Strongly coupled grand unification in higher dimensions

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We consider the scenario where all the couplings in the theory are strong at the cutoff scale, in the context of higher dimensional grand unified field theories where the unified gauge symmetry is broken by an orbifold compactification. In this scenario, the non-calculable correction to gauge unification from unknown ultraviolet physics is naturally suppressed by the large volume of the extra dimension, and the threshold correction is dominated by a calculable contribution from Kaluza-Klein towers that gives the values for $\sin^2\theta_w$ and α_s in good agreement with low-energy data. The threshold correction is reliably estimated despite the fact that the theory is strongly coupled at the cutoff scale. A realistic 5D supersymmetric $SU(5)$ model is presented as an example, where rapid $d=6$ proton decay is avoided by putting the first generation matter in the 5D bulk.

I. INTRODUCTION

The unification of the three gauge couplings around M_U $\sim 2 \times 10^{16}$ GeV [1] in the minimal supersymmetric standard model strongly suggests new physics at this energy scale. Conventionally, this new physics has been viewed as four-dimensional grand unified theories (GUTs) [2], in which all the standard model gauge interactions are unified into a single non-Abelian gauge group and quarks and leptons are unified into smaller numbers of representations under the gauge group. Grand unification in four dimensions (4D), however, raises several new questions, including how the GUT gauge symmetry is broken, why the doublet and triplet components of Higgs multiplets split, and why we have not already observed proton decay caused by color triplet Higgsino exchanges [3].

On the other hand, these questions have also been addressed in the context of higher dimensional theories in string theory. In this case, the grand unified group is broken by boundary conditions imposed on the gauge field, and the triplet Higgs fields are projected out from the zero-mode sector, leaving only the doublet Higgs fields as massless fields [4,5]. This is possible because there is no zero-mode gauge symmetry which transforms massless doublet Higgs fields into massless triplet Higgs fields. In this framework, however, there is no field theoretic unified symmetry remaining at low energy, so that we have to resort to string threshold calculations to tell whether the three gauge coupling constants are really unified at the string scale [6].

Recently, we have introduced a new framework in which the gauge coupling unification is realized in higher dimensional unified field theories compactified to 4D on orbifolds [7]. Kawamura first suggested an $SU(5)$ GUT in 5D [8], using an $S^1/(Z_2 \times Z_2')$ orbifold earlier introduced in the supersymmetry breaking context [9]. A completely realistic theory was obtained in Ref. [7], where it was shown that a special field theoretic symmetry called restricted gauge symmetry plays a crucial role in this type of theories. This restricted gauge symmetry arises from the fact that there is a moderately large energy interval where the physics is described by higher dimensional grand unified *field* theories. In

the higher dimensional picture it has gauge transformation parameters whose dependence on the extra dimensional co-ordinates is constrained by orbifold boundary conditions; in the 4D picture it is a symmetry that has different Kaluza-Klein (KK) decompositions for the "unbroken" and "broken" gauge transformations. Using these ideas, various higher dimensional GUT models have been constructed [8,7,10–15].

In the specific case of 5D $SU(5)$ models in Refs. [8,7], the 5D $SU(3)_C \times SU(2)_L \times U(1)_Y$ (3-2-1) gauge transformation has a KK decomposition in terms of $\cos[2ny/R]$, while the 5D $SU(5)/(SU(3)_C \times SU(2)_L \times U(1)_Y)$ ($X-Y$) has a decomposition in terms of $\cos[(2n+1)y/R]$. Since there is no zero mode for the $X-Y$ gauge transformation, the restricted gauge symmetry does not require that the doublet and triplet Higgs fields must have the same mass. However, due to higher KK tower gauge transformations, local operators written in the 5D bulk must still preserve the complete $SU(5)$ symmetry, and all the $SU(5)$-breaking local operators must be located on the fixed point where only 3-2-1 gauge symmetry is preserved [7]. In particular, $SU(5)$-violating effects from unknown ultraviolet physics must appear as boundary operators on this fixed point. This is crucial for guaranteeing the successful gauge coupling unification in this framework. Since the $SU(5)$-violating contributions to the gauge couplings which come from the fixed point are suppressed by the volume of the extra dimension compared with the $SU(5)$-preserving contribution from the bulk, we can argue that the gauge coupling is (approximately) unified, without invoking any string theory calculation, if the volume of the extra dimension is sufficiently large [7]. Then, small deviations from the case of exact unification at a single threshold scale become calculable and improve the agreement between the experimental value and theoretical prediction of $\sin^2\theta_w$ [7,16]. The gauge coupling unification in higher dimensional GUTs has been further studied using dimensional deconstruction [17,18] and dimensional regularization [19].

In view of the important role played by the large volume for the successful prediction of $\sin^2\theta_w$, in this paper we study the possibility that the theory has the maximally large vol-

ume allowed by strong coupling analysis. We consider the scenario where all the couplings in the theory are strong at the cutoff scale and show that it is consistent with observations. In this paper we restrict our analysis to an order-of-magnitude level, leaving detailed numerical studies for future work. We present a realistic 5D supersymmetric $SU(5)$ model as an explicit example. The model preserves the successful b/τ Yukawa unification and does not have the unwanted $SU(5)$ mass relations for the first two generations. It also partially explains fermion mass hierarchies due to the configuration of the matter fields in the extra dimension. We find that the observed values of the low-energy gauge couplings are well reproduced if we take the volume of the extra dimension to be large as suggested by the strong coupling analysis. Experimental signatures from $d=6$ proton decay are also discussed, and it is shown that the final state generically contains the second or third generation particles. Finally, the values of the cutoff and compactification scales obtained by analyzing gauge couplings give a 4D Planck scale close to the observation, giving a clue of how to solve the conventional problem in string theory of separating the string and the apparent unification scales.

The paper is organized as follows. In Sec. II we give a 5D supersymmetric $SU(5)$ model that can accommodate the large volume of the extra dimension without conflicting with the constraint from $d=6$ proton decay. In Sec. III, we consider the gauge coupling unification in this model and argue that the model is consistent with low-energy data. The $d=6$ proton decay and the 4D Planck scale are discussed in Sec. IV. Finally, our conclusions are drawn in Sec. V.

II. MINIMAL MODEL

In this paper we consider a minimal realization of the scenario where the theory is strongly coupled at the cutoff scale. Thus, we consider a single extra dimension and the smallest grand unified group, $SU(5)$. It should, however, be noted that we present this case as a representative example of a more general scenario. We begin with briefly reviewing the bulk structure of 5D supersymmetric $SU(5)$ theories [8,7]. The 5D spacetime is a direct product of 4D Minkowski spacetime M^4 and an extra dimension compactified on the $S^1/(Z_2 \times Z_2')$ orbifold, with coordinates x^μ ($\mu=0,1,2,3$) and $y(=x^5)$, respectively. The $S^1/(Z_2 \times Z_2')$ orbifold can be viewed as a circle of radius R divided by two Z_2 transformations; Z_2: $y \to -y$ and Z_2': $y' \to -y'$ where $y'=y-\pi R/2$. Here, R is around the GUT scale, $R \sim M_U^{-1}$. The physical space is an interval $y:[0,\pi R/2]$ which has two branes at the two orbifold fixed points at $y=0$ and $\pi R/2$.

Under the $Z_2 \times Z_2'$ symmetry, a generic 5D bulk field $\phi(x^\mu,y)$ has a definite transformation property

$$\phi(x^\mu,y) \to \phi(x^\mu,-y) = P\phi(x^\mu,y), \qquad (1)$$

$$\phi(x^\mu,y') \to \phi(x^\mu,-y') = P'\phi(x^\mu,y'), \qquad (2)$$

where the eigenvalues of P and P' must be ± 1. Denoting the field with $(P,P')=(\pm 1,\pm 1)$ by $\phi_{\pm\pm}$, we obtain the following mode expansions [9]:

TABLE I. The (Z_2,Z_2') transformation properties for the bulk gauge and Higgs multiplets.

(P,P')	4D $N=1$ superfield	Mass
$(+,+)$	V^a, H_F, $\bar{H}_F$	$2n/R$
$(+,-)$	$V^{\hat{a}}$, H_C, $\bar{H}_C$	$(2n+1)/R$
$(-,+)$	$\Sigma^{\hat{a}}$, H_C^c, $\bar{H}_C^c$	$(2n+1)/R$
$(-,-)$	Σ^a, H_F^c, $\bar{H}_F^c$	$(2n+2)/R$

$$\phi_{++}(x^\mu,y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{2^{\delta_{n,0}}\pi R}} \phi_{++}^{(2n)}(x^\mu)\cos\frac{2ny}{R}, \qquad (3)$$

$$\phi_{+-}(x^\mu,y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} \phi_{+-}^{(2n+1)}(x^\mu)\cos\frac{(2n+1)y}{R}, \qquad (4)$$

$$\phi_{-+}(x^\mu,y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} \phi_{-+}^{(2n+1)}(x^\mu)\sin\frac{(2n+1)y}{R}, \qquad (5)$$

$$\phi_{--}(x^\mu,y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} \phi_{--}^{(2n+2)}(x^\mu)\sin\frac{(2n+2)y}{R}, \qquad (6)$$

where the 4D fields $\phi_{++}^{(2n)}$, $\phi_{+-}^{(2n+1)}$, $\phi_{-+}^{(2n+1)}$, and $\phi_{--}^{(2n+2)}$ acquire masses $2n/R$, $(2n+1)/R$, $(2n+1)/R$, and $(2n+2)/R$ upon compactification. Zero modes are contained only in ϕ_{++} fields, so that the matter content of the massless sector is smaller than that of the full 5D multiplet.

In the 5D bulk, we have $SU(5)$ gauge supermultiplets and two Higgs hypermultiplets that transform as **5** and **5***. The 5D gauge supermultiplet contains a vector boson, A_M ($M=0,1,2,3,5$), two gauginos, λ and λ', and a real scalar, σ, which is decomposed into a vector supermultiplet, $V(A_\mu,\lambda)$, and a chiral multiplet in the adjoint representation, $\Sigma[(\sigma+iA_5)/\sqrt{2},\lambda']$, under $N=1$ supersymmetry in 4D. The hypermultiplet, which consists of two complex scalars, ϕ and ϕ^c, and two Weyl fermions, ψ and ψ^c, forms two 4D $N=1$ chiral multiplets, $\Phi(\phi,\psi)$ and $\Phi^c(\phi^c,\psi^c)$, transforming as representations conjugate to each other under the gauge group. Here Φ runs over the two Higgs hypermultiplets, H and $\bar{H}$. ($\{H,\bar{H}^c\}$ and $\{\bar{H},H^c\}$ transform as **5** and **5*** under the $SU(5)$, respectively.)

The 5D $SU(5)$ gauge symmetry is "broken" by the orbifold compactification to a 4D $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry by choosing $P=(+,+,+,+,+)$ and $P'=(-,-,-,+,+)$ acting on the **5** [8]. Each Z_2 reflection is taken to preserve the same 4D $N=1$ supersymmetry. The (Z_2,Z_2') charges for all components of the vector and Higgs multiplets are shown in Table I. Here, the indices a and $\hat{a}$ denote the unbroken and broken $SU(5)$ generators, T^a and $T^{\hat{a}}$, respectively. The C and F represent the color triplet and weak doublet components of the Higgs multiplets, respec-

tively: $H \supset \{H_C, H_F\}$, $\bar{H} \supset \{\bar{H}_C, \bar{H}_F\}$, $H^c \supset \{H_C^c, H_F^c\}$, and $\bar{H}^c \supset \{\bar{H}_C^c, \bar{H}_F^c\}$. Since only $(+,+)$ fields have zero modes, the massless sector consists of $N=1$ $SU(3)_C \times SU(2)_L \times U(1)_Y$ vector multiplets $V^{a(0)}$ with two Higgs doublet chiral superfields $H_F^{(0)}$ and $\bar{H}_F^{(0)}$. The higher modes for the vector multiplets $V^{a(2n)}$ ($n>0$) eat $\Sigma^{a(2n)}$ becoming massive vector multiplets, and similarly for the $V^{\hat{a}(2n+1)}$ and $\Sigma^{\hat{a}(2n+1)}$ ($n \geqslant 0$). Since the non-zero modes for the Higgs fields have mass terms of the form $H_F^{(2n)} H_F^{c(2n)}$, $\bar{H}_F^{(2n)} \bar{H}_F^{c(2n)}$, $H_C^{(2n+1)} H_C^{c(2n+1)}$, and $\bar{H}_C^{(2n+1)} \bar{H}_C^{c(2n+1)}$, there is no dimension five proton decay from color triplet Higgsino exchange [7].

Now, we consider the gauge couplings in our scenario. Here we roughly estimate various quantities at the tree level; more detailed discussions including radiative corrections are given in Sec. III. Since we require that the theory is strongly coupled at the cutoff scale M_*, the gauge kinetic terms are given by

$$S = \int d^4x \, dy \int d^2\theta \left[\frac{\eta M_*}{16\pi^3} \mathcal{W}^\alpha \mathcal{W}_\alpha \right.$$
$$\left. + \delta(y) \frac{\eta'}{16\pi^2} \mathcal{W}^\alpha \mathcal{W}_\alpha + \delta\left(y - \frac{\pi}{2}R\right) \frac{\eta_i'}{16\pi^2} \mathcal{W}_i^\alpha \mathcal{W}_{i\alpha} \right]$$
$$+ \text{H.c.}, \tag{7}$$

where we have used naive dimensional analysis (NDA) in higher dimensions.[1] Here, η, η', and η_i' are order one coefficients and i runs over $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$. The restricted gauge symmetry requires that the first two terms must preserve the $SU(5)$ symmetry. The last term, however, can have different coefficients for $i = SU(3)_C, SU(2)_L, U(1)_Y$, which encode $SU(5)$-violating effects from unknown physics above the cutoff scale. After integrating over the extra dimension, we obtain the zero-mode gauge couplings at the cutoff scale as

$$\frac{1}{g_i^2} = \frac{\eta M_* R}{16\pi^2} + \frac{\eta'}{16\pi^2} + \frac{\eta_i'}{16\pi^2}. \tag{8}$$

Since we know that $1/g_i^2 \sim 1$ from the observed values of the low-energy gauge coupling constants, the ratio between the compactification and the cutoff scales must be $M_* R$

$\sim 16\pi^2/g_i^2 = O(10^2 - 10^3)$.[2] We find that the threshold correction from unknown ultraviolet physics above M_* is suppressed by $1/(M_* R) \sim 1/(16\pi^2)$ and thus negligible in the present scenario. Therefore, the threshold correction to $\sin^2\theta_w$ is dominated by the calculable contribution coming from an energy interval between $1/R$ and M_*.

We next consider the configuration of matter fields. Since $M_* R \gtrsim 100$ corresponds to $1/R \lesssim 10^{15}$ GeV, it requires that the first generation matter must live in the bulk; otherwise $d=6$ proton decay occurs at a much faster rate than experimental constraints allow [7].[3] On the other hand, to preserve successful b/τ unification in supersymmetric GUTs [21], we have to put the third generation matter on the $SU(5)$-preserving brane located at $y=0$, since if we put quarks and leptons in the bulk there are no $SU(5)$ Yukawa relations [7].[4] These considerations almost fix the location of the matter fields. The remaining choices are only concerning where we put **10** and **5*** of the second generation. Since we do not want the $SU(5)$ relation, $m_s = m_\mu$, for the second generation, at least one of **10** and **5*** must be put in the bulk. Thus, we are left with three possibilities: (i) both **10** and **5*** in the bulk, (ii) **10** in the bulk and **5*** on the $y=0$ brane, (iii) **10** on the $y=0$ brane and **5*** in the bulk. As we will see later, the second possibility may be preferred in view of quark and lepton mass matrices, especially in view of the large mixing angle between the second and third generation neutrinos observed in the Super-Kamiokande experiment [22]. We therefore take this possibility as an illustrative purpose for the moment. We consider all three possibilities when we discuss $d=6$ proton decay later.[5]

We now explicitly present our model. The gauge and Higgs sectors are as discussed before. For the matter fields, we introduce the third generation matter chiral superfields $T_3(\mathbf{10})$, $F_3(\mathbf{5^*})$ and the second generation one $F_2(\mathbf{5^*})$ on the $y=0$ brane. In the 5D bulk, we have to introduce six hypermultiplets $\mathcal{T}_2 = \{T_2(\mathbf{10}), T_2^c(\mathbf{10^*})\}$, $\mathcal{T}_2' = \{T_2'(\mathbf{10}), T_2'^c(\mathbf{10^*})\}$, $\mathcal{T}_1 = \{T_1(\mathbf{10}), T_1^c(\mathbf{10^*})\}$, $\mathcal{T}_1' = \{T_1'(\mathbf{10}), T_1'^c(\mathbf{10^*})\}$, $\mathcal{F}_1 = \{F_1(\mathbf{5^*}), F_1^c(\mathbf{5})\}$, $\mathcal{F}_1' = \{F_1'(\mathbf{5^*}), F_1'^c(\mathbf{5})\}$, to obtain the correct low-energy matter content. The transformations for these bulk matter fields un-

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[1] In Ref. [20] a different coefficient of $M_*/24\pi^3$ was used for the bulk kinetic term, which was derived by considering loop expansions in the non-compactified 5D space. Here we use $M_*/16\pi^3$ instead, since it correctly reproduces the strong-coupling value for the 4D gauge coupling, $g \simeq 4\pi/(M_* R)^{1/2}$, after integrating out the extra dimension and is more appropriate in the case of the compactified space. The coefficients of brane-localized terms are determined by requiring that all loop expansion parameters are order one in the 4D picture.

[2] The actual value of $M_* R$ could be somewhat smaller than the naive estimate given here, due to a group theoretical factor C appearing in loop expansions: $M_* R \sim 16\pi^2/Cg_i^2$.

[3] The **5*** of the first generation may be located in the bulk without conflicting with the bound from proton decay.

[4] Models without the b/τ unification are obtained if we put the third generation **5*** in the bulk.

[5] In these modes, supersymmetry breaking may occur through the mechanism of Ref. [12] that uses small parameters appearing in boundary conditions. In this case, the first possibility of both **10** and **5*** in the bulk is preferred to suppress flavor violating contributions to the first-two generation sfermion masses. The flavor violation would then occur in the processes involving the third generation particles. One way of avoiding all these concerns is to consider 6D models in which gaugino mediation [23] works while suppressing $d=6$ proton decay [14]. We leave detailed phenomenologies of these models including supersymmetry breaking for future work.

der $Z_2 \times Z_2'$ are given by $P=(+,+,+,+,+)$ and $P'=(-,-,-,+,+)$ acting on the **5** for unprimed fields, but for primed fields the Z_2' quantum numbers are assigned to be the opposite of the corresponding unprimed fields [7] (for details, see Refs. [11,12]). Then, the quark and lepton zero modes come from various brane and bulk fields as

$$T_3 \supset Q_3,U_3,E_3, \quad F_3 \supset D_3,L_3, \tag{9}$$

$$T_2 \supset U_2,E_2, \quad T_2' \supset Q_2, \quad F_2 \supset D_2,L_2, \tag{10}$$

$$T_1 \supset U_1,E_1, \quad T_1' \supset Q_1, \quad F_1 \supset L_1, \quad F_1' \supset D_1. \tag{11}$$

Since the first generation quarks and leptons which would be unified into a single multiplet in the usual 4D GUTs come from different $SU(5)$ multiplets, proton decay from broken gauge boson exchange is absent at the leading order [7]. (This result is also obtained from KK momentum conservation in the fifth dimension.)

The Yukawa couplings are written on the $y=0$ brane. On this brane, all the operators of the form $[TTH]_{\theta^2}$ and $[TF\bar{H}]_{\theta^2}$ are written with the size of their coefficients dictated by naive dimensional analysis (NDA) in higher dimensions. Here, T and F runs over $\{T_3,T_2,T_2',T_1,T_1'\}$ and $\{F_3,F_2,F_1,F_1'\}$, respectively. Similar Yukawa couplings can also be written at $y=\pi R/2$ brane for matter in the bulk. After integrating over y, we obtain the Yukawa matrices for low-energy quarks and leptons. At the compactification scale, they take the form

$$\mathcal{L}_4 \simeq \sqrt{\frac{16\pi^2}{M_* R}}(\mathbf{10}_1 \quad \mathbf{10}_2 \quad \mathbf{10}_3)$$

$$\times \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix} \begin{pmatrix} \mathbf{10}_1 \\ \mathbf{10}_2 \\ \mathbf{10}_3 \end{pmatrix} H$$

$$+ \sqrt{\frac{16\pi^2}{M_* R}}(\mathbf{10}_1 \quad \mathbf{10}_2 \quad \mathbf{10}_3)$$

$$\times \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{5}_1^* \\ \mathbf{5}_2^* \\ \mathbf{5}_3^* \end{pmatrix} \bar{H}, \tag{12}$$

where $\epsilon \simeq (M_* R)^{-1/2} \sim 0.1$ and we have omitted order-one coefficients. Low-energy quark and lepton fields are defined as $\mathbf{10}_i \equiv \{Q_i,U_i,E_i\}$ and $\mathbf{5}_i^* \equiv \{D_i,L_i\}$ ($i=1,2,3$), so that they generically contain fields coming from different hypermultiplets [see Eqs. (9)–(11)]. Here, we have normalized these fields canonically in 4D. In the above equation, the matrix elements denoted as ϵ or ϵ^2 do not respect $SU(5)$ relations, while the ones denoted as 1 must respect $SU(5)$ relations since they entirely come from the Yukawa couplings among the matter fields localized on the $SU(5)$-preserving ($y=0$) brane. Therefore, the model does not have unwanted $SU(5)$ fermion mass relations for the first two generations, while preserving the b/τ unification.

Since $\sqrt{16\pi^2/M_* R} \sim g_i$, the present model predicts $y_t \sim g_i$ at the compactification scale, which is in reasonably good agreement with low-energy data. The over-all mass difference between up- and down-type quarks should be given by $\tan\beta \equiv \langle H_F \rangle / \langle \bar{H}_F \rangle \sim 50$.[6] This large value of $\tan\beta$ may also be compatible with the b/τ Yukawa unification [24]. The above mass matrices roughly explain the observed pattern of quark and lepton masses and mixings, for example, the presence of the mass hierarchy between the first two generations and the third generation fermions. To reproduce the detailed structure of fermion masses in the first two generations, however, there must be some cancellations among different elements and/or small numbers in coefficients of order $10^{-1} - 10^{-2}$. It will be interesting to look for the model where more complicated structure gives completely realistic fermion mass matrices [15].

How about neutrino masses? Small neutrino masses are obtained by introducing right-handed neutrino fields N through the seesaw mechanism [25]. They can be introduced either on the $y=0$ brane or in the 5D bulk, and have Yukawa couplings of the form $[FNH]_{\theta^2}$ and Majorana masses of the form $[NN]_{\theta^2}$ at the $y=0$ brane. After integrating out N fields, we obtain the mass matrix for the light neutrinos of the form

$$\mathcal{L}_4 \simeq \frac{1}{M_R} \left(\frac{16\pi^2}{M_* R} \right) (\mathbf{5}_1^* \quad \mathbf{5}_2^* \quad \mathbf{5}_3^*)$$

$$\times \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{5}_1^* \\ \mathbf{5}_2^* \\ \mathbf{5}_3^* \end{pmatrix} HH, \tag{13}$$

regardless of the configuration of the right-handed neutrino fields. The over-all mass scale M_R is given by right-handed neutrino Majorana masses, which we here assume to be provided by some other physics such as $U(1)_{B-L}$ breaking scale. An interesting point is that the present matter configuration naturally explains the observed large mixing angle between the second and third generation neutrinos, by putting both the second and third generation **5***'s on the $y=0$ brane.

III. GAUGE COUPLING UNIFICATION

In this section we show that the observed values of the low-energy gauge couplings are well reproduced if the volume of the extra dimension is large as is suggested by the strong coupling analysis. We also argue that the situation in the present scenario is better than in usual 4D GUTs, since the masses for the GUT-scale particles are completely determined by KK mode expansions.

Let us first estimate the radiative corrections to the gauge couplings coming from loops of KK towers whose masses lie between $1/R$ and M_*. In the 4D picture, the zero-mode gauge couplings g_i at the compactification scale $M_c (=1/R)$ are given by

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[6] In the case of $\mathbf{5}_3^*$ in the bulk, we obtain $\tan\beta \sim (m_t/m_b)\epsilon \sim 5$.

$$\frac{1}{g_i^2(M_c)} \simeq \frac{1}{g_0^2(M_*)} - \frac{b}{8\pi^2}(M_*R-1) + \frac{b_i'}{8\pi^2}\ln(M_*R),$$
(14)

where b and b_i' are constants of $O(1)$. The second and third terms on the right-hand side represent the pieces which run by power-law and logarithmically. A crucial observation made in Refs. [7,16] is that the coefficient b is necessarily $SU(5)$ symmetric, since the power-law contributions come from renormalizations of 5D kinetic terms which must be $SU(5)$ symmetric due to the restricted gauge symmetry. The logarithmic contributions come from renormalizations of 4D kinetic terms localized on the branes, and can be different for $SU(3)_C, SU(2)_L,$ and $U(1)_Y$. Thus, gauge coupling unification is logarithmic even above the compactification scale. This situation is quite different from the power-law unification scenario of Ref. [26].

Since the power-law piece is asymptotically non-free in the present setup, the ratio between the compactification and cutoff scales could be smaller than the purely classical estimate. This power-law contribution also has a sensitivity to the ultraviolet physics. However, it is expected that this does not change the order of magnitude of the tree-level estimate of M_*R, since the theory is strongly coupled only around the cutoff scale and is weakly coupled over a wide energy range from $1/R$ to M_*. Therefore, we here take $M_*R \simeq 100$ as a representative value. Note that we have ambiguities coming from η's in Eq. (8) in any case, so that the precise value is not very important at this stage.

To calculate the effect of the KK towers on the gauge coupling unification, we consider the one-loop renormalization group equations for the three gauge couplings [7,16]. Since the $SU(5)$-violating contribution to the gauge couplings from unknown ultraviolet physics above M_* is suppressed by the large volume, we set the three gauge couplings equal to a unified value g_* at M_*. Then, the equations take the following form:

$$\alpha_i^{-1}(m_Z) = \alpha_*^{-1}(M_*) + \frac{1}{2\pi}\left\{ a_i\ln\frac{m_{\rm SUSY}}{m_Z} + b_i\ln\frac{M_*}{m_Z} + c_i\sum_{n=0}^{N_l}\ln\frac{M_*}{(2n+2)M_c} + d_i\sum_{n=0}^{N_l}\ln\frac{M_*}{(2n+1)M_c}\right\},$$
(15)

where $(a_1,a_2,a_3)=(-5/2,-25/6,-4)$, $(b_1,b_2,b_3)=(33/5,1,-3)$, $(c_1,c_2,c_3)=(6/5+n_{5*}+3n_{10}, -2+n_{5*}+3n_{10}, -6+n_{5*}+3n_{10})$, and $(d_1,d_2,d_3)=(-46/5+n_{5*}+3n_{10}, -6+n_{5*}+3n_{10}, -2+n_{5*}+3n_{10})$. Here, we have assumed a common mass $m_{\rm SUSY}$ for the superparticles for simplicity, and the sum on n includes all KK modes below M_*, so that $(2N_l+2)M_c \le M_*$; n_{5*} and n_{10} represent the numbers of generations which are put in the bulk ($n_{5*}=1$ and $n_{10}=2$ in the present case). Taking a linear combination of the three equations, we obtain

$$(5\alpha_1^{-1}-3\alpha_2^{-1}-2\alpha_3^{-1})(m_Z) = \frac{1}{2\pi}\left\{ 8\ln\frac{m_{\rm SUSY}}{m_Z} + 36\ln\frac{(2N_l+2)M_c}{m_Z} - 24\sum_{n=0}^{N_l}\ln\frac{(2n+2)}{(2n+1)}\right\},$$
(16)

where we have set $M_*=(2N_l+2)M_c$. Note that n_{5*} and n_{10} drop out from this equation, since a combination of bulk hypermultiplets whose massless modes give a complete $SU(5)$ representation has $SU(5)$ symmetric matter content at each KK mass level. Since the corresponding linear combination in the usual 4D minimal supersymmetric $SU(5)$ GUT takes the form

$$(5\alpha_1^{-1}-3\alpha_2^{-1}-2\alpha_3^{-1})(m_Z)$$
$$= \frac{1}{2\pi}\left\{ 8\ln\frac{m_{\rm SUSY}}{m_Z} + 36\ln\frac{M_U}{m_Z}\right\},$$
(17)

where $M_U=(M_\Sigma^2 M_V)^{1/3}$ [27], we find the following correspondence between the two theories:

$$\ln\frac{M_c}{m_Z} = \ln\frac{M_U}{m_Z} + \frac{2}{3}\sum_{n=0}^{N_l}\ln\frac{(2n+2)}{(2n+1)} - \ln(2N_l+2),$$
(18)

as far as the running of the gauge couplings is concerned. An important point here is that this KK contribution improves the agreement between the experimental value and theoretical prediction of $\sin^2\theta_w$ and α_s [7,16]. This is because b_i''s in Eq. (14) are given by $b_i'=b_i-c_i/2$, and are not equal to the low-energy β-function coefficients, b_i, plus some universal pieces. At the leading order, the contributions from KK towers to $\sin^2\theta_w$ and α_s are given by $\Delta_{\sin^2\theta_w} \simeq -(1/5\pi)\alpha\ln(M_*R)$ and $\Delta_{\alpha_s} \simeq -(3/7\pi)\alpha_s^2\ln(M_*R)$, respectively, which well reproduce experimental values with $M_*R \simeq 10^2-10^3$. A more detailed analysis, including the next to leading order effect, has been given in Ref. [19], where it was shown that if $M_*R \simeq 100$ the KK contribution would indeed give the right values for $\sin^2\theta_w$ and α_s in a reasonable range of $m_{\rm SUSY}$.

Using the experimental values of the gauge couplings, we obtain 1×10^{16} GeV$\le M_U \le 3\times10^{16}$ GeV, and this translates into the range of M_c for a given N_l. Taking $M_*R \simeq 100$, we find that the compactification scale must be in the range

$$5\times10^{14} \text{ GeV} \le M_c \le 2\times10^{15} \text{ GeV},$$
(19)

which is considerably lower than the usual 4D unification scale $M_U \simeq 2 \times 10^{16}$ GeV. Since the mass for the broken gauge bosons is given by $1/R$, it induces the $d=6$ proton decay at a rate contradicting the bound from Super-Kamiokande [28], if quarks and leptons are localized on the $SU(5)$-preserving brane. In fact, this constraint was used in Ref. [19] to conclude that strict NDA assumption does not work, and the contribution from unknown ultraviolet physics is needed to obtain the right values for $\sin^2\theta_w$ and α_s. In other words, the contribution from KK towers alone is insufficient to explain the small difference of $\sin^2\theta_w$ between the experiment and naive 4D GUT prediction, since $M_* R$ must be smaller than ~ 10 from the proton decay constraint.[7] In the present case, however, the first generation matter lives in the bulk so that the constraint from $d=6$ proton decay is evaded even if the compactification scale is low. This allows us to consider larger values for $M_* R$, that is, the scenario where the theory is strongly coupled at the cutoff scale. Then, the calculable contribution from KK towers could completely explain the small discrepancy of $\sin^2\theta_w$ between the experimental and theoretical values that was present in the case of the minimal 4D GUT with a single threshold. Note that the non-calculable contribution from unknown physics above M_* is expected to be small in this case through NDA in higher dimensions.

We here consider uncertainties for the present analysis. Since the theory is assumed to be strongly coupled at the cutoff scale, higher order effects could be important around that energy scale. However, the logarithmic contribution from KK towers discussed above comes from an entire energy range from $1/R$ to M_*, and the theory is weakly coupled in most of this energy region. Actually, various interactions quickly become weak below M_*, suppressed by powers of (E/M_*) at energy scale E. This is because in the 5D picture the couplings in the theory have negative mass dimensions, and in the 4D picture the number of KK states circulating in the loop decreases with decreasing energies so that loop expansion parameters in the theory ('t Hooft couplings) become small by powers of (E/M_*). Therefore, we expect that the the leading logarithmic calculation of the threshold correction to $\sin^2\theta_w$ is reliable at least at the order of magnitude level, although the precise coefficients may receive corrections from this higher order effect. To be more precise, the difference of the gauge couplings runs logarithmically in all energy regions between $1/R$ and M_*, and the one-loop estimates are reliable only when the renormalization scale is at least a factor of a few smaller than M_*; higher loop effects would equally be important around the cutoff scale. This would give $O(10\%)$ uncertainties in the

calculations of the *threshold corrections* of $\sin^2\theta_w$ and α_s. A similar size of uncertainties is also expected from tree-level $SU(5)$-breaking boundary operators. We emphasize that the uncertainties are for the threshold corrections and are not $O(10\%)$ uncertainties for the values of $\sin^2\theta_w$ and α_s themselves.

We then find that the observed values for the gauge coupling constants are well reproduced by taking $M_* R = O(10^2 - 10^3)$. That is, we can explain the difference of $\sin^2\theta_w$ (and α_s) between the experimental value and the theoretical prediction obtained by assuming the exact gauge unification at a single threshold. We note that the situation is better in the present scenario than in usual 4D GUTs.[8] Let us consider, for example, predicting α_s from the observed values of $\sin^2\theta_w$ and e. It is known that if we calculate α_s without including any threshold correction, we obtain a somewhat larger value $\alpha_s|_{\rm th} \simeq 0.130$ [29] than the experimentally measured value $\alpha_s|_{\rm ex} \simeq 0.118 \pm 0.002$ [30]. Thus, we have to explain the difference $\alpha_s|_{\rm ex} - \alpha_s|_{\rm th} \simeq -0.012 \pm 0.002$ by the GUT-scale threshold correction $\Delta_{\alpha_s}^{\rm gut}$. (Here we ignore the weak-scale threshold corrections, which typically give $|\Delta_{\alpha_s}^{\rm weak}| \lesssim 0.004$.) In usual 4D GUTs, the size of the GUT-scale threshold correction is given by $|\Delta_{\alpha_s}^{\rm gut}| \lesssim 0.02$, but we cannot predict the value of $\Delta_{\alpha_s}^{\rm gut}$ in general since it strongly depends on the mass spectrum of the GUT scale particles.[9] On the other hand, in the present case, we completely know the pattern of the GUT-scale particle masses, so that we can calculate the threshold correction, $\Delta_{\alpha_s}^{\rm gut}$, for a given value of $M_* R$. It is given by $\Delta_{\alpha_s}^{\rm gut} \simeq -(3/7\pi)\alpha_s^2 \ln(M_* R)$. Numerically, we find $\Delta_{\alpha_s}^{\rm gut} \simeq -0.009 \pm 0.002$ ($\Delta_{\alpha_s}^{\rm gut} \simeq -0.013 \pm 0.003$) if $M_* R = 100$ ($M_* R = 1000$), where the errors represent the $O(10\%)$ uncertainties discussed before. We find that the observed value of α_s is well reproduced with the values of $M_* R$ suggested by the NDA analysis.

Of course, we cannot prove that these values of $M_* R$ exactly give a *truly* strongly coupled theory at the cutoff scale (all η's equal to 1), since there are many uncertainties in estimating the overall value for the gauge coupling (but not the differences between the three couplings) at the cutoff scale. For example, the contribution from $SU(5)$ symmetric power-law running (scheme dependence, in other words) could change the value. However, within the uncertainties in estimating various quantities, we can say that the scenario where the theory is (moderately) strongly coupled at the cutoff scale is consistent with low-energy observations. It is particularly interesting that the $M_* R$ value giving the desired low-energy gauge coupling values is consistent with the requirement that the theory is strongly coupled at the cutoff scale.

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[7]The constraint from $d=6$ proton decay was also used in Ref. [18] to conclude that $N_l (\simeq M_* R)$ must be smaller than ~ 20 and that the calculable contribution from KK towers cannot explain the discrepancy of the gauge coupling values between the experiment and the theoretical prediction of 4D GUT. Reference [17] also argues that N_l must be smaller than ~ 25 using $\alpha N_l \lesssim 1$ to estimate the strong coupling bound, while we here use $\alpha N_l / 4\pi \lesssim 1$ to estimate it.

[8]We thank Lawrence Hall for stressing this point.

[9]In the minimal supersymmetric $SU(5)$ GUT in 4D, $\Delta_{\alpha_s}^{\rm gut}$ is positive in most of the parameter space, due to the large mass for the triplet Higgs fields required to satisfy the bound from $d=5$ proton decay.

IV. OTHER ISSUES

In the model discussed in the previous sections, $d = 6$ proton decay occurs through mixings between the first and heavier generations occurring at the coupling to the heavy broken gauge bosons.[10] Thus, their rates are suppressed by mixing angles that are expected to have similar order of magnitudes to the corresponding Cabibbo-Kobayashi-Maskawa (CKM) angles. In the present case where $\mathbf{10}_2$ resides in the bulk and $\mathbf{5}_2^*$ lives on the brane, the dominant decay mode is $K^+ \nu_\mu$ or $\mu^+ \pi^0$. However, their amplitudes receive suppression of order $V_{ub} V_{cb} V_{e3}$ and $V_{ub}^2 V_{e2}$, respectively, which are $10^{-5} - 10^{-6}$. Therefore, the lifetime is roughly 10^{40} years, and there would be little hope for detection in the near future. In the case where both $\mathbf{10}_2$ and $\mathbf{5}_2^*$ are in the bulk, the dominant decay mode is $K^+ \nu_\tau$, whose amplitude also receives suppression of order $V_{ub} V_{cb} V_{e3} \sim 10^{-5}$. On the other hand, if $\mathbf{10}_2$ is on the brane and $\mathbf{5}_2^*$ is in the bulk, the $p \to K^+ \nu_\tau$ and $p \to \mu^+ K^0$ decays could occur with only $V_{us} V_{e3} \sim V_{us}^2 \sim 10^{-2}$ suppression in their amplitudes, that is, with the lifetime of $10^{33} - 10^{35}$ years. Incidentally, if $M_* R$ is somewhat larger than 100, the proton lifetime becomes shorter. In the case of $M_* R \simeq 500$ ($M_* R \simeq 1000$), for example, the lifetime becomes factor 70 (450) shorter compared with the case of $M_* R \simeq 100$. Thus, in the case where $\mathbf{10}_2$ is on the brane, it is probable that strange $d = 6$ proton decay, involving the second generation particles in the final state, could be discovered in future experiments.

Finally, we estimate 4D reduced Planck scale M_P assuming that the strength of the gravitational interaction is also dictated by the NDA analysis. Since the theory is strongly coupled at M_*, the kinetic term for the graviton is given by $S = \int d^4 x dy (M_*^3 / 16\pi^3) \mathcal{R}$, where $\mathcal{R}$ is the Ricci scalar. Thus, after integrating y, M_P is given by $M_P^2 = M_*^3 R / (16\pi^2)$. Substituting the value obtained in Eq. (19) with $M_* R \simeq 100$, we obtain $M_P \simeq 10^{17}$ GeV. This is substantially higher than the 4D unification scale $M_U \simeq 2 \times 10^{16}$ GeV, but still somewhat lower than the observed value $M_P \simeq 2 \times 10^{18}$ GeV. To reproduce the observed value, we need either an $O(10)$ coefficient, $M_* R \gtrsim 1000$, or n extra dimensions with radius $R \simeq O(10^{2/n})$ in which (only) gravity propagates. However, it is true that M_P is an order of magnitude separated from the apparent unification scale M_U by the presence of the large extra dimension necessary to break the GUT symmetry. The precise estimate is also dependent on the number of extra dimensions, gauge group and matter content, which here we used those of the minimal 5D $SU(5)$ model as a representative case. Thus, we expect that the existence of this type of dimension may provide a general way of separating the two scales in string theory.

V. CONCLUSIONS

In this paper, we have explored the scenario where all the couplings in the theory are strong at the cutoff scale, in the context of higher dimensional grand unified field theories.

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[10]Similar situations are also discussed in the context of dimensionally deconstructed models [17].

This provides a calculable framework for gauge coupling unification in higher dimensions. The non-calculable effect from unknown ultraviolet physics is suppressed by assuming that all the operators in the theory scale according to naive dimensional analysis in higher dimensions [7]. Then, the threshold correction to $\sin^2 \theta_w$ dominantly comes from the calculable contribution from KK towers, giving the values for $\sin^2 \theta_w$ and α_s in good agreement with low-energy data. Although the theory is strongly coupled at the cutoff scale M_*, it quickly becomes weakly coupled below M_*, allowing reliable estimates of threshold corrections to the gauge coupling unification. A crucial point is that we can have large values of $M_* R$ without conflicting with the constraint from proton decay by putting the first generation matter in the bulk. This enables us to consider the strong coupling scenario, in contrast with the previous work [19] where it was concluded that $M_* R$ must be smaller than ~ 10 due to the proton decay constraint and thus the calculable contribution from KK towers cannot fully explain the low-energy data.

We have shown that the ansatz where all the coupling constants are dictated by naive dimensional analysis in higher dimensions is consistent with low-energy observations. We have presented a completely realistic 5D supersymmetric $SU(5)$ model as an explicit example. This suggests that the higher dimensional grand unified theory is a low-energy effective theory of some more fundamental theory that is strongly coupled at the scale M_*. In the present scenario, the observed weakness of various couplings is attributed to the presence of a moderately large extra dimension(s). The hierarchy among various couplings arise from different numbers of dimensions in which various fields propagate. The presence of this large dimension(s) is required to solve the problems in conventional GUTs, such as doublet-triplet splitting and $d = 5$ proton decay problems, by extra dimensional mechanisms while preserving successful prediction of $\sin^2 \theta_w$ [7,8]. Therefore, in this framework, solving the many conventional problems in GUTs is transformed to finding a single mechanism of naturally getting such a large extra dimension(s) with the radius of order $10^2 - 10^3$ in units of the fundamental scale. It would be interesting to consider a mechanism of generating this type of large extra dimension(s) in the context of string theory.

Note added. While this work was being completed, we received Ref. [31] where it is hoped that $SU(5)$-breaking boundary operators may not exist when the gauge group is broken only by orbifold reflections (not translations). Even then, however, there are fixed points which do not preserve full $SU(5)$ symmetry. Thus, $SU(5)$-violating local operators can be written on these points, since they are not prohibited by the restricted gauge symmetry.

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