Minimal higher-dimensional extensions of the standard model and electroweak observables

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We consider minimal 5-dimensional extensions of the standard model compactified on an S^1/Z_2 orbifold, in which the $SU(2)_L$ and $U(1)_Y$ gauge fields and Higgs bosons may or may not all propagate in the fifth dimension while the observable matter is always assumed to be confined to a 4-dimensional subspace. We pay particular attention to consistently quantize the higher-dimensional models in the generalized R_ξ gauge and derive analytic expressions for the mass spectrum of the resulting Kaluza-Klein states and their couplings to matter. Based on recent data from electroweak precision tests, we improve previous limits obtained in the 5-dimensional standard model with a common compactification radius and extend our analysis to other possible 5-dimensional standard-model constructions. We find that the usually derived lower bound of ~ 4 TeV on a universal compactification scale may be considerably relaxed to ~ 3 TeV in a minimal scenario, in which the $SU(2)_L$ gauge boson is the only field that feels the presence of the fifth dimension.

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I. INTRODUCTION

String theory provides the only known theoretical framework within which gravity can be quantized and so undeniably plays a central role in our endeavours of unifying all fundamental forces of nature. A consistent quantum-mechanical formulation of a string theory requires the existence of additional dimensions beyond the four ones we experience in our everyday life. These new dimensions, however, must be sufficiently compact so as to escape detection. In the original string-theoretic considerations [1], the inverse length $1/R$ of the extra compact dimensions and the string mass M_s turned out to be closely tied to the 4-dimensional Planck mass $M_{\rm P}=1.9\times10^{16}$ TeV, with all involved mass scales being of the same order. More recent studies, however, have shown [2–6] that there could still be conceivable scenarios of a stringy nature where $1/R$ and M_s may be lowered independently of $M_{\rm P}$ by several or many orders of magnitude. In particular, Ref. [5] considers the radical possibility that M_s is of order TeV and represents the only fundamental scale in the universe at which unification of all forces of nature occurs. In this model, the compactification radius related to the higher-dimensional gravitational interactions lies in the submillimeter range, i.e., $1/R\lesssim10^{-3}$ eV, so Cavendish-type experiments may potentially test the model by observing deviations from Newton's law [5,7] at such small distances. The model also offers a wealth of phenomenological implications for high-energy colliders [8].

The above low string-scale framework could be embedded within, e.g., type I string theories [4], where the standard model (SM) may be described as an intersection of higher-dimensional Dp-branes [5,6,9]. As such intersections may naturally be higher dimensional, in addition to gravitons the SM gauge fields could also propagate independently within a higher-dimensional subspace, where the size of the new extra dimensions is of order TeV^{-1} for phenomenological reasons [11–16]. Since such low string-scale constructions may effectively result in different higher-dimensional extensions of the SM [9,10], the actual limits on the compactification radius are, to some extent, model dependent. Nevertheless, in the existing literature the derived phenomenological limits were obtained by assuming that the SM gauge fields propagate all freely in a common higher-dimensional space, in which the compactification radius is universal for all the extra dimensions.

In this paper we wish to lift the above restriction and extend the analysis to models which minimally depart from the assumption of a universal higher-dimensional scenario. Specifically, we will consider 5-dimensional extensions of the SM compactified on an S^1/Z_2 orbifold, where the $SU(2)_L$ and $U(1)_Y$ gauge bosons may not both live in the same higher-dimensional space, the so-called bulk. For example, one could imagine that the $SU(2)_L$ gauge field propagates in the bulk whilst the $U(1)_Y$ gauge boson is confined to our observable 4-dimensional subspace and vice versa. This observable 4-dimensional subspace is often termed 3-brane or simply brane and is localized at one of the two fixed points of the S^1/Z_2 orbifold, the boundary. In the aforementioned higher-dimensional scenarios, all SM fermions and the Higgs boson should necessarily be brane fields, such that an explicit breaking of the 4-dimensional gauge symmetry of the original (classical) Lagrangian is avoided.

Another issue of particular interest to us is related to our ability of consistently quantizing the higher-dimensional models under study in the so-called R_ξ gauge. In particular, it can be shown that higher-dimensional gauge-fixing conditions can always be found that reduce to the usual R_ξ gauge after the compact dimensions have been integrated out. Such a quantization procedure can be successfully applied to both Abelian and non-Abelian theories that include Higgs bosons living in the bulk and/or on the brane. The R_ξ gauge has the attractive theoretical feature that the unphysical sector decouples from the theory in the limit of the gauge-fixing parameter $\xi \to \infty$, thereby allowing for explicit checks of the gauge independence of physical observables, such as *S*-matrix elements.

After compactification of the extra dimensions, we obtain an effective 4-dimensional theory which is usually described by infinite towers of massive Kaluza-Klein (KK) states. In the 5-dimensional extensions of the SM under consideration, such infinite towers generically consist of KK excitations of the *W* boson, the *Z* boson, and the photon. Since the mass of the first excited KK state is typically set by the inverse of the compactification radius *R*, one expects that the KK effect on high-precision electroweak observables will become more significant for higher values of *R*. Thus, if all SM gauge bosons live in the bulk, compatibility of this model with the present electroweak data gives rise to a lower bound [13] of ~ 4 TeV on $1/R$ at the 2σ level.

On the other hand, the possibilities that the $\mathrm{SU}(2)_L$ gauge boson is a brane field with the $\mathrm{U}(1)_Y$ gauge boson living in the bulk and vice versa are phenomenologically even more challenging. In such cases, we find that the lower limit on the compactification scale $1/R$ can become significantly weaker, i.e., $1/R \gtrsim 3$ TeV. This new result emerges partially from the fact that some of the most constraining high-precision electroweak observables are getting differently affected by the presence of the KK states within these mixed brane-bulk scenaria. For example, the muon lifetime does not directly receive contributions from KK excitations if the *W* boson lives on the brane, but only indirectly when the analytic result is expressed in terms of the *Z*-boson mass in the context of our adopted renormalization scheme. Most interestingly, unlike in the frequently investigated model with all SM gauge fields in the bulk, other competitive observables, such as A_{FB}^b and A_{LR}^e [17], do now possess additional distinct analytic dependences on the compactification scale $1/R$ within these novel brane-bulk models. As a consequence, the results of the performed global-fit analysis become substantially different for these scenaria.

The paper is organized as follows. In Sec. II we consider a 5-dimensional Abelian model compactified on an S^1/Z_2 orbifold, in which the gauge field propagates in the bulk. The model is quantized by prescribing the proper higher-dimensional gauge-fixing condition which leads to the usual class of R_ξ gauges after the extra dimension has been integrated out. The same gauge-fixing procedure may successfully be implemented for Abelian models augmented by one Higgs boson which could either be a bulk or a brane field, or even for more general models with two Higgs bosons where the one Higgs boson can live on the brane and the other one in the bulk. In Sec. II we also present analytic expressions for the masses of the physical KK gauge bosons and for their mixings with the corresponding weak eigenstates. In Sec. III we extend our gauge-fixing procedure to a higher-dimensional non-Abelian theory and discuss the basic structure of the gauge sector after compactification. In Sec. IV we study 5-dimensional extensions of the SM, in which the $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ gauge fields and Higgs bosons may or may not all feel simultaneously the presence of the compact dimension while the fermionic matter is always assumed to be confined on the brane. In fact, we distinguish three cases: (i) both $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ gauge bosons are bulk fields, (ii) only the $\mathrm{U}(1)_Y$ gauge boson is a bulk field while the $\mathrm{SU}(2)_L$ one is a brane field, and (iii) only the $\mathrm{SU}(2)_L$ gauge boson resides in the bulk while the $\mathrm{U}(1)_Y$ one is restricted to the brane. Technical details of our study have been relegated to the Appendixes A and B. In Sec. V we perform a fully fledged global-fit analysis to the aforementioned 5-dimensional extensions of the SM, based on recent data on high-precision electroweak observables. Section VI summarizes our conclusions.

II. 5-DIMENSIONAL ABELIAN MODEL

To describe as well as motivate our higher-dimensional gauge-fixing quantization procedure, it is very instructive to consider first a simple Abelian 5-dimensional model, such as 5-dimensional (5D) quantum electrodynamics (QED) where the extra spatial dimension is compactified on an S^1/Z_2 orbifold. Then, we shall extend our quantization procedure to more general Abelian models with bulk and/or brane Higgs fields.

As a starting point, let us consider the 5D-QED Lagrangian given by

$$\mathcal{L}(x,y) = -\frac{1}{4} F_{MN}(x,y) F^{MN}(x,y) + \mathcal{L}_{\mathrm{GF}}(x,y) + \mathcal{L}_{\mathrm{FP}}(x,y), \tag{2.1}$$

where

$$F_{MN}(x,y) = \partial_M A_N(x,y) - \partial_N A_M(x,y) \tag{2.2}$$

denotes the 5-dimensional field strength tensor, and $\mathcal{L}_{\mathrm{GF}}(x,y)$ and $\mathcal{L}_{\mathrm{FP}}(x,y)$ are the gauge-fixing and the induced Faddeev-Popov ghost terms, respectively. In a 5-dimensional Abelian model, one may neglect the Faddeev-Popov ghost term $\mathcal{L}_{\mathrm{FP}}$ induced by $\mathcal{L}_{\mathrm{GF}}$, as the Abelian ghosts are noninteracting and hence they cannot occur in *S*-matrix elements. We shall return to this point in Sec. III, when discussing quantization of higher-dimensional non-Abelian theories.

Throughout the paper, Lorentz indices in 5 dimensions are denoted with capital Roman letters, e.g., $M,N = 0,1,2,3,5$, while the respective indices pertaining to the ordinary 4 dimensions are symbolized by Greek letters, e.g., $\mu,\nu = 0,1,2,3$. Furthermore, we use the abbreviations $x = (x^0, \vec{x})$ and $y = x^5$ to denote the coordinates of the usual $(1+3)$-dimensional Minkowski space and the coordinate of the fifth compact dimension, respectively.

In a 5-dimensional theory, the gauge-boson field A_M transforms as a vector under the Lorentz group SO(1,4). In the absence of the gauge-fixing and ghost terms $\mathcal{L}_{\mathrm{FP}}$ and $\mathcal{L}_{\mathrm{GF}}$ in Eq. (2.1), the 5D-QED Lagrangian is invariant under a U(1) gauge transformation

$$A_M(x,y)\to A_M(x,y)+\partial_M\Theta(x,y). \tag{2.3}$$

Being consistent with the above property of gauge symmetry, we can compactify the theory on an S^1/Z_2 orbifold, such that the following equalities are satisfied:

$$\begin{aligned}
A_M(x,y)&=A_M(x,y+2\pi R),\\
A_\mu(x,y)&=A_\mu(x,-y),\\
A_5(x,y)&=-A_5(x,-y),\\
\Theta(x,y)&=\Theta(x,y+2\pi R),\\
\Theta(x,y)&=\Theta(x,-y).
\end{aligned} \tag{2.4}$$

As we will see below, the fact that $A_\mu(x,y)$ is taken to be even under Z_2 results in the embedding of conventional QED with a massless photon into our 5D QED. Notice that all other constraints on the field $A_5(x,y)$ and the gauge parameter $\Theta(x,y)$ in Eq. (2.4) follow automatically if the theory is to remain gauge invariant after compactification.

Given the periodicity and reflection properties of A_M and Θ under y in Eq. (2.4), we can expand these quantities in a Fourier series as follows:

$$\begin{aligned}
A^\mu(x,y)&=\frac{1}{\sqrt{2\pi R}}A^\mu_{(0)}(x)+\sum_{n=1}^{\infty}\frac{1}{\sqrt{\pi R}}A^\mu_{(n)}(x)\cos\left(\frac{ny}{R}\right),\\
A^5(x,y)&=\sum_{n=1}^{\infty}\frac{1}{\sqrt{\pi R}}A^5_{(n)}(x)\sin\left(\frac{ny}{R}\right),\\
\Theta(x,y)&=\frac{1}{\sqrt{2\pi R}}\Theta_{(0)}(x)+\sum_{n=1}^{\infty}\frac{1}{\sqrt{\pi R}}\Theta_{(n)}(x)\cos\left(\frac{ny}{R}\right).
\end{aligned} \tag{2.5}$$

The Fourier coefficients $A^\mu_{(n)}(x)$, also called KK modes, turn out to be the mass eigenstates in 5D QED. However, this is not a generic feature of higher-dimensional models, namely, the Fourier modes cannot always be identified with the KK mass eigenstates. Below we will encounter examples, in which the Fourier modes will mix to form the KK mass eigenstates.

From Eqs. (2.3) and (2.5), one can now derive the corresponding gauge transformations for the KK modes [6]

$$\begin{aligned}
A_{(n)\mu}(x)&\to A_{(n)\mu}(x)+\partial_\mu\Theta_{(n)}(x),\\
A_{(n)5}(x)&\to A_{(n)5}(x)-\frac{n}{R}\Theta_{(n)}(x).
\end{aligned} \tag{2.6}$$

Integrating out the y dimension yields the effective 4-dimensional Lagrangian

$$\mathcal{L}(x)=-\frac{1}{4}F_{(0)\mu\nu}F^{\mu\nu}_{(0)}+\sum_{n=1}^{\infty}\left[-\frac{1}{4}F_{(n)\mu\nu}F^{\mu\nu}_{(n)}+\frac{1}{2}\left(\frac{n}{R}A_{(n)\mu}+\partial_\mu A_{(n)5}\right)\left(\frac{n}{R}A^\mu_{(n)}+\partial^\mu A_{(n)5}\right)\right]+\mathcal{L}_{\rm GF}(x), \tag{2.7}$$

where $\mathcal{L}_{\rm GF}(x)=\int_0^{2\pi R}dy\,\mathcal{L}_{\rm GF}(x,y)$. Note that the invariance of $\mathcal{L}(x)$ under the transformations (2.6) becomes manifest in the absence of the gauge-fixing term $\mathcal{L}_{\rm GF}(x,y)$.

In addition to the usual QED terms involving the massless field $A^\mu_{(0)}$, the other terms in the effective 4-dimensional Lagrangian (2.7) describe two infinite towers of massive vector excitations $A^\mu_{(n)}$ and (pseudo) scalar modes $A^5_{(n)}$ that mix with each other, for $n\geqslant 1$. The scalar modes $A^5_{(n)}$ play the role of the would-be Goldstone modes in a nonlinear realization of an Abelian Higgs model, in which the corresponding Higgs fields are taken to be infinitely massive.

As in usual Higgs models, one may seek for a higher-dimensional generalization of 't Hooft's gauge-fixing condition, for which the mixing terms bilinear in $A^\mu_{(n)}$ and $A^5_{(n)}$ are eliminated from the effective 4-dimensional Lagrangian (2.7). For instance, the covariant gauge-fixing term [6]

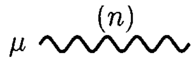
$$\mathcal{L}_{\rm GF}(x,y)=-\frac{1}{2\xi}(\partial_M A^M)^2 \tag{2.8}$$

does not lead to a complete cancellation of the bilinear operators $A^\mu_{(n)}\partial_\mu A^5_{(n)}$ in Eq. (2.7), with the exception of the Feynman gauge $\xi=1$. Taking, however, advantage of the fact that orbifold compactification generally breaks SO(1,4) invariance [18], one can abandon the requirement of covariance of the gauge fixing condition with respect to the extra dimension. In this context, we are free to choose the following noncovariant generalized R_ξ gauge:[1]

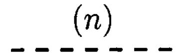
$$\mathcal{L}_{\rm GF}(x,y)=-\frac{1}{2\xi}(\partial^\mu A_\mu-\xi\,\partial_5 A_5)^2. \tag{2.9}$$

Nevertheless, the gauge-fixing term in Eq. (2.9) is still invariant under ordinary 4-dimensional Lorentz transformations. Upon integration over the extra dimension, it is not difficult to see that all mixing terms involving $A^\mu_{(n)}\partial_\mu A^5_{(n)}$ in Eq. (2.7) drop out up to irrelevant total derivatives. As a consequence, the propagators for the fields $A^\mu_{(n)}$ and $A^5_{(n)}$ take on their usual forms that describe massive gauge fields and their respective would-be Goldstone bosons of an ordinary 4-dimensional Abelian-Higgs model in the R_ξ gauge:

[1]For a related suggestion made recently, see Ref. [19].



$$\mu \overset{(n)}{\sim\sim\sim} \nu = \frac{i}{k^2-\left(\frac{n}{R}\right)^2}\left[-g^{\mu\nu} + \frac{(1-\xi)k^\mu k^\nu}{k^2-\xi\left(\frac{n}{R}\right)^2}\right] \tag{2.10}$$



$$\overset{(n)}{- - - - - - -} = \frac{i}{k^2-\xi\left(\frac{n}{R}\right)^2} \tag{2.11}$$

Therefore, we shall often refer to the $A^5_{(n)}$ fields as Goldstone modes, even though these KK modes do not directly result from a mechanism of spontaneous symmetry breaking in the usual sense.

Having defined the appropriate R_ξ gauge through the gauge-fixing term in Eq. (2.9), we can recover the usual unitary gauge in the limit $\xi\to\infty$. This limit is also equivalent to the gauge-fixing condition $A_5(x,y)=0$ or equivalently to $A^5_{(n)}(x)=0$, where all unphysical KK scalar modes are absent from the theory [20]. Thus, for the case at hand, we have seen how starting from a noncovariant higher-dimensional gauge-fixing condition, we can arrive at the known covariant 4-dimensional R_ξ gauge after compactification. As we will see below, the above quantization procedure can be extended to more elaborate higher-dimensional models that may include brane and/or bulk Higgs fields.

A. Abelian model with a bulk Higgs boson

Here, we shall discuss an extension of the Abelian model outlined above by adding a bulk Higgs scalar. The 5D Lagrangian of this theory reads

$$\mathcal{L}(x,y) = -\frac{1}{4}F^{MN}F_{MN} + (D_M\Phi)^*(D^M\Phi) - V(\Phi) + \mathcal{L}_{\rm GF}(x,y), \tag{2.12}$$

where $D_M=\partial_M + i\,e_5 A_M$ is the covariant derivative, e_5 denotes the 5-dimensional gauge coupling, and $\Phi(x,y)$ is the 5-dimensional complex scalar field

$$\Phi(x,y) = \frac{1}{\sqrt{2}}[h(x,y) + i\,\chi(x,y)] \tag{2.13}$$

that transforms under a U(1) gauge transformation as

$$\Phi(x,y) \to \exp[-i\,e_5\Theta(x,y)]\Phi(x,y). \tag{2.14}$$

In Eq. (2.12), the 5-dimensional Higgs potential is given by

$$V(\Phi) = \mu_5^2|\Phi|^2 + \lambda_5|\Phi|^4, \tag{2.15}$$

with $\lambda_5>0$.

After imposing the S^1/Z_2 compactification conditions $\Phi(x,y)=\Phi(x,y+2\pi R)$ and $\Phi(x,y)=\Phi(x,-y)$ on $\Phi(x,y)$, we can perform a Fourier decomposition of the scalar fields $h(x,y)$ and $\chi(x,y)$ in terms of cosines

$$h(x,y) = \frac{1}{\sqrt{2\pi R}}h_{(0)}(x) + \sum_{n=1}^{\infty}\frac{1}{\sqrt{\pi R}}h_{(n)}(x)\cos\left(\frac{ny}{R}\right),$$

$$\chi(x,y) = \frac{1}{\sqrt{2\pi R}}\chi_{(0)}(x) + \sum_{n=1}^{\infty}\frac{1}{\sqrt{\pi R}}\chi_{(n)}(x)\cos\left(\frac{ny}{R}\right). \tag{2.16}$$

As we will see below, our choice of an even Z_2 parity for the bulk Higgs scalar Φ ensures that the lowest lying KK modes describe a conventional 4-dimensional Abelian Higgs model. Instead, if Φ were odd under Z_2, this would not allow Yukawa interactions of the Higgs scalars with fermions localized on a brane $y=0$ and the generation of fermion masses through the Higgs mechanism would be impossible in this case.

Let us now turn our attention to the effective Higgs sector of our Abelian model. The effective 4-dimensional Lagrangian associated with the Higgs fields may conveniently be given by

$$\mathcal{L}_{\rm Higgs}(x) = \frac{1}{2}\sum_{n=0}^{\infty}\left[(\partial_\mu h_{(n)})(\partial^\mu h_{(n)}) - \frac{n^2}{R^2}h_{(n)}^2 - \mu^2 h_{(n)}^2 + (h\leftrightarrow\chi)\right] + \cdots, \tag{2.17}$$

where $\mu^2=\mu_5^2$ and the ellipses denote quartic interactions which involve the Higgs fields $h_{(n)}$ and $\chi_{(n)}$ and which all depend on $\lambda=\lambda_5/(2\pi R)>0$. In Eq. (2.17), the mass terms proportional to n^2/R^2 arise from compactifying the y-dimension. As in the usual 4-dimensional case, for $\mu^2<0$, the zero KK Higgs mode $\Phi_{(0)}=(h_{(0)}+i\chi_{(0)})/\sqrt{2}$ acquires a non-vanishing vacuum expectation value (VEV)

$$\langle\Phi_{(0)}\rangle = \frac{1}{\sqrt{2}}\langle h_{(0)}\rangle = \frac{v}{\sqrt{2}}, \tag{2.18}$$

which breaks the U(1) symmetry. Moreover, it can be shown that as long as the phenomenologically relevant condition $v<1/R$ is met, $h_{(0)}$ will be the only mode to receive a nonzero VEV, i.e., $v=\sqrt{|\mu|^2/\lambda}$.

After spontaneous symmetry breaking, the effective kinetic Lagrangian of the theory for the n-KK mode may be cast into the form

$$\mathcal{L}_{\rm kin}^{(n)}(x) = -\frac{1}{4}F_{(n)}^{\mu\nu}F_{(n)\mu\nu} + \frac{1}{2}\left(\frac{n^2}{R^2}+e^2v^2\right)A_{(n)\mu}A_{(n)}^{\mu} + \frac{1}{2}(\partial_\mu A_{(n)5})(\partial^\mu A_{(n)5}) + \frac{1}{2}(\partial_\mu \chi_{(n)})(\partial^\mu \chi_{(n)}) - \frac{1}{2}\left(\frac{n}{R}\chi_{(n)} - ev\,A_{(n)5}\right)^2 + A_{(n)}^{\mu}\partial_\mu\left(\frac{n}{R}A_{(n)5}+ev\,\chi_{(n)}\right) + \cdots, \qquad (2.19)$$

where $e = e_5/\sqrt{2\pi R}$ and the dots indicate the omission of bilinear terms involving $h_{(n)}$. From Eq. (2.19), it is evident that the mass spectrum of the zero KK modes is identical to that of a conventional Abelian Higgs model, i.e., $m_{A(0)} = ev$ and $m_{h(0)} = \sqrt{2\lambda}v$. This is so, because $A_{(0)5}$ is absent and we are left with the standard 4-dimensional terms only. To determine the complete mass spectrum for the higher KK modes, we first introduce the (pseudo)scalar KK modes $G_{(n)}$ and $a(n)$ through the orthogonal linear transformations

$$G_{(n)} = \left(\frac{n^2}{R^2}+e^2v^2\right)^{-1/2}\left(\frac{n}{R}A_{(n)5}+ev\,\chi_{(n)}\right),$$

$$a_{(n)} = \left(\frac{n^2}{R^2}+e^2v^2\right)^{-1/2}\left(ev\,A_{(n)5}-\frac{n}{R}\chi_{(n)}\right). \qquad (2.20)$$

Then, with the aid of Eq. (2.20), $\mathcal{L}_{\rm kin}^{(n)}$ in Eq. (2.19) can be rewritten in the more compact form

$$\mathcal{L}_{\rm kin}^{(n)}(x) = -\frac{1}{4}F_{(n)}^{\mu\nu}F_{(n)\mu\nu} + \frac{1}{2}(m_{A(n)}A_{(n)\mu}+\partial_\mu G_{(n)}) \times (m_{A(n)}A_{(n)}^{\mu}+\partial^\mu G_{(n)}) + \frac{1}{2}(\partial_\mu a_{(n)})(\partial^\mu a_{(n)}) - \frac{1}{2}m_{a(n)}^2 a_{(n)}^2 + \cdots, \qquad (2.21)$$

with $m_{A(n)}^2 = m_{a(n)}^2 = (n^2/R^2) + e^2v^2$. From this last expression for $\mathcal{L}_{\rm kin}^{(n)}$, we readily see that $G_{(n)}$ plays the role of a Goldstone mode in an Abelian Higgs model, while the pseudoscalar field $a_{(n)}$ describes a physical KK excitation degenerate in mass with the KK gauge mode $A_{(n)\mu}$. In particular, since the zero KK modes of the fields are expected to be much lighter than their first KK excitations, i.e., $ev \ll 1/R$, the masses of all higher n-KK gauge and Higgs modes are approximately $m_{(n)} = n/R$ and the Goldstone modes $G_{(n)}$ may almost be identified with $A_{(n)5}$, i.e., $G_{(n)} \approx A_{(n)5}$ as in 5D QED.

From the above discussion, it becomes now clear that the appropriate gauge-fixing Lagrangian in Eq. (2.12) for a 5-dimensional generalized R_ξ gauge should be

$$\mathcal{L}_{\rm GF}(x,y) = -\frac{1}{2\xi}\left[\partial_\mu A^\mu - \xi\left(\partial_5 A_5 + e_5\frac{v}{\sqrt{2\pi R}}\chi\right)\right]^2. \qquad (2.22)$$

Taking Eq. (2.22) into account, we arrive at the total effective kinetic Lagrangian

$$\mathcal{L}_{\rm kin}^{(n)}(x) = -\frac{1}{4}F_{(n)}^{\mu\nu}F_{(n)\mu\nu} + \frac{1}{2}m_{A(n)}^2 A_{(n)\mu}A_{(n)}^{\mu} - \frac{1}{2\xi}(\partial_\mu A_{(n)}^{\mu})^2 + \frac{1}{2}(\partial_\mu G_{(n)})(\partial^\mu G_{(n)}) - \frac{\xi}{2}m_{A(n)}^2 G_{(n)}^2 + \frac{1}{2}(\partial_\mu a_{(n)})(\partial^\mu a_{(n)}) - \frac{1}{2}m_{a(n)}^2 a_{(n)}^2 + \frac{1}{2}(\partial_\mu h_{(n)})(\partial^\mu h_{(n)}) - \frac{1}{2}m_{h(n)}^2 h_{(n)}^2. \qquad (2.23)$$

In the above, $m_{h(n)} = \sqrt{(n^2/R^2)+2\lambda v^2}$ are the KK Higgs boson masses and $m_{A(n)}$ and $m_{a(n)}$ are the KK masses of $A_{(n)}$ and $a_{(n)}$ given after Eq. (2.21). Observe finally that the limit $\xi \to \infty$ in Eq. (2.23) consistently corresponds to the unitary gauge.

B. Abelian model with a brane Higgs boson

A qualitatively different way of implementing the Higgs sector in a higher-dimensional Abelian model is to localize the Higgs field at the $y=0$ boundary of the S^1/Z_2 orbifold. The 5-dimensional Lagrangian of this theory reads

$$\mathcal{L}(x,y) = -\frac{1}{4}F^{MN}F_{MN} + \delta(y)[(D_\mu\Phi)^*(D^\mu\Phi) - V(\Phi)] + \mathcal{L}_{\rm GF}(x,y). \qquad (2.24)$$

Here, the covariant derivative $D_\mu = \partial_\mu + ie_5 A_\mu(x,y)$ and the Higgs potential $V(\Phi) = \mu^2|\Phi|^2 + \lambda|\Phi|^4$ have their familiar 4-dimensional forms, and the δ function $\delta(y)$ confines the Higgs sector on the brane $y=0$. Under a gauge transformation, the brane Higgs field $\Phi(x)$ transforms as

$$\Phi(x) \to \exp[-i\,e_5\Theta(x,0)]\Phi(x). \qquad (2.25)$$

Under Eq. (2.25) and the local transformation (2.3) of the gauge field $A_M(x,y)$, the theory exhibits U(1) invariance. Notice that the bulk scalar field $A_5(x,y)$ vanishes on the brane $y=0$ as a result of its odd Z_2 parity.

After compactification and integration over the y dimension, the effective Lagrangian of the model under discussion will be the sum of two terms: the effective Lagrangian (2.7) of 5D QED and the square bracket $[\cdots]_{y=0}$ in Eq. (2.24). Obviously, $\Phi = (h+i\chi)/\sqrt{2}$ being a brane field does not possess KK excitations and, for $\mu^2<0$ (with $\lambda>0$), acquires a VEV $\langle\Phi\rangle = \langle h\rangle/\sqrt{2} = v/\sqrt{2}$. After spontaneous symmetry breaking, masses are generated for all the KK gauge modes $A_{(n)}^{\mu}$. However, unlike in the Abelian model with a bulk

Higgs boson discussed in Sec. II A, the corresponding gauge-boson mass matrix here is no longer diagonal and has the form

$$M_A^2 = \begin{pmatrix} m^2 & \sqrt{2}m^2 & \sqrt{2}m^2 & \cdots \\ \sqrt{2}m^2 & 2m^2+(1/R)^2 & 2m^2 & \cdots \\ \sqrt{2}m^2 & 2m^2 & 2m^2+(2/R)^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \tag{2.26}$$

where $m^2 = e^2 v^2$ denotes the mass generated by the Higgs mechanism. The eigenvalues of M_A^2 follow from:

$$\det(M_A^2 - \lambda \mathrm{I}) = \left(\prod_{n=1}^{\infty} (n^2/R^2 - \lambda) \right) \times \left(m^2 - \lambda - 2\lambda m^2 \sum_{n=1}^{\infty} \frac{1}{(n/R)^2 - \lambda} \right) = 0. \tag{2.27}$$

Since $\lambda = (n/R)^2$ is not a solution as can be easily seen, the mass eigenvalues $m_{A(n)}$ are given by the zeros of the second big bracket in Eq. (2.27). This is equivalent to solving the transcendental equation

$$\sqrt{\lambda} = m_{(n)} = \pi\, m^2 R \cot(\pi\, m_{(n)} R), \tag{2.28}$$

with $m_{A(n)} = m_{(n)}$. The respective KK mass eigenstates $\hat{A}^{\mu}_{(n)}$ are given by

$$\hat{A}^{\mu}_{(n)} = \left(1 + \pi^2 m^2 R^2 + \frac{m_{(n)}^2}{m^2} \right)^{-1/2} \times \sum_{j=0}^{\infty} \frac{2 m_{(n)} m}{m_{(n)}^2 - (j/R)^2} \left(\frac{1}{\sqrt{2}} \right)^{\delta_{j,0}} A^{\mu}_{(j)}. \tag{2.29}$$

To find the appropriate form of the gauge-fixing term $\mathcal{L}_{\mathrm{GF}}(x,y)$ in Eq. (2.24), we follow Eq. (2.22), but restrict the scalar field χ to the brane $y=0$, viz.

$$\mathcal{L}_{\mathrm{GF}}(x,y) = -\frac{1}{2\xi} \{ \partial_\mu A^\mu - \xi [\partial_5 A_5 + e_5 v\, \chi\, \delta(y)] \}^2. \tag{2.30}$$

Then, the effective 4-dimensional gauge-fixing Lagrangian $\mathcal{L}_{\mathrm{GF}}(x)$ is given by

$$\mathcal{L}_{\mathrm{GF}}(x) = -\frac{1}{2\xi} (\partial_\mu A^\mu_{(0)})^2 - \frac{1}{2\xi} \sum_{n=1}^{\infty} \left(\partial_\mu A^\mu_{(n)} - \xi \frac{n}{R} A_{(n)5} \right)^2 + ev\, \chi (\partial_\mu A^\mu_{(0)}) + \sqrt{2}\, ev \chi \sum_{n=1}^{\infty} (\partial_\mu A^\mu_{(n)}) - \xi \sqrt{2} ev\, \chi \sum_{n=1}^{\infty} \frac{n}{R} A_{(n)5} - \frac{\xi}{2} e_5^2 v^2 \chi^2 \delta(0). \tag{2.31}$$

On the S^1/Z_2 orbifold, the δ function may be represented by

$$\delta(y) = \frac{1}{2\pi R} + \sum_{n=1}^{\infty} \frac{1}{\pi R} \cos\left(\frac{ny}{R} \right), \tag{2.32}$$

which implies

$$\delta(0) = \frac{1}{2\pi R} + \sum_{n=1}^{\infty} \frac{1}{\pi R}. \tag{2.33}$$

It is interesting to verify whether our 5-dimensional gauge-fixing term in Eq. (2.30) does consistently lead to the generalized R_ξ gauge after integration over the extra dimension. In doing so, we apply the R_ξ-gauge-fixing prescription individually to each KK gauge mode in the effective Lagrangian, instead of using Eq. (2.30). It is then not difficult to obtain

$$\mathcal{L}^{(n)}_{\mathrm{GF}}(x) = -\frac{1}{2\xi} \left[\partial_\mu A^\mu_{(n)} - \xi \left(\frac{n}{R} A_{(n)5} + \sqrt{2}^{(1-\delta_{n,0})} ev\, \chi \right) \right]^2. \tag{2.34}$$

This analytic result coincides with the one stated in Eq. (2.31), provided $e_5 = \sqrt{2\pi R}\, e$ and Eq. (2.33) are used. As is also expected from a generalized R_ξ gauge, all mixing terms of the gauge modes $A^{\mu}_{(n)}$ with $A_{(n)5}$ and χ disappear up to total derivatives. Hence, the eigenvalues $m_{(n)}$ as derived from Eq. (2.28) do represent the physical masses.

The unphysical mass spectrum of the Goldstone modes may be determined by diagonalizing the following ξ-dependent mass matrix of the fields χ and $A_{(n)5}$:

$$\mathcal{L}^{\xi}_{\mathrm{mass}}(x) = -\frac{\xi}{2} (\chi, A_{(1)5}, A_{(2)5}, \ldots) M_\xi^2 \begin{pmatrix} \chi \\ A_{(1)5} \\ A_{(2)5} \\ \vdots \end{pmatrix}, \tag{2.35}$$

with

$$M_\xi^2 = \begin{pmatrix} e^2 v^2 \left(1 + \sum_{n=1}^{\infty} 2 \right) & \sqrt{2}(1/R) ev & \sqrt{2}(2/R) ev & \cdots \\ \sqrt{2}(1/R) ev & (1/R)^2 & 0 & \cdots \\ \sqrt{2}(2/R) ev & 0 & (2/R)^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{2.36}$$

It can be shown that the characteristic polynomial of M_ξ^2 is formally identical to the one of M_A^2 given in Eq. (2.27):

$$\det(M_\xi^2 - \lambda \mathrm{I}) = \det(M_A^2 - \lambda \mathrm{I}). \tag{2.37}$$

Consequently, the mass eigenvalues of M_ξ^2 are given by $m_{(n)}$ in Eq. (2.28). Thus, as is expected from an R_ξ gauge, we find an one-to-one correspondence of each physical vector mode of mass $m_{(n)}$ to an unphysical Goldstone mode with gauge-dependent mass $\sqrt{\xi}\, m_{(n)}$. Moreover, the Goldstone mass eigenstates are given by

$$\hat{G}_{(n)} = \left(1 + \pi^2 m^2 R^2 + \frac{m_{(n)}^2}{m^2} \right)^{-1/2} \times \left(\sqrt{2}\chi + \sum_{j=1}^{\infty} \frac{2(j/R)m}{m_{(n)}^2 - (j/R)^2} A_{(j)5} \right). \tag{2.38}$$

In the unitary gauge $\xi \to \infty$, the fields $\hat{G}_{(n)}$, or equivalently the fields $A_{(n)5}$ and χ, are absent from the theory. Therefore, as opposed to the bulk-Higgs model of Sec. II A, the present brane-Higgs model does not predict other KK massive scalars apart from the physical Higgs boson h.

C. Abelian 2-Higgs model

It is now interesting to consider a model with two complex Higgs fields: one Higgs field $\Phi_1(x,y)$ propagating in the bulk and the other field $\Phi_2(x)$ localized on a brane at $y=0$. The 5-dimensional Lagrangian of this Abelian 2-Higgs model is given by

$$\mathcal{L}(x,y) = -\frac{1}{4} F^{MN} F_{MN} + (D_M \Phi_1)^* (D^M \Phi_1) + \delta(y) \times (D_\mu \Phi_2)^* (D^\mu \Phi_2) - V(\Phi_1, \Phi_2) + \mathcal{L}_{\mathrm{GF}}(x,y), \tag{2.39}$$

where V is the most general Higgs potential allowed by gauge invariance

$$\begin{aligned} V(\Phi_1,\Phi_2) = {} & \mu_1^2 (\Phi_1^\dagger \Phi_1) + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \delta(y) \bigg[\frac{1}{2} \mu_2^2 (\Phi_2^\dagger \Phi_2) \\ & + m_{12}^2 (\Phi_1^\dagger \Phi_2) + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \frac{1}{2} \lambda_3 (\Phi_1^\dagger \Phi_1) \\ & \times (\Phi_2^\dagger \Phi_2) + \frac{1}{2} \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\ & + \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \lambda_6 (\Phi_1^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2) \\ & + \lambda_7 (\Phi_2^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2) + \mathrm{H.c.} \bigg]. \end{aligned} \tag{2.40}$$

Note that all terms involving the brane field Φ_2 are multiplied by a δ function. Here, we shall restrict ourselves to a CP-conserving Higgs sector, i.e., the parameters m_{12}^2, λ_5, λ_6, and λ_7 in Eq. (2.40) are real. Furthermore, we assume that both complex scalar fields acquire real VEV's. Thus, we may linearly expand Φ_1 and Φ_2 around their VEV's as follows:

$$\Phi_1(x,y) = \frac{1}{\sqrt{2}} \left[\frac{v_1}{\sqrt{2\pi R}} + h_1(x,y) + i\,\chi_1(x,y) \right], \tag{2.41}$$

$$\Phi_2(x) = \frac{1}{\sqrt{2}} [v_2 + h_2(x) + i\,\chi_2(x)]. \tag{2.42}$$

Adopting the commonly used notation in 2-Higgs models, we define $v_1 = v\cos\beta$ and $v_2 = v\sin\beta$, i.e., $\tan\beta = v_2/v_1$.

In this 5-dimensional Abelian 2-Higgs model, the effective mass matrix M_A^2 of the Fourier modes $A_{(n)}^\mu$ is given by a sum of two matrices

$$M_A^2 = M_{\mathrm{brane}}^2 + M_{\mathrm{bulk}}^2. \tag{2.43}$$

The first matrix M_{brane}^2, which includes the KK masses, may be obtained by Eq. (2.26) after replacing $m^2 = e^2 v^2$ with $m^2 = e^2 v^2 \sin^2\beta$. The second matrix M_{bulk}^2 is proportional to unity, $M_{\mathrm{bulk}}^2 = e^2 v^2 \cos^2\beta\, \mathrm{I}$. Because of the particular structure of M_A^2 in this model, the mass eigenvalues of the KK gauge modes are given by

$$m_{A(n)}^2 = m_{(n)}^2 + \Delta m_{(n)}^2, \tag{2.44}$$

where $\Delta m_{(n)}^2 = e^2 v^2 \cos^2\beta$ and $m_{(n)}$ are the roots of the transcendental equation (2.28). The corresponding mass eigenstates $\hat{A}_{(n)}^\mu$ may in turn be determined by Eq. (2.29), after $m_{(n)}^2$ has been replaced with $m_{A(n)}^2 - \Delta m_{(n)}^2$.

Following a similar R_ξ-gauge-fixing prescription as above, we may eliminate the mixing terms between $A_{(n)}^\mu$ and the fields $A_{(n)5}$, $\chi_{1(n)}$ and χ_2 by choosing

$$\mathcal{L}_{\mathrm{GF}}(x,y) = -\frac{1}{2\xi} \left[\partial_\mu A^\mu - \xi \left(\partial_5 A_5 + e_5 \frac{v}{\sqrt{2\pi R}} \cos\beta\, \chi_1 + e_5 v \sin\beta\, \chi_2 \delta(y) \right) \right]^2. \tag{2.45}$$

In Appendix A, we show that the resulting Goldstone modes $\hat{G}_{(n)}$ in this model have masses $\sqrt{\xi} m_{A(n)}$. These Goldstone modes may be expressed in terms of the other pseudoscalar fields $A_{(n)5}$, $\chi_{1(n)}$ and χ_2 as follows:

$$\hat{G}_{(n)} = E_{\chi_2}^{(n)} \chi_2 + \sum_{j=0}^{\infty} \left(E_{\chi_{1(j)}}^{(n)} \chi_{1(j)} + E_{A_{(j)5}}^{(n)} A_{(j)5} \right), \tag{2.46}$$

with

$$E_{\chi_2}^{(n)} = \frac{1}{N}, \quad E_{A_{(j)5}}^{(n)} = -\frac{1}{N} \frac{\sqrt{2} e v \sin\beta (j/R)}{(j/R)^2 + e^2 v^2 \cos^2\beta - m_{A(n)}^2},$$

$$E^{(n)}_{\chi_{1(0)}} = -\frac{1}{N}\,\frac{e^2 v^2 \sin\beta\cos\beta}{e^2 v^2\cos^2\beta - m^2_{A(n)}},$$

$$E^{(n)}_{\chi_{1(j)}} = -\frac{1}{N}\,\frac{\sqrt{2}e^2 v^2 \sin\beta\cos\beta}{(j/R)^2 + e^2 v^2\cos^2\beta - m^2_{A(n)}}, \tag{2.47}$$

and

$$N^2 = \frac{1}{2}\,\frac{m^2_{A(n)}}{m^2_{A(n)} - e^2v^2\cos^2\beta}\Bigg(1+\pi^2 e^2 v^2 \sin^2\beta\, R^2 + \frac{m^2_{A(n)} - e^2v^2\cos^2\beta}{e^2v^2\sin^2\beta}\Bigg). \tag{2.48}$$

The masses of the lowest-lying KK Higgs scalars strongly depend on the details of the Higgs potential, whereas the masses of the higher n-KK Higgs modes are approximately n/R.

We conclude this section by remarking that even for the most general Abelian case, an appropriate higher-dimensional gauge-fixing condition analogous to Eq. (2.45) can always be found that leads after compactification to the usual R_ξ gauge as known from ordinary 4-dimensional theories. In the following, we shall see how the above gauge-fixing quantization procedure can be extended to non-Abelian models as well.

III. HIGHER-DIMENSIONAL NON-ABELIAN THEORY

In this section, we shall consider a pure non-Abelian theory, such as 5-dimensional Quantum Chromodynamics (5D QCD), without interactions to matter. The 5D-QCD Lagrangian takes on the simple general form

$$\mathcal{L}(x,y) = -\frac{1}{4}F^a_{MN}F^{aMN} + \mathcal{L}_{\rm GF} + \mathcal{L}_{\rm FP}, \tag{3.1}$$

where

$$F^a_{MN} = \partial_M A^a_N - \partial_N A^a_M + g_5 f^{abc} A^b_M A^c_N \tag{3.2}$$

and f^{abc} are the structure constants of the gauge group SU(N), with $N=3$ for 5D QCD. In Eq. (3.1), the gauge-fixing term $\mathcal{L}_{\rm GF}$ and the induced Faddeev-Popov Lagrangian $\mathcal{L}_{\rm FP}$ will be determined later in this section.

As we did for the Abelian case, we compactify each of the N gauge fields $A^a_M(x,y)$ separately on S^1/Z_2 through the constraints (2.4). Moreover, under a SU(N) gauge transformation, $A^a_M(x,y)$ transforms as

$$A^a_M(x,y) \to A^a_M(x,y) + \partial_M \Theta^a(x,y) - g_5 f^{abc}\Theta^b(x,y)A^c_M(x,y). \tag{3.3}$$

After a Fourier expansion of $A^a_\mu(x,y)$, $A^a_5(x,y)$, and $\Theta^a(x,y)$ according to Eq. (2.5), one finds that the local SU(N) transformation (3.3) amounts to [6]

$$A^a_{(0)\mu} \to A^a_{(0)\mu} + \partial_\mu \Theta^a_{(0)} - \frac{1}{2}\,\frac{g_5}{\sqrt{2\pi R}} f^{abc} \sum_{m=0}^{\infty} 2^{1-\delta_{m,0}}\Theta^b_{(m)} \times (1+\delta_{m,0})A^c_{(m)\mu},$$

$$A^a_{(n)\mu} \to A^a_{(n)\mu} + \partial_\mu \Theta^a_{(n)} - \frac{1}{2}\,\frac{g_5}{\sqrt{2\pi R}} f^{abc} \sum_{m=0}^{\infty} \sqrt{2}^{1-\delta_{m,0}}\Theta^b_{(m)} \times [\sqrt{2}^{-\delta_{m,n}}(1+\delta_{m,n})A^c_{(|m-n|)\mu} + A^c_{(m+n)\mu}],$$

$$A^a_{(n)5} \to A^a_{(n)5} - \frac{n}{R}\Theta^a_{(n)} - \frac{1}{2}\,\frac{g_5}{\sqrt{2\pi R}} f^{abc} \sum_{m=0}^{\infty} \sqrt{2}^{1-\delta_{m,0}}\Theta^b_{(m)} \times [\mathrm{sgn}(n-m)A^c_{(|m-n|)5} + A^c_{(m+n)5}], \tag{3.4}$$

where $n \geqslant 1$. As opposed to the Abelian case, the new feature here is that the KK modes can now mix with each other under a gauge transformation. As a result of this mixing, any attempt to truncate the theory at a given KK mode $n = n_{\rm trunc}$ will explicitly break gauge invariance.[2]

It is straightforward to generalize the gauge-fixing term of 5D QED given in Eq. (2.9) to the 5D-QCD case. The gauge-fixing term in 5D QCD is given by

$$\mathcal{L}_{\rm GF}(x,y) = -\frac{1}{2\xi}[F^a(A^a)]^2, \tag{3.5}$$

with

$$F^a(A^a) = \partial^\mu A^a_\mu - \xi\,\partial_5 A^a_5. \tag{3.6}$$

In this generalized R_ξ gauge, all mixing terms $A^a_{(n)\mu}\partial^\mu A^a_{(n)5}$ disappear, so the Fourier modes represent mass eigenstates. As in the Abelian case, the latter is spoiled by a Higgs mechanism involving brane interactions.

In non-Abelian theories, the R_ξ gauge induces an interacting ghost sector, which is described by the Faddeev-Popov Lagrangian

$$\mathcal{L}_{\rm FP}(x,y) = \overline{c}^a \frac{\delta F^a(A^a)}{\delta\Theta^b} c^b = \overline{c}^a[\partial^\mu(\partial_\mu\delta^{ab} - g_5 f^{abc}A^c_\mu) - \xi\,\partial_5(\partial_5\delta^{ab} - g_5 f^{abc}A^c_5)]c^b. \tag{3.7}$$

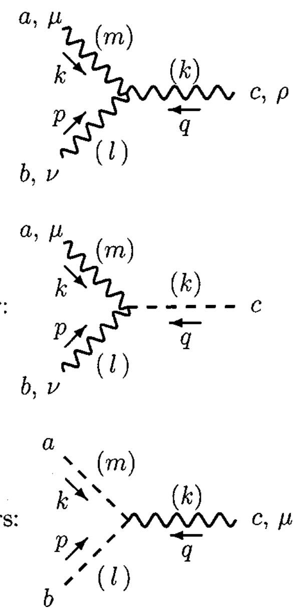
In the above, $c^a(x,y)$ denote the higher-dimensional ghost fields, which are even under Z_2: $c^a(x,y) = c^a(x,-y)$, i.e., they share the same transformation properties with the group parameters $\Theta^a(x,y)$.

[2] To overcome this difficulty, recent papers [21,22] suggested to match the truncated theory with a manifestly gauge-invariant non-Abelian chiral-type Lagrangian. Although the two theories agree well for $n \ll n_{\rm trunc}$, they have a significantly different mass spectrum close to the truncation energy scale, i.e., for KK modes $n \approx n_{\rm trunc}$.

3-boson vertex:

vertex with 1 scalar:

vertex with 2 scalars:



3-boson vertex: $g\left(\frac{1}{\sqrt{2}}\right)^{(\delta_{k,0}+\delta_{l,0}+\delta_{m,0}+1)}\delta_{k,l,m}\, f^{abc}\left[g^{\mu\nu}(k-p)^{\rho} + g^{\nu\rho}(p-q)^{\mu} + g^{\rho\mu}(q-k)^{\nu}\right]$

vertex with 1 scalar: $-i\,g\,f^{abc}g^{\mu\nu}\left[\left(\frac{m}{R}\right)\left(\frac{1}{\sqrt{2}}\right)^{(\delta_{l,0}+1)}\tilde{\delta}_{k,l,m} - \left(\frac{l}{R}\right)\left(\frac{1}{\sqrt{2}}\right)^{(\delta_{m,0}+1)}\tilde{\delta}_{k,m,l}\right]$

vertex with 2 scalars: $g\left(\frac{1}{\sqrt{2}}\right)^{(\delta_{k,0}+1)}\tilde{\delta}_{l,k,m}\, f^{abc}(p-k)^{\mu}$

FIG. 1. Feynman rules for the triple gauge boson coupling. $\delta_{k,l,m}$ and $\tilde{\delta}_{l,k,m}$ are defined in Eq. (3.8).

In Figs. 1 and 2, we exhibit the Feynman rules for the self-interactions of the KK modes $A^{a}_{(n)\mu}$ and $A^{a}_{(n)5}$ in the effective 4-dimensional theory in the R_{ξ} gauge (3.5). In the unitary gauge, i.e., $\xi\to\infty$, the 5D-QCD Feynman rules reduce to those presented in Ref. [23]. The factors $\delta_{k,l,m}$, $\tilde{\delta}_{l,k,m}$, $\delta_{k,l,m,n}$, and $\tilde{\delta}_{k,n,l,m}$ imply selection rules for the triple and quartic coupling of the KK modes $A^{a}_{(n)\mu}$ and $A^{a}_{(n)5}$, which are typical for the interactions between bulk fields. These factors are given by

$$\delta_{k,l,m} = \delta_{k+l+m,0} + \delta_{k+l-m,0} + \delta_{k-l+m,0} + \delta_{k-l-m,0},$$

$$\tilde{\delta}_{k,l,m} = -\delta_{k+l+m,0} + \delta_{k+l-m,0} - \delta_{k-l+m,0} + \delta_{k-l-m,0}, \tag{3.8}$$

for the triple gauge boson coupling and

$$\delta_{k,l,m,n} = +\delta_{k+l+m+n,0} + \delta_{k+l+m-n,0} + \delta_{k+l-m+n,0} + \delta_{k+l-m-n,0} + \delta_{k-l+m+n,0} + \delta_{k-l+m-n,0} + \delta_{k-l-m+n,0} + \delta_{k-l-m-n,0},$$

$$\tilde{\delta}_{k,l,m,n} = -\delta_{k+l+m+n,0} + \delta_{k+l+m-n,0} + \delta_{k+l-m+n,0} - \delta_{k+l-m-n,0} - \delta_{k-l+m+n,0} + \delta_{k-l+m-n,0} + \delta_{k-l-m+n,0} - \delta_{k-l-m-n,0}, \tag{3.9}$$

for the quartic gauge boson coupling.

IV. 5-DIMENSIONAL EXTENSIONS OF THE STANDARD MODEL

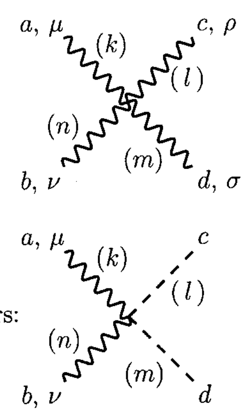
In this section we shall study minimal 5-dimensional extensions of the SM compactified on an S^1/Z_2 orbifold, in which the $SU(2)_L$ and $U(1)_Y$ gauge bosons as well as the Higgs doublets may not all propagate in the bulk. In all these higher-dimensional scenarios, we shall assume that the chiral fermions are localized on a brane at the $y=0$ fixed point of the S^1/Z_2 orbifold.

A. $SU(2)_L \otimes U(1)_Y$-bulk model

To start with, we shall first consider the most frequently investigated model, where all electroweak gauge fields propagate in the bulk and couple to both a brane and a bulk

4-boson vertex:

vertex with 2 scalars:



4-boson vertex: $-ig^{2}\delta_{k,l,m,n}\left(\frac{1}{\sqrt{2}}\right)^{(\delta_{k,0}+\delta_{l,0}+\delta_{m,0}+\delta_{n,0}+2)}\left[f^{ace}f^{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma})\right]$

vertex with 2 scalars: $i\,g^{2}\left(\frac{1}{\sqrt{2}}\right)^{(\delta_{k,0}+\delta_{n,0})}\tilde{\delta}_{k,n,l,m}\; 2\,g^{\mu\nu}\left[f^{ace}f^{bde} + f^{ade}f^{bce}\right]$

FIG. 2. Feynman rules for the quartic gauge boson coupling. $\delta_{k,l,m,n}$ and $\tilde{\delta}_{k,n,l,m}$ are defined in Eq. (3.9).

Higgs doublets. The Lagrangian of the gauge-Higgs sector of this higher-dimensional standard model (HDSM) is given by

$$\mathcal{L}(x,y) = -\frac{1}{4}B_{MN}B^{MN} - \frac{1}{4}F^a_{MN}F^{aMN} + (D_M\Phi_1)^\dagger(D^M\Phi_1) + \delta(y)(D_\mu\Phi_2)^\dagger(D^\mu\Phi_2) - V(\Phi_1,\Phi_2) + \mathcal{L}_{\rm GF}(x,y) + \mathcal{L}_{\rm FP}(x,y), \qquad (4.1)$$

where B_{MN} and F^a_{MN} $[a=1,2,3$ for SU(2)$]$ are the field strength tensors of the $\mathrm{U}(1)_Y$ and $\mathrm{SU}(2)_L$ gauge fields, respectively. As usual, we define the covariant derivative D_M as

$$D_M = \partial_M - i\, g_5 A^a_M \tau^a - i\frac{g'_5}{2}B_M . \qquad (4.2)$$

The Higgs potential $V(\Phi_1,\Phi_2)$ of this $\mathrm{SU}(2)_L\otimes\mathrm{U}(1)_Y$-bulk model has the very same analytic form as in Eq. (2.40), where $\Phi_1(x,y)$ is a bulk Higgs doublet and $\Phi_2(x)$ a brane one. After spontaneous symmetry breaking, the Higgs doublets will linearly be expanded about their VEVs, i.e.,

$$\Phi_1(x,y) = \begin{pmatrix} \chi_1^+ \\ \frac{1}{\sqrt{2}}\left(\frac{v_1}{\sqrt{2\pi R}} + h_1 + i\chi_1\right)\end{pmatrix},$$

$$\Phi_2(x) = \begin{pmatrix} \chi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + h_2 + i\chi_2)\end{pmatrix}. \qquad (4.3)$$

Here, we shall not repeat the calculational steps for determining the particle mass spectrum of the $\mathrm{SU}(2)_L\otimes\mathrm{U}(1)_Y$-bulk model, as they are analogous to those of the Abelian model discussed in Sec. II C (see also Appendix B). In fact, the above analogy in the derivation of the particle mass spectrum becomes rather explicit if the bulk gauge fields are written in terms of their higher-dimensional mass eigenstates

$$W^\pm_M = \frac{1}{\sqrt{2}}(A^1_M \mp i\, A^2_M),$$

$$Z_M = \frac{1}{\sqrt{g_5^2 + g_5'^2}}(g_5 A^3_M - g'_5 B_M), \qquad (4.4)$$

$$A_M = \frac{1}{\sqrt{g_5^2 + g_5'^2}}(g'_5 A^3_M + g_5 B_M).$$

Proceeding as in the Abelian case, we may easily determine the appropriate R_ξ-gauge-fixing functions for the $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ gauge bosons:

$$F^a(A^a) = \partial_\mu A^{a\mu} - \xi_A\Bigg[\partial_5 A^a_5 - i\frac{g_5}{\sqrt{2\pi R}}(\Phi_1^\dagger\tau^a\Phi_0 - \Phi_0^\dagger\tau^a\Phi_1)\cos\beta - i\, g_5(\Phi_2^\dagger\tau^a\Phi_0 - \Phi_0^\dagger\tau^a\Phi_2)\sin\beta\;\delta(y)\Bigg], \qquad (4.5)$$

$$F(B) = \partial_\mu B^\mu - \xi_B\Bigg[\partial_5 B_5 - i\frac{g'_5}{2\sqrt{2\pi R}}(\Phi_1^\dagger\Phi_0 - \Phi_0^\dagger\Phi_1)\cos\beta - i\frac{g'_5}{2}(\Phi_2^\dagger\Phi_0 - \Phi_0^\dagger\Phi_2)\sin\beta\;\delta(y)\Bigg], \qquad (4.6)$$

with

$$\Phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}, \qquad v = \sqrt{v_1^2 + v_2^2}. \qquad (4.7)$$

To avoid gauge-dependent photon-Z-mixing terms at the tree level, we will assume in the following that it is always $\xi_A = \xi_B = \xi$. Under this assumption, the gauge-fixing Lagrangian $\mathcal{L}_{\rm GF}(x,y)$ in Eq. (4.1) may be expressed in terms of the real gauge-fixing functions $F^a(A^a)$ and $F(B)$ as follows:

$$\mathcal{L}_{\rm GF}(x,y) = -\frac{1}{2\xi}[F^a(A^a)]^2 - \frac{1}{2\xi}[F(B)]^2. \qquad (4.8)$$

Furthermore, the Faddeev-Popov term $\mathcal{L}_{\rm FP}(x,y)$ in Eq. (4.1) is induced by the variations of $F^a(A^a)$ and $F(B)$ with respect to $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ gauge transformations. More explicitly, $\mathcal{L}_{\rm FP}(x,y)$ may be computed in the standard way by

$$\mathcal{L}_{\rm FP}(x,y) = \bar{c}^a\frac{\delta F^a(A^a)}{\delta\Theta^b}c^b + \bar{c}\frac{\delta F(B)}{\delta\Theta}c, \qquad (4.9)$$

where $c^a(x,y)$ and $c(x,y)$ are the 5-dimensional ghost fields associated with the $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ gauge groups, respectively. As in the 5D QCD, the ghost fields are even under Z_2.

In the above R_ξ-gauge-fixing prescription, the complete kinetic Lagrangian of the gauge sector written in terms of the fields defined in Eq. (4.4) becomes rather analogous to the corresponding one of the Abelian model investigated in Sec. II C. In Appendix B, we give the propagators of the KK gauge and Goldstone modes in the R_ξ gauge, together with the exact analytic results for the couplings of the gauge bosons to fermions to be discussed in Sec. IV D.

B. $\mathrm{SU}(2)_L$-brane, $\mathrm{U}(1)_Y$-bulk model

Let us now consider a new minimal 5-dimensional alternative to the SM, in which only the $\mathrm{U}(1)_Y$ gauge boson propagates in the bulk, while the $\mathrm{SU}(2)_L$ gauge field lives on the $y=0$ boundary of the S^1/Z_2 orbifold. The Lagrangian of this $SU(2)_L$-brane, $U(1)_Y$-bulk model is

$$\mathcal{L}(x,y)=-\frac{1}{4}B_{MN}B^{MN}+\delta(y)\bigg[-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}+(D_\mu\Phi)^\dagger(D^\mu\Phi)-V(\Phi)\bigg]+\mathcal{L}_{\rm GF}(x,y)+\mathcal{L}_{\rm FP}(x,y). \tag{4.10}$$

Observe that only a brane Higgs doublet

$$\Phi(x)=\begin{pmatrix}\chi_2^+\\ \frac{1}{\sqrt{2}}(v+h+i\chi)\end{pmatrix} \tag{4.11}$$

can be added in this model. The reason is that a bulk Higgs doublet would destroy the gauge invariance of the theory in the bulk if one coupled it to the covariant derivative $D_\mu=\partial_\mu-i\,g\,A^a_\mu(x)\tau^a-i(g_5'/2)B_\mu(x,0)$ on the $y=0$ brane. As a consequence, the Higgs potential of this model has the known SM form $V(\Phi)=\mu^2|\Phi|^2+\lambda|\Phi|^4$.

In the $SU(2)_L$-brane, $U(1)_Y$-bulk model, only the $B^\mu(x,y)$ boson has to be expanded in Fourier modes. Although the W-boson sector is completely standard, the neutral gauge sector gets complicated by the brane-bulk mixing of $B_\mu(x,y)$ with $A^3_\mu(x)$ through the VEV of the brane Higgs field $\Phi(x)$. To be more precise, we find the effective mass-matrix Lagrangian of the neutral gauge sector

$$\mathcal{L}^N_{\rm mass}(x)=\frac{1}{2}(A^{3\mu},B^\mu_{(0)},B^\mu_{(1)},\dots)M_N^2\begin{pmatrix}A^3_\mu\\ B_{(0)\mu}\\ B_{(1)\mu}\\ \vdots\end{pmatrix} \tag{4.12}$$

with

$$M_N^2=\begin{pmatrix} m^2\frac{g^2}{g'^2} & -m^2\frac{g}{g'} & -\sqrt{2}m^2\frac{g}{g'} & \cdots\\ -m^2\frac{g}{g'} & m^2 & \sqrt{2}m^2 & \cdots\\ -\sqrt{2}m^2\frac{g}{g'} & \sqrt{2}m^2 & 2m^2+(1/R)^2 & \cdots\\ \vdots & \vdots & \vdots & \ddots\end{pmatrix} \tag{4.13}$$

and $g_5'=g'\sqrt{2\pi R}$, $m^2=g'^2v^2/4$. The mass matrix M_N^2 contains a zero eigenvalue which corresponds to a massless photon $\hat{A}_\mu$, i.e.,

$$\hat{A}_\mu=s_wA^3_\mu+c_wB_{(0)\mu}, \tag{4.14}$$

where $s_w=\sqrt{1-c_w^2}=g'/\sqrt{g^2+g'^2}$ is the sine of the weak mixing angle. The other nonzero mass eigenvalues $m_{Z(n)}$ of M_N^2 in Eq. (4.13) may be determined by the roots of the transcendental equation

$$m_{Z(n)}=\pi\,m^2R\cot(\pi\,m_{Z(n)}R)+\frac{g^2}{g'^2}\frac{m^2}{m_{Z(n)}}. \tag{4.15}$$

The respective mass eigenstates are given by

$$\hat{Z}^\mu_{(n)}=\frac{1}{N}\Bigg[\frac{m_Z}{m_{Z(n)}}c_wA^{3\mu}-\sum_{j=0}^{\infty}\frac{\sqrt{2}m_{Z(n)}m_Z}{m^2_{Z(n)}-(j/R)^2}\left(\frac{1}{\sqrt{2}}\right)^{\delta_{j,0}}s_wB^\mu_{(j)}\Bigg], \tag{4.16}$$

where $m_Z=\sqrt{g^2+g'^2}v/2$,

$$N^2=\frac{1}{2}\left[\frac{c_w^2}{s_w^2}\left(\frac{m_Z^2}{m^2_{Z(n)}}-2\right)+s_w^2\pi^2m_Z^2R^2+\frac{m^2_{Z(n)}}{m_Z^2s_w^2}+1\right]. \tag{4.17}$$

Notice that the KK mass eigenmode $\hat{Z}_{(0)}$ has to be identified with the observable Z boson.

In analogy to the SM-bulk model, the appropriate R_ξ gauge-fixing functions for this brane-bulk model are written

$$F^a(A^a)=\partial_\mu A^{a\mu}+\xi\,ig(\Phi^\dagger\tau^a\Phi_0-\Phi_0^\dagger\tau^a\Phi), \tag{4.18}$$

$$F(B)=\partial_\mu B^\mu-\xi\left[\partial_5B_5-i\frac{g_5'}{2}(\Phi^\dagger\Phi_0-\Phi_0^\dagger\Phi)\,\delta(y)\right], \tag{4.19}$$

with

$$\Phi_0=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}. \tag{4.20}$$

Nevertheless, because of the specific brane-bulk structure of the higher-dimensional model, the corresponding gauge-fixing Lagrangian has now the form

$$\mathcal{L}_{\rm GF}(x,y)=-\frac{1}{2\xi}[F^a(A^a)]^2\delta(y)-\frac{1}{2\xi}[F(B)]^2. \tag{4.21}$$

Like the charged gauge sector, the charged scalar sector is completely standard in this model. The neutral scalar sector, however, has a structure very similar to the one of the Abelian model discussed in Sec. II B. Again, one can show the existence of an one-to-one correspondence between the KK gauge modes with mass $m_{Z(n)}$ and their associate would-be Goldstone modes with mass $\sqrt{\xi}m_{Z(n)}$. The latter KK modes are given by

$$\hat{G}^0_{(n)}=\frac{1}{N}\left(\chi-\frac{g'v}{\sqrt{2}}\sum_{j=1}^{\infty}\frac{j/R}{m^2_{Z(n)}-(j/R)^2}B_{(j)5}\right), \tag{4.22}$$

where the normalization factor N is defined in Eq. (4.17).

The Faddeev-Popov Lagrangian $\mathcal{L}_{\rm FP}$ can also be obtained in the standard fashion. Taking the brane-bulk structure of the model into account, we may determine $\mathcal{L}_{\rm FP}$ by

$$\mathcal{L}_{\rm FP}(x,y)=\overline{c^a}(x)\,\frac{\delta F^a[A^a(x)]}{\delta\Theta^b(x)}\,c^b(x)\,\delta(y)+\overline{c}(x,y)\frac{\delta F[B(x,y)]}{\delta\Theta(x,y)}c(x,y), \tag{4.23}$$

where the (x,y) dependence of the different quantities involved is explicitly indicated.

C. SU$(2)_L$-bulk, U$(1)_Y$-brane model

Another minimal 5-dimensional extension of the SM, complementary to the one discussed in Sec. IV B, emerges if the SU$(2)_L$ gauge boson is the only field that feels the presence of the fifth compact dimension. By analogy, the Lagrangian of this model reads

$$\mathcal{L}(x,y)=-\frac{1}{4}F^a_{MN}F^{aMN}+\delta(y)\bigg[-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}+(D_\mu\Phi)^\dagger(D^\mu\Phi)-V(\Phi)\bigg]+\mathcal{L}_{\rm GF}(x,y)+\mathcal{L}_{\rm FP}(x,y), \tag{4.24}$$

with $D_\mu=\partial_\mu-i\,g_5A^a_\mu(x,0)\,\tau^a-i(g'/2)B_\mu(x)$. As in the model discussed in the previous section, there is only one Higgs field on the brane $y=0$ and the Higgs potential is of the SM form. Because only the SU$(2)_L$ gauge boson lives in the bulk, the charged gauge sector of this higher-dimensional standard model is equivalent to that of the SM-bulk model discussed in Sec. IV A in the limit $\sin\beta\to 1$, i.e., only the Higgs field restricted to the brane $y=0$ acquires a nonvanishing VEV. Thus, the SU$(2)_L$-bulk, U$(1)_Y$-brane model predicts a KK tower of W-boson excitations, while the neutral gauge sector is quite analogous to the one discussed in the previous section. Specifically, the effective mass-matrix Lagrangian of the neutral gauge sector is given by

$$\mathcal{L}^N_{\rm mass}(x)=\frac{1}{2}(B^\mu,A^{3\mu}_{(0)},A^{3\mu}_{(1)},\ldots)M^2_N\begin{pmatrix}B_\mu\\ A^3_{(0)\mu}\\ A^3_{(1)\mu}\\ \vdots\end{pmatrix}, \tag{4.25}$$

with

$$M^2_N=\begin{pmatrix} m^2\dfrac{g'^2}{g^2} & -m^2\dfrac{g'}{g} & -\sqrt{2}m^2\dfrac{g'}{g} & \cdots\\ -m^2\dfrac{g'}{g} & m^2 & \sqrt{2}m^2 & \cdots\\ -\sqrt{2}m^2\dfrac{g'}{g} & \sqrt{2}m^2 & 2m^2+(1/R)^2 & \cdots\\ \vdots & \vdots & \vdots & \ddots\end{pmatrix}, \tag{4.26}$$

$g_5=g\sqrt{2\pi R}$ and $m^2=g^2v^2/4$. Again, we find that the zero KK mode given by the linear combination $\hat{A}_\mu=s_wA^3_{(0)\mu}+c_wB_\mu$ represents a massless vector field, the photon. The higher KK modes are massive and their masses may be obtained by the solutions of the transcendental equation

$$m_{Z(n)}=\pi\, m^2R\,\cot(\pi\, m_{Z(n)}R)+\frac{g'^2}{g^2}\,\frac{m^2}{m_{Z(n)}}. \tag{4.27}$$

The Z boson, denoted as $Z_{(0)}$, and its heavier KK mass eigenmodes may be conveniently expressed in terms of the gauge eigenstates as

$$\hat{Z}^\mu_{(n)}=\frac{1}{N}\left[\sum_{j=0}^{\infty}\frac{\sqrt{2}m_{Z(n)}m_Z}{m^2_{Z(n)}-(j/R)^2}\left(\frac{1}{\sqrt{2}}\right)^{\delta_{j,0}}c_wA^{3\mu}_{(j)}-\frac{m_Z}{m_{Z(n)}}s_wB^\mu\right], \tag{4.28}$$

where

$$N^2=\frac{1}{2}\left[\frac{s_w^2}{c_w^2}\left(\frac{m_Z^2}{m^2_{Z(n)}}-2\right)+c_w^2\pi^2m_Z^2R^2+\frac{m^2_{Z(n)}}{m_Z^2c_w^2}+1\right]. \tag{4.29}$$

In close analogy to the previous section, the higher-dimensional gauge-fixing functions leading to the generalized R_ξ gauge are given by

$$F^a(A^a)=\partial_\mu A^{a\mu}-\xi[\partial_5A^a_5-ig_5(\Phi^\dagger\tau^a\Phi_0-\Phi_0^\dagger\tau^a\Phi)\,\delta(y)], \tag{4.30}$$

$$F(B)=\partial_\mu B^\mu+\xi\, i\frac{g'}{2}(\Phi^\dagger\Phi_0-\Phi_0^\dagger\Phi), \tag{4.31}$$

giving rise to the gauge-fixing Lagrangian

$$\mathcal{L}_{\rm GF}(x,y)=-\frac{1}{2\xi}[F^a(A^a)]^2-\frac{1}{2\xi}[F(B)]^2\,\delta(y). \tag{4.32}$$

The charged scalar sector of this model is identical to that of the SM-bulk model of Sec. IV A, with the presence of a Higgs field on the $y=0$ boundary only. On the other hand, the neutral scalar sector predicts a KK tower of would-be Goldstone modes associated with the longitudinal polarization degrees of the massive KK gauge modes $\hat{Z}_{(n)}$. The would-be KK Goldstone modes are determined by

$$\hat{G}^0_{(n)}=\frac{1}{N}\left(\chi+\frac{gv}{\sqrt{2}}\sum_{j=1}^{\infty}\frac{j/R}{m^2_{Z(n)}-(j/R)^2}A^3_{(j)5}\right), \tag{4.33}$$

with N as defined in Eq. (4.29). The Faddeev-Popov Lagrangian can be calculated as in the model described earlier in Sec. IV B [see Eq. (4.23)], by considering the obvious modifications that take account of the complementary brane-bulk structure of the model.

D. Localization of fermions on the brane

In the minimal 5-dimensional extensions of the SM we have been studying, we have assumed that all the SM fermions are localized at the $y=0$ fixed point of the S^1/Z_2 orbifold. Therefore, upon integrating out the y dimension, both the effective kinetic terms of fermions and the effective Yukawa sector take on the usual 4-dimensional SM structure. Clearly, the SM fermions do not have KK modes. Under a gauge transformation, the left- and right-handed fermions transform according to

$$\Psi_L(x)\to\exp[ig_5\Theta^a(x,0)\tau^a+ig_5'Y^L\Theta(x,0)]\Psi_L(x),$$

$$\Psi_R(x)\to\exp[ig_5'Y^R\Theta(x,0)]\Psi_R(x). \tag{4.34}$$

The corresponding covariant derivatives that couple the chiral fermions to the gauge fields are given by

$$D_\mu^L=\partial_\mu-i\,g_5A_\mu^a\tau^a-i\,g_5'Y^LB_\mu,$$

$$D_\mu^R=\partial_\mu-i\,g_5'Y^RB_\mu. \tag{4.35}$$

It is obvious that the effective coupling of a fermion to a gauge boson restricted to the same brane $y=0$ has its SM value. On the other hand, the effective interaction Lagrangian describing the coupling of a fermion to a gauge boson living in the bulk has the generic form

$$\mathcal{L}_{\rm int}(x)=\bar{\Psi}\gamma^\mu(g_V+g_A\gamma^5)\Psi\left(A_{(0)\mu}+\sqrt{2}\sum_{n=1}^{\infty}A_{(n)\mu}\right). \tag{4.36}$$

Again, the coupling parameters g_V and g_A are set by the quantum numbers of the fermions and receive their SM values. Because the KK mass eigenmodes generally differ from the Fourier modes, their couplings to fermions $g_{V(n)}$ and $g_{A(n)}$ have to be calculated for each model individually by taking into account the appropriate weak-basis transformations. The precise values of $g_{V(n)}$ and $g_{A(n)}$ will be very important for our phenomenological discussion in the next section. The Feynman rules for the interactions of the KK gauge mass eigenmodes to fermions are exhibited in Appendix B.

Likewise, the Feynman rules for the interaction of the Goldstone modes to fermions can also be obtained from the SM Yukawa sector by relating the KK weak modes to the respective KK mass eigenmodes. It is worth remarking here that although the Z_2-odd fifth component of a bulk gauge boson A_M, A_5, does not couple directly to the brane fermions, A_5 is involved in fermionic couplings due to its mixing with the CP-odd Higgs fields which are even under Z_2. In particular, one can show that the resulting Goldstone couplings to fermions have the proper analytic structure to assure gauge invariance in the computation of S-matrix elements.

V. GLOBAL-FIT ANALYSIS

In this section, we shall evaluate the bounds on the compactification scale $1/R$ of minimal higher-dimensional extensions of the SM by analyzing a large number of high precision electroweak observables. To be specific, we proceed as follows. We relate the SM prediction $\mathcal{O}^{\rm SM}$ for an electroweak observable to the prediction $\mathcal{O}^{\rm HDSM}$ for the same observable obtained in the higher-dimensional SM under investigation through

$$\mathcal{O}^{\rm HDSM}=\mathcal{O}^{\rm SM}(1+\Delta_{\mathcal{O}}^{\rm HDSM}). \tag{5.1}$$

Here, $\Delta_{\mathcal{O}}^{\rm HDSM}$ is the tree-level modification of a given observable $\mathcal{O}$ from its SM value due to the presence of one extra dimension. To enable a direct comparison of our predictions with the electroweak precision data [17], we include SM radiative corrections to $\mathcal{O}^{\rm SM}$. However, we neglect SM- as well as KK-loop contributions to $\Delta_{\mathcal{O}}^{\rm HDSM}$ as higher order effects.

As input SM parameters for our theoretical predictions, we choose the most accurately measured ones, namely, the Z-boson mass M_Z, the electromagnetic fine structure constant α and the Fermi constant G_F. In all the 5-dimensional models under study, the tree-level Z-boson mass $m_{Z(0)}$ generally deviates from its SM form $m_Z=\sqrt{g^2+g'^2}v/2$. Therefore, we parametrize this deviation as follows:

$$m_{Z(0)}^2=m_Z^2(1+\Delta_ZX), \tag{5.2}$$

where

$$X=\frac{\pi^2}{3}\frac{m_Z^2}{M^2} \tag{5.3}$$

(with $M=1/R$) represents the typical scale factor quantifying the higher-dimensional effect and Δ_Z is a model-dependent parameter of order unity. Since the massless photon retains its SM properties through the entire process of compactification, the electromagnetic fine structure constant is still given by its SM value

$$\alpha=\frac{e^2}{4\pi}. \tag{5.4}$$

Instead, the Fermi constant G_F as determined by the muon lifetime may receive direct as well as indirect contributions due to KK states. We may account for this modification of G_F by writing

$$G_F=\frac{\pi\alpha}{\sqrt{2}s_w^2c_w^2m_{Z(0)}^2}(1+\Delta_GX), \tag{5.5}$$

where the order unity parameter Δ_G strongly depends on the details of the 5-dimensional model under consideration.

In the computation of the electroweak precision observables, it will be necessary to express the weak mixing angle θ_w in terms of the input parameters α, $m_{Z(0)}^2$, and G_F, by means of Eq. (5.5). In this respect, it is useful to define an effective weak mixing angle $\hat{\theta}_w$ using the tree-level SM relation

$$G_F = \frac{\pi\alpha}{\sqrt{2}\hat{s}_w^2\hat{c}_w^2 m_{Z(0)}^2}. \tag{5.6}$$

With the above definition for $\hat{\theta}_w$ and Eq. (5.5), we may relate the squared sines of the two weak mixing angles by

$$\hat{s}_w^2 = s_w^2(1+\Delta_\theta X). \tag{5.7}$$

Again, Δ_θ in Eq. (5.7) is a model-dependent parameter of order unity to be determined below.

A. $\mathbf{SU(2)_L \otimes U(1)_Y}$-bulk model

Before we present predictions for the electroweak observables in the $SU(2)_L\otimes U(1)_Y$-bulk model, let us first give the KK modifications for some of the fundamental parameters of the theory. The KK modifications of the Z- and W-boson masses are found to be

$$\Delta_Z = -s_\beta^4,$$

$$\Delta_W = -s_\beta^4\hat{c}_w^2, \tag{5.8}$$

where Δ_W is defined in analogy to Eq. (5.2). In Eq. (5.8) and in the following, we will often use the following short-hand notations for trigonometric functions: $s_x=\sin x$, $c_x=\cos x$, $s_{2x}=\sin 2x$, $c_{2x}=\cos 2x$.

KK effects also cause tree-level shifts to the W- and Z-boson gauge couplings. The physical gauge-boson couplings are given by

$$g_{W(0)} = g(1-s_\beta^2\hat{c}_w^2 X),$$

$$g_{Z(0)} = g(1-s_\beta^2 X). \tag{5.9}$$

These last two relations are approximate, i.e., they are obtained by expanding the exact analytic results for the masses and couplings, stated in Appendix B, to leading order in the parameter X defined in Eq. (5.3). Finally, the KK tree-level shift Δ_G of the Fermi constant G_F is

$$\Delta_G = \hat{c}_w^2\left(1-2s_\beta^2-\frac{\hat{s}_w^2}{\hat{c}_w^2}s_\beta^4\right), \tag{5.10}$$

which implies

$$\Delta_\theta = -\frac{\hat{c}_w^4}{\hat{c}_{2w}}\left(1-2s_\beta^2-\frac{\hat{s}_w^2}{\hat{c}_w^2}s_\beta^4\right). \tag{5.11}$$

Notice that Δ_θ determining the difference between s_w and $\hat{s}_w$ is a key parameter in the computation of many precision observables, as it additionally enters via the vector coupling of the Z boson.

In our calculations of the electroweak observables to leading order in X, we consistently use $m_{Z(n)}\approx m_{W(n)}\approx n/R$ and $g_{Z(n)}\approx g_{W(n)}\approx\sqrt{2}g$ for $n\geqslant 1$. Within this approximative framework, we compute the following high precision observables: the W-boson mass M_W, the Z-boson invisible width $\Gamma_Z(\nu\bar{\nu})$, Z-boson leptonic widths $\Gamma_Z(l^+l^-)$, the Z-boson hadronic width $\Gamma_Z(\text{had})$, the weak charge of cesium Q_W measuring atomic parity violation, various ratios R_l and R_q involving partial Z-boson widths and the Z-boson hadronic width, fermionic asymmetries A_f at the Z pole, and various fermionic forward-backward asymmetries $A_{\rm FB}^{(0,f)}$ at vanishing polarization. A complete list of the considered observables along with the SM predictions and their experimental values is given in Appendix C.

TABLE I. Predictions for $\Delta_{\mathcal{O}}^{\rm HDSM}/X$ in the $SU(2)_L\otimes U(1)_Y$-bulk model. The auxiliary parameters Δ_V, Δ_f, and Δ_h are defined in Eq. (5.12).

Observable	$\Delta_{\mathcal{O}}^{\rm HDSM}/X$
M_W	$\frac{1}{2}\left(s_\beta^4\hat{s}_w^2+\frac{\hat{s}_w^2}{\hat{c}_w^2}\Delta_\theta\right)$
$\Gamma_Z(\nu\bar{\nu})$	$\hat{s}_w^2(s_\beta^2-1)^2-1$
$\Gamma_Z(l^+l^-)$	$\hat{s}_w^2(s_\beta^2-1)^2-1+\Delta_l$
$\Gamma_Z(\text{had})$	$\hat{s}_w^2(s_\beta^2-1)^2-1+\Delta_h$
$Q_W(\text{Cs})$	$[(1-s_\beta^2)^2+4Z(Q_W^{\rm SM})^{-1}\Delta_\theta]\hat{s}_w^2$
R_l	$-\Delta_l+\Delta_h$
R_q	$\Delta_q-\Delta_h$
A_f	$\Delta_V-\Delta_f$
$A_{\rm FB}^{(0,f)}$	$\Delta_V-\Delta_f+f\leftrightarrow e$

In Table I, we present predictions for the parameter $\Delta_{\mathcal{O}}^{\rm HDSM}/X$ in the SM-bulk model, where $\Delta_{\mathcal{O}}^{\rm HDSM}$ and X are defined by Eqs. (5.1) and (5.3), respectively. Moreover, the auxiliary parameters that occur in Table I are given by

$$\Delta_V = \frac{4\,Q_f\hat{s}_w^2}{2T_{3f}-4Q_f\hat{s}_w^2}\Delta_\theta,$$

$$\Delta_f = \frac{8\,\hat{s}_w^2Q_f(2T_{3f}-4Q_f\hat{s}_w^2)}{(2T_{3f}-4Q_f\hat{s}_w^2)^2+(2T_{3f})^2}\Delta_\theta,$$

$$\Delta_h = \frac{8\,\hat{s}_w^2\sum_q Q_q(2T_{3q}-4Q_q\hat{s}_w^2)}{\sum_q[(2T_{3q}-4Q_q\hat{s}_w^2)^2+(2T_{3q})^2]}\Delta_\theta,$$

$$Q_W^{\rm SM} = (Z-N)-4\,Z\hat{s}_w^2, \tag{5.12}$$

where Q_f and T_{3f} are the electric charge and the third component of the weak isospin of a fermion f, respectively, $q=u,d,c,s,b$ and $N=78$ is the number of neutrons and $Z=55$ the number of protons in the cesium nucleus. In Eq. (5.12), the parameters Δ_V, Δ_f and Δ_h are all proportional to Δ_θ, since they arise from substituting s_w^2 by $\hat{s}_w^2$ into the different electroweak observables. In detail, Δ_V parameter-

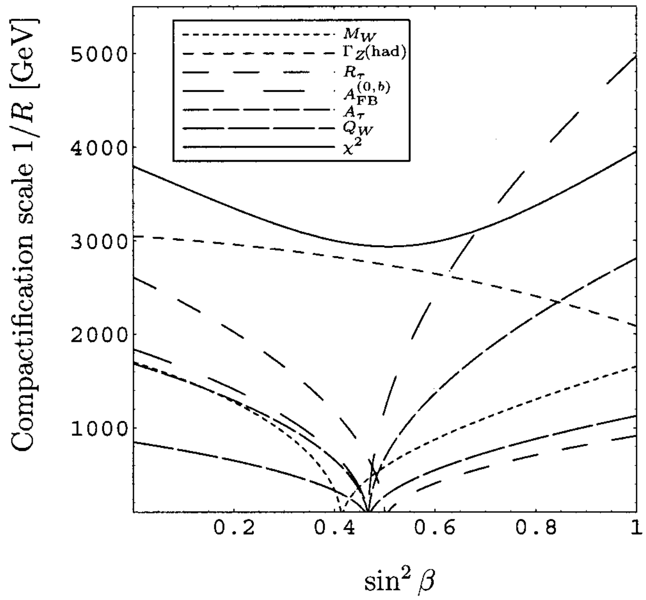


FIG. 3. Lower bounds on the compactification scale $M=1/R$ at the 3σ level from precision observables and a χ^2 analysis in the SM-bulk model.

izes the KK shift in the vector coupling of the Z boson to fermions. Δ_f results from an analogous KK shift in the sum of the squared vector and axial vector couplings for a given fermion f. Similarly, Δ_h gives the KK shift in the total hadronic width of the Z boson.

Employing the results of $\Delta_{\mathcal{O}}^{\mathrm{HDSM}}$ in Table I, we can compute the predictions for all the electroweak observables listed in Appendix C, by virtue of Eq. (5.1). We will confront these predictions with the respective experimental values, which are also listed in Appendix C. To do so, we perform a χ^2 test to obtain bounds on the compactification scale $M=1/R$ as a function of the bulk-brane angle $\sin\beta$. Thus, in our global-fit analysis (ignoring correlation effects between the observables to first approximation), a compactification radius is considered to be compatible at the $n\sigma$ confidence level (C.L.), if $\chi^2(R)-\chi^2_{\mathrm{min}}<n^2$, where

$$\chi^2(R)=\sum_i \frac{(\mathcal{O}_i^{\mathrm{exp}}-\mathcal{O}_i^{\mathrm{HDSM}})^2}{(\Delta\mathcal{O}_i)^2} \tag{5.13}$$

and χ^2_{min} is the minimum of χ^2 for a compactification radius in the physical region, i.e., for $R^2>0$. In Eq. (5.13), i runs over all the observables listed in Table VI in Appendix C. From this table, one easily sees that the total experimental and theoretical uncertainty $(\Delta\mathcal{O}_i)^2$ of an observable $\mathcal{O}_i$ is dominated by its experimental uncertainty.

Figure 3 shows lower bounds on the compactification scale $1/R$ coming from different types of observables as functions of $\sin^2\beta$, where we take into account only one observable at a time. In addition, Fig. 3 displays the result obtained by a global χ^2 fit. For a model dominated by a brane Higgs field ($\sin\beta=1$), the most stringent bound on $1/R$ is set by the forward-backward asymmetry involving b quarks, while for a bulk-Higgs dominated model (with $\sin\beta=0$) $1/R$ is most severely constrained by the hadronic Z-boson width. A global χ^2 analysis yields a lower bound on $1/R$ of about 4 TeV at the 3σ C.L., for the two limiting cases for which only one Higgs field that either lives in the bulk or on the $y=0$ brane has a nonvanishing VEV. The lower bound on $1/R$ may decrease to ~ 3 TeV, for a mixed brane-bulk Higgs scenario with $\sin^2\beta\sim 0.5$. This is so, because all the observables but the various Z-boson widths do not lead to useful lower limits on $1/R$ in the region of $\sin^2\beta\sim 0.5$.

B. $\mathrm{SU}(2)_L$-brane, $\mathrm{U}(1)_Y$-bulk model

Next we shall investigate the model, in which only the $\mathrm{U}(1)_Y$ gauge boson feels the presence of the extra dimension, whereas the $\mathrm{SU}(2)_L$ gauge boson is confined on the $y=0$ brane. In this case, we have

$$\Delta_Z=-\hat{s}_w^2, \qquad \Delta_W=0. \tag{5.14}$$

Obviously, the W-boson mass does not change by KK effects. However, the modification of the Z-boson coupling to fermions becomes more involved in this model. Specifically, KK effects induce nonfactorizable shifts both in the vector and axial part of the $Z\bar{f}f$ coupling, when the result is expressed in terms of the Z-boson mass eigenstate. To leading order in X, we can account for these new nonfactorizable modifications by parametrizing the $Z\bar{f}f$-coupling in terms of an effective electric charge $Q_{f(0)}$ and an effective third component of the weak isospin $T_{3(0)}$:

$$Q_{f(0)}=Q_f(1-X), \qquad T_{3f(0)}=T_{3f}(1-\hat{s}_w^2X), \tag{5.15}$$

with $Q_f=T_{3f}+Y_f$. The exact relations between $Q_{f(0)}$ and Q_f and between $T_{3f(0)}$ and T_{3f} are given in Appendix B.

Taking the above results into account, we find

$$\Delta_G=-\hat{s}_w^2 \tag{5.16}$$

and, thereby,

$$\Delta_\theta=\frac{\hat{s}_w^2\hat{c}_w^2}{\hat{c}_{2w}^2}. \tag{5.17}$$

The simplicity of the above results is a consequence of the fact that the charged gauge sector lives on the brane and hence is not affected by KK effects.

With the help of the new auxiliary parameters, we exhibit in Table II the tree-level KK shifts $\Delta_{\mathcal{O}}^{\mathrm{HDSM}}$ to the different electroweak observables. The parameters δ_V and δ_A give the KK modifications in the vector and axial-vector part of the $Z\bar{f}f$ coupling, except of the modifications which are purely due to the difference between θ_w and $\hat{\theta}_w$; i.e.,

TABLE II. Predictions for $\Delta_{\mathcal{O}}^{\rm HDSM}/X$ in the $SU(2)_L$-brane, $U(1)_Y$-bulk model. See text for the definition of the delta parameters.

Observable	$\Delta_{\mathcal{O}}^{\rm HDSM}/X$
M_W	$\frac{1}{2}(\hat{s}_w^2\hat{c}_w^2/\hat{c}_{2w})$
$\Gamma_Z(\nu\bar{\nu})$	$-\hat{s}_w^2$
$\Gamma_Z(l^+l^-)$	$\hat{s}_w^2+\Delta_l+\delta_l$
$\Gamma_Z(\text{had})$	$\hat{s}_w^2+\Delta_h+\delta_h$
$Q_W(\text{Cs})$	$4\,Z(Q_W^{\rm SM})^{-1}\hat{s}_w^2\Delta_\theta$
R_l	$-\Delta_l+\Delta_h-\delta_l+\delta_h$
R_q	$\Delta_q-\Delta_h+\delta_q-\delta_h$
A_f	$\Delta_V-\Delta_f-\delta_f+\delta_V+\delta_A$
$A_{\rm FB}^{(0,f)}$	$\Delta_V-\Delta_f-\delta_f+\delta_V+\delta_A+f\leftrightarrow e$

$$\delta_V=\frac{-2T_{3f}\hat{s}_w^2+4Q_f\hat{s}_w^2}{2T_{3f}-4Q_f\hat{s}_w^2},$$

$$\delta_A=-\hat{s}_w^2. \tag{5.18}$$

The parameter δ_f quantifies the KK shift in the sum of the squared vector and axial vector couplings of a given fermion f to the Z boson in this $SU(2)_L$-brane, $U(1)_Y$-bulk model. The parameter δ_f is given by

$$\delta_f=\frac{(-16T_{3f}^2+16T_{3f}Q_f)\hat{s}_w^2+(16T_{3f}Q_f-32Q_f^2)\hat{s}_w^4}{(2T_{3f}-4Q_f\hat{s}_w^2)^2+(2T_{3f})^2}. \tag{5.19}$$

In analogy with Δ_h, we finally define $(q=u,d,c,s,b)$

$$\delta_h=\frac{\sum_q[(-16T_{3q}^2+16T_{3q}Q_q)\hat{s}_w^2+(16T_{3q}Q_q-32Q_q^2)\hat{s}_w^4]}{\sum_q[(2T_{3q}-4Q_q\hat{s}_w^2)^2+(2T_{3q})^2]}. \tag{5.20}$$

Moreover, the parameters Δ_V, Δ_f, and Δ_h are defined in Eq. (5.12) with Δ_θ given by Eq. (5.17).

Following the procedure outlined in the previous section, we can now evaluate the lower bounds on the compactification scale $M=1/R$ in the $SU(2)_L$-brane, $U(1)_Y$-bulk model. In Table III, we display the lower limits on $1/R$ for each observable separately, together with that found by a global analysis. The most restrictive bound is obtained by the b-quark forward-backward asymmetry, giving rise to a lower limit on $1/R$ of ~ 4.4 TeV at the 3σ C.L. Finally, our global-fit analysis leads to the slightly less restrictive lower bound $1/R\gtrsim 3.5$ TeV.

C. $SU(2)_L$-bulk, $U(1)_Y$-brane model

Let us finally consider the complementary scenario, in which only the $SU(2)_L$ gauge boson propagates in the higher-dimensional space. In this case, the KK-mass shifts for the Z and W bosons are computed to be

TABLE III. Lower bounds (in TeV) on the compactification scale $1/R$ at the 3σ C.L. in models where either only the $U(1)_Y$ or only the $SU(2)_L$ gauge boson propagates in the higher-dimensional space.

Observable	$U(1)_Y$ in bulk	$SU(2)_L$ in bulk
M_W	1.2	1.2
$\Gamma_Z(\text{had})$	0.8	2.3
$Q_W(\text{Cs})$	0.4	0.8
$A_{\rm FB}^{(0,b)}$	4.4	2.4
A_τ	2.5	1.4
R_τ	1.0	0.5
global analysis	3.5	2.6

$$\Delta_Z=\Delta_W=-\hat{c}_w^2. \tag{5.21}$$

By analogy, the KK effects on the $Z\bar{f}f$ coupling can also be taken into account by introducing an effective third component of the weak isospin

$$T_{3f(0)}=T_{3f}(1-\hat{c}_w^2X). \tag{5.22}$$

Unlike in the model discussed in the previous section, the electric-charge term in the $Z\bar{f}f$-coupling remains unaffected by KK effects, i.e., $Q_{f(0)}=Q_f$. Thus, from the muon decay, we calculate

$$\Delta_G=-\hat{c}_w^2, \tag{5.23}$$

which leads to

$$\Delta_\theta=\frac{\hat{c}_w^4}{\hat{c}_{2w}^2}. \tag{5.24}$$

As in the previous section, we introduce the auxiliary parameters Δ_V, Δ_f, Δ_h, δ_V, δ_A, δ_f, and δ_h, which enables us to cast the tree-level KK shifts $\Delta_{\mathcal{O}}^{\rm HDSM}$ to the electroweak observables in Table IV. The meaning of these auxiliary parameters are the same as in Secs. V A and V B. In particular,

TABLE IV. Predictions for $\Delta_{\mathcal{O}}^{\rm HDSM}/X$ in the $SU(2)_L$-bulk, $U(1)_Y$-brane model. See text for the definition of the auxiliary parameters.

Observable	$\Delta_{\mathcal{O}}^{\rm HDSM}/X$
M_W	$\frac{1}{2}(\hat{s}_w^2\hat{c}_w^2/\hat{c}_{2w})$
$\Gamma_Z(\nu\bar{\nu})$	$-\hat{c}_w^2$
$\Gamma_Z(l^+l^-)$	$\hat{c}_w^2+\Delta_l+\delta_l$
$\Gamma_Z(\text{had})$	$\hat{c}_w^2+\Delta_h+\delta_h$
$Q_W(\text{Cs})$	$4\,Z(Q_W^{\rm SM})^{-1}\hat{s}_w^2\Delta_\theta$
R_l	$-\Delta_l+\Delta_h-\delta_l+\delta_h$
R_q	$\Delta_q-\Delta_h+\delta_q-\delta_h$
A_f	$\Delta_V-\Delta_f-\delta_f+\delta_V+\delta_A$
$A_{\rm FB}^{(0,f)}$	$\Delta_V-\Delta_f-\delta_f+\delta_V+\delta_A+f\leftrightarrow e$

TABLE V. Lower bounds (in TeV) on the compactification scale $1/R$ at 2σ, 3σ, and 5σ C.L.s.

Model	2σ	3σ	5σ
$SU(2)_L$-brane, $U(1)_Y$-bulk	4.3	3.5	2.7
$SU(2)_L$-bulk, $U(1)_Y$-brane	3.0	2.6	2.1
$SU(2)_L$-bulk, $U(1)_Y$-bulk (brane Higgs)	4.7	4.0	3.1
$SU(2)_L$-bulk, $U(1)_Y$-bulk (bulk Higgs)	4.6	3.8	3.0

Δ_V, Δ_f, Δ_h are given by Eq. (5.12), with Δ_θ in Eq. (5.24), while δ_V, δ_A, δ_f, and δ_h are, respectively, found to be ($q=u,d,c,s,b$)

$$\delta_V=-\frac{2T_{3f}\hat{c}_w^2}{2T_{3f}-4Q_f\hat{s}_w^2},$$

$$\delta_A=-\hat{c}_w^2,$$

$$\delta_f=\frac{(-16T_{3f}^2+16T_{3f}Q_f\hat{s}_w^2)\hat{c}_w^2}{(2T_{3f}-4Q_f\hat{s}_w^2)^2+(2T_{3f})^2},$$

$$\delta_h=\frac{\sum_q\,(-16T_{3q}^2+16T_{3q}Q_q\hat{s}_w^2)\hat{c}_w^2}{\sum_q\,[(2T_{3q}-4Q_q\hat{s}_w^2)^2+(2T_{3q})^2]}. \tag{5.25}$$

In Table III, we also present the lower bounds on $1/R$ for the different type of observables. In the present model, the b-quark forward-backward asymmetry offers the most stringent lower bound on the compactification scale as well: $1/R \gtrsim 2.4$ TeV at the 3σ C.L. Most interestingly, we observe that this lower bound on $1/R$ is much more relaxed than the one found in the previous models. The same observation applies to our global fit as well, i.e., a χ^2 analysis constrains the compactification scale $M=1/R$ to be higher than about 2.6 TeV at the 3σ C.L.

In Table V, we summarize the lower bounds on $1/R$ obtained by our global fits in the minimal higher-dimensional extensions of the SM under discussion. We find that the χ^2 values increase rapidly as the compactification scale decreases, such that the lower bounds on $1/R$ at higher confidence levels are relatively stable. Thus, from Table V, we see again that the lower bound on the compactification scale is the smallest in the $SU(2)_L$-bulk, $U(1)_Y$-brane model.

VI. CONCLUSIONS

We have studied new possible 5-dimensional extensions of the SM compactified on an S^1/Z_2 orbifold, in which the $SU(2)_L$ and $U(1)_Y$ gauge fields and Higgs bosons may or may not all experience the presence of the fifth dimension. Moreover, the fermions in these models are considered to be confined to one of the two boundaries of the S^1/Z_2 orbifold. We have paid special attention to consistently quantize the higher-dimensional models in the generalized R_ξ gauges. Specifically, we have been able to identify the appropriate higher-dimensional gauge-fixing conditions which should be imposed on the theories so as to yield the known R_ξ gauge after the fifth dimension has been integrated out. Based on the so-quantized effective Lagrangians, we have derived analytic expressions for the KK-mass spectrum of the gauge bosons and for their interactions to the fermionic matter.

The aforementioned analytic expressions have proven very essential to obtain accurate predictions for low-energy as well as high-energy electroweak observables measured at CERN e^+e^- collider LEP and SLAC linear collider (SLC). In particular, we have performed an extensive global-fit analysis of recent high-precision electroweak data to three different 5-dimensional extensions of the SM: (i) the $SU(2)_L \otimes U(1)_Y$-bulk model, where all SM gauge bosons are bulk fields, (ii) the $SU(2)_L$-brane, $U(1)_Y$-bulk model, where only the W bosons are restricted to the brane, and (iii) the $SU(2)_L$-bulk, $U(1)_Y$-brane model, where only the $U(1)_Y$ gauge field is confined to the brane. After carrying out a χ^2 test, we obtain different sensitivities to the compactification radius R for the above three models. For the often-discussed first model, we find the 2σ (3σ) lower bounds on $1/R$: $1/R \gtrsim 4.6$ (3.6) and 4.7 (4.0) TeV, for a Higgs boson living in the bulk and on the brane, respectively. For the second and third models, the corresponding 2σ (3σ) lower limits are 4.3 (3.5) and 3.0 (2.6) TeV. Consequently, we observe that the bounds on $1/R$ may be reduced by even up to 1 TeV, if the W bosons are the only fields that propagate in the bulk.

The analysis presented here involves a number of assumptions which are inherent in any nonstringy field-theoretic treatment of higher-dimensional theories. Although the results obtained in the higher-dimensional models with one compact dimension are convergent at the tree level, they become divergent if more than one extra dimensions are considered. Also, the analytic results are ultraviolet (UV) divergent at the quantum level, since the higher-dimensional theories are not renormalizable. Within a string-theoretic framework, the above UV divergences are expected to be regularized by the string mass scale M_s. Therefore, from an effective field-theory point of view, the phenomenological predictions will depend to some extend on the UV cutoff procedure [24] related to the string scale M_s. Nevertheless, assuming validity of perturbation theory, we expect that quantum corrections due to extra dimensions will not exceed the 10% level of the tree-level effects we have been studying here. Finally, we have ignored possible model-dependent winding-number contributions, which become relevant when the compactification scale $1/R$ and M_s turn out to be of comparable size [25].

The lower limits on the compactification scale derived by the present global analysis indicate that resonant production of the first KK state may only be accessed at the CERN Large Hadron Collider (LHC), at which heavy KK masses up to 6–7 TeV [9,15] might be explored. In particular, if the W bosons propagate in the bulk with a compactification radius $R \sim 3$ TeV^{-1}, one may still be able to probe resonant effects originating from the second KK state, and so differentiate the model from other 4-dimensional new-physics scenaria.

Note added. Shortly after completion of our paper, we became aware of Refs. [26] and [27]. The focus of these papers is the SM-bulk model, in which KK effects on high-energy scattering processes at LEP2 and other colliders were analyzed. In addition to being complementary by concentrating on high-precision electroweak observables, we have investigated new minimal higher-dimensional extensions of the SM, where the $SU(2)_L$ and $U(1)_Y$ gauge bosons may not both propagate in the higher-dimensional space. In particular, we find that the lower limits on $1/R$ may be substantially relaxed in one of these scenarios. Finally, we address the issue of a consistent quantization of the higher-dimensional field theory in the generalized R_ξ gauge.

Finally, after our paper had been communicated, Ref. [28] has appeared, which also discusses the R_ξ gauge before compactification in fermionless non-Abelian theories.

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APPENDIX A: GOLDSTONE MODES IN THE ABELIAN 2-HIGGS MODEL

In this appendix, we wish to show that the KK Goldstone modes given in Eq. (2.46) have the properties of true Goldstone particles as these are known from spontaneous symmetry breaking models. The higher-dimensional gauge-fixing Lagrangian in Eq. (2.45) induces at each KK level n the gauge fixing terms

$$\mathcal{L}_{\rm GF}^{(n)} = -\frac{1}{2\xi}\left[\partial_\mu A^\mu_{(n)} - \xi\left(\sqrt{\frac{n^2}{R^2}+e^2v^2\cos^2\beta}\,G_{(n)} + \sqrt{2}^{\,1-\delta_{n,0}}\,ev\sin\beta\,\chi_2\right)\right]^2, \tag{A1}$$

where the factor of $\sqrt{2}$ stems from the δ function [see Eq. (2.32)]. In the Abelian 2-Higgs model, the fields $G_{(n)}$ are defined analogously with Eq. (2.20) as

$$G_{(n)} = \left(\frac{n^2}{R^2}+e^2v^2\cos^2\beta\right)^{-1/2}\left(\frac{n}{R}A_{(n)5} + ev\cos\beta\,\chi_{1(n)}\right). \tag{A2}$$

Thus, the ξ-dependent mass terms of the scalar modes in the $\chi_2 G_{(n)}$ basis are given by

$$\mathcal{L}_{\rm mass}^{\xi}(x) = -\frac{\xi}{2}(\chi_2, G_{(0)}, G_{(1)}, \ldots)\, M_\xi^2 \begin{pmatrix}\chi_2\\ G_{(0)}\\ G_{(1)}\\ \vdots\end{pmatrix}, \tag{A3}$$

with

$$M_\xi^2 = \begin{pmatrix} e^2v^2\left(1+\sum_{n=1}^{\infty}2\right)\sin^2\beta & e^2v^2\sin\beta\cos\beta & \sqrt{2}evc_1\sin\beta & \cdots \\ e^2v^2\sin\beta\cos\beta & e^2v^2\cos^2\beta & 0 & \cdots \\ \sqrt{2}evc_1\sin\beta & 0 & (1/R)^2+e^2v^2\cos^2\beta & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{A4}$$

and $c_n = \sqrt{(n/R)^2+(ev\cos\beta)^2}$. The infinite sum in the upper left entry of M_ξ^2 is due to $\delta(0)$ according to Eq. (2.33). We expect that only the Goldstone modes of the theory acquire gauge-dependent masses coming from the gauge-fixing terms. Computing the characteristic polynomial of M_ξ^2, we find

$$\det(M_\xi^2 - \lambda \mathrm{I}) = -\lambda\,\det(M_A^2 - \lambda \mathrm{I}), \tag{A5}$$

where M_A^2 is the gauge-boson mass matrix given in Eq. (2.43). As a consequence, we may assign a Goldstone mass eigenstate $\hat{G}_{(n)}$ with mass $\sqrt{\xi}m_{A(n)}$ for each KK gauge eigenmode with mass $m_{A(n)}$. This constitutes a necessary condition in order to obtain gauge-invariant S-matrix elements within the R_ξ class of gauges. From Eq. (A5), we observe the existence of an additional degree of freedom which does not acquire a ξ-dependent mass with no correspondence to a KK gauge mode. This additional CP-odd scalar field will generally receive a gauge-independent mass that will entirely depend on the parameters of the Higgs potential. Additionally, it may mix with the other physical CP-odd states to form mass eigenstates (see discussion below).

On the other hand, in a consistent theory, the KK Goldstone modes should not acquire any gauge-independent mass term apart from their ξ-dependent mass mentioned above. In addition to the KK mass terms, the physical mass matrix of the KK scalar modes is determined by the Higgs kinetic terms in Eq. (2.39) and the Higgs potential (2.40). Since the CP-even Higgs modes do not mix with $A_{(n)5}$ in the CP-conserving case, the scalar mass matrix is block diagonal and we can concentrate on the CP-odd mass matrix $M^2_{CP\ \rm odd}$, as it appears in the original Lagrangian

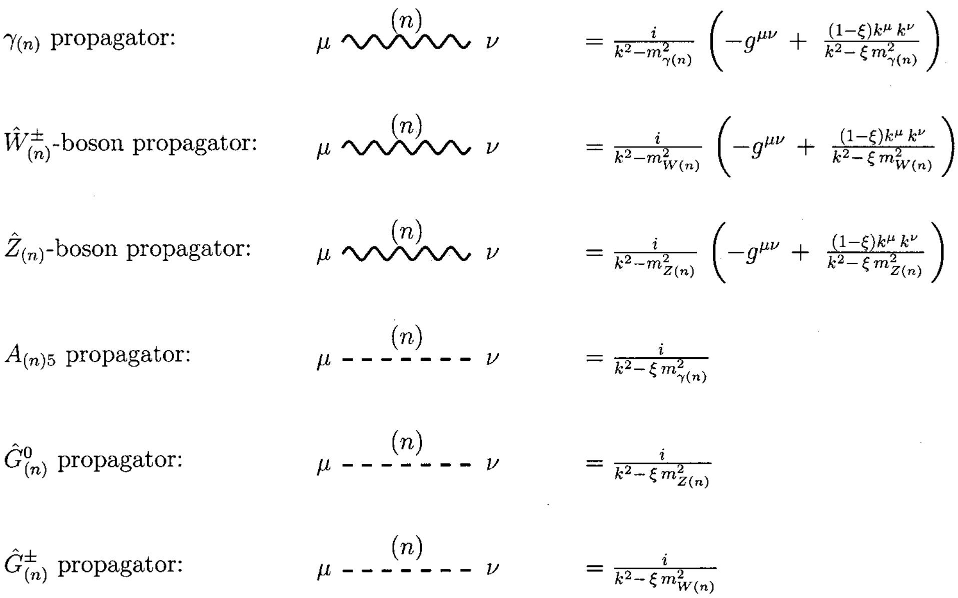

FIG. 4. KK gauge- and Goldstone-boson propagators in the 5-dimensional extensions of the SM in the generalized R_ξ gauge.

$$\mathcal{L}^{CP\text{ odd}}_{\text{mass}}(x,y)=-\frac{1}{2}(\chi_1,\chi_2)M^2_{CP\text{ odd}}\begin{pmatrix}\chi_1\\ \chi_2\end{pmatrix}, \tag{A6}$$

before integrating out the y dimension. After a straightforward computation from Eq. (2.40), this CP-odd mass matrix may be cast into the form

$$M^2_{CP\text{ odd}}=\delta(y)\begin{pmatrix}m^2_{\chi 11} & m^2_{\chi 12}\\ m^2_{\chi 12} & m^2_{\chi 22}\end{pmatrix}, \tag{A7}$$

where

$$m^2_{\chi 11}=-\tan\beta m^2_{12}+2v^2\sin^2\beta\,\lambda_5+\frac{1}{2}v^2\sin\beta\cos\beta\lambda_6 + \frac{1}{2}v^2\sin^2\beta\tan\beta\lambda_7. \tag{A8}$$

The other entries of the CP-odd mass matrix $M^2_{CP\text{ odd}}$ can be related to $m^2_{\chi 11}$ via

$$m^2_{\chi 22}=m^2_{\chi 11}/\tan^2\beta \quad\text{and}\quad m^2_{\chi 12}=m^2_{\chi 21}=-m^2_{\chi 11}/\tan\beta. \tag{A9}$$

In deriving Eqs. (A7), (A8), and (A9), we have made use of the minimization conditions on the Higgs potential, i.e., $\langle\partial V/\partial\Phi_i\rangle=0$, with $i=1,2$. In particular, the latter enabled us to cast the CP-odd mass matrix into the simple form of Eq. (A7), where all entries are proportional to an overall δ-function. Note that the absence of bulk mass terms originating from the Higgs potential is a characteristic of the CP-odd scalar sector of the model under consideration.

After integrating out the y dimension in Eq. (A7), we obtain the effective mass matrix for all the CP-odd KK modes $\chi_{1(n)}$, χ_2, and $A_{(n)5}$. From this effective CP-odd mass matrix including the KK mass terms, it is straightforward, although somehow tedious, to show that the would-be Goldstone modes (2.46) do not receive indeed any gauge-independent mass from the Higgs potential, whereas all physical CP-odd mass eigenstates should acquire high enough mass eigenvalues to avoid conflict with experimental data.

APPENDIX B: MASSES, COUPLINGS AND FEYNMAN RULES

Here, we shall present exact analytic results for the masses and the couplings of the KK gauge modes to fermions in the minimal 5-dimensional extensions of the SM discussed in Sec. IV.

To start with, we display in Fig. 4 the propagators for the KK gauge and Goldstone modes in the R_ξ gauge. In addition, the masses of the KK gauge bosons may be determined as follows

(i) $SU(2)_L\otimes U(1)_Y$-bulk model:

$$m_{\gamma(n)}=\frac{n}{R}, \tag{B1}$$

$$\sqrt{m^2_{W(n)}-m^2_W\cos^2\beta} =\pi m^2_W\sin^2\beta R\cot\left(\pi R\sqrt{m^2_{W(n)}-m^2_W\cos^2\beta}\right), \tag{B2}$$

$$\sqrt{m^2_{Z(n)}-m^2_Z\cos^2\beta} =\pi m^2_Z\sin^2\beta R\cot\left(\pi R\sqrt{m^2_{Z(n)}-m^2_Z\cos^2\beta}\right), \tag{B3}$$

where $n=0,1,2,\ldots$, $m_W=gv/2$ and $m_Z=\sqrt{g^2+g'^2}\,v/2$.

(ii) $SU(2)_L$-brane, $U(1)_Y$-bulk model:

$$m_{Z(n)}=\pi m^2_Z\sin^2\theta_w R\cot(\pi R m_{Z(n)})+\frac{m^2_Z}{m_{Z(n)}}\cos^2\theta_w. \tag{B4}$$

Note that there are no KK excitations for the photon and W boson in this model.

(iii) $SU(2)_L$-bulk, $U(1)_Y$-brane model:

$$m_{W(n)} = \pi m_W^2 R \cot(\pi R m_{W(n)}), \tag{B5}$$

$$m_{Z(n)} = \pi m_Z^2 \cos^2\theta_w R \cot(\pi R m_{Z(n)}) + \frac{m_Z^2}{m_{Z(n)}} \sin^2\theta_w . \tag{B6}$$

There are no KK excitations for the photon field in this model.

In the following, we will give the exact analytic expressions for the couplings of KK gauge bosons to fermions. To this end, we first define the following generic interaction Lagrangian

$$\mathcal{L}_{\rm int} = \sum_n g_{W(n)} (\hat{W}^+_{(n)\mu} J_W^{+\mu} + \hat{W}^-_{(n)\mu} J_W^{-\mu}) + \sum_n g_{Z(n)} \hat{Z}_{(n)\mu} J_Z^\mu + \sum_n e_{(n)} \hat{A}_{(n)\mu} J^\mu_{\rm EM}, \tag{B7}$$

with

$$J_W^{+\mu} = \frac{1}{2\sqrt{2}} [\overline{\nu_i} \gamma^\mu (1-\gamma^5) e_i + \overline{u_i} \gamma^\mu (1-\gamma^5) d_j V_{ij}],$$

$$J_Z^\mu = \frac{1}{4\cos\theta_w} \bar{f} \gamma^\mu [(2T_{3f(n)} - 4Q_{f(n)} \sin^2\theta_w) - 2T_{3f(n)} \gamma^5] f, \tag{B8}$$

$$J^\mu_{EM} = \bar{f} Q_f \gamma^\mu f$$

and $\nu_i = (\nu_e, \nu_\mu, \nu_\tau)$, $e_i = (e,\mu,\tau)$, $u_i = (u,c,t)$, and $d_i = (d,s,b)$. In addition, f denotes all the 12 SM fermions. After a basis transformation from the weak to the mass eigenstates, we obtain the following effective gauge and quantum couplings related to the three different higher-dimensional models ($n = 0,1,2,\ldots$).

(i) $SU(2)_L \otimes U(1)_Y$-bulk model:

$$e_{(0)} = e, \qquad e_{(n\geq 1)} = \sqrt{2} e,$$

$$g_{Z(n)} = \sqrt{2} g \left(1 + \frac{m_Z^2 \sin^2\beta}{m_{Z(n)}^2 - m_Z^2 \cos^2\beta} + \frac{\pi^2 m_Z^4 \sin^4\beta}{M^2 (m_{Z(n)}^2 - m_Z^2 \cos^2\beta)} \right)^{-1/2},$$

$$g_{W(n)} = \sqrt{2} g \left(1 + \frac{m_W^2 \sin^2\beta}{m_{W(n)}^2 - m_W^2 \cos^2\beta} + \frac{\pi^2 m_W^4 \sin^4\beta}{M^2 (m_{W(n)}^2 - m_W^2 \cos^2\beta)} \right)^{-1/2}, \tag{B9}$$

$$T_{3f(n)} = T_{3f}, \qquad Q_{f(n)} = Q_f,$$

with $M = 1/R$.

(ii) $SU(2)_L$-brane, $U(1)_Y$-bulk model:

$$g_{Z(n)} = g,$$

$$T_{3f(n)} = \frac{T_{3f}}{c_w} \frac{m_{Z(n)}^2}{m_Z^2} \left[\frac{1}{s_w^2} \left(\frac{1}{2} - \frac{m_{Z(n)}^2}{m_Z^2} \right) + \frac{s_w^2}{2c_w^2} \times \left(\pi^2 \frac{m_{Z(n)}^2}{M^2} + \frac{m_{Z(n)}^2}{m_Z^2 s_w^2} + \frac{m_{Z(n)}^4}{m_Z^4 s_w^4} \right) \right]^{-1/2},$$

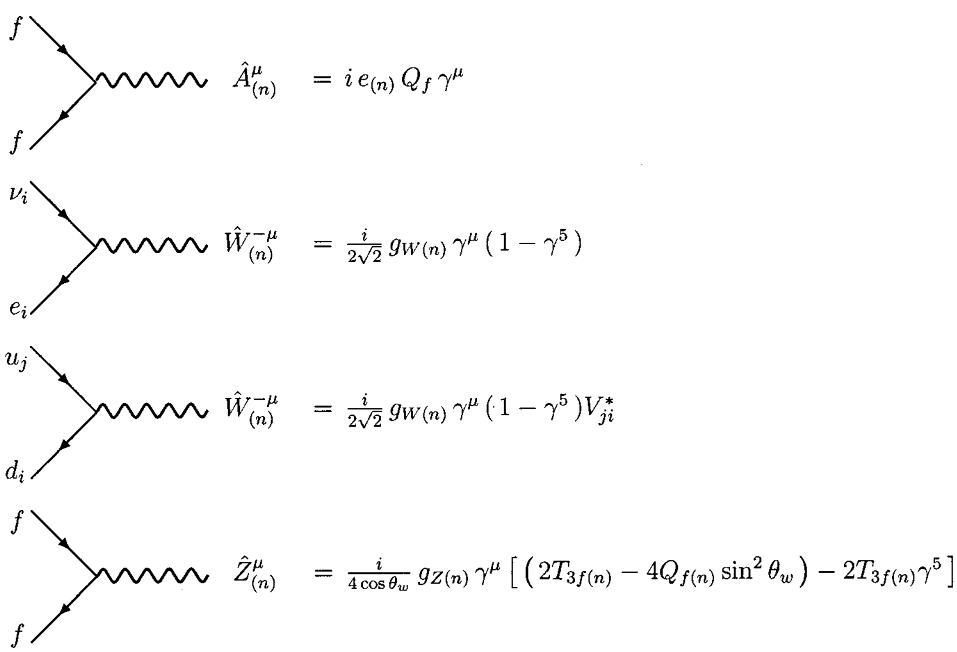

FIG. 5. Feynman rules for couplings of the KK gauge bosons to fermions in the minimal 5-dimensional extensions of the SM.

$$Q_{f(n)}=\frac{Q_f}{c_w}\left(\frac{m_{Z(n)}^2}{m_Z^2 s_w^2}-\frac{c_w^2}{s_w^2}\right)\left[\frac{1}{s_w^2}\left(\frac{1}{2}-\frac{m_{Z(n)}^2}{m_Z^2}\right)+\frac{s_w^2}{2c_w^2}\left(\pi^2\frac{m_{Z(n)}^2}{M^2}+\frac{m_{Z(n)}^2}{m_Z^2 s_w^2}+\frac{m_{Z(n)}^4}{m_Z^4 s_w^4}\right)\right]^{-1/2}. \quad \text{(B10)}$$

(iii) $SU(2)_L$-bulk, $U(1)_Y$-brane model:

$$g_{Z(n)}=g,\quad g_{W(n)}=\sqrt{2}g\left(1+\frac{m_W^2}{m_{W(n)}^2}+\frac{\pi^2 m_W^4}{M^2 m_{W(n)}^2}\right)^{-1/2},$$

$$T_{3f(n)}=\frac{T_{3f}}{s_w}\frac{m_{Z(n)}^2}{m_Z^2}\left[\frac{1}{c_w^2}\left(\frac{1}{2}-\frac{m_{Z(n)}^2}{m_Z^2}\right)+\frac{c_w^2}{2s_w^2}\left(\pi^2\frac{m_{Z(n)}^2}{M^2}+\frac{m_{Z(n)}^2}{m_Z^2 c_w^2}+\frac{m_{Z(n)}^4}{m_Z^4 c_w^4}\right)\right]^{-1/2}, \quad \text{(B11)}$$

$$Q_{f(n)}=\frac{Q_f}{s_w}\left[\frac{1}{c_w^2}\left(\frac{1}{2}-\frac{m_{Z(n)}^2}{m_Z^2}\right)+\frac{c_w^2}{2s_w^2}\left(\pi^2\frac{m_{Z(n)}^2}{M^2}+\frac{m_{Z(n)}^2}{m_Z^2 c_w^2}+\frac{m_{Z(n)}^4}{m_Z^4 c_w^4}\right)\right]^{-1/2}.$$

In Fig. 5 we display the Feynman rules for the couplings of the KK gauge bosons to fermions that pertain to the above minimal 5-dimensional extensions of the SM.

APPENDIX C: INPUT PARAMETERS, OBSERVABLES, AND SM PREDICTIONS

In this appendix, we list the numerical values of the input parameters and electroweak observables, along with their SM predictions. These numerical values were used in Sec. V to constrain the parameters of the 5-dimensional models.

As input parameters for our theoretical predictions, we use the most accurately determined ones, namely the Fermi constant G_F measured in muon decay, the fine structure constant α determined by the quantum Hall effect and the Z-boson mass M_Z [17]:

$$G_F=1.16637(1)\times 10^{-5}\ \text{GeV}^{-2},$$
$$\alpha=1/137.0359895(61), \quad \text{(C1)}$$
$$M_Z=91.1872(21)\ \text{GeV},$$

where the numbers in parentheses indicate the 1σ uncertainties.

TABLE VI. Precision measurements and the corresponding SM predictions for all observables considered in our analysis (notation as in Ref. [17]).

Observable	Exp. value ($\mathcal{O}^{\text{EXP}}$)	SM prediction ($\mathcal{O}^{\text{SM}}$)
M_W	80.448(62) GeV	80.378(20) GeV
Γ_Z(had)	1.7439(20) GeV	1.7422(15) GeV
$\Gamma_Z(l^+l^-)$	83.96(9) MeV	84.00(3) MeV
$\Gamma_Z(\nu\bar{\nu})$	498.8(15) MeV	501.65(15) MeV
Q_W(Cs)	−72.06(44)	−73.09(03)
R_e	20.803(49)	20.740(18)
R_μ	20.786(33)	20.741(18)
R_τ	20.764(45)	20.786(18)
R_b	0.21642(73)	0.2158(2)
R_c	0.1674(38)	0.1723(1)
A_e	0.15108(218)	0.1475(13)
A_μ	0.137(16)	0.1475(13)
A_τ	0.1425(44)	0.1475(13)
A_b	0.911(25)	0.9348(1)
A_c	0.630(26)	0.6679(6)
A_s	0.85(9)	0.9357(1)
$A_{\text{FB}}^{(0,e)}$	0.0145(24)	0.0163(3)
$A_{\text{FB}}^{(0,\mu)}$	0.0167(13)	0.0163(3)
$A_{\text{FB}}^{(0,\tau)}$	0.0188(17)	0.0163(3)
$A_{\text{FB}}^{(0,b)}$	0.0988(20)	0.1034(9)
$A_{\text{FB}}^{(0,c)}$	0.0692(37)	0.0739(7)
$A_{\text{FB}}^{(0,s)}$	0.0976(114)	0.1035(9)

Given the above input parameters, predictions can be made for a number of high-precision observables within the SM framework. The results of these predictions may be found in Ref. [17], together with experimental values of the observables. For the reader's convenience, the actual values taken into account in our analysis are also listed in Table VI. The theoretical values in this table are obtained by assuming a light SM Higgs boson.

As was already discussed in Sec. V, we introduce an effective weak mixing angle $\hat{\theta}_w$ by enforcing the tree-level SM relation

$$G_F=\frac{\pi\alpha}{\sqrt{2}\sin^2\hat{\theta}_w\cos^2\hat{\theta}_w M_Z^2}. \quad \text{(C2)}$$

If renormalization-group running of the parameters is included, e.g., $\alpha(M_Z)=1/128.92(3)$, we find

$$\sin^2\hat{\theta}_w=0.23105(8), \quad \text{(C3)}$$

which is the value used for the Z-pole observables in Sec. V.

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