

Solitogenesis of Q -balls

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We study the formation of Q -balls in the early universe, concentrating on potentials with a cubic or quartic attractive interaction. Large Q -balls can form via solitogenesis, a process of gradual charge accretion, provided some primordial charge asymmetry and initial “seed” Q -balls exist. We find that such seeds are possible in theories in which the attractive interaction is of the form $AH\psi^*\psi$, with ψ a complex scalar and H a light Higgs field. Condensate formation and fragmentation is only possible for masses m_ψ in the sub-eV range; these Q -balls may survive until present.

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I. INTRODUCTION

Q -balls are lumps of coherent scalar field that can be described semiclassically as non-topological solitons. They can arise in scalar field theories with a conserved global $U(1)$ charge and some kind of attractive interaction [1]. In a sector of fixed charge, the Q -ball is the ground state of the theory. Q -balls generically occur in supersymmetric extensions of the standard model [2]. In these theories, baryon and lepton number play the role of conserved charge.

Q -balls come in two types. Type II Q -balls are associated with the flat directions of the potential, which are a generic feature of supersymmetric theories. The vacuum expectation value (VEV) inside the Q -ball depends upon its charge. The formation of this type of Q -balls through fragmentation of an Affleck-Dine- (AD-) like condensate has been studied extensively in the literature [3–5]. Type I Q -balls on the other hand are characterized by a potential that is minimized at a finite VEV, independent of the charge of the Q -ball. We have analyzed under which conditions this type of Q -ball can be formed in the early universe. In this paper we present the results.

Large Q -balls can form via solitogenesis, a process of gradual charge accretion similar to nucleosynthesis, provided some primordial charge asymmetry exists [6,7]. The bottleneck for this process to occur then is the presence of initial “seed” Q -balls. Most potentials do not allow for small Q -balls which makes solitogenesis improbable. The exceptions are theories in which the attractive interaction is provided by a cubic term in the Lagrangian of the form $AH\psi^*\psi$, with a light “Higgs” mass. Condensate formation does occur for light fields, for masses in the range $m_\psi \lesssim \text{eV}$. If this leads to fragmentation, the thus formed Q -balls can survive evaporation if their binding energies are large. Q -balls formed during a phase transition do not survive evaporation, unless the phase transition takes place at extremely low temperatures $T \lesssim 10^{-6} m_\psi$.

If Q -balls survive until present they can be part of the dark matter of the universe. Recently it was proposed that the dark matter could be self-interacting; this would overcome various discrepancies between observations and predictions based on collisionless dark matter, such as weakly interacting massive particles (WIMPs) and axions [8]. Because of their extended nature Q -balls have relatively large cross sections,

and therefore can naturally satisfy the required self-interactions [9].

Another cosmologically interesting feature of Q -balls is that solitogenesis in the false vacuum can result in a phase transition [10]. Accretion of charge proceeds until a critical charge is reached, at which point it becomes energetically favorable for the Q -ball to expand, filling space with the true vacuum phase.

II. Q -BALLS

Consider a theory of an interacting scalar field ϕ that carries unit charge under some conserved $U(1)$ charge. The potential has a global minimum $U(0)=0$ at $\phi=0$. We also require that the potential admits Q -ball solutions, i.e.,

$$\mu_0 = \sqrt{\frac{2U(\phi)}{\phi^2}} = \min, \quad \text{for } \phi = \phi_0 \neq 0. \quad (1)$$

The Q -balls solutions are of the form $\phi(\vec{x}, t) = e^{i\omega t} \bar{\phi}(\vec{x})$. The charge and energy of such a configuration is

$$Q = \omega \int d^3x \bar{\phi}^2, \quad (2)$$

and

$$E = \int d^3x \left[\frac{1}{2} |\nabla \bar{\phi}|^2 + U_\omega(\bar{\phi}) \right] + \omega Q, \quad (3)$$

with

$$U_\omega(\phi) = U(\phi) - \frac{1}{2} \omega^2 \phi^2. \quad (4)$$

Minimizing the energy for a fixed ω is equivalent to finding a 3D bounce for tunneling in the potential U_ω . The bounce solution exists for $\mu_0 < \omega < \sqrt{U''(0)}$ by virtue of Eq. (1), and is spherically symmetric [1]. The equations of motion are

$$\frac{d^2 \bar{\phi}}{dr^2} + \frac{2}{r} \frac{d\bar{\phi}}{dr} - \frac{\partial U_\omega(\bar{\phi})}{\partial \bar{\phi}} = 0, \quad (5)$$

with boundary conditions $\phi'(0)=0$ and $\phi(\infty)=0$.

We will consider scalar potentials of the form

$$U_1(\phi) = \frac{1}{2}m_\phi^2\phi^2 - A\phi^3 + \lambda\phi^4, \quad (6)$$

$$U_2(\phi) = \frac{1}{2}m_\phi^2\phi^2 - A\phi^4 + \lambda\phi^6; \quad (7)$$

both have $\phi_0 = A/2\lambda$ and $\mu_0^2 = m_\phi^2 - A^2/2\lambda$. U_2 is a typical potential that arises in effective field theories. U_1 is a non-polynomial potential, as the cube term is of the form $(\phi^*\phi)^{3/2}$. It is a typical potential in finite temperature theories; this is however not interesting in the current context since at high temperatures Q -balls evaporate quickly. But it can also arise as an effective field theory. Consider for example the potential

$$U'_1(\psi) = m_\psi^2\psi^*\psi + m_H^2H^*H - A'H\psi^*\psi + \text{H.c.} \\ + \frac{\lambda_1}{4}\psi^*\psi H^*H + \frac{\lambda_2}{4}(\psi^*\psi)^2 + \frac{\lambda_3}{4}(H^*H)^2, \quad (8)$$

where the ‘‘Higgs’’ field H is uncharged under $U(1)$, whereas ψ carries unit charge. Further, we take A' real. Now make the field redefinitions

$$\text{Re } H = \frac{1}{\sqrt{2}}\varphi \sin \theta, \quad \psi = \frac{1}{\sqrt{2}}\varphi e^{i\Omega} \cos \theta, \quad (9)$$

then U'_1 becomes of the U_1 form, with ϕ some particular direction in (H, ψ) -space. We can also calculate $\mu_0^2 = 2U'_1/(\varphi^2 \cos^2 \theta)$ in terms of the U'_1 parameters. Taking $m_H = 0$ and all quartic couplings equal $\lambda_1 = \lambda_2 = \lambda_3 = \lambda'$ to simplify the algebra, this yields

$$\mu_0^2 = m_\psi^2 - \frac{A'^2}{3\lambda'} \quad (10)$$

at $\theta_0 = \pi/4$ and $\varphi_0 = 4A'/3\sqrt{2}\lambda'$.

Potentials of the form U'_1 are present in the scalar sector of the MSSM, where the Higgs field couples to sparticle fields through a cubic interaction [2]. The sparticles carry a conserved $U(1)$ charge in the form of baryon or lepton number. However, the sparticles and also possibly formed Q -balls are unstable, as they can decay into light fermions [11]. Stable Q -balls can be obtained in a model where the standard model (SM) Higgs field is coupled to a charged SM singlet [12]. The SM singlet ψ is charged under a hidden sector $U(1)_\psi$ global symmetry, under which none of the SM particles are charged. The Q -balls in this model interact with the SM particles only weakly, through the Higgs boson. Another possibility is that both the H and ψ field are hidden sector fields, interacting only gravitationally or through some other suppressed interaction with the visible sector. Hidden sectors appear in a variety of models, such as technicolor, mirror symmetry, hidden sector SUSY breaking, and brane world models. They also arise naturally from string theory; in het-

erotic $E_8 \times E_8$ string theory compactified on a Calabi-Yau manifold, one of the E_8 's contains the SM whereas the other is some hidden sector.

We will assume an initial charge asymmetry, i.e., an excess of particles over anti-particles. This asymmetry may be created through a mechanism similar to those invoked to explain the baryon asymmetry in the universe, such as the Affleck-Dine mechanism [13].

A. Large Q -balls—thin wall approximation

For large Q the Q -ball solution can be analyzed using a thin wall approximation, which consists of neglecting the effect of the surface compared to the bulk. The Q -ball may be approximated by a sphere of radius R_Q with $\phi = \phi_0$ inside and zero field value outside. The mass and radius of the soliton are

$$M_Q = \mu Q, \quad (11)$$

and

$$R_Q = \frac{\beta_Q}{m_\phi} Q^{1/3}, \quad \beta_Q = \left(\frac{3m_\phi^3}{4\pi\omega\phi_0^2} \right)^{1/3}, \quad (12)$$

with $\mu, \omega \rightarrow \mu_0$ for $Q \rightarrow \infty$. The soliton is large and its cross section is given by the geometrical area

$$\sigma_Q = \pi R_Q^2. \quad (13)$$

B. Small Q -balls

The limit of small charge corresponds to $\omega \rightarrow m_\phi$. In this limit the solution of the bounce equation (5) is of the form [14]

$$\bar{\phi} \sim (m_\phi^2 - \omega^2)^{2-a} e^{-\sqrt{m_\phi^2 - \omega^2} r}, \quad (14)$$

with a the power of the attractive term in the potential. This solution has the right behavior for $r \rightarrow \infty$ where $\phi \rightarrow 0$ and the quadratic term in the effective potential dominates, and for $\omega \rightarrow m_\phi$ where the zero of U_ω is near the origin. Using the solution to compute the conserved charge (2), and taking the limit $\omega \rightarrow m_\phi$, one finds a finite, non-zero value only for $4 + 2D - aD > 0$, with D the number of spatial dimensions. In $D = 3$ dimensions, U_1 admits small Q -balls but U_2 does not. Therefore, we will only consider U_1 in the remainder of this section.

In the limit of large ω , or equivalently very non-degenerate minima, one can neglect the quartic terms in $U_\omega(\phi)$. This is the thick wall approximation [15]. The approximation is valid for Q -balls with charge Q that satisfies, for potential U_1 :

$$Q \ll 14.6 m_\phi / \sqrt{\lambda} A, \\ Q < 7.3 m_\phi^2 / A^2. \quad (15)$$

If the above conditions are met one can define an expansion parameter

$$\epsilon \equiv Q \frac{A^2}{3S_\psi m_\phi^2} < \frac{1}{2}, \quad (16)$$

with $S_\psi \approx 4.85$. The mass of the soliton is

$$M_Q = Qm_\phi \left[1 - \frac{1}{6}\epsilon^2 + \mathcal{O}(\epsilon^4) \right]. \quad (17)$$

The radius of the Q -ball can be parametrized:

$$R_Q = \frac{\beta_Q}{m_\phi} Q^{1/3}, \quad (18)$$

with $\beta_Q \sim \mathcal{O}(1)$.

The Q -balls described above are classically stable for arbitrary small charge Q . However, one expects quantum fluctuations to become important in this regime. Indeed, numerical calculations indicate that this is the case, and only configurations with $Q \geq 7$ are quantum mechanically stable [16]. All these calculations are based on the potential U_1 . This potential is an effective potential which is well suited for a semi-classical description of large Q -balls. But for small Q -balls the degrees of freedom of the underlying potential U'_1 should be taken into account. In this regime a treatment in terms of quantum bound states is more appropriate. Solving the bound state problem in full generality is not an easy task. However in the limit that all quartic interactions can be neglected, the theory becomes identical to the Wick-Cutkosky model. This model can be solved analytically for the case of a massless exchange particle, i.e., $m_H = 0$. The various approaches used in the literature, e.g. the ladder approximation, Feshbach-Villars formulation, and variational-perturbative calculations [17–19], all lead to the same result that the bound state spectrum is discrete with the n th state having an energy (to lowest order in α):

$$E_n = 2m_\psi \left(1 - \frac{\alpha^2}{8n^2} \right), \quad \alpha = \frac{1}{16\pi} \frac{A'^2}{m_\psi^2}. \quad (19)$$

The above result for the binding energy is derived in the limit of a massless boson exchange. No analytic results are known for massive scalar exchange. However, numerical studies show that bound states still form, provided that α is larger than some critical value. We estimate, based on the results in [19], that bound states exist for

$$\alpha > \alpha_{\min} \approx 2 \frac{m_H}{m_\psi} + \mathcal{O}\left(\frac{m_H}{m_\psi}\right)^2. \quad (20)$$

That is, the Higgs mass needs to be sufficiently small $m_H \lesssim 10^{-2}(A'/m_\psi)^2 m_\psi$. The energy of the bound state is of the same parametric form as for the massless case.

The other assumption that went into the derivation of Eq. (19) is the absence of quartic couplings. We expect this to be a good approximation in the regime where quartic interactions are negligibly small. The cross section for

$\psi\psi$ -scattering is $\sigma_{\psi\psi \rightarrow \psi\psi} = S|\mathcal{M}|^2/16\pi E_{\text{cm}}^2$. For scattering through Higgs boson exchange, governed by the cubic interaction, this gives at tree level

$$\sigma^{\text{cubic}} \approx \frac{1}{128\pi m_\psi^2} \frac{A'^4}{E^4} \xrightarrow{T \lesssim A'} \sim \pi R_\psi^2. \quad (21)$$

Here $E = \max\{T, m_H\}$. At low temperatures $T \lesssim A'$, which are the temperatures of interest, the cross section quickly approaches the unitarity bound and higher order diagrams cannot be neglected. In this regime we will approximate the cross section by $\sigma \sim \pi R_\psi^2$ with $R_\psi = 2\pi/m_\psi$ the Compton wavelength. Scattering through the quartic point interaction has an amplitude $|\mathcal{M}| = \lambda'$. And thus the requirement that the repulsive quartic interactions are negligibly small, $\sigma^{\text{quartic}} \ll \sigma^{\text{cubic}}$, is satisfied for all quartic couplings $\lambda' \lesssim 1$. It may be that also for non-perturbative values of the quartic couplings bound states persist; but this certainly cannot be analyzed perturbatively. As it seems unnatural to have quartic couplings larger than one, we will ignore this possibility.

On the other hand, the quartic couplings cannot be arbitrarily small or else no Q -ball solution exists: for the case of zero Higgs mass and all quartic couplings equal, $\mu_0^2 > 0$ translates into $\lambda' > A'^2/3m_\psi^2$, as follows from Eq. (10).¹ Non-zero, but small Higgs mass $m_H < 10^{-2}m_\psi$ does not alter this result noticeably. The quartic couplings do not have to be all equal, but at least one of them has to be $\mathcal{O}(A'/m_\psi)^2$. For $A' = m_\psi$, Q -ball solutions exist for example for $(\lambda_1, \lambda_2, \lambda_3) = (0.4, 0.4, 0.4), (1, 0.01, 0.01)$ and $(0.05, 0.8, 0.3)$.

Both the quantum bound states discussed above and Q -balls describe the same objects—stable bound states with a fixed global charge—but in a different language. In both descriptions the existence and stability of the bound state relies on the trilinear coupling and the conserved global charge (that is, conserved particle number). For large bound states quantum fluctuations can be neglected, and a semi-classical description as a Q -ball becomes a good approximation. The trilinear coupling makes it possible for the energy of a bound state with a fixed charge to be less than a collection of free particles with the same charge. In the limit of small particle number (global charge), it becomes necessary to treat the full quantum problem because the semiclassical approximation breaks down. The trilinear term can be viewed as an attractive interaction between the ϕ -particles, which makes it possible for bound states to form. The lowest level bound state is the stable ground state, as charge conservation forbids it to lower its energy through annihilation of ϕ particles.

¹Condition $\mu_0^2 > 0$ corresponds to the requirement that $\phi=0$ is the global minimum of the potential. Q -ball solutions do exist for $\phi=0$ a local minimum. In the potentials U_1 and U_2 this possibility is not realized, since at low temperatures the field will end up in the true vacuum. [U_1 : at the temperature T that the minimum at $\phi \neq 0$ becomes global the energy barrier is $\sim 10^{-2}\lambda T^4$. U_2 : at high temperature $m^2(T) < 0$.]

It is tempting to compare the ground state result ($n=1$) of Eq. (19) with the $q=2$ result obtained in the thick wall approximation (17): both mass formulas give the same parametric dependence. However, in the overlapping regime both approximations are taken beyond their domain of validity: for equal masses $m_H=m_\psi$ bound states can only form for large α , and for $q=2$ a semi-classical treatment breaks down. Of course both approximations are similar in that they neglect the quartic interactions.

In conclusion, the potential U'_1 admits stable, two-particle bound states at low temperatures (below the binding energy), provided the Higgs mass is sufficiently light, and the quartic repulsive interactions small. We repeat that our assumption here is that non-zero quartic couplings do not destabilize the bound state provided $\sigma_{(\phi\phi\rightarrow\phi\phi)}^{\text{quartic}} \ll \sigma_{(\phi\phi\rightarrow\phi\phi)}^{\text{cubic}}$; this should be checked by an explicit calculation. For the potential to have a global minimum at $\phi=0$, or equivalently for Q -ball solutions to exist into which the bound states can grow, the couplings cannot be too small:

$$\lambda' \lesssim 1 \quad \text{repulsive forces small,}$$

$$m_H \lesssim 10^{-2} \left(\frac{A'^2}{m_\psi^2} \right) m_\psi \quad \text{small Higgs boson mass,} \quad (22)$$

$$\lambda' > \frac{A'^2}{3m_\psi^2} \quad Q\text{-balls exist.}$$

A possible set of parameters is $\lambda' \sim 0.5$, $A' \sim m_\psi$ and $m_H \lesssim 10^{-2} m_\psi$. The binding energy for the bound state is then $B_2 = \alpha^2/8 \sim 5 \times 10^{-5} m_\psi$, and $\mu_0 \sim 0.6 m_\psi$.

We will further assume that similar bound states of more than two particles can exist, and that they have energies

$$M_Q = Q m_\psi \left(1 - f_Q \frac{\alpha^2}{8} \right), \quad \alpha = \frac{1}{16\pi} \frac{A'^2}{m_\psi^2} \quad (23)$$

with f_Q some unknown factor depending on the charge Q , the mass of the exchange particle and the strength of the quartic interactions.

The binding energy of a Q -ball is $B_Q = Q m_\psi - M_Q$. Two-particle bound states are only stable at temperatures below the binding energy $T < B_2 \sim \alpha^2/8$. From then on they can grow by capturing charged particles. A non-relativistic particle with kinetic energy $E_k \sim T$ has energy $T + B_Q$ inside the Q -ball/bound state. For it to be captured it has to lose an amount larger than T in the collision. Assuming isotropy, on average a particle will lose half of its energy. Therefore, for temperatures $T < B_Q$ a considerable amount of the particles scattering with the Q -ball will be captured. We approximate the absorption cross section $\sigma_{\text{abs}}(Q)$ for a Q -ball with charge Q by the scattering cross section: $\sigma_{\text{abs}}(Q) \sim \pi R_Q^2$.

III. SOLITOSYNTHESIS

In thermal equilibrium, the production of large Q -balls through gradual charge accretion is very efficient. This process is dubbed solitosynthesis for its similarity with nucleo-

synthesis. It requires an initial charge asymmetry not unlike the baryon asymmetry of the universe. Freeze out of any of the reactions involved will put a halt to solitosynthesis.

In this section we will describe the thermodynamics of Q -balls in terms of a gas of non-relativistic ψ particles in thermal equilibrium. The ψ particles can bind together through the exchange of a light scalar particle, as given by the cubic interaction in U'_1 . For large Q -balls a semi-classical description in terms of U_1 suffices, and ψ can be replaced by ϕ in all the formulas.

A. Q -balls in thermal equilibrium

At non-relativistic temperatures $T < m_\psi$, the number densities of a distribution of Q -balls and free ψ particles in thermal equilibrium are governed by the Boltzmann distribution:

$$n_Q(T) = g_Q \left(\frac{M_Q T}{2\pi} \right)^{3/2} e^{(\mu_Q - M_Q)/T}, \quad (24)$$

and

$$n_\psi(T) = g_\psi \left(\frac{m_\psi T}{2\pi} \right)^{3/2} e^{(\mu_\psi - m_\psi)/T}. \quad (25)$$

Here g_Q is the internal partition function of the Q -ball, and $g_\psi=2$, the number of degrees of freedom of a complex field. Solitosynthesis is only possible if capture rates are large compared to the expansion rate of the universe, otherwise the densities are frozen. If so, the gas of ψ particles and Q -balls is in chemical equilibrium, and the accretion and absorption reactions

$$(Q) + \psi \leftrightarrow (Q+1) \quad (26)$$

enforce a relation between the various chemical potentials: $\mu_Q = Q \mu_\psi$. This allows one to express the Q -ball number density in terms of the ψ -number density

$$n_Q(T) = \frac{g_Q}{g_\psi} n_\psi \left(\frac{M_Q}{m_\psi} \right)^{3/2} \left(\frac{2\pi}{m_\psi T} \right)^{3(Q-1)/2} e^{B_Q/T}, \quad (27)$$

with $B_Q = Q m_\psi - M_Q > 0$ the binding energy of a Q -ball. Similar equations can be written down for the number densities of anti- ψ 's and anti- Q -balls.

We will assume a primordial asymmetry of ψ 's over ψ^* 's, $\eta \equiv (n_\psi - n_{\psi^*})/n_\gamma$, where $n_\gamma = 2.4 T^3/\pi^2$ is the photon number density. Initially one has both ψ and ψ^* particles. For large asymmetry the anti-particle density can be neglected. Also, if the Higgs mass is light then pair annihilation occurs, and at non-relativistic temperature anti-particles deplete rapidly. The annihilation reaction enforces $\mu_\psi = -\mu_{\psi^*}$, which in the non-relativistic limit leads to

$$n_{\psi^*} = n_\psi e^{-2\mu_\psi/T}. \quad (28)$$

For temperature $T \lesssim m_\psi$ the chemical potential $\mu \sim m_\psi$; otherwise the Boltzmann suppression $\exp[(\mu - m)/T]$ is tremendous and the charge conservation equation

$$\eta n_\gamma = n_\psi - n_{\psi^*} + \sum Q n_Q + \sum Q^* n_{Q^*} \quad (29)$$

can never be satisfied. Annihilation is efficient until the density of anti-particles is negligibly small. The number density of stable ψ -particles is then

$$n_\psi \approx \eta n_\gamma, \quad \eta = 2.5 \times 10^{-8} \Omega_\psi h^2 \frac{\text{GeV}}{m_\psi}. \quad (30)$$

The local density can be higher if clustering occurs. This is not to be expected until the matter dominated era, $T_\gamma < T_{\text{eq}} \approx 5.5(\Omega_0 h^2)^{-1} \text{eV}$.

The photon temperature may in general be different from the temperature of the Q -ball system. Particle species that decouple from the heat bath when they are highly relativistic maintain an equilibrium distribution with temperature $T \propto R^{-1}$. The photon temperature redshifts as $T_\gamma \propto g_{*s}^{-1/3} R^{-1}$, and thus the difference in temperatures is given by a factor $\zeta \equiv [g_{*s}(T_D) - g_{*s}(T)]^{1/3}$, with T_D the temperature at which the Q -ball system decouples. When the ψ -particles only interact gravitationally $\zeta \sim 10$, whereas it can be much lower for more general interactions $\psi\psi^* \leftrightarrow X$, where X are light particles that do not carry the same $U_Q(1)$ charge as the ψ -particles. We parametrize $T_\gamma = \zeta T$, with $\zeta \sim 1 - 10$.

The Q -ball densities can start growing when the exponent in Eq. (27) dominates over the potentially small factor in front. Since B_Q grows with Q , formation of large Q -balls is favored. The evolution of a single Q -ball is given by the absorption and evaporation rates of ψ particles by a Q -ball of charge Q [6]. These can be found using detailed balance arguments. In chemical equilibrium we have, for the process in Eq. (26),

$$n_\psi v_\psi \sigma_{\text{abs}}(Q) = n_{Q+1} r_{\text{evap}}(Q+1), \quad (31)$$

and

$$\begin{aligned} \frac{dQ}{dt} &= r_{\text{abs}}(Q) - r_{\text{evap}}(Q) \\ &= n_\psi v_\psi \left[\sigma_{\text{abs}}(Q) - \frac{n_{Q-1}}{n_Q} \sigma_{\text{abs}}(Q-1) \right]. \end{aligned} \quad (32)$$

The Q -ball starts growing when $r_{\text{abs}}(Q) > r_{\text{evap}}(Q)$. This happens for $T \leq T_g$ with

$$\frac{T_g}{m_\psi} = \frac{I_Q/m_\psi}{-\frac{3}{2} \ln\left(\frac{T_g}{m}\right) - \ln(c\zeta^3\eta)}, \quad (33)$$

and

$$I_Q = m_\psi + M_{Q-1} - M_Q. \quad (34)$$

Here $c = (Q/Q-1)^{(13/6)} g_Q/g_{Q-1}$, which goes to one for large Q , and $c \sim 10$ in the limit $Q \rightarrow 2$. I_Q is the binding energy with which a single ψ -particle is bound to the Q -ball. For very small Q -balls $I_Q/m_\psi = f_Q \alpha^2/8$. Accretion of the smallest (Q_{min})-ball starts when $T \sim I_{Q_{\text{min}}} < 10^{-6} m_\psi$ for A'

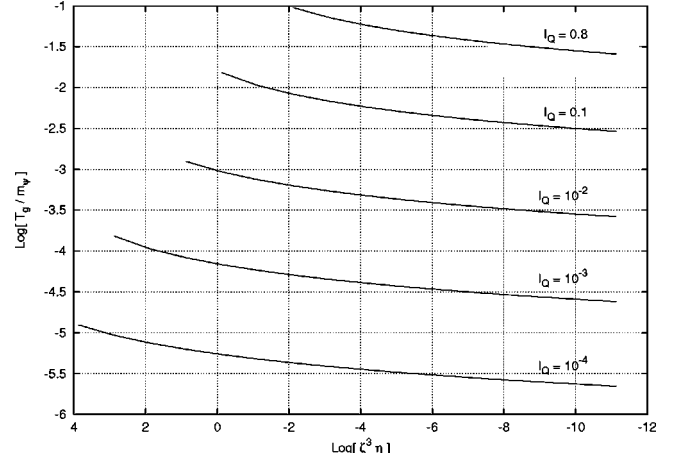


FIG. 1. Accretion time T_g plotted as a function of $c\zeta^3\eta$ for various values of I_Q/m_ψ . For large values of $c\zeta^3\eta$ —to the left of where the lines stop—accretion dominates over evaporation at all (non-relativistic) temperatures.

$\lesssim 1$. For large Q -balls $I_Q = m_\psi - \mu_0$. Figure 1 shows T_g as a function of $c\zeta^3\eta$ for various values of I_Q . For large $c\zeta^3\eta$, Eq. (33) has no solution; here absorption dominates at all non-relativistic temperatures.

For large H mass and early freeze out the charge asymmetry may be small, as then both ψ and ψ^* -densities are large at freeze out while annihilation is negligible. Both particle and anti-particle number are conserved, and one can in principle have growing Q - and anti- Q -balls at the same time. In this case however, the formation of small seed Q -balls, which are necessary to start the fusion process, appears to be a major obstacle.

B. Freeze out

For solitosynthesis to work T_g must be higher than T_{FO} , the temperature at which the absorption reactions Eq. (26) freeze out. This occurs when the reaction rate for accretion becomes smaller than the expansion rate of the universe:

$$\Gamma[(Q) + \psi \rightarrow (Q+1)] \lesssim H. \quad (35)$$

The Hubble constant during the radiation dominated era is $H = 1.7 g_{*s}^{1/2} T_\gamma^2 / M_{\text{pl}}$; it is $H = 1.7 g_{*s}^{1/2} T_\gamma^{3/2} T_{\text{eq}}^{1/2} / M_{\text{pl}}$ in the matter dominated era. The effective degrees of freedom are $g_{*s} \lesssim 10$ for $T_\gamma \lesssim 10$ MeV and $g_{*s} \sim 100$ for $0.1 \text{ GeV} \leq T_\gamma \leq 10^3 \text{ GeV}$. The accretion rate is $\Gamma = n_\psi v_\psi \sigma_{\text{abs}}(Q)$. Neglecting the charge density subsiding in Q -balls, then for stable ψ particles n_ψ is given by Eq. (30). At non-relativistic temperatures $v_\psi = (T/2\pi m_\psi)^{1/2}$.

We are interested in temperatures $T < T_g$; then the cross section is $\sigma_{\text{abs}} \sim \pi R_Q^2$ and freeze out occurs for temperatures, for $T_{\text{FO}} > T_{\text{eq}} \approx 5.5(\Omega_0 h^2)^{-1} \zeta^{-1} \text{eV}$:

$$\frac{T_{\text{FO}}}{m_\psi} \lesssim \left[\frac{10^{-9}}{\zeta \beta_Q Q^{2/3}} \left(\frac{m_\psi}{\text{GeV}} \right)^2 \left(\frac{0.3}{\Omega_\psi h^2} \right) \left(\frac{g_{*s}^{1/2}}{10} \right) \right]^{2/3}. \quad (36)$$

And for $T_{\text{FO}} < T_{\text{eq}}$:

$$\frac{T_{\text{FO}}}{m_\psi} \lesssim \left[\frac{10^{-13}}{\zeta \beta_Q Q^{2/3}} \left(\frac{m_\psi}{\text{GeV}} \right)^{3/2} \left(\frac{0.3}{\Omega_\psi h^2} \right)^{1/2} \left(\frac{g_*^{1/2}}{10} \right) \right]^{1/2}. \quad (37)$$

For $A' \sim m_\psi$, $\zeta \sim 10$ and $m_\psi \lesssim \text{GeV}$ freeze out of the accretion reactions for the smallest Q -balls ($Q=2$) occurs after the accretion phase, $T_{\text{FO}} \lesssim T_g$. In this case solitosynthesis can start at photon temperature $T_\gamma = \zeta T_g \sim 10^{-5} m_\psi \lesssim 10^4$ eV. Note that $I_Q \propto (A'/m_\psi)^4$ decreases rapidly for smaller cubic couplings and $T_{\text{FO}} < T_g$ can only be satisfied for increasingly low ψ -mass. In the matter dominated era the reaction rate, and thus the freeze out temperature, can be increased through clustering. For an overdensity of $\sim 10^5$ in galaxies, we find that for $A' \sim 0.1 m_\psi$ solitosynthesis occurs for $m_\psi \lesssim \text{GeV}$, starting at temperatures $T_\gamma \lesssim \text{eV}$. For smaller values $A' \lesssim 0.1$ or $m_\psi \lesssim \text{MeV}$, solitosynthesis occurs in the future, at temperatures smaller than the present day temperature $T_\gamma < T_0 = 2.35 \times 10^{-4} \text{eV}$.

With the accumulation of charge in Q -balls the number density of ψ particle decreases and the system freezes out. Since the accretion is such an explosive process, this will generally not happen until almost all charge resides in Q -balls. More quantitatively, when the ψ -density decreases to 10% of its original value, T_{FO} decreases only by a factor $10^{-2/3}$. The back reaction is only important when $T_g \approx T_{\text{FO}}$, and it shuts off the growth of Q -balls immediately; in all other cases most charge will end up in Q -balls.

The accretion rate of a single Q -ball is limited by the diffusion rate. However, diffusion of charge is only important when $l \lesssim R_Q$, with $l \sim \Gamma_{\psi\psi}^{-1}$ the mean free path. The radius R_Q of a Q -ball becomes equal to the mean free path for a large charge:

$$Q_{\text{diff}}(T) \sim 10^{87} \left[\left(\frac{m_\psi/T}{10^7} \right)^{7/2} \left(\frac{10}{\zeta} \right)^3 \frac{1}{\beta} \frac{0.3}{\Omega h^2} \frac{m_\psi}{\text{GeV}} \right]^3. \quad (38)$$

For $Q > Q_{\text{diff}}$ diffusion is important. The total amount of charge inside a Hubble volume is $Q_{\text{total}} = n_\psi H^{-3}$

$$Q_{\text{total}}(T) \sim 10^{63} \left(\frac{m_\psi/T}{10^7} \frac{10^2}{\zeta g_*^{1/2}} \right)^3 \left(\frac{\text{GeV}}{m_\psi} \right)^4 \left(\frac{\Omega_\psi h^2}{0.3} \right). \quad (39)$$

For small masses Q_{diff} may be lower than the total charge inside a Hubble volume; in this case Q_{diff} will be an upper limit on the charge of the Q -balls formed during solitosynthesis.

IV. SEEDS

Solitosynthesis is a very efficient way to form large Q -balls, provided there are some initial seed Q -balls at temperatures above freeze out. These seeds may be remnants of an earlier epoch, formed during a phase transition or via the decay of a Bose-Einstein condensate. Another option is that small stable Q -balls can form in the gas of ϕ -particles.

A. Formation of small Q -balls

As discussed in Sec. II B, small Q -ball solutions are only stable for potentials with a cubic interaction. Two-particle bound states can form through scalar exchange, provided the mass of the exchange boson is sufficiently small and the quartic interactions can be neglected. In this case seed Q -balls can be formed copiously and solitosynthesis can start. If the mass of the scalar mass is of the same order as the mass of the charged particles, two-particle bound states do not form, but it may still be that small Q -balls with charge $Q_{\text{min}} > 2$ are stable. Numerical calculations indicate that in the thick wall approximation (which has $m_H = m_\psi$) Q -balls are quantum mechanically stable for $Q_{\text{min}} \gtrsim 7$ [16].

For $Q > 2$, Q -ball formation is suppressed compared to the two-particle bound state, by the requirement that Q charges should be simultaneously in a volume of radius $\sim R_q$. Define $P(q)$ to be the probability to find a charge Q in the volume of a Q -ball, $V_q \approx R_q^3$. The mean charge in V_q is $\bar{q} = n_\psi V_q$, whereas the variance is $\sigma^2 = \langle (\Delta q)^2 \rangle = T(\partial \bar{q} / \partial \mu)_{T,V} = \bar{q}$. Since

$$\bar{q} \approx 1.0 \zeta^3 \eta q \left(\frac{T}{m_\psi} \right) \ll 1 \quad (40)$$

a discrete distribution is needed, the Poisson distribution:

$$P(q) = \frac{e^{-\bar{q}} \bar{q}^q}{q!} \approx \frac{(n_\psi V_q)^q}{q!}. \quad (41)$$

The density of lumps with charge Q in a volume V_q is $n_q = P(q)/V_q$. The reaction rate for the bound state is $\Gamma_q^{\text{bnd}} \sim \sigma_q^{\text{bnd}} n_q$ so that the chance that in a Hubble time a “ Q -lump” forms a bound state is $\sim n_q \sigma_q^{\text{bnd}} H^{-1}$. Multiplying this with the total number of Q -lumps in a Hubble volume gives the number of Q -ball seeds $N_q \sim n_q^2 \sigma_q^{\text{bnd}} H^{-4}$. Taking $R_Q \sim 1/m_\psi$, this yields

$$N_q \sim \eta^q (\sigma_{\text{bnd}} m_\psi^2) \left(\frac{T}{m_\psi} \right)^{6Q-8} \left(\frac{M_{\text{pl}}}{m_\psi} \right)^4. \quad (42)$$

Assuming $\sigma_q^{\text{bnd}} \lesssim \sigma_{\psi\psi}$ this gives an upper bound on N_q . Only for small $q=2,3,4$ or so is N_q larger than unity, and there is seed-forming.

B. Primordial seeds

The seed Q -balls may also be Q -balls formed at an earlier epoch. For this to be possible the initial Q -balls should be large enough to survive the period of evaporation. The evaporation rate is given by the detailed balance equation (31). Ignoring absorption, which is subdominant for $T < T_g$ (note that the evaporation rate decreases exponentially with temperature), one gets that the smallest Q -ball to survive has charge [6]

$$Q_s \sim 10^{57} \frac{\beta^6}{g_*^{3/2}} \left(\frac{\text{GeV}}{m_\psi} \right)^3 \left(\int_{T_g}^{T_i} dT \frac{e^{-I_Q/T}}{I_Q/T} \right)^3, \quad (43)$$

with T_i the temperature at formation. For $T_{\text{FO}}, T_g < T_i$, the integral can be approximated by $\sim \exp(-I_Q/T_i)/(1+I_Q/T_i)$. Only for masses $m_\psi \leq \text{eV}$ is Q_s smaller than the total number of particles available in a Hubble volume at $T_i \sim m_\psi$, Eq. (39), and is there a change for very large primordial Q -balls to survive the period of thermal evaporation.

Another possibility is that formation happens at the onset or during the accretion phase: $T_i \lesssim T_g$. For large binding energy $I_Q \rightarrow m_\psi$ (which is possible for large Q -balls) and large η' , accretion dominates over evaporation at non-relativistic temperatures, see Fig. 1.

Primordial Q -balls may also form during a first order phase transition [20] from the false “ Q -ball vacuum” to the true vacuum. At the Ginzburg temperature thermal transitions between regions of false and true vacuum freeze out; any region of false vacuum with a charge larger than the minimum charge of a stable Q -ball surviving below this temperature will become a Q -ball. The potentials under consideration do not exhibit the required first order phase transition (see footnote 1). One could add additional terms to the potential to get a phase transition. However, the survival of regions of false vacuum is exponentially suppressed with size, and correspondingly Q -ball formation is exponentially suppressed with charge. If formed, the Q -balls are expected to be small $Q \sim Q_{\text{min}}$. Unless there is a mechanism to delay the phase transition to very low temperatures $T \lesssim 10^{-6} m_\psi$, these Q -balls quickly evaporate and are cosmologically unimportant.

Formation of primordial Q -balls through fragmentation of a condensate [3] is studied in the next section.

V. BOSE-EINSTEIN CONDENSATION

We will now study whether there will be condensation. A condensate that is unstable under fluctuation can fragment into possibly large Q -balls. We will consider the effective potentials U_1 and U_2 . In this section $m_\phi = 1$, i.e., all quantities are expressed in units of mass.

We will assume that the number density of anti-particles can be neglected and $\rho \approx n_\phi$. The state of the system is given by the minimum of the effective potential for a fixed charge Q :

$$V(q, \phi) = V(\mu, \phi) + \mu\rho, \quad (44)$$

with $V(\mu, \phi)$ the effective potential for a fixed chemical potential. In this section ϕ denotes the classical background field, and ϕ_0 its value at the minimum of $V(q, \phi)$. A non-zero value of ϕ_0 signals the existence of a condensate. At finite temperature the frequency ω of the Q -ball can be identified with the chemical potential μ [21]. The charge density can be solved from

$$\frac{dV(q, \phi)}{d\mu} = 0 \quad \Rightarrow \quad \rho = \rho(\mu). \quad (45)$$

Eliminating μ in Eq. (44) then gives the effective potential in a fixed charge section. A stable configuration lies at the minimum of $V(q, \phi)$.

To analyze the stability of the condensate one can consider fluctuations in the homogeneous background. From the dispersion relation it follows that fluctuations are amplified for wavelengths smaller than k_{max} [3,22]:

$$k_{\text{max}}^2 = \frac{\rho^2}{\phi_0^4} - U''(\phi_0). \quad (46)$$

For $\rho^2 - \phi_0^4 U''(\phi_0) < 0$ the above equation does not have a physical solution and the condensate is stable.

We parametrize the charge density is $\rho = \eta n_\gamma$

$$\rho \approx 3 \times 10^{-9} \zeta^3 \left(\frac{\Omega_\phi h^2}{0.3} \right) \left(\frac{\text{GeV}}{m_\phi} \right) T^3 \equiv \eta' T^3. \quad (47)$$

A. Non-relativistic limit

At zero temperature the charge density $\rho \propto T^3$ is zero, and there is no condensate. At non-zero temperature condensate formation will occur if the charge is larger than the number of excited states.

In the non-relativistic limit the finite-temperature corrections to the potential are small, and as a first approximation we can use the zero temperature result $V(\mu, \phi) = U(\phi) - 1/2 \mu^2 \phi^2$ with $U(\phi)$ the classical potential, Eqs. (6),(7). Equation (45) gives $\rho = \mu \phi^2$. The minimum of $V(q, \phi)$ is at

$$\rho_1^2 = \phi_0^4 - 3A\phi_0^5 + 4\lambda\phi_0^6, \quad \text{for } U_1$$

$$\rho_2^2 = \phi_0^4 - 4A\phi_0^6 + 6\lambda\phi_0^8, \quad \text{for } U_2. \quad (48)$$

At low temperatures $\mu \rightarrow 1$ and $\rho \approx \phi_0^2 \ll 1$. In this limit a possible condensate is unstable against decay for values $\phi_0 < 3A/8\lambda$ for U_1 and $\phi_0 < A/3\lambda$ for U_2 , as follows from Eq. (46).

To see whether a condensate actually forms one has to compute the density of thermal states. At low temperatures the cubic and quartic terms in the potential become negligibly small, and the theory approaches the free theory. In this limit the number of thermal states is

$$n_\beta^{\text{NR}} = \frac{\zeta(3/2)}{(2\pi)^{3/2}} T^{3/2}. \quad (49)$$

Since $\rho \propto T^3$, at low temperature all charge will be in the excited states and the condensate is empty. The only chance to have a filled condensate is for $T \rightarrow 1$ and η' large, so that $\rho > n_\beta$ or $\eta' T^{3/2} > \zeta(3/2)/(2\pi)^{3/2} \approx 0.17$. Note however that in the limit $T \rightarrow 1$ the non-relativistic approximation breaks down, whereas in the limit $\eta' T^3 \rightarrow 1$ the free field approximation breaks down.

B. Relativistic limit

We will first consider the potential U_1 . The effective potential for fixed chemical potential to highest order in T is

$$V(\mu, \phi) = 1/2(1 + \lambda T^2/3 - \mu^2)\phi^2 - A\phi^3 + \lambda\phi^4 - \mu^2 \frac{T^2}{6} + c(T) + \mathcal{O}(T), \quad (50)$$

with $c(T)$ some temperature dependent constant which we will drop. From this it follows that

$$\rho = \mu\phi^2 + \mu T^2. \quad (51)$$

The first term in the above equation is the charge in the condensate, the second term represents the charge in excited states. The charge fraction in the condensate is $\phi_0^2/(\phi_0^2 + T^2)$, which is small for $\phi_0 \ll T$. The effective potential for fixed charge density in the relativistic limit becomes

$$V(q, \phi) = \frac{1}{2} \left(1 + \frac{\lambda}{3} T^2 \right) \phi^2 - A\phi^3 + \lambda\phi^4 + \frac{3\rho^2}{2(3\phi^2 + T^2)}. \quad (52)$$

Consider the case $\phi_0 \ll T$; then the potential is minimized at $\phi_0 = 0$ (and thus the approximation is consistent), provided

$$\eta'^2 < \frac{\lambda}{27} + \frac{1}{9T^2}. \quad (53)$$

This can also be seen from the second derivative $V''(q, 0) = 1 + \lambda/3T^2 - 9\eta'^2 T^2$, which becomes negative for large η' . Thus if condition (53) is obeyed there is no condensate. For fine-tuned values of A^2/λ a second minimum of the potential may develop, but since in the limit of large temperature the only minimum is at $\phi_0 = 0$ the field will not end up there.

Consider then the potentially more interesting case that η' is large, and condition (53) is not satisfied. Then $V''(0) < 0$ and the potential is minimized at non-zero field value. Minimization in the limit $T \gg \phi_0$ as well as in the limit $T \ll \phi_0$ does not yield a consistent solution. It follows that the minimum is at $\phi_0 \sim T$. This is confirmed by numerical calculations. The charge density in the condensate is comparable to that in excited states. The condensate is unstable for $k_{\max}^2 > 0$, Eq. (46), with

$$k_{\max}^2 = \frac{\eta'^2 T^6}{\phi_0^4} - \frac{1}{3} \lambda (T^2 + 36\phi_0^2) + 6A\phi_0 - 1. \quad (54)$$

At large temperatures $\phi_0 \propto \lambda^{-1}$ and the second term in the above equation dominates, as can be verified numerically. The condensate is stable for large T . The condensate becomes unstable in the limit $T \rightarrow 1$. As this is also the limit in which the high temperature expansion breaks down, it is unclear whether the condensate really fragments.

The analysis for potential U_2 is similar. At high temperature the effective potential becomes

$$V(\mu, \phi) = \frac{1}{2} (1 - \frac{3}{4} A T^2 - \mu^2) \phi^2 + (\frac{3}{2} \lambda T^2 - A) \phi^4 + \lambda \phi^6 - \mu^2 \frac{T^2}{6} + c(T) + \mathcal{O}(T). \quad (55)$$

For this case, the equivalent of Eq. (53) is

$$\eta'^2 + \frac{A}{12} < \frac{1}{9T^2}. \quad (56)$$

At large temperature a stable condensate will form with $\phi_0 \sim \sqrt{A/2 + 6\eta'^2/2\sqrt{\lambda}}$ for small asymmetry or $\phi_0 \sim T$ for large asymmetry $\eta' \gtrsim 1/9$. The condensate only survives in the limit $T \rightarrow 1$ for large η' . The condensate may be unstable in this limit.

To conclude this section, at non-relativistic temperatures there is no condensate and all charge resides in excited states. At temperatures $T \gtrsim m_\phi$ condensation occurs for large asymmetries $\eta' \gtrsim 1/9$, corresponding to masses $m_\psi \lesssim \text{eV}$. The condensate becomes unstable in the limit $T \rightarrow m_\phi$ and fragments into Q -balls. Caution should be taken, as the high temperature expansion breaks down in this limit. If the binding energy of the Q -balls is sufficiently large $A^2/2\lambda \gtrsim 10^{-2}$ the period of evaporation is absent, see Fig. 1, and these Q -balls survive.

VI. CONCLUSIONS

To summarize, solitosynthesis is a very efficient way to form large Q -balls provided some primordial charge asymmetry and initial seed Q -balls exist. Most theories do not allow small stable Q -ball or bound state solutions, and solitosynthesis does not start. The exceptions are theories in which the attractive interaction is provided by a cubic term of the form $AH\psi^*\psi$. Bound states can form if the Higgs mass is light ($m_H/m_\psi \lesssim 10^{-3} A^2/m_\psi^2$). No bound state calculations have been done in the presence of quartic coupling. We assume that for quartic interactions that are small compared to interactions governed by the cubic term bound states persist ($\lambda \lesssim 1$). We note that if this assumption turns out to be too optimistic, and stable bound states require smaller quartic couplings, then small bound states and large Q -balls become mutually exclusive. This is because for the potential to admit Q -ball solutions the quartic coupling cannot be too small ($\lambda \gtrsim A^2/m_H^2$). Successful solitosynthesis will occur if the accretion phase happens before the system falls out of equilibrium. All these conditions together limit the parameter space severely.

For solitosynthesis to have happened in the early universe one needs $A = 0.1 - 1 m_\psi$, at least one of the quartic couplings $\lambda \sim 1$, $m_H \lesssim 10^{-2} m_\psi$, and $\text{MeV} \lesssim m_\psi \lesssim \text{GeV}$. This rules out models in which the H field is the standard model Higgs field, such as the MSSM and the model studied in [12]. The temperature at which Q -balls start growing decreases very rapidly with A : $T_g/m_\psi \propto (A/m_\psi)^4$. For smaller values of the masses or of the cubic coupling than given above, solitosynthesis may still happen in the future.

Q -balls that survive until present can be part of the dark matter in the Universe. For them to play a role during structure formation they must have been formed before the universe became matter dominated, that is at temperatures $T_\gamma \gtrsim T_{\text{eq}} = 5.5(\Omega_0 h^2)^{-1}$ eV. This is only possible for $A' \sim 1$ and $m_\psi \sim \text{GeV}$. Whether the Q -balls can fulfill the required cross section to mass ratio to overcome the problems with cold dark matter as proposed in [9] remains another question. More (numerical) studies are needed to determine if solitosynthesis results in a few Q -balls with a very large charge, or in a large number of Q -balls with lesser charge.

Condensate formation is only possible for large asymmetries, or equivalently $m_\psi \lesssim \text{eV}$. Symmetries of the order one can be generated through the Affleck-Dine mechanism [13]. Early decoupling increases the number of charged particles by a factor ζ^3 with $\zeta = [g_{*s}(T_D) - g_{*s}(T)]^{1/3}$, which favors condensation. The condensate becomes unstable against fluctuations in the limit $T \rightarrow m_\psi$, i.e., the limit in which all the used approximations break down. Evidently, better approximations are needed to settle the matter. Q -balls formed through a possible fragmentation of the condensate survive

until present if accretion dominates over evaporation at non-relativistic temperatures. This is possible for Q -balls with a large binding energy, $I_Q = m_\psi - (m_\psi^2 - A^2/2\lambda)^{1/2} \gtrsim 10^{-2} m_\psi$.

The potentials studied do not allow for a first order phase transition from the false “ Q -ball vacuum” to the true vacuum. One could try and add terms to the potential so that such a phase transition occurs. However, the Q -balls that may form during the phase transition are small and will evaporate quickly.

Solitosynthesis can lead to a phase transition from the false to true vacuum. This will not happen for the potentials studied in this paper, as for these the field will always end up in the true vacuum.

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