

Strong and weak phases from time-dependent measurements of $B \rightarrow \pi\pi$

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Time dependence in $B^0(t) \rightarrow \pi^+ \pi^-$ and $\bar{B}^0(t) \rightarrow \pi^+ \pi^-$ is utilized to obtain a maximal set of information on strong and weak phases. One can thereby check theoretical predictions of a small strong phase δ between penguin and tree amplitudes. A discrete ambiguity between $\delta \approx 0$ and $\delta \approx \pi$ may be resolved by comparing the observed charge-averaged branching ratio predicted for the tree amplitude alone, using measurements of $B \rightarrow \pi l \nu$ and factorization, or by direct comparison of Cabibbo-Kobayashi-Maskawa matrix parameters with those determined by other means. It is found that with 150 fb^{-1} from BaBar and Belle, this ambiguity will be resolvable if no direct CP violation is found. In the presence of direct CP violation, the discrete ambiguity between δ and $\pi - \delta$ becomes less important, vanishing altogether as $|\delta| \rightarrow \pi/2$. The role of measurements involving the lifetime difference between neutral B eigenstates is mentioned briefly.

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I. INTRODUCTION

The observation of CP violation in decays of B mesons to J/ψ and neutral kaons [1,2] has inaugurated a new era in the study of matter-antimatter asymmetries. Previously, such asymmetries had been manifested only in the decays of neutral kaons and in the baryon asymmetry of the Universe. CP violation in B and neutral kaon decays is described satisfactorily in terms of phases in the Cabibbo-Kobayashi-Maskawa (CKM) matrix, but the baryon asymmetry of the Universe apparently requires sources of CP violation beyond the CKM phases. There is thus great interest in testing the self-consistency of the CKM description through a variety of processes.

One key test of the CKM picture involves the decays $B^0 \rightarrow \pi^+ \pi^-$. The time dependence of $B^0|_{\text{initial}} \rightarrow \pi^+ \pi^-$ and $\bar{B}^0|_{\text{initial}} \rightarrow \pi^+ \pi^-$ involves quantities $S_{\pi\pi}$ and $C_{\pi\pi}$ which are, respectively, coefficients of terms involving $\sin \Delta mt$ and $\cos \Delta mt$, and which depend in different ways on strong and weak phases. The BaBar Collaboration reported the first measurement of these quantities [3], recently updated to $S_{\pi\pi} = -0.05 \pm 0.37 \pm 0.07$ and $C_{\pi\pi} = -0.02 \pm 0.29 \pm 0.07$ [4]. The Belle Collaboration reports $S_{\pi\pi} = -1.21^{+0.38+0.16}_{-0.27-0.13}$ and $C_{\pi\pi} = -0.94^{+0.31}_{-0.25} \pm 0.09$ [2], using BaBar's sign convention for $C_{\pi\pi}$. The averages are $S_{\pi\pi} = -0.66 \pm 0.26$ and $C_{\pi\pi} = -0.49 \pm 0.21$.

Both model-independent considerations [5,6] and explicit calculations in QCD-improved factorization [7] indicate that a crude measurement of $S_{\pi\pi}$ around zero implies a significant restriction on CKM parameters if the strong phase difference δ between two amplitudes contributing to $B^0 \rightarrow \pi^+ \pi^-$ is small ($\delta \approx 10^\circ$ in [7]; see, however, [8].) The quantity $C_{\pi\pi}$ provides information on δ if the phase and $C_{\pi\pi}$ are both near zero, but a discrete ambiguity allows the phase to be near π instead.

In the present paper we reexamine the decays $B^0 \rightarrow \pi^+ \pi^-$ to extract the maximum amount of information

directly from data rather than relying on theoretical calculations of strong phases. We find that if $\sin \delta$ is small one can resolve a discrete ambiguity between $\delta \approx 0$ and $\delta \approx \pi$ by comparing the measured branching ratio of $B^0 \rightarrow \pi^+ \pi^-$ (averaged over B^0 and \bar{B}^0) with that predicted in the absence of the penguin amplitude. The latter can be obtained using information on the semileptonic process $B \rightarrow \pi l \nu$ assuming factorization for color-favored processes, which appears to hold well under general circumstances [9].

We find that with data foreseen within the next two years it should be possible to reduce theoretical and experimental errors to the level that a clear-cut choice can be made between the theoretically-favored prediction of small δ and the possibility of $\delta \approx \pi$, assuming that the parameter $C_{\pi\pi}$ describing direct CP violation in $B^0 \rightarrow \pi^+ \pi^-$ remains consistent with zero. If $C_{\pi\pi} \sim \sin \delta$ is found to be nonzero, direct CP violation will have been demonstrated in B decays, a significant achievement in itself. The sign of $C_{\pi\pi}$ will then determine the sign of δ . While the discrete ambiguity between δ and $\pi - \delta$ then becomes harder to resolve, its effect on CKM parameters becomes less important.

We recall notation for $B^0 \rightarrow \pi^+ \pi^-$ decays in Sec. II. The dependence of $S_{\pi\pi}$ and $C_{\pi\pi}$ on weak and strong phases is exhibited in Sec. III. It is seen that when $|C_{\pi\pi}|$ is maximal, there is little effect of any discrete ambiguity, since the strong phase δ is close to $\pm \pi/2$, while when $C_{\pi\pi} \approx 0$ the discrete ambiguity between $\delta \approx 0$ and $\delta \approx \pi$ results in very different inferred weak phases. The use of the flavor-averaged $B^0 \rightarrow \pi^+ \pi^-$ branching ratio to resolve this ambiguity is discussed in Sec. IV, while the CKM parameter restrictions implied by the observed $S_{\pi\pi}$ range are compared in Sec. V for $\delta = 0$ and $\delta = \pi$.

One more observable, which we call $D_{\pi\pi}$, obeys $S_{\pi\pi}^2 + C_{\pi\pi}^2 + D_{\pi\pi}^2 = 1$, so its magnitude is fixed by $S_{\pi\pi}$ and $C_{\pi\pi}$, but its sign provides new information. In principle, it is measurable in the presence of a detectable width difference between neutral B meson mass eigenstates, as is shown in Sec.

VI. However, we find that the sign of $D_{\pi\pi}$ is always negative for the allowed range of CKM parameters, and does not help to resolve the discrete ambiguity. A positive value of $D_{\pi\pi}$ would signify new physics. We conclude in Sec. VII.

II. NOTATION

We use the same notation as in Ref. [5], to which the reader is referred for details. We define T to be a color-favored tree amplitude in $B^0 \rightarrow \pi^+ \pi^-$ and P to be a penguin amplitude [10]. Using standard definitions of weak phases (see, e.g., [11]) $\alpha = \phi_2$, $\beta = \phi_1$, and $\gamma = \phi_3$, the decay amplitudes to $\pi^+ \pi^-$ for B^0 and \bar{B}^0 are

$$\begin{aligned} A(B^0 \rightarrow \pi^+ \pi^-) &= -(|T|e^{i\delta_T}e^{i\gamma} + |P|e^{i\delta_P}), \\ A(\bar{B}^0 \rightarrow \pi^+ \pi^-) &= -(|T|e^{i\delta_T}e^{-i\gamma} + |P|e^{i\delta_P}), \end{aligned} \quad (1)$$

where δ_T and δ_P are strong phases of the tree and penguin amplitudes, and $\delta \equiv \delta_P - \delta_T$. Our convention will be to take $-\pi \leq \delta \leq \pi$.

The coefficients of $\sin \Delta m_{dt}$ and $\cos \Delta m_{dt}$ measured in time-dependent CP asymmetries of $\pi^+ \pi^-$ states produced in asymmetric e^+e^- collisions at the $Y(4S)$ are [12]

$$S_{\pi\pi} \equiv \frac{2 \operatorname{Im}(\lambda_{\pi\pi})}{1 + |\lambda_{\pi\pi}|^2}, \quad C_{\pi\pi} \equiv \frac{1 - |\lambda_{\pi\pi}|^2}{1 + |\lambda_{\pi\pi}|^2}, \quad (2)$$

where

$$\lambda_{\pi\pi} \equiv e^{-2i\beta} \frac{A(\bar{B}^0 \rightarrow \pi^+ \pi^-)}{A(B^0 \rightarrow \pi^+ \pi^-)}. \quad (3)$$

In addition we may define the quantity

$$D_{\pi\pi} \equiv \frac{2 \operatorname{Re}(\lambda_{\pi\pi})}{1 + |\lambda_{\pi\pi}|^2}, \quad (4)$$

for which it is easily seen that

$$\begin{aligned} S_{\pi\pi}^2 + C_{\pi\pi}^2 + D_{\pi\pi}^2 &= 1, \\ \text{implying } S_{\pi\pi}^2 + C_{\pi\pi}^2 &\leq 1. \end{aligned} \quad (5)$$

The significance of $D_{\pi\pi}$ will be discussed in Sec. VI.

When $\delta=0$ or π the quantity $\lambda_{\pi\pi}$ becomes a pure phase:

$$\lambda_{\pi\pi} = e^{2i\alpha_{\text{eff}}}, \quad \alpha_{\text{eff}} = \alpha + \Delta\alpha, \quad (6)$$

$$\Delta\alpha = \begin{cases} \arctan \frac{|P/T| \sin \gamma}{1 + |P/T| \cos \gamma} & (\delta=0), \\ -\arctan \frac{|P/T| \sin \gamma}{1 - |P/T| \cos \gamma} & (\delta=\pi). \end{cases} \quad (7)$$

In such cases $S_{\pi\pi} = \sin(2\alpha_{\text{eff}})$, $D_{\pi\pi} = \cos(2\alpha_{\text{eff}})$.

The expressions (1) employ the phase convention in which top quarks are integrated out in the short-distance effective Hamiltonian and the unitarity relation $V_{ub}^* V_{ud}$

+ $V_{cb}^* V_{cd} = -V_{tb}^* V_{td}$ is used, with the $V_{ub}^* V_{ud}$ piece of the penguin operator included in the tree amplitude [5]. Using these expressions and substituting $\alpha = \pi - \beta - \gamma$, we then may write

$$\lambda_{\pi\pi} = e^{2i\alpha} \left(\frac{1 + |P/T| e^{i\delta} e^{i\gamma}}{1 + |P/T| e^{i\delta} e^{-i\gamma}} \right). \quad (8)$$

The consequences of assuming δ small, as predicted in Ref. [7], were explored in Refs. [5,6]. In the former, it was shown that even an earlier crude measurement [3] of $S_{\pi\pi}$, taken at 1σ , drastically reduced the allowed CKM parameter space. In the latter, where a slightly different convention for penguin amplitudes was used, it was shown how to use $S_{\pi\pi}$ and $C_{\pi\pi}$ to determine weak and strong phases.

One needs a value of $|P/T|$ to apply these expressions to data. In Refs. [5] and [6] $|P|$ was estimated using experimental data on $B^+ \rightarrow K^0 \pi^+$ (a process dominated by the penguin amplitude aside from small annihilation contributions) and flavor SU(3) including SU(3) symmetry breaking, while $|T|$ was estimated using factorization and data on $B \rightarrow \pi l \nu$. We shall use the result of Ref. [5], $|P/T| = 0.276 \pm 0.064$. Ref. [7] found 0.285 ± 0.076 , which included an estimate of annihilation, and Ref. [6] obtained 0.26 ± 0.08 , based on a different phase convention for the penguin amplitude, without including SU(3) breaking effects. The individual amplitudes of Ref. [5], in a convention in which their square gives a B^0 branching ratio in units of 10^{-6} , are $|T| = 2.7 \pm 0.6$ and $|P| = 0.74 \pm 0.05$. We shall make use of them in Sec. IV.

It is most convenient to express $S_{\pi\pi}$, $C_{\pi\pi}$, and $D_{\pi\pi}$ in terms of α , β , and δ , using $\gamma = \pi - \alpha - \beta$, since when $P=0$ one has $S_{\pi\pi} = \sin 2\alpha$ and $D_{\pi\pi} = \cos 2\alpha$. The value of β is fairly well known as a result of the recent measurements by BaBar [1] and Belle [2]: $\sin 2\beta = 0.78 \pm 0.08$, $\beta = (26 \pm 4)^\circ$. Defining

$$\bar{B}(B^0 \rightarrow \pi^+ \pi^-) \equiv [B(B^0 \rightarrow \pi^+ \pi^-) + B(\bar{B}^0 \rightarrow \pi^+ \pi^-)]/2, \quad (9)$$

$$\begin{aligned} R_{\pi\pi} &\equiv \frac{\bar{B}(B^0 \rightarrow \pi^+ \pi^-)}{\bar{B}(B^0 \rightarrow \pi^+ \pi^-)|_{\text{tree}}} \\ &= 1 - 2|P/T| \cos \delta \cos(\alpha + \beta) + |P/T|^2, \end{aligned} \quad (10)$$

explicit expressions for $S_{\pi\pi}$, $C_{\pi\pi}$, and $D_{\pi\pi}$ are then

$$S_{\pi\pi} = [\sin 2\alpha + 2|P/T| \sin(\beta - \alpha) \cos \delta - |P/T|^2 \sin 2\beta]/R_{\pi\pi} \quad (11)$$

$$C_{\pi\pi} = [2|P/T| \sin(\alpha + \beta) \sin \delta]/R_{\pi\pi}, \quad (12)$$

$$\begin{aligned} D_{\pi\pi} &= [\cos 2\alpha - 2|P/T| \cos(\beta - \alpha) \cos \delta \\ &\quad + |P/T|^2 \cos 2\beta]/R_{\pi\pi}. \end{aligned} \quad (13)$$

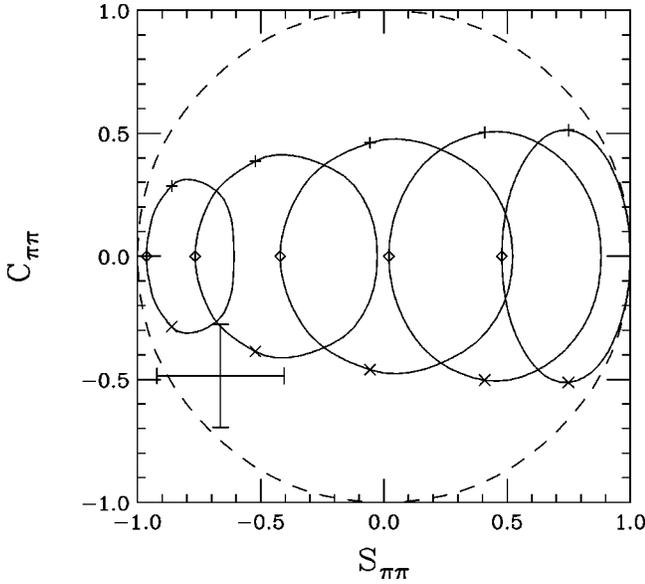


FIG. 1. Values of $S_{\pi\pi}$ and $C_{\pi\pi}$ for representative values of α lying roughly in the physical region. Closed curves correspond, from right to left, to $\alpha=60^\circ, 75^\circ, 90^\circ, 105^\circ,$ and 120° . Plotted points on curves correspond to $\delta=90^\circ$ (+ signs), 0 (diamonds), and -90° (crosses). The dashed circle denotes the bound $S_{\pi\pi}^2 + C_{\pi\pi}^2 \leq 1$. The plotted point with large errors corresponds to the average of the measurements [2,4] of $S_{\pi\pi}$ and $C_{\pi\pi}$. The central values $\beta=26^\circ, |P/T|=0.28$ have been taken.

The quantity $R_{\pi\pi}$ itself will be used in Sec. IV to resolve a discrete ambiguity, while the usefulness of the sign of $D_{\pi\pi}$ will be described in Sec. VI.

Note that $C_{\pi\pi}$ is odd in δ while $S_{\pi\pi}$ and $D_{\pi\pi}$ are even in δ . Within the present CKM framework one has $0 < \alpha + \beta < \pi$, implying $\sin(\alpha + \beta) > 0$, so that a measurement of non-zero $C_{\pi\pi}$ will specify the sign of δ (predicted in some theoretical schemes [7]).

We shall concentrate for the most part on a range of CKM parameters allowed by fits to weak decays, disregarding the possibility of new physics effects. Aside from the constraints associated with $S_{\pi\pi}$, it was found in Ref. [13] (quoting [14] and [15]; see also [5]) that $\sin 2\alpha = -0.24 \pm 0.72$, implying $\alpha = (97_{-21}^{+30})^\circ$, which we shall take as the “standard-model” range.

One could regard the three equations for $R_{\pi\pi}$, $S_{\pi\pi}$, and $C_{\pi\pi}$ as specifying the three unknowns $|P/T|$, δ , and α (given the rather good information on β). In what follows we shall, rather, use the present constraints on $|P/T|$ mentioned above, first concentrating on what can be learned from $S_{\pi\pi}$ and $C_{\pi\pi}$ alone and then using the information on $R_{\pi\pi}$ both as a consistency check and to resolve discrete ambiguities. The information provided by the sign of $D_{\pi\pi}$ will be treated separately.

III. DEPENDENCE OF $S_{\pi\pi}$ AND $C_{\pi\pi}$ ON α AND δ

We display in Fig. 1 the values of $S_{\pi\pi}$ and $C_{\pi\pi}$ for α roughly in the physical region, with $-\pi \leq \delta \leq \pi$. For any fixed α , the locus of such points is a closed curve with the

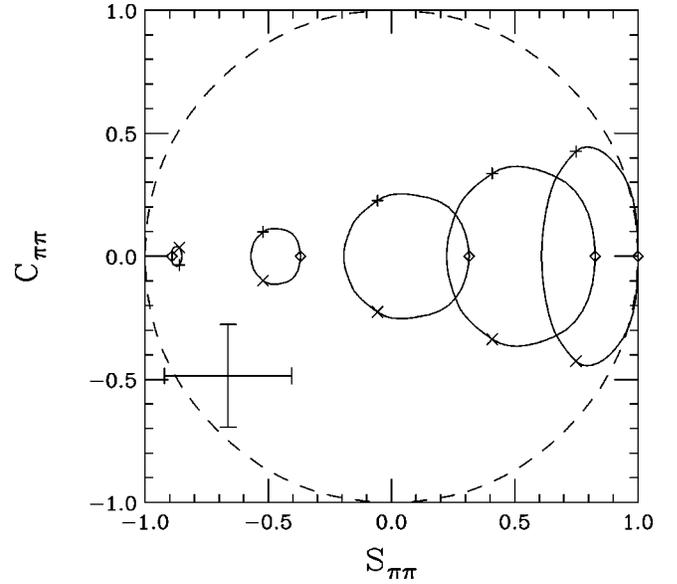


FIG. 2. Values of $S_{\pi\pi}$ and $C_{\pi\pi}$ for representative values of α lying mostly outside the physical region. Closed curves correspond, from left to right, to $\alpha=-30^\circ, -15^\circ, 0^\circ, 15^\circ,$ and 30° . Other notation as in Fig. 1.

points $\delta=0$ and $\delta=\pm\pi$ corresponding to $C_{\pi\pi}=0$ and with $C_{\pi\pi}(-\delta) = -C_{\pi\pi}(\delta)$. A large negative value of $S_{\pi\pi}$, as seems to be indicated by the Belle measurement [2], favors large values of α . Negative values of $C_{\pi\pi}$ imply a negative δ . The sum of squares of $S_{\pi\pi}$ and $C_{\pi\pi}$ is always bounded by 1, and one can show that for any value of δ and $\alpha + \beta$ one has $|C_{\pi\pi}| \leq 2|P/T|/(1+|P/T|^2)$. For a given value of $\alpha + \beta$ the bound is stronger:

$$|C_{\pi\pi}| \leq \frac{2|P/T||\sin(\alpha + \beta)|}{\sqrt{(1+|P/T|^2)^2 - 4|P/T|^2 \cos^2(\alpha + \beta)}}. \quad (14)$$

The corresponding plot for (mostly) unphysical values of α is shown in Fig. 2. If desired, one may map negative values of α into the interval $[0, \pi]$ by the replacement $\alpha \rightarrow \alpha + \pi$, $\delta \rightarrow \delta \pm \pi$, which leaves all expressions invariant. The conventional physical region is bounded by $0 \leq \alpha \leq \pi - \beta$.

The closed curves in Fig. 1 have considerable dependence on δ for α around $\pi/2$. One can show that $S_{\pi\pi}$ becomes independent of δ when $\cos 2\alpha = |P/T|^2 \cos 2\beta$. Since $|P/T|^2$ is small, these points are $\alpha \approx \pi/4, 3\pi/4$. At such critical values of α the curves degenerate into vertical lines. For $\alpha = \pi - \beta$, one has $\gamma = 0, C_{\pi\pi} = 0, S_{\pi\pi} = \sin 2\alpha$, and the curves collapse to a point.

The curves in Figs. 1 and 2 were plotted for the central values $\beta = 26^\circ, |P/T| = 0.28$. Their dependence on $\pm 1\sigma$ variations of β is quite mild for α in the physical region, while they are more sensitive to $\pm 1\sigma$ excursions of $|P/T|$, as shown in Fig. 3.

Let us imagine a measurement of $S_{\pi\pi}$ and $C_{\pi\pi}$ which reduces present errors by a factor of $\sqrt{3}$. Given that the present measurements are based on around a total of 100 fb^{-1} , one could envision such an improvement when both BaBar and Belle report values based on 150 fb^{-1} . Then

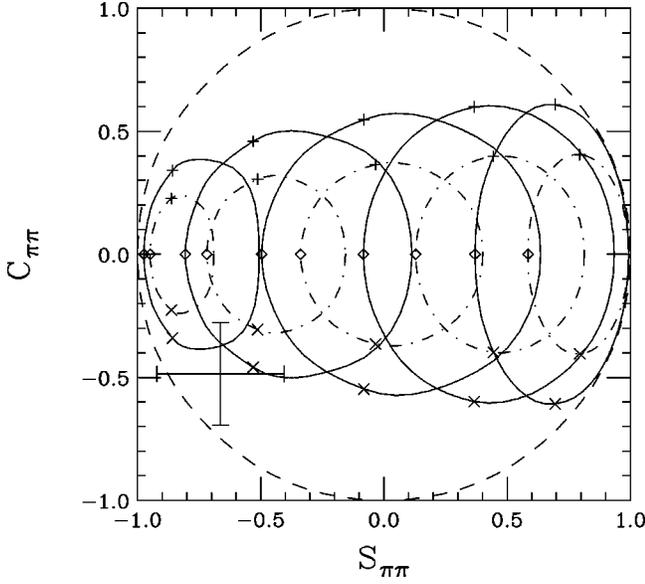


FIG. 3. Values of $S_{\pi\pi}$ and $C_{\pi\pi}$ as functions of α and δ ; same as Fig. 1 except $|P/T|=0.34$ (solid curves) and $|P/T|=0.21$ (dot-dashed curves).

the size of the error ellipse associated with $S_{\pi\pi}$ and $C_{\pi\pi}$ will be small in comparison with that of the closed curves for α in the vicinity of 90° , and measurement of these quantities could provide useful information were it not for the fact that every point in the $S_{\pi\pi}, C_{\pi\pi}$ plane corresponds to several pairs α, δ . The most important of these pairs occurs when both values of α are in the physical region but one corresponds to a certain value of δ and the other (roughly) to $\pi - \delta$. This discrete ambiguity is most severe (corresponding to the most widely separated values of α) when $C_{\pi\pi}=0$, corresponding to $\delta=0$ or π . For example, in Fig. 1, $S_{\pi\pi}=C_{\pi\pi}=0$ corresponds to both $\alpha \approx 76^\circ$ (when $\delta=0$) and to $\alpha \approx 105^\circ$ (when $\delta=\pi$). These values of α are separated by nearly 30° . We shall see in the next section how a measurement of the branching ratio $\bar{B}(B^0 \rightarrow \pi^+ \pi^-)$ can help resolve this ambiguity.

Measuring a nonzero value for $C_{\pi\pi}$ determines the sign of δ , but leaves an ambiguity between δ and $\pi - \delta$. The corresponding ambiguity in determining α becomes smaller when δ moves away from 0 and π . For maximal direct CP violation, corresponding to $|\delta|=\pi/2$, one has $\sin \delta = \pm 1$, $\cos \delta = 0$, and no discrete ambiguity. These cases correspond to the envelope of the curves in Figs. 1–3, joining the points labeled with $+$ ($\delta=\pi/2$) or \times ($\delta=-\pi/2$).

IV. INFORMATION FROM DECAY RATE

The quantity $R_{\pi\pi}$, defined in Eq. (10), can help resolve the discrete ambiguity between $\delta=0$ and $\delta=\pi$ in the case $C_{\pi\pi}=0$, where such an ambiguity is most serious. It has been frequently noted [16] that the central value of this quantity is less than 1, suggesting the possibility of destructive interference between tree and penguin amplitudes. With the estimate $|T|=2.7 \pm 0.6$ mentioned above, and with the experimental average [17] of CLEO, Belle, and BaBar branch-

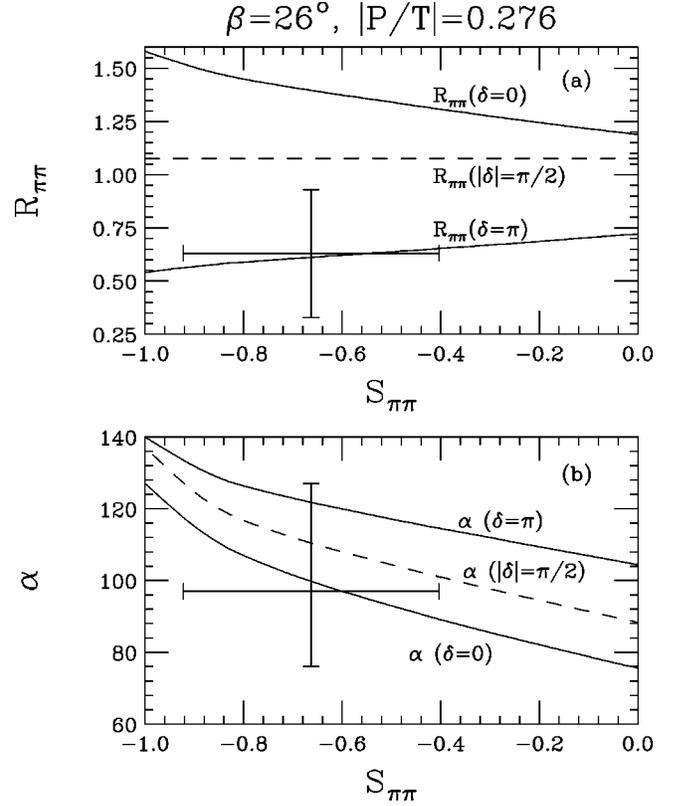


FIG. 4. Values of (a) $R_{\pi\pi}$ and (b) α as functions of $S_{\pi\pi}$ for the cases $\delta=0$ and $\delta=\pi$ leading to $C_{\pi\pi}=0$ (solid lines), and for $|\delta|=\pi/2$ (dashed lines). The plotted points correspond to experimental values of $S_{\pi\pi}$ and (a) $R_{\pi\pi}$ or (b) α . Other parameters as in Fig. 1. For these sets of parameters $D_{\pi\pi} < 0$; when $C_{\pi\pi}=0$ one has $D_{\pi\pi} = -(1 - S_{\pi\pi}^2)^{1/2}$.

ing ratios equal to $\bar{B}(B^0 \rightarrow \pi^+ \pi^-) = (4.6 \pm 0.8) \times 10^{-6}$, we have $R_{\pi\pi} = 0.63 \pm 0.30$, which lies suggestively but not conclusively below 1. A value of $R_{\pi\pi} < 1$ would imply $\cos \delta < 0$ within the CKM framework, since all currently allowed values of γ correspond to $\cos \gamma > 0$. Furthermore, a value of $R_{\pi\pi}$ below 1 permits one to set a bound on $\alpha + \beta$ or on γ , which is independent of δ ,

$$R_{\pi\pi} = 1 + (|P/T| + \cos \delta \cos \gamma)^2 - \cos^2 \delta \cos^2 \gamma \geq \sin^2 \gamma, \quad (15)$$

similar to the Fleischer-Mannel bound in $B \rightarrow K\pi$ [18]. At the 1σ level, this already implies $\gamma \leq 71^\circ$ in the CKM framework. In a more general framework, $\gamma \geq 109^\circ$ is also allowed.

We show in Fig. 4 the dependence of $R_{\pi\pi}$ and α on $S_{\pi\pi}$ for the extreme cases $\delta=0$ and $\delta=\pi$. For reference we also exhibit the curves for $|\delta|=\pi/2$. As mentioned, only $C_{\pi\pi}$ depends on the sign of δ . Also shown are experimental points corresponding to present ranges of $R_{\pi\pi}$, α , and $S_{\pi\pi}$. If errors on $S_{\pi\pi}$ and $C_{\pi\pi}$ are reduced by about a factor of $\sqrt{3}$, and on $R_{\pi\pi}$ by a factor of about three, as would be possible with a sample of 150 fb^{-1} for each experiment, one can see a constraint emerging which would favor one or the other choices for δ . We discussed reduction of errors on $S_{\pi\pi}$

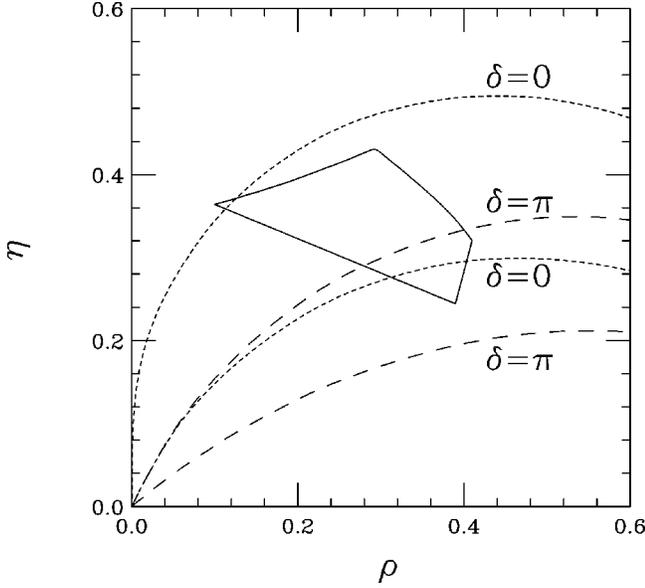


FIG. 5. Plot in the (ρ, η) plane of regions allowed by the observed 1σ ranges $-0.92 \leq S_{\pi\pi} \leq -0.40$ and $0.21 \leq |P/T| \leq 0.34$ for $\delta=0$ (small dashes) and $\delta=\pi$ (large dashes), compared with the region allowed by other constraints (solid lines). Bottom solid line: lower bound on β . Upper left solid line: upper bound on ϵ_K . Upper right solid line: upper bound on $|V_{ub}|$. Right-hand solid line: lower bound on Δm_d .

and $C_{\pi\pi}$ already. The corresponding reduction for $R_{\pi\pi}$ requires reduction of errors on $|T|^2$ and $\bar{B}(B^0 \rightarrow \pi^+ \pi^-)$ from their present values of 44% and 17%, respectively, each to about 10%, which was shown in Ref. [6] to be possible with 100 fb^{-1} .

V. COMPARISON WITH CKM PARAMETERS DETERMINED BY OTHER MEANS

In Ref. [5] we compared the region of CKM parameters allowed by data on various weak transitions with that implied by the first observed range of $S_{\pi\pi}$ [3] and $|P/T|$ for the case $\delta=0$. In Fig. 5 we reproduce that plot, corresponding to present 1σ limits on $S_{\pi\pi}$ and $|P/T|$ values in the range $0.21 \leq |P/T| \leq 0.34$, along with the case $\delta=\pi$.

The case $\delta=\pi$ is seen to exclude a large region of the otherwise-allowed parameter space, while $\delta=0$ is compatible with nearly the whole otherwise-allowed range. Of course this does not permit a distinction at present between the two solutions, but it illustrates the potential of improved data. Turning things around, the examples in Fig. 5 corresponding to $\delta=0$ and $\delta=\pi$ illustrate the importance of excluding one of these two values by means of the ratio $R_{\pi\pi}$. Values $0 < |\delta| < \pi$ with $C_{\pi\pi} \neq 0$ correspond to constraints intermediate between those for $\delta=0$ and $\delta=\pi$.

The present (ρ, η) constraints differ from those in Refs. [5,6] based on the earlier BaBar data [3], which were consistent (as are the present BaBar data [4]) with vanishing $S_{\pi\pi}$ and $C_{\pi\pi}$. In that case $\delta=0$ led to a significant restriction in the (ρ, η) plane, permitting only low values of ρ , while $\delta \approx \pi$ would have been consistent with nearly the whole al-

lowed (ρ, η) region (as well as with the present data on $R_{\pi\pi}$).

VI. INFORMATION FROM WIDTH DIFFERENCE

The quantity $D_{\pi\pi}$ appears with equal contributions in the time-dependent decay rates of B^0 or \bar{B}^0 to a CP eigenstate, when the width difference $\Delta\Gamma_d \equiv \Gamma_L - \Gamma_H$ between neutral B mass eigenstates is nonzero [19]:

$$\begin{aligned} \Gamma(B^0(t) \rightarrow \pi^+ \pi^-) \propto e^{-\Gamma_d t} [& \cosh(\Delta\Gamma_d t/2) \\ & - D_{\pi\pi} \sinh(\Delta\Gamma_d t/2) \\ & + C_{\pi\pi} \cos(\Delta m_d t) - S_{\pi\pi} \sin(\Delta m_d t)]. \end{aligned} \quad (16)$$

Width difference effects in the $B_s - \bar{B}_s$ system were investigated some time ago in time-dependent B_s decays [19,20]. The feasibility of measuring corresponding $\Delta\Gamma_d$ effects in B^0 decays, expected to be much smaller but having a well-defined sign ($\Delta\Gamma_d > 0$) in the CKM framework, was studied very recently [21]. While a measurement of $D_{\pi\pi}$ in $B \rightarrow \pi^+ \pi^-$ is unfeasible in near-future experiments because of the very small value of $\Delta\Gamma_d$ ($\Delta\Gamma_d/\Gamma_d < 1\%$), we will discuss the theoretical consequence of such a measurement. This brief study and its conclusion seem to be generic to a broad class of processes, including the U-spin related decay $B_s(t) \rightarrow K^+ K^-$ [22], in which width difference effects are much larger [23].

In the absence of P , one just has $S_{\pi\pi} = \sin(2\alpha)$, $D_{\pi\pi} = \cos(2\alpha)$, so the two quantities are out of phase with respect to one another by $\pi/4$ in α . This reduces part of the ambiguity in determining α from the mixing-induced asymmetry. The same is true when $\delta=0$ or π , since then α is replaced by α_{eff} as noted in the previous section.

The dependences of $S_{\pi\pi}$ and $D_{\pi\pi}$ on δ for fixed α also are out of phase with respect to one another, in the following sense. When $S_{\pi\pi}$ is most sensitive to δ , $D_{\pi\pi}$ is least sensitive, and vice versa. One can show, for example, that $D_{\pi\pi}$ is completely independent of δ when

$$\sin 2\alpha = -|P/T|^2 \sin 2\beta, \quad (17)$$

which corresponds, since $|P/T|^2$ is small, to values of α near 0, $\pi/2$, and π . Recall that the corresponding values for $S_{\pi\pi}$ were near $\pi/4$ and $3\pi/4$. Conversely, whereas $S_{\pi\pi}$ is maximally sensitive to δ near $\alpha = \pi/2$, $D_{\pi\pi}$ is maximally sensitive to δ near $\alpha = \pi/4$ and $3\pi/4$.

In the absence of the penguin amplitude $D_{\pi\pi}$ would just be $\cos 2\alpha$. Since α is not too far from $\pi/2$ in its currently allowed range, $D_{\pi\pi}$ remains negative in this entire range also in the presence of the penguin amplitude. Positive values of $D_{\pi\pi}$ are obtained for values of α which are excluded in the CKM framework. For the values $\delta=0$ and $\delta=\pi$, when $C_{\pi\pi}=0$, one has $D_{\pi\pi} = -(1 - S_{\pi\pi}^2)^{1/2}$. For these values of δ , $S_{\pi\pi}$ is seen in Fig. 1 to lie in the range $-1.0 < S_{\pi\pi} \leq 1.0$, implying $-1.0 \leq D_{\pi\pi} \leq 0$. Since in the standard

model one expects $D_{\pi\pi}$ to be negative, positive $D_{\pi\pi}$, obtained for unphysical values of α , would signify new physics.

VII. CONCLUSIONS

We have investigated the information about the weak phase α and the strong phase δ between penguin (P) and tree (T) amplitudes which can be obtained from the quantities $S_{\pi\pi}$ and $C_{\pi\pi}$ measured in the time-dependent decays $B^0 \rightarrow \pi^+ \pi^-$ and $\bar{B}^0 \rightarrow \pi^+ \pi^-$. One has a number of discrete ambiguities associated with the mapping $(S_{\pi\pi}, C_{\pi\pi}) \rightarrow (\alpha, \delta)$. These appear to be most severe when $C_{\pi\pi} \simeq 0$, since very different values of α can be associated with $\delta \simeq 0$ and $\delta \simeq \pi$. We have shown that under such circumstances these ambiguities are resolved by sufficiently accurate measurements of the ratio $R_{\pi\pi}$ of the flavor-averaged $B^0 \rightarrow \pi^+ \pi^-$ branching ratio to its predicted value due to the tree amplitude alone. At present this ratio appears to be less than 1, but with large errors. Reduction of present errors on $S_{\pi\pi}$ and $C_{\pi\pi}$ by a factor of $\sqrt{3}$ and on $R_{\pi\pi}$ by a factor of three will have significant impact on these phase determinations. If a nonzero value of $C_{\pi\pi}$ is found, the discrete ambiguity becomes less important, vanishing altogether when $|\delta| = \pi/2$.

A small value of $R_{\pi\pi}$, around its present central value, would favor $\delta = \pi$ over $\delta = 0$, as shown in Fig. 4(a). A large negative value of $S_{\pi\pi}$, as indicated by the Belle measurement [2], favors large values of α , in particular if $\delta \simeq \pi$. This is demonstrated in Fig. 3 and Fig. 4(b). Correspondingly, Fig. 5 shows that low values of ρ are excluded in the latter case. This figure, drawn also for the case $\delta = 0$, illustrates the important role of the measurement of $S_{\pi\pi}$ and the knowledge of δ in determining the CKM parameters ρ and η .

Another parameter, called $D_{\pi\pi}$ here, equal to $\pm(1$

$-S_{\pi\pi}^2 - C_{\pi\pi}^2)^{1/2}$, is measurable in principle in time-dependent $B^0 \rightarrow \pi^+ \pi^-$ decays if effects of the difference between widths of mass eigenstates can be discerned. The sign of $D_{\pi\pi}$ is enough to resolve a discrete ambiguity between values of α expected in the standard model (corresponding to $D_{\pi\pi}$ negative) and unphysical α (corresponding to $D_{\pi\pi}$ positive).

As has been noted previously [16], there are hints of destructive tree-penguin interference in $B^0 \rightarrow \pi^+ \pi^-$, which may be difficult to reconcile with the favored range of CKM parameters without invoking large values of δ . If this interesting situation persists, one may for the first time encounter an inconsistency in the CKM description of CP violation, which often assumes small strong phases. Improved time-dependent measurements of $B^0 \rightarrow \pi^+ \pi^-$ will be of great help in resolving this question. Given that standard model fits [13–15] prefer $\cos \gamma > 0$, a value of $R_{\pi\pi}$ significantly less than 1 in the absence of any other evidence for large δ also could call into question the applicability of factorization to $B^0 \rightarrow \pi^+ \pi^-$ [5]. More accurate measurements of the spectrum in $B \rightarrow \pi l \nu$ [6] and more accurate tests of factorization in other color-favored processes [9] will help to check this possibility.

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