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Holography and the large number hypothesis

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Dirac's large number hypothesis is motivated by certain scaling transformations that relate the parameters of macro and microphysics. We show that these relations can actually be explained in terms of the holographic N bound conjectured by Bousso and a series of purely cosmological observations, namely, that our universe is spatially homogeneous, isotropic, and flat to a high degree of approximation and that the cosmological constant dominates the energy density at present.

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Explaining the value of the constants of nature is one of the most exciting challenges of theoretical physics. Some of these constants play a fundamental role in the foundations of the scientific paradigms. This is the case of Planck's constant $\hbar$ in quantum mechanics, and of Newton's constant G and the speed of light c in general relativity. These three constants provide a natural system of units for all physical quantities. For instance, the length and mass units are $l_P = \sqrt{\hbar G/c^3} = 1.6\times 10^{-35}$ m and $m_P = \sqrt{\hbar c/G} = 2.2\times 10^{-8}$ kg. In terms of these Planck units, the other constants of nature become dimensionless numbers.

Already in the 1920s, Eddington tried unsuccessfully to deduce the value of all constants of physics from theoretical considerations [1]. Most importantly, he pointed out the existence of relations between the parameters of fields that at first sight seem unconnected, such as nuclear physics and cosmology. Among these, perhaps the most intriguing relation is the apparent coincidence between the present number of baryons in the universe, known as the Eddington number, and the squared ratio of the electric to the gravitational force between the proton and the electron. This coincidence between large numbers can also be expressed in the alternative form [2]

$$\hbar^2 H_0 \approx G c m_N^3. \tag{1}$$

This approximate identity is sometimes called the Eddington-Weinberg relation. Here, m_N is the proton mass and $H_0 \approx 70$ km/(s Mpc) is the present value of the Hubble constant [3].

Actually, the Hubble parameter is not a true constant, but varies as the inverse of the cosmological time in standard Friedmann-Robertson-Walker (FRW) cosmology [2]. This fact led Dirac [4] to put forward the hypothesis that Newton's constant must depend on time as H_0, $G \propto t^{-1}$, so that relation (1) is always valid. In spite of its attractive features, Dirac's large number hypothesis turns out to be incompatible with the experimental bounds that exist on the time variation of G [5]. Therefore, the explanation of the Eddington-Weinberg relation still remains a mystery.

Recently, the determination of cosmological parameters has experienced a considerable revolution. The observation of type Ia supernovae (SNe Ia) at high redshift has provided evidence in favor of a positive cosmological constant [6]. In addition, accurate measurements of the angular power spectrum of anisotropies in the cosmic microwave background (CMB) have shown that the curvature of the universe is close to flat [7]. These CMB and SNe Ia data, together with other cosmological information, have been combined in a consistent (nearly) flat FRW model whose values of the cosmological constant Λ and matter density ρ_0 are, approximately, $c^2\Lambda = 16\pi G\rho_0 = 2H_0^2$ [8].

This value of the cosmological constant poses two puzzles. On the one hand, one would expect that Λ emerged from vacuum fluctuations. In a theory of quantum gravity, these fluctuations would have Planck energy density. The discrepancy from this theoretical expectations is of nearly 120 orders of magnitude, since, in Planck units, $H_0 \approx 10^{-60}$. This is the so-called cosmological constant problem [9]. The value of Λ, on the other hand, is constant, whereas the density of matter decreases with expansion. As a consequence, the relation $16\pi G\rho_0 \approx c^2\Lambda$ is not valid in most of the history of the universe. Why is it precisely now that the matter content and Λ provide similar contributions to the energy? This additional puzzle is known as the cosmic coincidence problem [10].

A new perspective on the cosmological constant problem, which puts the emphasis on fundamental aspects of gravity rather than in purely quantum field theory (QFT) considerations, has recently emerged with the advent of holography [11]. In an oversimplified version, the holographic principle states that the entropy S [12] of a physical system subject to gravity is bounded from above by a quarter of its boundary area in Planck units, $S \leqslant A/(4l_p^2)$. From this point of view, the physical degrees of freedom are not proportional to the volume in the presence of the gravitational field, but reside in the bounding surface.

A more rigorous, covariant formulation of the holographic conjecture has been elaborated by Bousso, providing in principle an entropy bound on null hypersurfaces [13,14]. Other less general holographic proposals that find straightforward application to spatial volumes in cosmology have also been suggested [15–17]. In this respect, an issue of debate has been the largest region of the universe in which an entropy bound may be feasible. Fischler and Susskind [15] originally proposed considering the particle horizon, at least for adiabatic evolution, but other possibilities that appear more natural were soon suggested. One such possibility is the use of the cosmological apparent horizon, which bounds an anti-trapped region and has an associated notion of gravitational entropy [13,16]. Another proposal that has found considerable support is the restriction to the Hubble radius cH_0^{-1} [17], since this supplies the scale of causal connection beyond which gravitational perturbations on a flat background cannot grow with time. It is worth noting, anyway, that for a flat FRW model like the one that possibly describes our universe the apparent and Hubble horizons do in fact coincide [16].

For any spacetime with a positive cosmological constant, Bousso [18] has argued that the holographic principle leads to the prediction that the number of degrees of freedom N available in the universe is related to Λ by

$$N=\frac{3\pi}{\Lambda l_P^2 \ln 2}. \tag{2}$$

The observable entropy S is then bounded by $N \ln 2$. This conjecture is called the N bound. Under quantization, the system would be describable by a Hilbert space of finite dimension (equal to 2^N). Bousso's conjecture is largely influenced by Banks' ideas about the cosmological constant [19]. According to Banks, Λ should not be considered a parameter of the theory; rather, it is determined by the inverse of the number of degrees of freedom. From this viewpoint, the cosmological constant problem disappears, because N can be regarded as part of the data that describe the system at a fundamental level. Based also on holography, other possible explanations have been proposed for the value of Λ that are closer in spirit to the standard methods of QFT [20].

Since the cosmological constant affects the large scale structure of the universe but should originate from effective local vacuum fluctuations, it may provide a natural connection between macro- and microphysics. In addition, Λ is related to the number of degrees of freedom by the holographic principle. As a consequence, one could expect that holography would play a fundamental role in explaining the coincidence of the large numbers arising in cosmology and particle physics. A first indication that this intuition may work was provided by Zizzi [21], who recovered Eddington number starting with a discrete quantum model for the early universe that saturates the holographic bound. The main aim of the present paper is to prove that the large number hypothesis and the holographic conjecture are in fact not fully independent. To be more precise, we will show that, in a homogeneous, isotropic, and (quasi)flat universe like ours, the relations between large numbers can be explained by the holographic principle assuming that the present energy density is nearly dominated by Λ.

The scaling relations that lie behind the large number hypothesis can be expressed in the form

$$l_N \approx \Omega l_P, \tag{3}$$

$$m_N \approx \Omega^{-1} m_P, \tag{4}$$

$$l_U \equiv cH_0^{-1} \approx \Omega^3 l_P, \tag{5}$$

$$m_U \approx \Omega^3 m_P. \tag{6}$$

The scale Ω has the value $10^{19}-10^{20}$. Here, m_N and l_N are the mass and radius of a nucleon, e.g., the proton. The symbol l_U denotes the observable radius of the universe, which we define as the distance that light can travel in a Hubble time H_0^{-1}. This time is roughly the age of our universe. Finally, the mass of the universe m_U is the energy contained in a spatial region of radius l_U.

In fact, relations (3) and (4) are not independent. For an elementary particle governed by quantum mechanics, the typical effective size should be of the order of its Compton wavelength, $l_N \approx \hbar/(cm_N)$. It therefore suffices to explain, for instance, why $m_P m_N^{-1}$ is of order Ω.

Something similar happens with the scaling laws (5) and (6). Assuming homogeneity and isotropy, m_U is defined as $4\pi l_U^3 \rho_0^T/3$. Here, $\rho_0^T \equiv \rho_0 + c^2\Lambda/(8\pi G)$ is the total energy density. Hence, given the relation between l_U and l_P, formula (6) amounts to the approximate equality $\rho_0^T \approx \rho_0^C$, where $\rho_0^C \equiv 3H_0^2/(8\pi G)$ is the critical density of a FRW model at present. In a universe like ours, the scaling equation for m_U is thus a consequence of Eq. (5) and spatial flatness.

Examining relations (3)–(6), a length scale l_S of order Ω^2 in Planck units appears to be missing. Roughly, this scale corresponds to the size of stellar gravitational collapse determined by the Chandrasekhar limit (or any other similar mass limit) [22]. Actually, for such stellar-mass black holes, the formulas of the Schwarzschild radius and the Chandrasekhar mass [2] lead to

$$l_S \approx \Omega^2 l_P, \qquad m_S \approx \Omega^2 m_P. \tag{7}$$

At this stage of our discussion, the only scaling laws that remain unexplained are relations (4) and (5). In fact, one of these approximate identities can be viewed as the definition of Ω, e.g., the equation for l_U. The appearance of large numbers in our relations may then be understood, following Dirac [4], as a purely cosmological issue. Since H_0^{-1} is essentially the age of the universe, the fact that $\Omega \gg 1$ is just a consequence of the universe being so old. In addition, it is easy to check that, given formula (5), the scaling transformation for m_N is equivalent to Eq. (1). Therefore, the only coincidence of large numbers that needs explanation is the Eddington-Weinberg relation.

Suppose now that nucleons (or hadronic particles in general) can be described as elementary excitations of typical size l_N in an effective quantum theory. The number of physical degrees of freedom in a spatial region of volume V will be of the order of $3V/(4\pi l_N^3)$. In a cosmological setting, it seems natural to consider the Hubble radius as the largest size of the region in which such an effective quantum description of particles may exist, because it provides the scale of causal connection where the microphysical interactions take place. For a homogeneous and isotropic universe with negligible curvature, like the one we inhabit, the FRW equations imply that $8\pi G\rho_0 + c^2\Lambda \approx 3H_0^2$ [2]. Given the positivity of ρ_0, guaranteed by the dominant energy condition, the maximum Hubble radius is thus close to $\sqrt{3/\Lambda}$. For an almost flat FRW universe, the volume of the corresponding spatial region is nearly $4\pi\sqrt{3/\Lambda^3}$. As a consequence, the maximum number of observable degrees of freedom N in this kind of cosmological scenarios should roughly be $\sqrt{27/(\Lambda^3 l_N^6)}$. Taking into account the holographic N bound (2), we then conclude

$$l_N \approx (l_P^4 \Lambda^{-1})^{1/6}. \tag{8}$$

Using the relation $l_N m_N \approx l_P m_P$, which we have already justified, we immediately obtain

$$m_N^3 \approx m_P^3 (l_P^2 \Lambda)^{1/2}. \tag{9}$$

This approximate identity reproduces Eq. (1) provided that the present Hubble radius cH_0^{-1} is close to $\Lambda^{-1/2}$. Therefore, the so far unexplained Eddington-Weinberg relation can be understood from a holographic perspective, assuming an almost flat FRW cosmology, if and only if the cosmological constant has a nearly dominant contribution to the present energy density. This is ensured, e.g., by cosmic coincidence.

Note that the result $c^2\Lambda \approx H_0^2$ can be regarded as a partial solution to the cosmological constant problems (the value of Λ and cosmic coincidence) in our (quasi)flat universe if, adopting a different viewpoint, we take for granted Bousso's proposal and Eq. (1). Alternatively, if we use the Eddington-Weinberg relation and $c^2\Lambda \approx H_0^2$, the arguments given above about the relation between N and l_N allow us to reach an approximate version of the N bound for our spacetime. Thus, we see that in a nearly homogeneous, isotropic, and flat universe like ours, the cosmological constant problems, the N bound, and the coincidence of large numbers are interrelated.

In our application of the N bound, we have argued that the Hubble radius is the largest scale in which microphysics can act. Nonetheless, our conclusions would not have changed if, as proposed in Ref. [16] for cosmic holography, we had employed the cosmological apparent horizon instead of the Hubble radius, because they are approximately equal in quasiflat FRW models. We have also made use of the fact that, for this kind of model, the maximum Hubble radius is nearly $\sqrt{3/\Lambda}$ if Λ is positive. This is also the size of the cosmological horizon of the de Sitter space with the same value of Λ. In (almost) flat FRW cosmologies with a dominant Λ term at late times, a situation that apparently applies to our universe, any observer has a future event horizon that tends asymptotically to such a de Sitter horizon. Hence, our results would not have been altered had we replaced the maximum Hubble radius with the asymptotic event horizon in all our considerations.

The fact that the N bound provides an effective length scale for microphysics, given by Eq. (8), has played a central role in our arguments. This fact has allowed us to understand the origin of the Eddington-Weinberg relation. According to the explanation that we have put forward, such a relation does not hold at all times, but only when the cosmological constant dominates the energy density. Although we expect this condition to be satisfied at present and in the future, it excludes the early stages of the evolution of the universe. In our theoretical framework, the constants of nature G, $\hbar$, and c do not vary with time, and so we do not recover Dirac's cosmology [4].

In obtaining relation (8), we have actually supposed that the total number of degrees of freedom N available in the universe is roughly of the same order as the maximum number of degrees observable in its baryonic content. It should be clear that this assumption does not conflict with the fact that the present energy density is not dominated by baryonic matter. More importantly, since the number of baryonic degrees of freedom cannot exceed N, the quantity $(l_P^4\Lambda^{-1})^{1/6}$ provides, in any case, a lower bound to the typical size of nucleons l_N. Further discussion of this point will be presented elsewhere.

The length scale (8) has also been deduced by Ng, although replacing Λ^{-1} with the square of the observable radius of the universe [23]. However, he proposed to interpret l_N as the minimum resolution length in the presence of quantum gravitational fluctuations, instead of as the typical size of particles in the effective QFT that describes the baryonic content. From our viewpoint, this scale does not provide a fundamental length limiting the resolution of spacetime measurements, but rather restricts the number of degrees of freedom available in the effective QFT. Concerning the value of l_N, Ng proposes two ways to deduce it. In one of them, a spatial region is considered as a Salecker-Wigner clock able to discern distances larger than its Schwarzschild radius [23]. The question arises whether this interpretation is applicable to the observable universe, because its Schwarzschild and Hubble radii are of the same order of magnitude. The other line of reasoning employs holographic arguments related to those presented here. Nevertheless, since Ng uses the present size of the universe instead of $\Lambda^{-1/2}$, it is not clear whether the resolution scale that he obtains must be viewed as time independent.

Let us return to expression (5) for the present Hubble radius, which we have interpreted as the definition of Ω. We have argued that the fact that $\Omega \gg 1$ can be regarded as a consequence of the old age of the universe, which is a cosmological problem and not a numerical coincidence between microscopic and macroscopic parameters. Nonetheless, using the N bound and the present dominance of Λ, it is actually possible to explain the appearance of the large scale Ω along very similar lines to those proposed by Banks for the resolution of the cosmological constant problem [19]. As we have seen, when the energy density is nearly dominated by Λ, the Hubble radius is close to $\sqrt{3/\Lambda}$. In addition, the N bound implies that this latter length is equal to $l_P\sqrt{N \ln 2/\pi}$. Recalling Eq. (5), we then obtain

$$\Omega \approx N^{1/6}. \tag{10}$$

So Ω is a large number because our universe contains a huge amount of degrees of freedom. From this perspective, the value of Ω is fixed by N, which can be considered an input of the theory that describes our world.

Finally, we want to present some brief comments about the entropy of the universe. If the only entropic contribution is baryonic, we can estimate it as $S_b \approx n_N$. Here, we have supposed that each baryon has an associated entropy of order unity, and n_N is the Eddington number, which can be calculated as the ratio of the baryonic mass of the universe to the typical mass of a nucleon. In a rough approximation (valid for our estimation of orders of magnitude), we can identify the matter and the baryonic energy densities. Taking into account cosmic coincidence, we can then approximate n_N by $m_U m_N^{-1}$. In this way, we get $S_b \approx n_N \approx \Omega^4$. This is much less than the maximum allowed entropy, which, from relation (10) and the definition of N, is of the order of Ω^6. An intermediate entropic regime would be reached if the matter of the universe collapsed into stellar-mass black holes. As we have commented, this regime corresponds to the length scale $l_S \approx \Omega^2 l_P$. One can check that, in this case, the entropy would be $S_S \approx \Omega^5$. It is rather intriguing that S_S matches relatively well what seems to be the actual entropy of the universe, S_0. The main contribution to this entropy comes from supermassive black holes in galactic nuclei. Assuming that a typical galaxy contains $10^{11}-10^{12}$ stellar masses m_S and that its central black hole mass is $(10^6-10^7)m_S$, it is straightforward to find that $S_0 \approx (1-10^3)S_S$.

Summarizing, we have proved that, in the light of the holographic principle, the relations between large numbers constructed from microscopic and cosmological parameters are not independent of other fine-tuning and coincidence problems that have a purely cosmological nature. More explicitly, provided that the universe can be approximately described by a spatially homogenous, isotropic, and flat cosmological model and that the main contribution to the present energy density comes from the cosmological constant, it is possible to explain all the scaling relations that motivated Dirac's large number hypothesis by appealing exclusively to basic principles and to the N bound conjecture.

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