PHYSICAL REVIEW D, VOLUME 65, 093014

Muon anomalous magnetic moment in string inspired extended family models

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(Received 27 February 2001; published 15 May 2002)

We propose a standard model minimal extension with two lepton weak $SU(2)$ doublets and a scalar singlet to explain the deviation of the measured anomalous magnetic moment of the muon from the standard model expectation. This scheme can be naturally motivated in string inspired models such as E_6 and AdS conformal field theory.

DOI: 10.1103/PhysRevD.65.093014 PACS number(s): 13.40.Em, 12.60.Cn

The recent result of the $g-2$ Collaboration at BNL may be the overdue first direct signal of physics beyond the standard model (SM). The data seem to indicate a 2.6σ deviation from the theoretical expectation of the SM [1]:

$$a_\mu^{exp} - a_\mu^{SM} = 426 \pm 165 \times 10^{-11}. \tag{1}$$

Several logical possibilities have been considered [2] to explain the effect, such as supersymmetry [3], leptoquarks [4], muon substructure [5], technicolor [6], large extra dimensions [7], new interactions and fermions [8]; for a review see [2]. In this article we propose a minimal extension of the SM, which to date has not been discussed in this context.

It is sufficient to extend the second fermion family (extending the other families is optional but aesthetically appealing) by adding an extra pair of lepton doublets and a scalar singlet:

$$\begin{pmatrix} M \\ N \end{pmatrix}_L + \begin{pmatrix} M \\ N \end{pmatrix}_R, \quad S^0 \tag{2}$$

with no other states or extension of the gauge group necessary. There are several ways to motivate this choice and we will focus on two.

First if the standard model is embedded in an E_6 model, then the fermion families contain extra states. For the symmetry breaking chain $E_6 \to SO(10) \to SU(5) \to SU(3) \times SU(2) \times U(1)$, an E_6 family of fermions decomposes as

$$\begin{aligned} 27 &\to 16 + 10 + 1 \\ &\to (\overline{5} + 10 + 1) + (5 + \overline{5}) + 1 \\ &\to (\text{SM family}) + (3,1) + (\overline{3},1) + (1,2) + (1,\overline{2}) \\ &\quad + 2(1,1). \end{aligned} \tag{3}$$

The conjugate pair of doublets are just those required above. The extra singlet quarks and SM singlets can be made relatively heavy by proper choice of vacuum expectation values (VEVs) and coupling constants. Extra gauge bosons can also be sufficiently heavy that they have decoupled from the subsequent analysis. For other discussions of a_μ in the E_6 model see [9,10].

The second possibility is a non-supersymmetric (SUSY) model with extra lepton doublets based on either an Abelian or a non-Abelian orbifold AdS conformal field theory (CFT) type IIB string theory [11–14]. These models have gauge groups that are the product of $SU(d_i N)$ groups where the d_i are the dimensions of the irreducible representations (irreps) of the orbifolding group Γ and N can be chosen. Thus the gauge group G can be of the form $G = SU(3) \times SU(3) \times SU(3) \times G'$ (where G' can be either broken or ignored for our purposes). All the fermions reside in bifundamental representations of the gauge group and so there can be N_F $SU^3(3)$ families plus other states:

$$N_F[(3,\overline{3},1) + (1,3,\overline{3}) + (\overline{3},1,3)] + \cdots. \tag{4}$$

Note that these contain the same fermions one acquires from breaking E_6 to its maximal subgroup $SU^3(3)$ so we are again naturally led to

$$\begin{aligned} &N_F[(\text{SM family}) + (3,1) + (\overline{3},1) + (1,2) + (1,\overline{2}) \\ &+ 2(1,1)] + \cdots, \end{aligned} \tag{5}$$

which provides the necessary pair of extra lepton doublets. A typical length scale for orbifold AdS-CFT models is a few TeV, so fermions in this mass region or somewhat lighter fit easily into the scheme proposed here.

Scalar singlets also arise in both schemes.

The contribution of the new muon type heavy charged lepton M^- to a_μ is due to the Feynman diagram, Fig. 1.

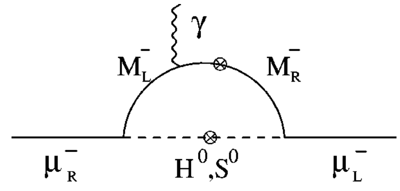

FIG. 1. Feynman graph contributing to the muon anomalous magnetic moment in the extended family model.

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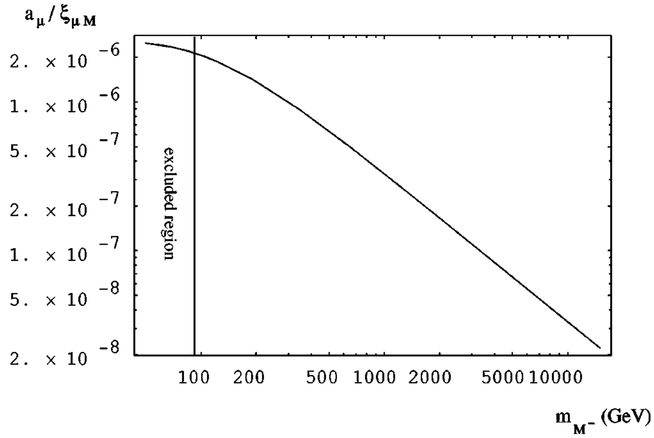

FIG. 2. Muon magnetic moment a_μ in terms of $\xi_{\mu M}$ as a function of the new muon type heavy charged lepton mass M^-.

Similar diagrams with SM gauge boson instead of Higgs-boson–scalar-singlet exchange have been considered, but cannot provide a correction to a_μ to allow agreement with experiment [10]. The flavor changing neutral current (FCNC) contribution with the τ (instead of M^-) exchange is forbidden in the standard model, since the mass matrix is directly proportional to the Yukawa coupling matrix. The viable extended model with a non-SM Higgs doublet has been discussed in [15]. In our case the proportionality is broken by assigning a bare mass to M^-.

The contribution to $a_\mu \equiv (g_\mu - 2)/2$ is given by [16]

$$a_\mu = \frac{\xi_{\mu M}}{16\pi^2}\int_0^1 \frac{x^2(1-x)+x^2\dfrac{m_M}{m_\mu}}{x(x-1)+x\dfrac{m_M^2}{m_\mu^2}+(1-x)\dfrac{m_1^2}{m_\mu^2}}dx. \tag{6}$$

Here $\xi_{\mu M}$ refers to the product

$$\xi_{\mu M} = h_{\mu M} h'_{\mu M} O_{H^0 1} O_{S^0 1}, \tag{7}$$

where $h_{\mu M}$, $h'_{\mu M}$ are the couplings to the real fields H^0 and S^0, respectively, and $O_{H^0 1} O_{S^0 1}$ denotes the product of orthogonal doublet-singlet mixing matrix elements with the light scalar eigenstate ($m_1 \ll m_2$ has been assumed).

In Fig. 2, $a_\mu/\xi_{\mu M}$ is plotted as a function of the heavy charged lepton mass m_M. The plot shows the entire perturbative region, i.e., $\xi_{\mu M} \lesssim 0.2$, so that $m_M \lesssim 15$ TeV. With $\xi_{\mu M} \gtrsim 0.01$–0.1, assuming a physical Higgs boson mass (light scalar mass eigenstate) of $m_1 = 115$ GeV and m_M of order 1–10 TeV, the anomaly can be explained. (The result is insensitive to the Higgs boson mass when $m_M \gg m_1$.) This is well above the recent limits of heavy charged leptons, $m_M \gtrsim 95$ GeV [17], but lies in the range to be explored at future accelerators such as the CERN Large Hadron Collider (LHC) [18].

Since the $\mu^- - M^- - H^0$ coupling $h_{\mu M}$ induces off-diagonal terms in the muon mass matrix, the model may have interesting implications for weak universality, which will be discussed in the following. The masses of the charged leptons are obtained by diagonalizing the mass term

$$\overline{(\mu_R^C, \mu_L, M_R^C, M_L)}\mathcal{M}\begin{pmatrix}\mu_R\\ \mu_L^C\\ M_R\\ M_L^C\end{pmatrix} = \overline{(\mu_R^C, \mu_L, M_R^C, M_L)}\begin{pmatrix}0 & gv & 0 & hv\\ gv & 0 & h'v' & 0\\ 0 & h'v' & 0 & M\\ hv & 0 & M & 0\end{pmatrix} \times \begin{pmatrix}\mu_R\\ \mu_L^C\\ M_R\\ M_L^C\end{pmatrix} \tag{8}$$

via

$$V^\dagger \mathcal{M} V = \mathrm{diag}(m_\mu, m_\mu, m_M, m_M). \tag{9}$$

Here v, v' are the VEVs of the Higgs doublet and singlet, respectively, and g is the muon Yukawa coupling. A stringent bound on the mixing matrix elements V_{11}, V_{12} determining the admixture of the heavy state in the muon flavor eigenstate is obtained from its contribution to the pion decay. The agreement of the measured pion decay rate ratio,

$$R_\pi = \frac{\Gamma(\pi^+ \to e^+ \nu)}{\Gamma(\pi^+ \to \mu^+ \nu)} = (1.235 \pm 0.004) \times 10^{-4}, \tag{10}$$

with the SM value $R_\pi(SM) = (1.230 \pm 0.008) \times 10^{-4}$ implies $G_N^\mu \lesssim 4 \times 10^{-3} G_F$ for any new physics contribution G_N, i.e., $\sqrt{V_{11}^2 + V_{12}^2} < 4 \times 10^{-3}$ (see, e.g., [19]). This constrains the off-diagonal entry hv to be small, while the product of coupling constants and mixing matrices $\xi_{\mu M}$ in Eq. (7) has to be large to yield the correct value for the anomalous magnetic muon moment.

Eigenvalues $m_\mu \simeq gv = 100$ MeV and $m_M = 1$–10 TeV as well as the right anomalous contribution to the muon magnetic moment are obtained by assigning, e.g., $gv \simeq 100$ MeV, $hv \simeq 3-30$ GeV, $v' < vh/h'$, $h' \simeq 2$, and $M \simeq m_M = 1$–10 TeV and maximal scalar mixing $O_{H_0 1} O_{S^0 1} \simeq 0.5$. The mixing matrix elements are $V_{11} \simeq V_{12} \simeq 2 \times 10^{-3}$ and $V_{21} \simeq V_{22} \ll 10^{-3}$. They are close to but compatible with the bounds obtained from the pion decay rate ratio.

The one-loop correction to the muon mass is obtained by removing the photon line from the diagram Fig. 1, and can be estimated to be [2]

$$m_{\mu}^{loop} \simeq \frac{1}{16\pi^2} \cdot \xi_{\mu M} \cdot m_M \cdot \left(\frac{m_M^2 \cdot ln\, m_M^2 - m_1^2 \cdot ln\, m_1^2}{m_M^2 - m_1^2} - \frac{m_M^2 \cdot ln\, m_M^2 - m_2^2 \cdot ln\, m_2^2}{m_M^2 - m_2^2} \right), \tag{11}$$

which can be small as long as $m_2 \ll m_M$, i.e., $m_2 \lesssim 300\text{–}500$ GeV for $m_M \simeq 1\text{–}10$ TeV.

To conclude, we have shown an extended muon family is sufficient to provide the contribution needed to bring theory back in line with the present $g-2$ data for the muon. The proposed model is motivated from more fundamental theory and the results are simply obtained. The new particles required by the model are potentially within the reach of the LHC and large violations of weak universality are predicted.

Note added in proof. After submission of this article a new calculation of the SM contribution to a_μ reduced the non-SM contribution to $a_\mu^{\exp} - a_\mu^{SM} = (25 \pm 16) \times 10^{10}$, which reduces the evidence for non-SM physics to 1.6σ, but also requires less fine-tuning to be accommodated in the present model.

We would like to thank L. Clavelli, M. Sher and T. J. Weiler for useful discussions and correspondence. This work was supported in part by the DOE grant no. DE-FG05-85ER40226.

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