Small note on *pp*-wave vacua in 6 and 5 dimensions

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We discuss Kowalski-Glikman type pp-wave solutions with unbroken supersymmetry in 6 and 5 dimensional supergravity theories.

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In this Brief Report we want to discuss pp-wave solutions with unbroken supersymmetry, the so-called Kowalski-Glikman (KG) solutions, in lower dimensional supergravity theories. The known KG solutions [1–3] consist of some covariantly constant field strength and a metric which has the form of a pp-wave: i.e.,

$$ds_d^2 = 2\,du(dv + A\,du) + dx^i\,dx_i\,, \tag{1}$$

where $A = x^i A_{ij} x^j$ $(i,j = 1,\ldots,d-2)$. For this metric, the only nonvanishing component of the Ricci curvature is $R_{uu} = 2\eta^{ij}A_{ij}$ and by introducing light-cone coordinates in the tangent space we find that the only nonvanishing component of the spin connection is $\omega\!\!\!/_\mu = 2\delta_{u\mu}\partial_i A\,\gamma^{i+}$.[1]

The basic problem one is faced with when looking for nontrivial solutions of supergravity theories that preserve all supersymmetry are the dilatino variations, since they are algebraic in nature. Clearly, the easiest way to avoid such trouble is by not having dilatino equations in the first place. Sometimes, however, one can make use of special properties of the dilatino equations in order to find nontrivial vacua. The first example is $d=10$, $N=1$ supergravity where one can use the chirality of the theory [4] in order to find a solution that preserves all supersymmetry. Another example is type IIB supergravity, where one can find such solutions, notably the $\mathrm{ADS}_5 \otimes S^5$ solution and the Kowalski-Glikman type solution presented in [2], with a Ramond-Ramond (RR) five-form flux since the dilatino equation does not contain a contribution from the five-form field strength [5]. The fact that the type IIB dilatino variation does not depend on the five-form field strength, however, is due to the fact that it is self-dual and that the spinors are chiral. One is therefore tempted to say that nontrivial solutions with unbroken supersymmetry exist whenever there are no dilatinos or when the theory is chiral, and it is these kinds of theories we are going to examine.

There are not many supergravity theories that are chiral or have no dilatinos, so that the investigation of the existence of KG solutions is rather limited. The highest dimensional possibilities have already been presented in the literature, namely, by Kowalski-Glikman in the case of M theory [1] and by Blau *et al.* for type IIB [2]. The next on the list is $N=1$, $d=10$ supergravity. Such an investigation was carried out by Kowalski-Glikman [4] who showed that the solution is not of the pp-wave type, but rather has geometry $\mathrm{ADS}_3 \otimes S^3 \otimes \mathbb{E}_4$. A similar analysis was performed on the $N=2$, $d=4$ supergravity [3], showing that the only supersymmetric solutions are the Robinson-Bertotti and the KG solutions. This means that the only remaining candidates are $d=6$ (2,0) or (4,0) supergravity and $d=5$, $N=2$ supergravity. Although $N=1$, $d=4$ supergravity matches the profile, it can be discarded since the integrability condition for the Killing spinor equation implies that the space must be Riemann flat, i.e., Minkowski.

The $d=6$ (2,0) supergravity is comprised of the graviton e_μ^a, two symplectic Majorana-Weyl (MW) Rarita-Schwinger fields, combined into the USp(2) vector Ψ_μ, and a two-form B whose field strength $H=dB$ is self-dual. As such one is faced with the same problem as in type IIB supergravity. A Lorentz invariant action can, however, be written down by introducing a Lagrange multiplier field [6], by writing a non-self-dual action as in [7], or by adding an antisymmetric tensor multiplet [8]. However, using the conventions of [9] the equation of motion for the metric reads

$$R_{\mu\nu} = \tfrac{1}{4} H_{\mu\kappa\rho} H_\nu{}^{\kappa\rho}. \tag{2}$$

Choosing the self-dual ansatz $H = \lambda\,du \wedge (dx^1\,dx^2 + dx^3\,dx^4)$ the above equation is solved by choosing $2\eta^{ij}A_{ij} = -\lambda^2$.

The Killing spinor equations in this case read

$$0 = \delta\Psi_\mu = \nabla_\mu \epsilon - \frac{1}{8\times 3!} H\!\!\!/\,\gamma_\mu \epsilon. \tag{3}$$

By observing that due to the self-duality of H we have $H\!\!\!/\,\gamma_\mu\epsilon = 2\times 3!\,\lambda\,\gamma^+\gamma^{12}\gamma_\mu\epsilon$, we can see that Eq. (3) is automatically satisfied in the v direction. The equations in the i direction read

$$0 = \partial_i \epsilon - \Omega_i \epsilon, \quad \Omega_i = \tfrac{1}{4}\lambda\,\gamma^+\gamma^{12}\gamma_i\,. \tag{4}$$

Following [2,10] these equations, since $\Omega_i\Omega_j = 0$, are solved by $\epsilon = (1 + x^i\Omega_i)\xi(u)$. In the u direction Eq. (3) reduces to

$$\partial_u \epsilon - x^i A_{ij}\gamma^j\gamma^+\epsilon - \Omega^-\epsilon = 0, \tag{5}$$

[1] We use the mostly minus signature for the metric, the γ matrices satisfy $\{\gamma^\mu,\gamma^\nu\} = 2g^{\mu\nu}$, and the covariant derivative on spinors is taken to be $\nabla\epsilon = d\epsilon - 4^{-1}\omega\!\!\!/\,\epsilon$. We also introduce the light-cone combinations $\gamma^+ = \gamma^u$ and $\gamma^- = \gamma^v + A\gamma^u$, which satisfy $\{\gamma^+,\gamma^-\} = 2$ and $\{\gamma^\pm,\gamma^\pm\} = \{\gamma^\pm,\gamma^i\} = 0$.

where the combination $\Omega^- = \frac{1}{4}\lambda\gamma^+\gamma^{12}\gamma^-$ was used. By making the ansatz $\xi = \exp(\Omega^- u)\epsilon_0$, with ϵ_0 an unconstrained constant symplectic MW spinor, all x-independent terms are canceled, leaving

$$x^i A_{ij}\gamma^j\gamma^+\epsilon_0 = -\tfrac{1}{8}\lambda^2 x^i\gamma_i\gamma^+\epsilon_0. \tag{6}$$

This equation is readily solved by $A_{ij} = -\frac{1}{8}\lambda^2\eta_{ij}$, which is compatible with the equations of motion.

The $d=6$ (4,0) supergravity is invariant under global $USp(4) \sim SO(5)$ (see Ref. [11] and references therein) and its field content is a Sechsbein, $e_\mu{}^a$, four symplectic MW spinors, which are combined into a SO(5) vector Ψ_μ, and five two-forms B^I which have self-dual field strengths and transform as a vector under SO(5). The equations of motion and the supersymmetry transformation are

$$0 = R_{\mu\nu} - \tfrac{1}{4}H^I_{\mu\kappa\rho}H_\nu{}^{\kappa\rho I},$$
$$0 = \delta\Psi_\mu = \nabla_\mu\epsilon - \frac{1}{8\times 3!}\not{H}\gamma_\mu\Gamma^I\epsilon, \tag{7}$$

where the Γ's belong to the five-dimensional Euclidean Clifford algebra. The (2,0) solution can be embedded into the (4,0) theory by taking only the $I=1$ component to be different from zero. The calculations are just the same as in the (2,0) case, the only difference being that every Ω has to be multiplied by Γ^1. The result, however, is the same: (4,0) supergravity admits a KG-type wave solution that breaks no supersymmetry whatsoever.

The last on the shortlist is $d=5, N=2$ supergravity [12]. Its field content consists of a Fünfbein, e^a_μ, two symplectic Majorana Rarita-Schwinger fields Ψ_μ and a vector V_μ whose field strength will be taken to be

$$F = du \wedge \lambda_i\, dx^i. \tag{8}$$

Since the chosen form for the field strength is, given the metric (1), covariantly constant and has an overall dependence on the differential du, the equation of motion for the vector field is automatically satisfied. The equation of motion for the metric reads

$$R_{\mu\nu} = \frac{1}{2}\left[F_{\mu\kappa}F^\kappa_\nu - \frac{1}{6}g_{\mu\nu}F^2\right], \tag{9}$$

and leads to the condition $4\eta^{ij}A_{ij} = \lambda_i\lambda^i$. The supersymmetry variation of the gravitino reads [12]

$$0 = \delta\Psi_\mu = \nabla_\mu\epsilon + \frac{1}{8\sqrt{3}}\not{F}\gamma_\mu\epsilon - \frac{1}{4\sqrt{3}}F_{\mu\nu}\gamma^\nu\epsilon. \tag{10}$$

The analysis is completely analogous to the one for the (2,0) theory, but for the definitions

$$\Omega_i = -\frac{1}{4\sqrt{3}}\gamma^+[\lambda_j\gamma^j\gamma_i + \lambda_i], \quad \Omega^- = \frac{1}{4\sqrt{3}}\lambda_i\gamma^i(\gamma^+\gamma^- + 1). \tag{11}$$

One finds that the analogous condition to Eq. (6) reads

$$x^i A_{ij}\gamma^j\gamma^+\epsilon_0 = x^i[\Omega_i, \Omega^-]\epsilon_0, \tag{12}$$

where, as before, ϵ_0 is an unconstrained symplectic Majorana spinor. After some γ manipulations, one finds the matrix A to be

$$A_{ij} = \frac{1}{24}\{3\lambda_i\lambda_j + \lambda_l\lambda^l\eta_{ij}\}, \tag{13}$$

which is compatible with the equations of motion. Of course we could make use of the SO(3) invariance to put $\lambda_{2,3} = 0$ and we find A to be proportional to diag(4,1,1).

In this Brief Report we have presented Kowalski-Glikman type solutions, solutions that do not break any supersymmetry and that look like pp waves, in chiral six-dimensional supergravities and in $N=2, d=5$ supergravity. Since we used rather restrictive criteria, the absence of dilatinos or chirality, it would be nice to consider other theories and see whether they admit KG solutions. Work in this direction is in progress.

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