41 ± 0.35	$2.0^{+0.9}_{-0.8} \pm 0.4$
$\bar{B}^0 \rightarrow D^0 \eta'$	0.89 ± 0.15	1.05 ± 0.18	1.05 ± 0.18	1.05 ± 0.18	
$\bar{B}^0 \rightarrow D^{*0} \eta'$	0.72 ± 0.18	0.85 ± 0.21	0.85 ± 0.21	0.85 ± 0.21	

color-allowed modes to the differential semileptonic distribution at the appropriate q^2 [19]:

$$S_h^{(*)} \equiv \frac{\mathcal{B}(\bar{B}^0 \rightarrow D^{(*)+} h^-)}{d\mathcal{B}(\bar{B}^0 \rightarrow D^{(*)+} l^- \bar{\nu})/dq^2|_{q^2=m_h^2}} = 6\pi^2 a_1^2 f_h^2 |V_{ij}|^2 Y_h^{(*)}, \quad (24)$$

where V_{ij} is the relevant CKM matrix element and the expression of $Y_h^{(*)}$ can be found in [8]. Since the ratio $S_h^{(*)}$ is independent of V_{cb} and form factors, its experimental measurement can be utilized to fix a_1 in a model-independent manner, provided that $Y_h^{(*)}$ is also independent of form-factor models. Based on the earlier CLEO data, it is found that $a_1(D\pi) = 0.93 \pm 0.10$ and $a_1(D^*\pi) = 1.09 \pm 0.07$ [7]. Needless to say, the forthcoming measurements from BaBar, Belle, and CLEO will enable us to extract the model independent a_1 more precisely. Note that QCD factorization predicts $a_1(D^{(*)}\pi) \approx 1.05$ in the heavy quark limit [20].

Assuming $a_2(D^{(*)} \eta^{(\prime)}) = a_2(D^{(*)} \pi)$ we see from Table IV that the predicted branching ratios of $\bar{B}^0 \rightarrow D^{(*)0} \eta$ are consistent with experiment. Note that the predicted rates of $D^{(*)0}(\eta, \eta')$ are the same for LF, MS, and NS models since $a_2(D\pi)F_0^{B\pi}(m_D^2)$ is model independent [see Eq. (4)] and the form factors $F_0^{B\eta_0}$ and $F_0^{B\eta_8}$ are assumed to be proportional to $F_0^{B\pi}$ in these models.

B. Effective parameters a_1^{eff} and a_2^{eff}

Thus far we have assumed that quark-diagram topologies include all strong-interaction effects including FSIs. It is equally good to take a different point of view on the quark-diagram topologies, namely, their original forms do not include FSIs from the outset. In this case, there is no relative strong phase between the isospin amplitudes $A_{1/2}$ and $A_{3/2}$ given by Eq. (17). Next, one puts isospin phase shifts into Eq. (16) to get

$$\begin{aligned} A(\bar{B}^0 \rightarrow D^+ \pi^-)_{\text{FSI}} &= \sqrt{\frac{2}{3}} A_{1/2} e^{i\delta_{1/2}} + \sqrt{\frac{1}{3}} A_{3/2} e^{i\delta_{3/2}}, \\ A(\bar{B}^0 \rightarrow D^0 \pi^0)_{\text{FSI}} &= \sqrt{\frac{1}{3}} A_{1/2} e^{i\delta_{1/2}} - \sqrt{\frac{2}{3}} A_{3/2} e^{i\delta_{3/2}}, \\ A(B^- \rightarrow D^0 \pi^-)_{\text{FSI}} &= \sqrt{3} A_{3/2} e^{i\delta_{3/2}}, \end{aligned} \quad (25)$$

where the subscript ‘‘FSI’’ indicates that the physical amplitudes take into account the effects of FSIs. This is motivated by comparing the experimental results with the calculated isospin amplitudes under the factorization approximation. Neglecting inelastic scattering, one can then extract the coefficients $a_{1,2}^{\text{eff}}$ from a comparison of the measured and calculated isospin amplitudes [8]. It is straightforward to show that

$$\begin{aligned} A(\bar{B}^0 \rightarrow D^0 \pi^0)_{\text{FSI}} &= A(\bar{B}^0 \rightarrow D^0 \pi^0) \\ &\quad + \frac{2\mathcal{T} - \mathcal{C} + 3\mathcal{E}}{3\sqrt{2}} (e^{i(\delta_{1/2} - \delta_{3/2})} - 1), \\ A(\bar{B}^0 \rightarrow D^+ \pi^-)_{\text{FSI}} &= A(\bar{B}^0 \rightarrow D^+ \pi^-) \\ &\quad + \frac{2\mathcal{T} - \mathcal{C} + 3\mathcal{E}}{3} (e^{i(\delta_{1/2} - \delta_{3/2})} - 1), \end{aligned} \quad (26)$$

where we have dropped the overall phase $e^{i\delta_{3/2}}$. The quark-diagram amplitudes \mathcal{T} , \mathcal{C} , \mathcal{E} in Eq. (26) have the same expressions as before except that $a_{1,2}$ in Eqs. (4) and (21) are replaced by the real parameters $a_{1,2}^{\text{eff}}$. The latter do not contain FSI effects and are defined for $\delta_{1/2} = \delta_{3/2} = 0$ [16].³ In other words, the parameters $a_{1,2}^{\text{eff}}$ are defined when FSIs are not imposed to the topological quark diagram amplitudes.

The isospin phase difference in Eq. (26) is 29° for both $\bar{B} \rightarrow D\pi$ and $\bar{B} \rightarrow D^*\pi$. It is easily seen that $a_2^{\text{eff}}/a_1^{\text{eff}}$ is determined from the second line of Eqs. (22) and (23) but without a phase for the ratio $A_{1/2}/(\sqrt{2}A_{3/2})$. For example, $a_2^{\text{eff}}/a_1^{\text{eff}}$ for $\bar{B} \rightarrow D\pi$ is given by

$$\frac{a_2^{\text{eff}}}{a_1^{\text{eff}}}\bigg|_{D\pi} = \frac{1 - \left| \frac{A_{1/2}}{\sqrt{2}A_{3/2}} \right|_{D\pi} f_\pi m_B^2 - m_D^2 F_0^{BD}(m_\pi^2)}{\frac{1}{2} + \left| \frac{A_{1/2}}{\sqrt{2}A_{3/2}} \right|_{D\pi} f_D m_B^2 - m_\pi^2 F_0^{B\pi}(m_D^2)}. \quad (27)$$

³The distinction of hard and soft FSI phases in principle cannot be done in a systematical way. For example, a sizable ‘‘hard’’ strong-interaction phase for a_2 in $B \rightarrow \pi\pi$ decay is calculable in the QCD factorization approach. However, a_2 is not computable for $\bar{B} \rightarrow D\pi$ and hence its strong phase is most likely soft.

TABLE V. Extraction of the parameters a_1^{eff} and a_2^{eff} from the measured $B \rightarrow D^{(*)} \pi$ rates. Note that $a_2^{\text{eff}}(D\pi)$ and $a_2^{\text{eff}}(D^* \pi)$ should be multiplied by a factor of (200 MeV/ f_D) and (230 MeV/ f_{D^*}), respectively.

Model	$a_1^{\text{eff}}(D\pi)$	$a_2^{\text{eff}}(D\pi)$	$a_2^{\text{eff}}(D\pi)/a_1^{\text{eff}}(D\pi)$	$a_1^{\text{eff}}(D^* \pi)$	$a_2^{\text{eff}}(D^* \pi)$	$a_2^{\text{eff}}(D^* \pi)/a_1^{\text{eff}}(D^* \pi)$
NRSX	0.88 ± 0.06	0.23 ± 0.08	0.26 ± 0.09	0.97 ± 0.04	0.16 ± 0.04	0.17 ± 0.04
LF	0.87 ± 0.06	0.25 ± 0.09	0.29 ± 0.10	0.83 ± 0.03	0.18 ± 0.05	0.22 ± 0.05
MS	0.91 ± 0.06	0.27 ± 0.10	0.30 ± 0.10	0.87 ± 0.03	0.20 ± 0.05	0.23 ± 0.06
NS	0.96 ± 0.06	0.32 ± 0.12	0.34 ± 0.11	0.94 ± 0.04	0.22 ± 0.06	0.24 ± 0.06

The results are shown in Table V. Obviously $a_2^{\text{eff}}/a_1^{\text{eff}}$ and a_2^{eff} are smaller than the previous solution.

C. Comparison

We are ready to compare the above two different types of approaches. In the type-I solution, $D^{(*)0} \pi^0$ rates are accommodated because of an enhanced $|a_2(D^{(*)0} \pi)|$. The branching ratio of $D^{(*)0} \pi^-$ is not overestimated owing to a relative strong phase between a_1 and a_2 . In the type-II solution, although a_2^{eff} is smaller than the magnitude of a_2 , the $D^{(*)0} \pi^0$ states gain a feedback from $D^{(*)+} \pi^-$ via FSIs.⁴ More precisely, elastic FSIs will enhance the decay rate of $D^0 \pi^0$ by a factor of about 3 and suppress $D^+ \pi^-$ slightly.

It has been realized that the isospin analysis proves to be useful only if a few channels are open as the case of two-body nonleptonic decays of kaons and hyperons. The isospin phases there (or decay amplitude phases) are related to strong-interaction eigenphases (for a recent discussion, see [22]). For example, one can identify the isospin phase shift in $K \rightarrow \pi\pi$ with the measured $\pi\pi$ strong-interaction phase at the energy $\sqrt{s} = m_K$. However, when there are many channels open and some channels coupled, as in D and especially B decays, the decay phase is no longer the same as the eigenphase in the S matrix. Indeed, the S matrix in general contains a parameter describing inelasticity. Consider the decay $\bar{B}^0 \rightarrow D^+ \pi^-$ as an example. The state $D^+ \pi^-$ couples to not only $D^0 \pi^0$, but also $D^0 \eta$, $D^0 \eta'$, $D\pi\pi\pi$ channels, . . . , etc. It has been argued that in the heavy quark limit the B decay is dominated by multiparticle inelastic rescattering [23]. As a consequence, even if elastic $D^{(*)} \pi$ scattering is measured at energies $\sqrt{s} = m_B$, the isospin phases appearing in Eq. (16) or (25) cannot be identified with the measured strong phases. Moreover, the isospin amplitudes are not conserved by inelastic FSIs. Therefore, the isospin analysis presented before should be regarded as an intermediate step for describing physical decay amplitudes.

Nevertheless, the isospin decomposition of $\bar{B} \rightarrow D\pi$ amplitudes in Eq. (16) or (25) is still valid. The isospin analysis is useful in some aspects. First, it provides an independent check on the relative phases among three decay

amplitudes. Second, the deviation of $|A_{1/2}/(\sqrt{2}A_{3/2})|$ from unity measures the degree of departure from the heavy quark limit [17]. Third, the deviation of a_2 from a_2^{eff} characterizes the importance of (soft) FSI contributions to the color-suppressed quark diagram, recalling that $a_{1,2}^{\text{eff}}$ are defined for the topologies without FSIs. This point will be elucidated more below.

As stressed in [15], the topological quark graphs are meant to have all strong interactions included. Hence, they are *not* Feynman graphs. For example, the genuine W -exchange topology in $\bar{B} \rightarrow D\pi$ decay consists of not only the short-distance W -exchange diagram but also the rescattering graph in which $\bar{B}^0 \rightarrow D^+ \pi^-$ is followed by the strong interaction process $(D^+ \pi^-)_{I=1/2} \rightarrow$ scalar resonances $\rightarrow D^0 \pi^0$. Likewise, the process with inelastic rescattering from the leading \mathcal{T} amplitude into $D^0 \pi^0$ via quark exchange has the same topology as the color-suppressed tree diagram \mathcal{C} [24]. Therefore, color-suppressed tree and W -exchange topologies receive short-distance and long-distance contributions.

From Tables III and V we see that $R_{D\pi} = |a_2(D\pi)/a_2^{\text{eff}}(D\pi)| \approx 1.75$ and $R_{D^*\pi} = |a_2(D^*\pi)/a_2^{\text{eff}}(D^*\pi)| \approx 1.95$. The corresponding quantities in $D \rightarrow \bar{K}\pi$ decays are $R_{\bar{K}\pi} \approx 2.0$ and $R_{\bar{K}^*\pi} \approx 1.7$, respectively. Therefore, although the relative phase 59° (63°) between $B^0 \rightarrow D^{(*)0} \pi^0$ and $B^0 \rightarrow D^{(*)+} \pi^-$ is significantly reduced from the phase 150° between $D^0 \rightarrow \bar{K}^{0(*)} \pi^0$ and $D^0 \rightarrow K^{-(*)} \pi^+$ [18], the ratio R does not decrease sizably from charm to bottom and, in contrast, it increases for the VP case. It is thus anticipated that in both $D \rightarrow \bar{K}\pi$ and $\bar{B} \rightarrow D\pi$ decays, the soft FSI contributions to the color-suppressed topology \mathcal{C} are dominated by inelastic rescattering [23].⁵ Since η and ω are isospin singlets, the conventional isospin analysis of FSIs is no longer applicable to the final states involving η or ω . The fact that the predicted $\bar{B}^0 \rightarrow D^{(*)0} \eta$ rates based on the assumption $a_2(D^{(*)} \eta) = a_2(D^{(*)} \pi)$ are consistent with experiment (see Table IV) supports the notion that FSIs in B decay are indeed highly inelastic.

⁴Recently, it has been suggested in [21] that quasielastic scatterings of $D^{(*)} P \rightarrow D^{(*)} P$ and $DV \rightarrow DV$, for example, $DP = D^+ \pi^-, D^0 \pi^0, D^0 \eta_8, D_s^+ K^-$, can explain the enhancement of not only $D^0 \pi^0$ but also $D^0 \eta$ via inelastic rescattering from the class-I mode $\bar{B}^0 \rightarrow D^+ \pi^-$.

⁵The quark diagram W exchange in $D \rightarrow \bar{P}P$ decays and its phase relative to the topological amplitude \mathcal{T} are dominated by nearby resonances in the charm mass region [25], as shown explicitly in [26].

IV. DISCUSSION AND CONCLUSION

Beyond the phenomenological level, it is desirable to have a theoretical estimate of $a_2(D\pi)$. Unfortunately, contrary to the parameter $a_1(D\pi)$, $a_2(D\pi)$ is not calculable in the QCD factorization approach owing to the presence of infrared divergence caused by the gluon exchange between the emitted D^0 meson and the $(\bar{B}^0\pi^0)$ system. In other words, the nonfactorizable contribution to a_2 is dominated by nonperturbative effects. Nevertheless, a rough estimate of a_2 by treating the charmed meson as a light meson while keeping its highly asymmetric distribution amplitude yields $a_2(D\pi)\approx 0.25\exp(-i40^\circ)$ [20]. Evidently, large power corrections from long-distance FSI effects are needed to account for the discrepancy between theory and experiment for $a_2(D\pi)$. The rescattering contribution via quark exchange, $D^+\pi^-\rightarrow D^0\pi^0$, to the topology C in $\bar{B}^0\rightarrow D^0\pi^0$ has been estimated in [27] using the ρ trajectory Regge exchange. It was found that the additional contribution to $D^0\pi^0$ from rescattering is mainly imaginary: $a_2(D\pi)/a_2(D\pi)_{\text{without FSI}}=1+0.61\exp(73^\circ)$. This analysis suggests that the rescattering amplitude can bring a large phase to $a_2(D\pi)$ as expected.

In QCD factorization, $a_2(\pi\pi)$ or $a_2(K\pi)$ is found to be of order 0.20 with a small strong phase (see, e.g., [28]). The fact that the magnitude of $a_2(D\pi)$ is larger than the short-distance one, $a_2(K\pi)$ or $a_2^{\text{eff}}(D\pi)$, should not be surprising because the former includes all possible FSIs, while the latter is defined without long-distance FSIs. In other words, $a_2(D\pi)$ include many possible long-distance effects. In the language of isospin analysis, we see from Eq. (26) that

$$a_2(D\pi)=a_2^{\text{eff}}(D\pi)-\frac{2ha_1^{\text{eff}}(D\pi)-a_2^{\text{eff}}(D\pi)}{3}\times(e^{i(\delta_{1/2}-\delta_{3/2})}-1), \quad (28)$$

where we have neglected the W exchange and

$$h=\frac{f_\pi m_B^2-m_D^2}{f_D m_B^2-m_\pi^2}\frac{F_0^{BD}(m_\pi^2)}{F_0^{B\pi}(m_D^2)}. \quad (29)$$

It follows from Eq. (19) and Table V that $a_2(D\pi)/a_2^{\text{eff}}(D\pi)\approx 1.65\exp(56^\circ)$. It is worth remarking that $a_2(J/\psi K)$ in $B\rightarrow J/\psi K$ decay is calculable in QCD factorization; the theoretical result $|a_2(J/\psi K)|=0.19^{+0.14}_{-0.12}$ [29] is consistent with the data 0.26 ± 0.02 [7]. Hence it remains to understand why $|a_2(D\pi)|$ is larger than $|a_2(D^*\pi)|$ and $|a_2(J/\psi K)|$ or why (soft) final-state interaction effects are more important in $D\pi$, $D^*\pi$ than in $J/\psi K$ final states.

To conclude, the recent measurements of the color-suppressed modes $\bar{B}^0\rightarrow D^{(*)0}\pi^0$ imply non-vanishing relative FSI phases among various $\bar{B}\rightarrow D\pi$ decay amplitudes. Depending on whether or not FSIs are implemented in the topological quark-diagram amplitudes, two solutions for the parameters a_1 and a_2 are extracted from data using various form-factor models. It is found that a_2 is not universal: $|a_2(D\pi)|\sim 0.40-0.55$ and $|a_2(D^*\pi)|\sim 0.30-0.45$ with a relative phase of order 60° between a_1 and a_2 . If FSIs are not included in quark-diagram amplitudes from the outset, we have $a_2^{\text{eff}}(D\pi)\sim 0.23-0.32$, $a_2^{\text{eff}}(D^*\pi)\sim 0.16-0.22$. The large value of $|a_2(D\pi)|$ compared to $a_2^{\text{eff}}(D\pi)$ or naive expectation implies the importance of long-distance FSI contributions to color-suppressed internal W emission via final-state rescatterings of the color-allowed tree amplitude.

ACKNOWLEDGMENTS

We would like to thank Hsiang-nan Li, Alexey A. Petrov, Zhi-zhong Xing, and Kwei-Chou Yang for delighting discussions. We also wish to thank the Physics Department, Brookhaven National Laboratory for its hospitality. This work was supported in part by the National Science Council of R.O.C. under Grant No. NSC90-2112-M-001-047.

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