Vacuum replicas in QCD

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The properties of the vacuum are addressed in two- and four-dimensional quark models for QCD. It is demonstrated that two-dimensional QCD (the 't Hooft model) possesses only one possible vacuum state—the solution to the mass-gap equation, which provides spontaneous breaking of chiral symmetry (SBCS). On the contrary, the four-dimensional theory with confinement modeled by the linear potential with a Coulomb OGE interaction not only has a chirally noninvariant ground vacuum state, but possesses an excited vacuum replica, which also exhibits SBCS and can be realized as a metastable intermediate state of hadronic systems. We discuss the influence of this vacuum replica on physical observables as well as on the possibility of probing the vacuum background fields in QCD.

I. INTRODUCTION

Potential models play an important role in studies of QCD. In spite of their obvious shortcomings, such as the loss of Lorentz invariance and causality, they offer a simple and intuitive tool for investigation of the chiral properties of the theory, hadronic spectra, decays, and so on. The Hamiltonian approach, taken together with the Bogoliubov technique for diagonalization, provides a powerful method to analyze such potential models. The two-dimensional 't Hooft model for QCD, in addition to being a genuine quantum field theory, constitutes an example of an exactly solvable theory with an instantaneous linear potential yielded by the two-dimensional gluon [1].

In this paper we address the question of whether QCD can possess excited vacuum states, here called replicas, on top of the usual chirally nonsymmetric vacuum. The problem of different vacuum states, their coexistence and stability, is not new and has been addressed many times in many contexts (as an example, see, e.g., [2] or [3] where the "history" of the vacuum is briefly outlined). Here we address the theoretical possibility of the existence of replicas in the framework of quark QCD models which have otherwise been successful in describing hadronic phenomenology, and will argue that if one uses these types of model—which indeed can be traced back to the Gaussian approximation for the cumulant expansion of QCD_4—for their phenomenological success then one must also consider the quite possible existence of these replicas. They come in the same package. Then it should not come as a surprise that the number of spatial dimensions together with the strength and form of the interquark potential should play a role of paramount importance in the existence of such replicas. We start from two-dimensional QCD and demonstrate that, for two dimensions, only one chirally nonsymmetric state (the vacuum) may exist. Therefore in two dimensions there are no replicas. This statement holds

true for both linear and harmonic oscillator potentials. On the contrary, an infinite set of solutions exists for the mass-gap equation in four dimensions, if the interquark interaction is chosen to be quadratic in the interquark distance, as was found in [4]. In the present paper we confirm this result. However, for the more realistic linear potential [5] the situation in QCD_4 changes dramatically. We study the corresponding mass-gap equation and find that the *pure* linearly rising interaction *just* fails to be sufficiently "binding" to hold any replicas, so that only one chirally nonsymmetric solution to the mass-gap equation may exist in this case (the trivial chirally symmetric solution is always present in 3 +1 dimensions, as opposed to the two-dimensional case). On the contrary, for a more physical interaction with realistic values for both the string tension σ_0 and the strong coupling constant α_s of the Coulomb interaction, together with a constant term $\gamma_\mu U \gamma_\mu$ to adjust for the right chiral condensate, we find one vacuum replica to exist in addition to the usual vacuum state. We give qualitative arguments that the excited vacuum state should exist in real QCD and discuss its possible influence on physical observables.

This paper is organized as follows. In the second section we consider the two-dimensional 't Hooft model and briefly review the formalism, going through the concepts of the Hamiltonian approach, the Bogoliubov transformation from bare to dressed quarks, and the mass-gap equation for the chiral angle. We argue that the mass-gap equation for the 't Hooft model has only one solution, found numerically in [6], which defines the chirally noninvariant phase of the theory, whereas the phase with unbroken chiral symmetry possesses an infinite energy and, hence, is unphysical. These results are found to remain for a similar, but technically simpler, situation when using the harmonic oscillator instead of the linear potential. In the third section we deal with the case of four-dimensional QCD and study the mass-gap equation numerically, for both harmonic and linear confining quark

kernels. We find that an enumerable infinite set of replicas always exists in the case of the quadratic potential, whereas the linear potential appears to be just not quite "binding" enough, so that it has to be added to either a Coulomb or a constant potential (or to a combination of both) to support the existence of an excited vacuum state. It turns out that for a realistic set of parameters $\{\sigma_0, \alpha_s, U\}$ simultaneously describing hadronic spectra *and* yielding a quark condensate in the ballpark of $\langle \bar{q}q \rangle = -(250 \text{ MeV})^3$ it is *impossible* to avoid the existence of one replica. The fourth section is devoted to a discussion of the hadronic processes in the presence of the excited vacuum. We give arguments on how the excited vacuum replica can affect the physical observables and serve as a probe of the QCD vacuum structure. Our conclusions and the outlook are the subject of the fifth section. Throughout this paper we consider the chiral limit, i.e., we always put the mass of the quark m equal to zero from the very beginning.

II. TWO-DIMENSIONAL QCD ('T HOOFT MODEL)

A. Introduction to the model

The two-dimensional 't Hooft model, suggested in 1974 [1] and widely discussed in the literature as a toy model for QCD, is described by the Lagrangian density

$$L(x) = -\frac{1}{4} F^a_{\mu\nu}(x) F^a_{\mu\nu}(x) + \bar{q}(x) i \hat{D} q(x),$$

$$\hat{D} = (\partial_\mu - i g A^a_\mu t^a) \gamma_\mu, \tag{1}$$

and the large-N_C limit implies that[1]

$$\gamma = \frac{g^2 N_C}{4\pi} \underset{N_C \to \infty}{\longrightarrow} \text{const.} \tag{2}$$

We fix the axial gauge imposing the condition $A_1(x_0, x) = 0$ [7,8], so that the vector-field propagator takes the form

$$D^{ab}_{\mu\nu}(x_0 - y_0, x - y) = -\frac{i}{2} \delta^{ab} g_{\mu 0} g_{\nu 0} |x - y| \delta(x_0 - y_0), \tag{3}$$

and integrate out gluonic degrees of freedom to arrive at the Hamiltonian

$$H = \int dx\, q^\dagger(x) \left(-i\gamma^5 \frac{\partial}{\partial x} \right) q(x)$$
$$- \frac{g^2}{2} \int dx \int dy\, q^\dagger(x) \frac{\lambda^a}{2} q(x) q^\dagger(y) \frac{\lambda^a}{2} q(y) \frac{|x - y|}{2}. \tag{4}$$

Following the standard BCS technique, we introduce the dressed quark field[2] [7]

$$q_\alpha(t, x) = \int \frac{dp}{2\pi} e^{ipx} [b_\alpha(p, t) u(p) + d^\dagger_\alpha(-p, t) v(-p)], \tag{5}$$

$$b_\alpha(p)|\tilde{0}\rangle = d_\alpha(-p)|\tilde{0}\rangle = 0, \quad b^\dagger_\alpha(p)|\tilde{0}\rangle = |q\rangle,$$

$$d^\dagger_\alpha(-p)|\tilde{0}\rangle = |\bar{q}\rangle, \tag{6}$$

$$\{b_\alpha(p, t) b^\dagger_\beta(q, t)\}$$
$$= \{d_\alpha(-p, t) d^\dagger_\beta(-q, t)\}$$
$$= 2\pi \delta(p - q) \delta_{\alpha\beta}, \tag{7}$$

$$u(p) = T(p)\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad v(-p) = T(p)\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{8}$$

$$T(p) = e^{-\theta(k)\gamma_1/2}, \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2},$$

$$\theta(-p) = -\theta(p),$$

where the Bogoliubov-Valatin angle θ is subject to the mass-gap equation. In the new basis the Hamiltonian (4) takes the form

$$H = LN_C \mathcal{E}_v + :H_2: + :H_4:, \tag{9}$$

where $LN_C \mathcal{E}_v$ stands for the vacuum energy (L being the one-dimensional volume) and the second and third terms on the right-hand side (RHS) are quadratic and quartic in the quark (antiquark) creation (annihilation) operators. In the remainder of this paper we shall concentrate on the energy density $\mathcal{E}_v$.

B. The vacuum energy and chiral symmetry breaking

The vacuum energy $LN_C \mathcal{E}_v$ can be calculated as an average of the time ordered Hamiltonian (4) over the BCS vacuum state,

$$E_{\text{vac}} = \langle \tilde{0} | TH(b, b^\dagger, d, d^\dagger) | \tilde{0} \rangle$$
$$\equiv LN_C \mathcal{E}_{\text{vac}} = LN_C (\mathcal{E}^{\text{free}}_{\text{vac}} + \Delta \mathcal{E}_{\text{vac}}), \tag{10}$$

where we have introduced the excess of the vacuum energy density over the free-theory energy density $\mathcal{E}^{\text{free}}_{\text{vac}}$. We have

$$\Delta \mathcal{E}_{\text{vac}} = \int \frac{dp}{2\pi} \text{Tr} \left\{ \gamma^5 p \Lambda_-(p) \right.$$
$$\left. + \frac{\gamma}{4\pi} \int \frac{dk}{(p-k)^2} \Lambda_+(k) \Lambda_-(p) \right\} - \mathcal{E}^{\text{free}}_{\text{vac}},$$

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[1]We consider the weak limit of the theory $m \gg g \sim 1/\sqrt{N_C}$, so that the limit $N_C \to \infty$ is to be taken prior to $m \to 0$ [9].

[2]We put tilde in the notation of the BCS vacuum, $|\tilde{0}\rangle$, to distinguish it from the trivial one, $|0\rangle$, annihilated by the bare operators b_0 and d_0.

$$\Lambda_{\pm}(p) = T(p) \frac{1 \pm \gamma^0}{2} T^+(p), \qquad (11)$$

or, in terms of the angle θ,

$$\Delta\mathcal{E}_{\text{vac}}[\theta] = -\int \frac{dp}{2\pi} [p \sin\theta(p) - |p|]$$

$$- \frac{\gamma}{4\pi} \int \frac{dp\,dk}{(p-k)^2} \cos[\theta(p) - \theta(k)], \quad (12)$$

where for the free massless theory we substituted $\gamma = 0$ and $\theta_{\text{free}}(p) = (\pi/2)\text{sgn}(p)$. The excess $\Delta\mathcal{E}_{\text{vac}}[\theta]$ is the main object of our study. The true angle θ should minimize the vacuum energy, that is, it is a solution to the mass-gap equation

$$\frac{\delta\Delta\mathcal{E}_{\text{vac}}[\theta]}{\delta\theta(p)} = 0. \qquad (13)$$

First, we perform a simple qualitative analysis of Eq. (12). Let $\theta(p)$ be the solution of Eq. (13) corresponding to the minimum of $\Delta\mathcal{E}_{\text{vac}}[\theta]$. Then for $\theta(p/A)$, with an arbitrary parameter $A \neq 1$, $\Delta\mathcal{E}_{\text{vac}}[\theta/A] > \Delta\mathcal{E}_{\text{vac}}[\theta]$. Naive dimensional analysis shows that $\Delta\mathcal{E}_{\text{vac}}$ scales with A as

$$\Delta\mathcal{E}_{\text{vac}} = \frac{1}{2} C_1 A^2 - \gamma C_2, \qquad (14)$$

with $C_{1,2}$ being positive constants. Then the stable solution is provided by minimizing the energy (14) with respect to A, that is, by $A_0 = 0$, which corresponds either to the free massless theory

$$\theta_{\text{free}}(p) = \frac{\pi}{2}\text{sgn}(p), \quad E_{\text{free}}(p) = |p|, \qquad (15)$$

with $E_{\text{free}}(p)$ being the free-quark dispersive law, or to the analytical solution of the mass-gap equation found in [7], which reads

$$\theta(p) = \frac{\pi}{2}\text{sgn}(p), \quad E(p) = |p| - P\frac{\gamma}{|p|}, \qquad (16)$$

where the symbol P stands for the principal-value prescription. Both solutions yield unbroken chiral symmetry. Thus one would arrive at the erroneous conclusion that no chirally nonsymmetric solution of the mass-gap equation may exist, were it not for the fact that the vacuum energy (12) is logarithmically infrared divergent and the correct form of the relation (14) should be

$$\Delta\mathcal{E}_{\text{vac}} = \frac{1}{2} C_1 A^2 - \gamma C_2 \ln A + \gamma C_3 \qquad (17)$$

instead of Eq. (14), with C_3 proportional to the logarithm of the cutoff.[3] The function (17) always has a minimum at

$$A_0 = \sqrt{\gamma \frac{C_2}{C_1}} \neq 0, \qquad (18)$$

which corresponds to a nontrivial solution of the mass-gap equation, found numerically in [6]. From the form of the function (17) one can see the logarithmic growth of the energy when approaching the solution (16), which corresponds to $A_0 = 0$.[4] Therefore two conclusions can be deduced. First, there is no need to demand $\Delta\mathcal{E}_{\text{vac}}$ to be negative as the nontrivial vacuum energy defined by Eq. (18) is always compared with the infinite energy of the chirally symmetric phase. Second, no phase transition of chiral symmetry restoration is possible in the 't Hooft model, its phase diagram being trivial. A similar conclusion is made in [10].

The chiral condensate $\langle\bar{q}q\rangle$ is given by

$$\langle\bar{q}q\rangle = -\frac{N_C}{\pi} \int_0^\infty dp \cos\theta(p). \qquad (19)$$

Thus the nontrivial solution, found in [6], provides a nonvanishing chiral condensate, whereas the solution (16) gives $\langle\bar{q}q\rangle = 0$, but with an *infinite* energy. Thus we conclude that in the 't Hooft model there can be only one phase with spontaneously broken chiral symmetry. Note that there is no contradiction with the Coleman theorem [11], which precludes spontaneous breaking of symmetries in two-dimensional theories, if one considers the infinite-N_C limit [9,12].

The contribution of the infrared logarithmic divergence in Eq. (17) has yet another consequence for the 't Hooft model—namely, that the nonvanishing chiral condensate and spontaneous chiral symmetry breaking already happen at the BCS level, whereas going beyond the BCS approximation just provides corrections suppressed by powers of N_C. Indeed, staying at the BCS level, one diagonalizes the theory in the one-particle sector, introducing a nontrivial vacuum, full of correlated $q\bar{q}$ pairs. It was demonstrated in [8] that one can go beyond the BCS level in the 't Hooft model by performing a second Bogoliubov-like transformation and diagonalizing the theory in the mesonic sector, that is, in the sector of the quark-antiquark bound states. The true vacuum of the theory $|\Omega\rangle$ is connected to the BCS vacuum $|\tilde{0}\rangle$ via a unitary operator. One should use the true vacuum state for calculations of all matrix elements of operators, like the chiral con-

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[3]Notice that there is no way to get rid of the logarithmic divergence in $\Delta\mathcal{E}_{\text{vac}}$ defined by Eq. (12). If the infrared behavior of the integrand is improved, for example, by changing $\cos[\theta(p) - \theta(k)]$ to $\cos[\theta(p) - \theta(k)] - 1$ (principal-value prescription), then an ultraviolet logarithmic divergence appears. Thus one cannot remove both divergences in Eq. (12) simultaneously.

[4]Since the logarithmically divergent term in Eq. (17) is proportional to the coupling constant γ, then the theory still possesses the free limit (15) (which also corresponds to $A_0 = 0$) when γ tends to zero and the logarithmic term disappears.

densate, but as shown in [8] the difference between the two averages is of next-to-leading order in N_C:

$$\langle \Omega | \bar{q}q | \Omega \rangle = \langle \tilde{0} | \bar{q}q | \tilde{0} \rangle + O(\sqrt{N_C}) \sim O(N_C) + O(\sqrt{N_C}). \tag{20}$$

It is instructive to compare the 't Hooft model with other two-dimensional models, e.g., the Gross-Neveu one [13]. The latter is given by the interaction Lagrangian

$$L_{\text{int}} = \frac{1}{2} g(\bar{\psi}_i \psi_i)^2, \quad g > 0, \tag{21}$$

where the flavor index i runs from 1 to $N \gg 1$, whereas $\lambda = g^2 N$ remains finite. The positive sign of the constant g chosen in Eq. (21) is known to lead to condensation in the singlet $\bar{\psi}\psi$ channel [13], whereas the negative sign leads to charged $\psi\psi$ and $\bar{\psi}\bar{\psi}$ condensates [14]. It was shown that the chiral symmetry is also broken in the Gross-Neveu model, but in order to have a nontrivial chiral condensate one has to go beyond the BCS level, summing fermionic bubbles. Bosonization of the model, in terms of the compound state $\sigma = g\bar{\psi}\psi$, appears to be the most convenient formalism for doing this and the resulting renormalized "potential" reads

$$V(\sigma, \sigma_0) = \frac{1}{2}\sigma^2 + \frac{\lambda}{4\pi}\sigma^2 \left[\ln\left(\frac{\sigma}{\sigma_0}\right)^2 - 3 \right] \tag{22}$$

with σ_0 being the renormalization point, which brings the mass scale into the theory. Only the first term in Eq. (22) appears at the BCS level, leading to the trivial solution $\sigma_{\text{min}} = 0$. On the other hand, the full potential (22) also provides a nontrivial solution,

$$\sigma_{\text{min}} = \pm \sigma_0 \exp\left(1 - \frac{\pi}{\lambda} \right). \tag{23}$$

From Eq. (22) one can see that the logarithmic term, which scales as $\ln A$ under the above mentioned transformation, is multiplied by $\sigma^2 \sim A^2$ which cancels the logarithmic growth of the potential at the origin. Thus the two phases, chirally symmetric and nonsymmetric, coexist in the model, whereas the latter, with nonvanishing σ_{min}, is energetically preferable.

The chiral condensate of the Gross-Neveu model scales as

$$\langle \sigma \rangle \sim \langle g\bar{\psi}\psi \rangle \sim gN \sim \lambda\sqrt{N} \sim \sqrt{N} \tag{24}$$

for large N. Relation (24) is the analogue of Eq. (20) with a vanishing leading term of order $O(N)$, which is in agreement with the statement made above that the chiral condensate in the Gross-Neveu model vanishes at the BCS level. One can arrive at the same conclusion from simple dimensional considerations. Indeed, the Gross-Neveu model describes a scale-invariant renormalizable theory, so that there is no parameter with the dimension of mass in the bare Lagrangian and one has to deal with the mechanism of the dimensional transmutation to get a scale. In contrast, the 't Hooft Lagrangian (1) defines a super-renormalizable theory and the

dimensional parameter, the coupling constant g, is present in the theory from the very beginning.

Formula (17) can be given a more physically transparent form if one notices that the chiral condensate $\Sigma \equiv \langle \bar{q}q \rangle$ from Eq. (19) scales linearly with A, $\Sigma \to A\Sigma$, so that it can be used instead of A,

$$\Delta \mathcal{E}_{\text{vac}} = C_1' \left[\frac{1}{2}\left(\frac{\Sigma}{\Sigma_0} \right)^2 - \ln\left| \frac{\Sigma}{\Sigma_0} \right| \right] + \gamma C_3', \tag{25}$$

where Σ_0 stands for the actual value of the chiral condensate.

C. Harmonic-oscillator-type interaction

In this subsection we complete the study of two-dimensional Hamiltonians using a harmonic oscillator kernel in Eq. (4), instead of the linear kernel,

$$\gamma |x - y| \to K_0^3 (x - y)^2, \tag{26}$$

or, in momentum space,

$$\frac{\gamma}{(p - k)^2} \to \pi K_0^3 \delta''(p - k), \tag{27}$$

with K_0 being the new constant with the dimension of mass. Because of the presence of the delta function in k in the interaction kernel, all integrals can be considerably simplified. For example, the excess of the vacuum energy $\Delta \tilde{\mathcal{E}}_{\text{vac}}[\theta]$, similar to Eq. (12), reads now

$$\Delta \tilde{\mathcal{E}}_{\text{vac}}[\theta] = -\int \frac{dp}{2\pi} \left\{ [p \sin \theta(p) - |p|] - \frac{\pi K_0^3}{4} [\theta'(p)]^2 \right\}. \tag{28}$$

The integral on the RHS in Eq. (28) is convergent and scales as

$$\Delta \tilde{\mathcal{E}}_{\text{vac}}[\theta] = \tilde{C}_1 A^2 + K_0^3 \frac{\tilde{C}_2}{A} \tag{29}$$

under $\theta(p) \to \theta(p/A)$, so that qualitatively the same conclusion holds true, that there exists only one phase of the theory with spontaneously broken chiral symmetry and there is no unbroken phase. As for Eq. (25), we can rewrite Eq. (29) using the chiral condensate instead of A:

$$\Delta \tilde{\mathcal{E}}_{\text{vac}} = \tilde{C}_1' \left[\frac{1}{2}\left(\frac{\Sigma}{\tilde{\Sigma}_0} \right)^2 + \left| \frac{\tilde{\Sigma}_0}{\Sigma} \right| \right] \tag{30}$$

with the minimum given by $\Sigma = \tilde{\Sigma}_0$.

D. Numerical solutions for the chiral angle

In the previous subsections we found that there must be nontrivial functions $\theta(p)$ providing the minimum of the vacuum energies (12) and (28). As was mentioned above, these functions are solutions to the mass-gap equations
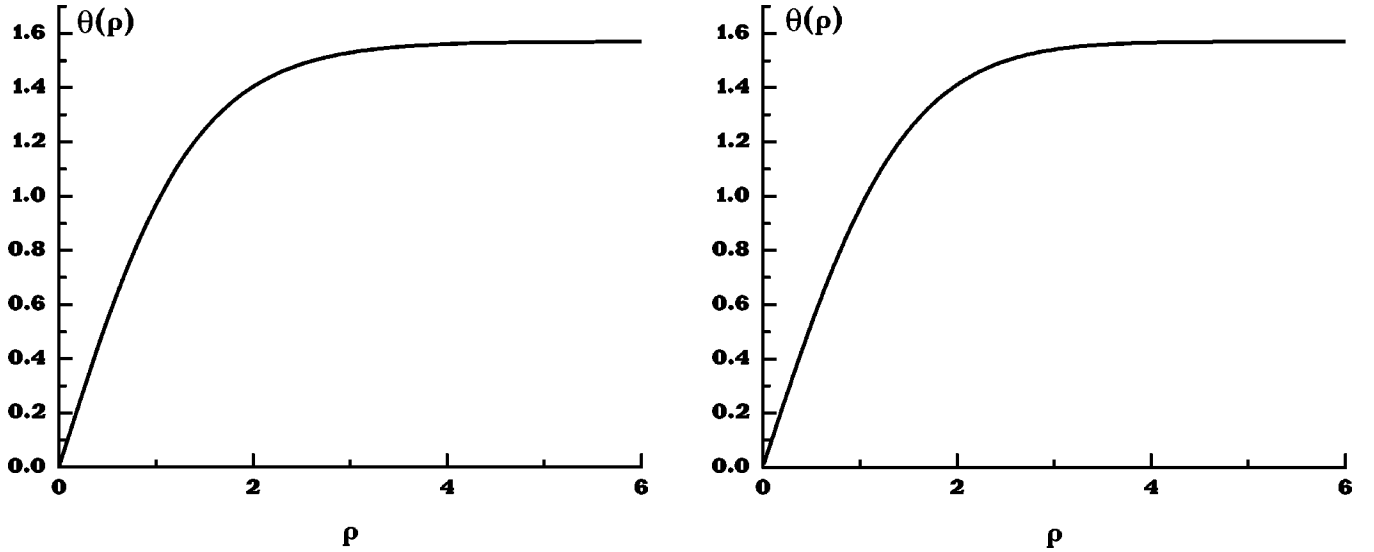
FIG. 1. The ground-state solutions of the two-dimensional mass-gap equations (32) that correspond to the linear (left plot) and the oscillator (right plot) potentials, respectively. The momentum p is given in units of $\sqrt{\gamma}$ for the left plot and in units of K_0 for the right one.

$$\frac{\delta \Delta \mathcal{E}_{\text{vac}}[\theta]}{\delta \theta(p)} = 0, \quad \frac{\delta \Delta \tilde{\mathcal{E}}_{\text{vac}}[\theta]}{\delta \theta(p)} = 0, \tag{31}$$

which take the following forms:

$$p \cos \theta(p) = \frac{\gamma}{2} \int \frac{dk}{(p-k)^2} \sin[\theta(p) - \theta(k)],$$

$$p \cos \theta(p) = -\frac{\pi K_0^3}{2} \theta''(p) \tag{32}$$

for the linear and harmonic oscillator potentials, respectively.

Now we turn to numerical studies of these two equations. Note that the qualitative analysis performed in the previous subsection cannot answer the question of how many solu-

tions of a mass-gap equation exist. Indeed, a second solution, if it exists, would just lead to different constants C ($\tilde{C}$) and a different value of the chiral condensate Σ_0 ($\tilde{\Sigma}_0$), since this solution is expected to have a different profile and it cannot be obtained from the ground-state solution by dilatations.

As the first step, we reproduce the solution of the first equation in (32) found in [6] and construct a similar solution for the other (see Fig. 1). Notice that we define the chiral angle $\theta(p)$ from $-\pi/2$ to $\pi/2$ with the boundary conditions $\theta(p=0)=0$, $\theta(p \to \infty) \to \pi/2$ [as was mentioned before, $\theta(-p) = -\theta(p)$]. It turns out that under such conditions no excited solutions of the mass-gap equations (32) exist. Only if both requirements are relaxed can one find the solutions depicted in Fig. 2, which start at 0 for $p=0$ and tend to $\pi/2 + 2\pi n$, $n \geq 1$, for $p \to \infty$. From Fig. 1 one can easily see that the solutions for the ground state in both types of con-
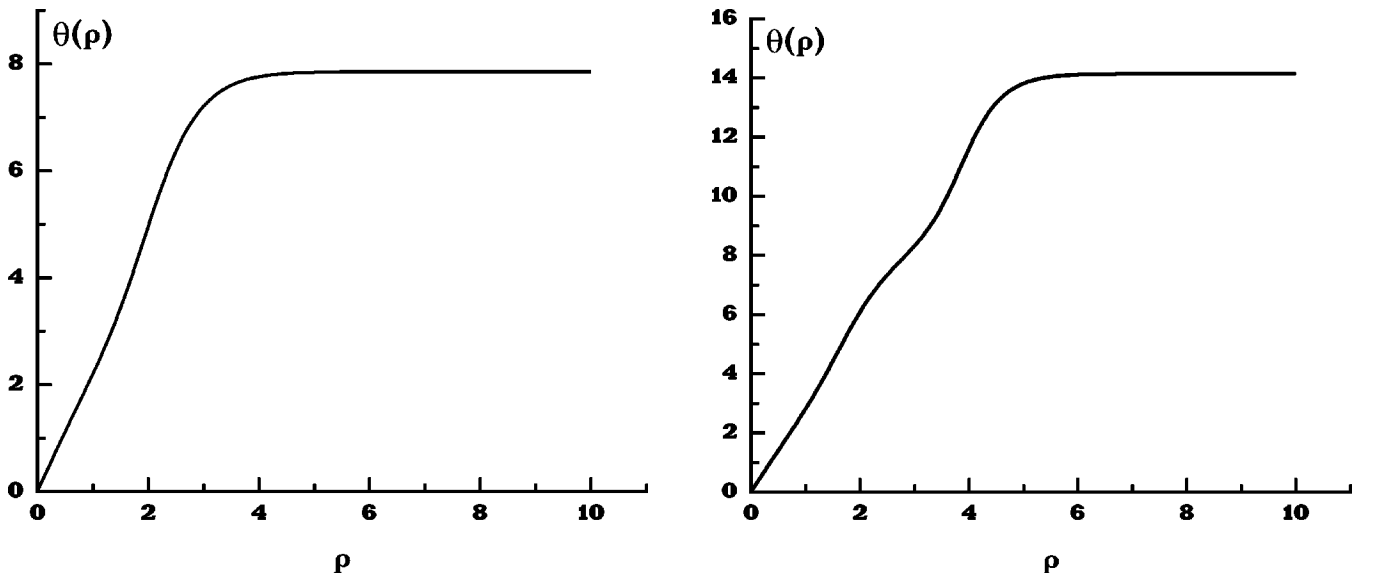

FIG. 2. The unphysical excited solutions to the mass-gap equation (32) (oscillator confinement). The momentum p is given in units of K_0.

fining potential have a very similar shape, so it would be natural to expect similar excited solutions to exist for the linear potential as well. However, such solutions lead to maximal condensation of the $\bar{q}q$ pairs not only at $p=0$, but for some finite p's, where $\cos\theta=1$, which is unphysical, so we disregard these solutions.

Thus we conclude that only the ground-state solution may exist for the mass-gap equation in the 't Hooft model, defining the only possible phase of the theory with spontaneously broken chiral symmetry and the chiral condensate

$$\Sigma_0 = -0.410 N_C \sqrt{\gamma}, \quad (33)$$

which coincides with the value found in a sequence of papers [6,8,9]. A similar result for the theory with the oscillator-type interaction reads

$$\widetilde{\Sigma}_0 = -0.405 N_C K_0, \quad (34)$$

which is very close to Eq. (33) if the scales $\sqrt{\gamma}$ and K_0 are chosen close to one another.

Notice that in two-dimensional QCD the energy of the vacuum is an apparently meaningless quantity by itself, although it leads to the meaningful mass-gap equation, as discussed above. Indeed, the expression (12) contains a divergent integral, so that the result depends on the cutoff. On the other hand, in view of the fact that the physical solution of the mass-gap equations is unique, one cannot define a difference of two energies, where the divergent term would cancel.

Now, when the numerical methods are tuned and tested on the simple example of two-dimensional QCD, we turn to the main subject of the present paper—namely, studies of the vacuum replicas in QCD_4.

III. FOUR-DIMENSIONAL QCD

A. Instantaneous interaction and forms of the potential: The mass-gap equation

In this subsection we consider the case of QCD_4 and study the problem of existence of the vacuum replicas for various types of gluonic correlator. In contrast to QCD_2, we put the number of colors equal to three from the very beginning.

Following the standard procedure, we start from the QCD partition function and perform its cluster expansion, so that, after integrating out the gluonic field, the theory contains effective four-quark, six-quark, and so on, vertices with the form factors given by the corresponding correlators of the gluonic fields,

$$\langle\langle A_{\mu_1}^{a_1}(x_1)A_{\mu_2}^{a_2}(x_2)\rangle\rangle, \quad \langle\langle A_{\mu_1}^{a_1}(x_1)A_{\mu_2}^{a_2}(x_2)A_{\mu_3}^{a_3}(x_3)\rangle\rangle, \ldots, \quad (35)$$

where the irreducible average $\langle\langle\cdots\rangle\rangle$, is defined in the standard way,

$$\langle\langle 1\rangle\rangle=\langle 1\rangle, \quad \langle\langle 12\rangle\rangle=\langle 12\rangle-\langle 1\rangle\langle 2\rangle, \ldots \quad (36)$$

We fix the Fock-Schwinger gauge $(x-z_0)_\mu A_\mu^a(x)=0$ with an arbitrary fixed point z_0 [17] that allows us to connect the gluonic field with the field-strength tensor $F_{\mu\nu}^a(x)$,

$$A_\mu^a(x)=\int_0^1 \alpha\,d\alpha(x-z_0)_\nu F_{\nu\mu}^a(\alpha(x-z_0)+z_0), \quad (37)$$

and, finally, to deal with the set of gauge-invariant cumulants [18],

$$\langle\langle F_{\mu_1\nu_1}^{a_1}(x_1,z_0)F_{\mu_2\nu_2}^{a_2}(x_2,z_0)\rangle\rangle,$$

$$\langle\langle F_{\mu_1\nu_1}^{a_1}(x_1,z_0)F_{\mu_2\nu_2}^{a_2}(x_2,z_0)F_{\mu_3\nu_3}^{a_3}(x_3,z_0)\rangle\rangle, \ldots,$$

$$F_{\mu\nu}^a(x,z_0)\equiv\Phi(z_0,x)F_{\mu\nu}^a(x)\Phi(x,z_0), \quad (38)$$

with $\Phi(x,y)$ being the standard parallel transporter from the point x to the point y. Now we apply a set of approximations.

On one hand, Gaussian dominance is used, which means that we keep only the lowest-order correlator $\langle\langle FF\rangle\rangle$, whereas all others are considered suppressed [18]. Such an approximation can be justified, first, by Casimir scaling, suggested long ago [19] and recently confirmed by lattice calculations [20]. Casimir scaling is exact in the Gaussian approximation. On the other hand, the minimal-area law asymptotic for an isolated Wilson loop can be saturated by the Gaussian correlator. Finally, numerical calculations of hadronic observables within this method show good agreement with experimental and lattice data. The details of the method can be found, e.g., in the review [21] and references therein.

As far as the form of the nonperturbative part of the bilocal correlator is concerned, we choose it in the form

$$\langle\langle A_\mu^a(x)A_\nu^b(y)\rangle\rangle=\int_0^1\alpha\,d\alpha\int_0^1\beta\,d\beta(x-z_0)_\lambda(x-z_0)_\sigma$$
$$\times\langle\langle F_{\lambda\mu}^a(\alpha(x-z_0)+z_0)$$
$$\times F_{\sigma\nu}^b(\beta(y-z_0)+z_0)\rangle\rangle$$
$$=\delta^{ab}g_{\mu 0}g_{\nu 0}V_0(|\vec{x}-\vec{y}|)\delta(x_0-y_0), \quad (39)$$

which implies that the gluonic correlation length T_g (also playing the role of the QCD string radius) is small enough that the stringlike interaction can be approximated by the instantaneous one.[5] For the confining potential $V_0(|\vec{x}|)$, we consider two cases, the quadratic and linear forms

$$V_0(|\vec{x}|)=K_0^3|\vec{x}|^2, \quad (40)$$

$$V_0(|\vec{x}|)=\sigma_0|\vec{x}|. \quad (41)$$

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[5]Lattice calculations [22] give the value $T_g\approx 0.2$–0.3 fm which is small enough at the hadronic scale.

The Lorentz structure of confinement is taken to be $\gamma_0 \times \gamma_0$. In the linear case we shall add to $V_0(|\vec{x}|)$ a Coulomb term and a constant,

$$V_1(|\vec{x}|) = -\frac{\alpha_s}{|\vec{x}|} + U, \qquad (42)$$

with the Lorentz structure being $\gamma_\mu \times \gamma_\mu$. Notice that because the color structure of the interaction is $\lambda^a/2 \times \lambda^a/2$, the constant term in the potential (42) yields a force between singlets that is proportional to the resonating group method normalization overlap of these singlets. This is a simple example of the Wigner-Eckart theorem. The Coulomb potential is necessary for this class of model to possess a heavy-quark limit compatible with the known spectroscopy.

Thus our model is defined by the parameter set $\{\sigma_0, \alpha_s, U\}$. We shall choose the usual literature values for two of them, $\sigma_0 = 0.135$ GeV^2, $\alpha_s = 0.3$, and adjust U to obtain a quark condensate $\langle \bar{q}q \rangle \simeq -(250$ MeV$)^3$. Next we turn to the study of the vacuum structure following the same steps used in QCD$_2$.

First, we introduce the dressed quark field [15,23]

$$q_\alpha(t,\vec{x}) = \sum_{\zeta=\uparrow,\downarrow} \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p}\vec{x}} [b_{\alpha\zeta}(\vec{p},t) u_\zeta(\vec{p})$$

$$+ d_{\alpha\zeta}^\dagger(-\vec{p},t) v_\zeta(-\vec{p})], \qquad (43)$$

with α and ζ being the quark color and the projection of the spin, respectively, and the amplitudes u and v defined as

$$u(\vec{p}) = \left[\sqrt{\frac{1+\sin\varphi}{2}} + \sqrt{\frac{1-\sin\varphi}{2}} \hat{p}\cdot\vec{\alpha} \right] u_0(\vec{p}),$$

$$v(-\vec{p}) = \left[\sqrt{\frac{1+\sin\varphi}{2}} - \sqrt{\frac{1-\sin\varphi}{2}} \hat{p}\cdot\vec{\alpha} \right] v_0(-\vec{p}).$$

$$(44)$$

Here the chiral angle φ is used instead of the QCD$_2$ θ. They are simply related by

$$\varphi(p) = \frac{\pi}{2} - \theta(p). \qquad (45)$$

It is convenient to define the new angle φ from $-\pi/2$ to $\pi/2$ with the boundary conditions $\varphi(p=0) = \pi/2$ and $\varphi(p \to \infty) \to 0$.

If the chiral angle $\varphi(p)$ is not identically zero, then the true vacuum state is populated by quark-antiquark pairs with $^3P_0$ coupling,

$$|\tilde{0}\rangle = \prod_{cfp} dp \left(\frac{1+\cos 2\phi(p)}{2} + \frac{\sin 2\phi(p)}{2} C_{cfp}^\dagger \right.$$

$$\left. + \frac{1-\cos 2\phi(p)}{4} C_{cfp}^{\dagger 2} \right) |0\rangle \equiv S_0|0\rangle, \qquad (46)$$

where the indices c and f indicate the color and flavor, respectively; the angle $\phi(k)$ is connected to the chiral one $\varphi(k)$ via the simple relation

$$\varphi(p) = \arctan\frac{m}{p} + 2\phi(p) \underset{m=0}{=} 2\phi(p), \qquad (47)$$

and defines the above mentioned unitary operator S_0:

$$C_{cfp}^\dagger = (b_\uparrow^\dagger(\vec{p}), b_\downarrow^\dagger(\vec{p}))_{cf} \mathfrak{M} \begin{pmatrix} d_\uparrow^\dagger(\vec{p}) \\ d_\downarrow^\dagger(\vec{p}) \end{pmatrix}_{cf}, \qquad (48)$$

$$\mathfrak{M}_{\sigma_1\sigma_2} = (-\sqrt{6}) \sum_{\sigma,m} \begin{pmatrix} 1 & 1 & 0 \\ m & \sigma & 0 \end{pmatrix} \hat{p}_{1m} \begin{pmatrix} 1/2 & 1/2 & 1 \\ \sigma_1 & \sigma_2 & \sigma \end{pmatrix}$$

$$= \begin{pmatrix} -\hat{\vec{p}}_x + i\hat{\vec{p}}_y & \hat{\vec{p}}_z \\ \hat{\vec{p}}_z & \hat{\vec{p}}_x + i\hat{\vec{p}}_y \end{pmatrix}. \qquad (49)$$

The operators annihilating the new vacuum $|\tilde{0}\rangle$ are

$$\tilde{b}(\vec{p}) = S_0 b(\vec{p}) S_0^\dagger, \quad \tilde{d}(-\vec{p}) = S_0 d(-\vec{p}) S_0^\dagger. \qquad (50)$$

The interested reader can find the details of the formalism in [16].

The next step consists of arranging the normal ordering of the QCD$_4$ Hamiltonian,

$$H = \int d^3x [H_0(\vec{x}) + H_I^{(1)}(\vec{x}) + H_I^{(2)}(\vec{x})], \qquad (51)$$

$$H_0(\vec{x}) = q^\dagger(\vec{x})(-i\vec{\alpha}\cdot\vec{\nabla}) q(\vec{x}), \qquad (52)$$

$$H_I^{(1)}(\vec{x}) = \frac{1}{2} \int d^3y \, q^\dagger(\vec{x}) \frac{\lambda^a}{2} q(\vec{x})$$

$$\times [V_0(|\vec{x}-\vec{y}|) + V_1(|\vec{x}-\vec{y}|)] q^\dagger(\vec{y}) \frac{\lambda^a}{2} q(\vec{y}),$$

$$(53)$$

$$H_I^{(2)}(\vec{x}) = -\frac{1}{2} \int d^3y \, q^\dagger(\vec{x}) \vec{\alpha}\frac{\lambda^a}{2} q(\vec{x}) V_1(|\vec{x}-\vec{y}|)$$

$$\times q^\dagger(\vec{y}) \vec{\alpha}\frac{\lambda^a}{2} q(\vec{y}). \qquad (54)$$

In the new basis, we obtain an expression similar to Eq. (9) and we again concentrate on the minimization of the vacuum energy density—namely, on the mass-gap equation, which takes the form

$$A(p)\cos\varphi(p) - B(p)\sin\varphi(p) = 0,$$

$$A(p) = \frac{1}{2} C_F \int \frac{d^3k}{(2\pi)^3} [V_0(\vec{p}-\vec{k})$$

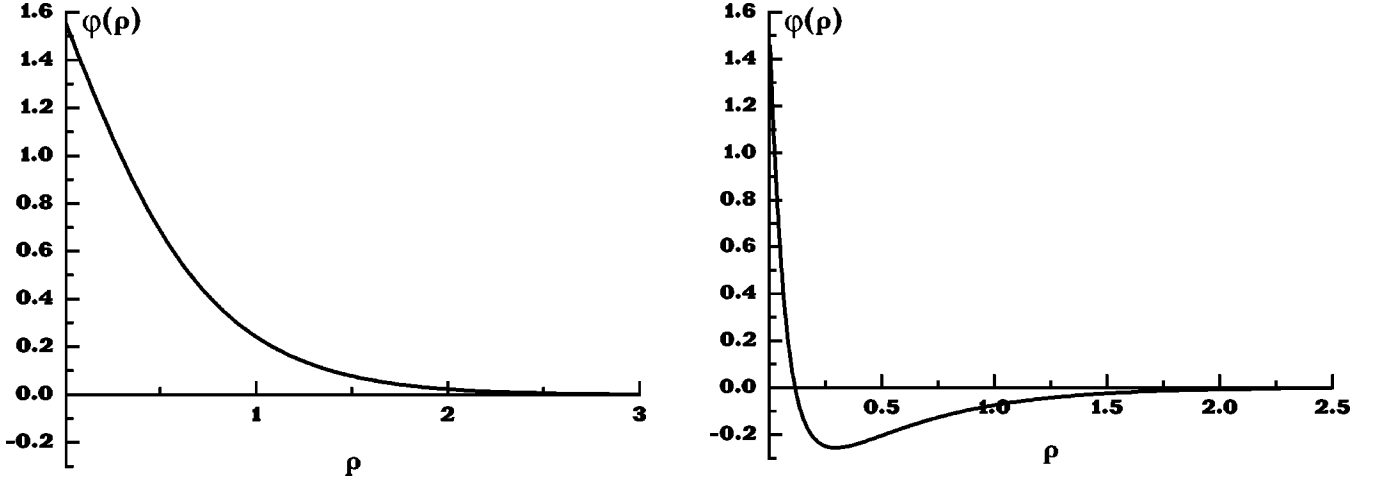$$+ 4V_1(\vec{p}-\vec{k})]\sin\varphi(k), \qquad (55)$$

FIG. 3. The first two solutions of Eq. (57) for the chiral angle $\varphi(p)$ found with the Runge-Kutta method. The momentum p is given in units of K_0.

$$B(p) = p + \frac{1}{2} C_F \int \frac{d^3k}{(2\pi)^3} (\hat{\vec{p}} \cdot \hat{\vec{k}})$$

$$\times [V_0(\vec{p} - \vec{k}) + 2V_1(\vec{p} - \vec{k})] \cos \varphi(k),$$

where $C_F = \frac{4}{3}$ is the Casimir operator in the fundamental representation.

Thus, equipped with the general form of the mass-gap equation, we are in a position to perform the numerical search for the excited vacua for the oscillator and linear potentials.

B. Solutions of the mass-gap equation for the oscillator-type interaction

In this subsection we solve numerically the mass-gap equation (55) for the oscillator-type confining potential (40) with V_1 put to zero. Similarly to the two-dimensional case, the Fourier transform of the potential (40) is the second derivative of a delta function,

$$V_0(\vec{p}) = -(2\pi)^3 K_0^3 \frac{\partial^2}{\partial \vec{p}^2} \delta^{(3)}(\vec{p}), \qquad (56)$$

which leads to the mass-gap equation in differential form,

$$\varphi'' = -\frac{2}{p} \varphi' + \frac{2p}{C_F K_0^3} \sin \varphi - \frac{1}{p^2} \sin 2\varphi, \qquad (57)$$

and we solve it numerically using the Runge-Kutta method. In Fig. 3 we show the ground-state and the first excited solutions of Eq. (57).

Thus we conclude that at least one replica exists for the case of a quadratic confining potential. Moreover, using the same numerical technique, one can build the third, fourth, and so on, excited solutions, as was found in [4]. In fact, the harmonic oscillator supports an infinite tower of such replicas.

C. Vacuum replica for the linear potential

Finally, we turn to the main goal of the present paper—namely, whether the linear confining potential supports the existence of replicas.

In contrast to the case of the oscillator potential, the mass-gap equation for the linear confinement, given by Eq. (41), is integral:

$$p \sin \varphi(p) = \frac{C_F \sigma_0}{2\pi^2} \int \frac{d^3k}{(\vec{p} - \vec{k})^4} [\sin \varphi(k) \cos \varphi(p)$$

$$- \hat{\vec{p}} \cdot \hat{\vec{k}} \sin \varphi(p) \cos \varphi(k)], \qquad (58)$$

or, equivalently,

$$p \sin \varphi(p) = \frac{\sigma}{4\pi} \int_0^\infty dk\, k^2 \left[\frac{4}{(p^2 - k^2)^2} \sin \varphi(k) \cos \varphi(p) \right.$$

$$- \left(\frac{2k(k^2 + p^2)}{p(p^2 - k^2)^2} - \frac{1}{p^2} \ln \left| \frac{p+k}{p-k} \right| \right)$$

$$\left. \times \cos \varphi(k) \sin \varphi(p) \right], \qquad (59)$$

where $\sigma = C_F \sigma_0$ is the string tension in the fundamental representation, and the angular variables are integrated out. One can check that the integral on the RHS of Eq. (59) is convergent. Equation (59) possesses only one nontrivial eigenstate: the ground-state vacuum. Everything happens as if the confining linearly rising potential just fails to be "binding" enough to produce an excited vacuum replica. In addition, the chiral condensate appears strongly underestimated in the case of the pure linear confining potential [5]. Thus we supply it with the extra interaction given by Eq. (42), which reads, in momentum space,

$$V_1(\vec{p}) = -\frac{4\pi \alpha_s}{\vec{p}^2} + (2\pi)^3 U \delta^{(3)}(\vec{p}), \qquad (60)$$

and modify the mass-gap equation accordingly:

$$p \sin \varphi(p) = \frac{\sigma}{4\pi} \int_0^\infty dk\, k^2 \left[\frac{4}{(p^2-k^2)^2} \sin \varphi(k) \cos \varphi(p) - \left(\frac{2(k^2+p^2)}{pk(p^2-k^2)^2} - \frac{1}{p^2 k^2} \ln\left| \frac{p+k}{p-k} \right| \right) \cos \varphi(k) \sin \varphi(p) \right]$$

$$- \frac{C_F \alpha_s}{4\pi} \int_0^\infty dk\, k^2 \left[\frac{4}{pk} \ln\left| \frac{p+k}{p-k} \right| \sin \varphi(k) \cos \varphi(p) + \left(\frac{p^2+k^2}{p^2 k^2} \ln\left| \frac{p+k}{p-k} \right| - \frac{2}{pk} \right) \cos \varphi(k) \sin \varphi(p) \right]$$

$$+ \frac{1}{4} C_F U \sin 2\varphi(p). \qquad (61)$$

In contrast to Eq. (59), the mass-gap equation (61) is not divergence-free because of the Coulomb part, proportional to α_s. If we considered the case of a nonzero quark mass m, then the given divergence would be only logarithmic; however, for Eq. (61) we have $m=0$ and the divergence becomes linear. We regularize it by using a modified potential V_1,

$$V_1(\vec{p}) = 4\pi \alpha_s \left[\frac{1}{\vec{p}^2 + \Lambda^2} - \frac{1}{\vec{p}^2} \right] + (2\pi)^3 U \delta^{(3)}(\vec{p}), \quad (62)$$

with a cutoff Λ. Notice that the mass-gap equation for the single Coulomb potential does not contain any scale (it would be given by the quark mass m beyond the chiral limit), which appears only after the regularization and is given by Λ. Then all quantities with the dimension of mass are directly proportional to Λ.

Now let us assume, for a moment, the general case of an arbitrary nonvanishing current quark mass. Then the quarks acquire an effective mass m_{eff}, which can be written as a sum of two components,

$$m_{\text{eff}} = m + \Sigma(m,\Lambda), \qquad (63)$$

where m and $\Sigma(m,\Lambda)$ are the current quark mass and the quark self-energy due to the contribution of the chiral condensate, respectively. The latter is obviously Λ dependent. To evaluate m_{eff} one can use the relations

$$\sin \varphi(p) = \frac{m(p)}{\sqrt{p^2 + m^2(p)}}, \quad \cos \varphi(p) = \frac{p}{\sqrt{p^2 + m^2(p)}}, \qquad (64)$$

similar to the free-case relations, but with $\varphi(p)$ being the solution to the mass-gap equation. Now the effective mass m_{eff} can be associated with $m(p \sim 0)$. Let us consider the heavy-quark limit, $m \to \infty$, first. A large quark current mass is known to destroy the chiral condensate. As a result, the self-energy part of the relation (63) must vanish, so that the effective mass is basically given by the current mass,

$$m_{\text{eff}} \underset{m \to \infty}{\simeq} m. \qquad (65)$$

In particular, $\Sigma(m,\Lambda)/m \underset{m \to \infty}{\to} 0$. That is, for large m, we have to go up in Λ for the system to become more and more Coulombic, whereas the chiral condensate goes to zero. In

other words the potential of Eq. (62) naturally possesses the Coulomb limit for very heavy current quark masses.

In the opposite limit, $m=0$, the effective mass appears entirely due to the chiral symmetry breaking,

$$m_{\text{eff}} = \Sigma(0,\Lambda) = \Lambda \times \text{const}, \qquad (66)$$

where the last equality holds true provided we choose $\Lambda \sim \sqrt{\sigma_0}$, the only scale of the problem. Thus in *either* case $m_{\text{eff}} \gtrsim \Lambda$, and it is remarkable that such a potential should equally address both heavy- and light-quark sectors.

Therefore we adopt the following strategy. We choose the standard values of the string tension, $\sigma_0 = 0.135$ GeV^2, and the strong coupling constant, $\alpha_s = 0.3$. As discussed above, we put Λ to be of the order of $\sqrt{\sigma_0}$ and solve the resulting mass-gap equation numerically for the ground state, fitting the only free parameter U in such a way as to have the chiral condensate

$$\langle \bar{q}q \rangle = -\frac{3}{\pi^2} \int_0^\infty dp\, p^2 \sin \varphi(p) \qquad (67)$$

around its standard value, about $-(250 \text{ MeV})^3$. With the solution found for the chiral angle $\varphi(p)$, we evaluate the effective quark mass m_{eff} and check if it has a value of order 200–300 MeV, that is, a value of the same order of magnitude as the regulator Λ, and also close to the value usually adopted for the constituent quark mass. For self-consistency we also need to ensure that the fitting parameter U appears to be of the order of the interaction scale and hence does not bring a new scale into the problem. In Table I, as well as the known values of σ_0 and α_s, we give the best set of parameters U and Λ consistent with the standard value of the chiral condensate.

Now, when all parameters of the model are totally fixed, we perform our search for the excited vacuum replica which, indeed, appears to exist, and we give its profile, together with

TABLE I. Parameters of the model fixed from the fit for the ground vacuum state.

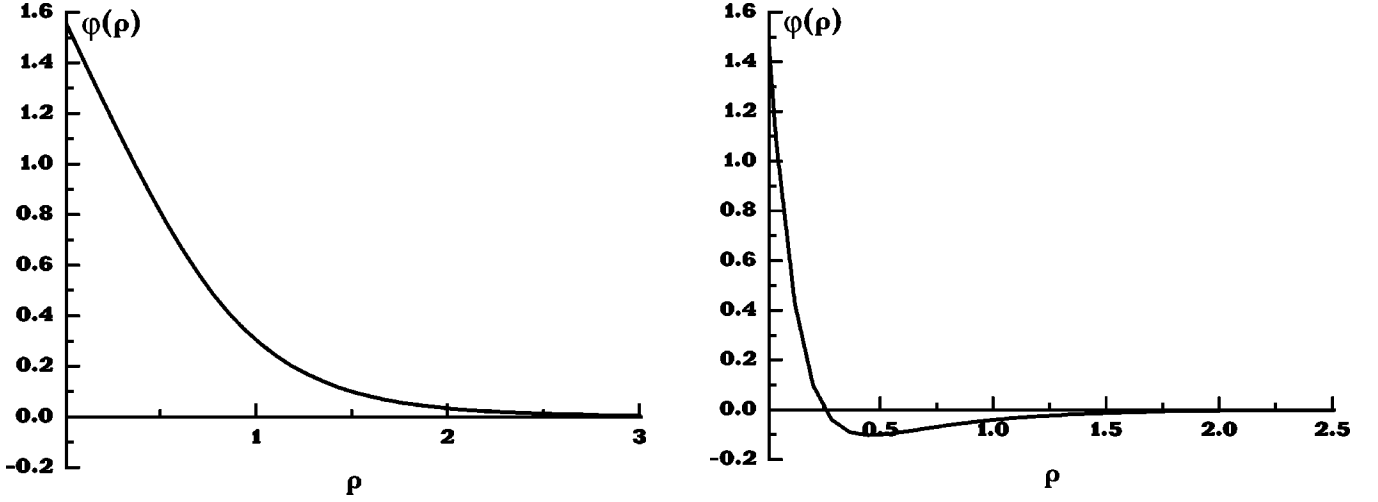| σ_0 (GeV^2)	α_s	U (MeV)	Λ (MeV)	m_{eff} (MeV)	$\langle \bar{q}q \rangle$ (MeV^3)
0.135	0.3	220	250	230	$-(250)^3$

FIG. 4. The ground-state (left plot) and the first excited (right plot) solution of Eq. (61) for the set of parameters given in Table I. The momentum p is given in units of $\sqrt{\sigma_0}$.

the ground-state one, in Fig. 4. We tried many other combinations of U and Λ and, whatever values we used, we always found this replica to persist.

The chiral condensate for the excited vacuum state,

$$\langle \widetilde{\overline{q}q} \rangle = (126 \text{ MeV})^3, \tag{68}$$

is smaller in the absolute value and has a "wrong" sign compared to the ground-state one, given in Table I. For massless quarks this sign can be easily reversed by the change $\varphi \to - \varphi$ everywhere, which also satisfies the mass-gap equation and does not affect the vacuum energy. However, a nonvanishing quark mass breaks this symmetry, since an extra term $m \cos \varphi(p)$ appears in the mass-gap equation (58). It follows immediately from the Gell-Mann–Oakes–Renner relation that the sign of the condensate given in Eq. (68) leads to a pion with an imaginary mass, that is, to the tachyon. However, the latter does not lead to a disaster since the excited vacuum state is, indeed, unstable and it tends to decay to the ground state.

Now we calculate the density of the vacuum energy defined as

$$\Delta \mathcal{E}_{\text{vac}} = - \frac{g}{2} \int \frac{d^3 p}{(2\pi)^3} \{ A(p) \sin \varphi(p) + [B(p) + p] \cos \varphi(p) \}$$
$$- \mathcal{E}_{\text{vac}}^{\text{trivial}}, \tag{69}$$

where $A(p)$ and $B(p)$ are given in Eq. (55) and the degeneracy factor g counts the number of independent quark degrees of freedom,

$$g = (2s+1)N_C N_f, \tag{70}$$

with $s = \frac{1}{2}$ being the quark spin; the number of colors N_C is put to three, and the number of light flavors N_f is two. Thus we find that $g = 12$. The trivial solution of the mass-gap equation is $\varphi_{\text{trivial}}(p) \equiv 0$ (in contrast to the two-dimensional

case, this solution does not lead to difficulties with the vacuum energy). Then for the two nontrivial solutions depicted in Fig. 4 one has

$$\Delta \mathcal{E}_{\text{vac}}^{\text{ground}} = - (128 \text{ MeV})^4 \tag{71}$$

and

$$\Delta \mathcal{E}_{\text{vac}}^{\text{excited}} = - (36 \text{ MeV})^4. \tag{72}$$

The negative sign ensures that they are both energetically preferable to the trivial vacuum.

It is instructive to rewrite Eqs. (71),(72) in the form of the vacuum energy per fermi cubed:

$$\Delta \mathcal{E}_{\text{vac}}^{\text{ground}} = - 34.7 \text{ MeV/fm}^3, \tag{73}$$

$$\Delta \mathcal{E}_{\text{vac}}^{\text{excited}} = - 0.2 \text{ MeV/fm}^3. \tag{74}$$

Now, if the vacuum is locally excited, then the extra energy stored in the volume $\mathcal{V}_0 = 1 \text{ fm}^3$ is

$$\Delta E_0 = (\Delta \mathcal{E}_{\text{vac}}^{\text{excited}} - \Delta \mathcal{E}_{\text{vac}}^{\text{ground}}) \mathcal{V}_0 \approx 34.5 \text{ MeV}. \tag{75}$$

We have checked the result (75) against slight variations of the parameters of the model and found it to be very stable, changing only by several percent.

IV. PHYSICAL PROCESSES IN THE PRESENCE OF THE EXCITED VACUUM REPLICA

In this section we show how the excited vacuum might be "seen" in hadronic processes.

As follows from Eq. (75), the energy stored in the bubble with the excited vacuum inside is proportional to its volume. At the moment we can give only hand-waving qualitative arguments concerning the most probable value of this volume. Indeed, the energy scale of the interaction we have in QCD is about 300 MeV, corresponding to distances about 0.5 fm, which can be expanded up to the Compton length of the pion, about 1.5 fm or so—the standard hadronic scale. With

a bubble of such a radius one can achieve an energy around 300 MeV, which is sufficient to produce an S-wave pair of pions when the excited vacuum decays back to the ground-state one. This value looks quite natural since all parameters of the model, like the interaction scale $\sqrt{\sigma_0}$ and the one for spontaneous breaking of chiral symmetry, given by the chiral condensate or by m_{eff} (see Table I), have the same order of magnitude. As was discussed above, all three scales should always appear self-consistent, regardless of the details of the model used for numerical calculations. In the present paper we give just an example of such a model, which we find rather realistic.

It is instructive to see how the excited vacuum enters the amplitudes of the hadronic processes. Usually one is to evaluate a matrix element of the form

$$\langle \widetilde{0}, \mathrm{hadrons} | \mathcal{O} | \widetilde{0}, \mathrm{hadrons}' \rangle \qquad (76)$$

with $\mathcal{O}$ being a field operator. Note that this is the true vacuum of the theory, $|\widetilde{0}\rangle$, to enter the matrix element, and we put it explicitly in Eq. (76). If the excited vacuum comes into the game, then it can appear either in the bra or in the ket vector, or in both. We use the notation $|\widetilde{1}\rangle$ for it. Let us illustrate this statement by the correlator of two quark electromagnetic currents,

$$\langle \widetilde{0} | T J_\mu(x) J_\nu(y) | \widetilde{1} \rangle$$
$$= \langle \widetilde{0} | T e^{i\hat{P}x} J_\mu(0) e^{-i\hat{P}x} e^{i\hat{P}y} J_\nu(0) e^{-i\hat{P}y} | \widetilde{1} \rangle, \qquad (77)$$

where $\hat{P}$ is the operator of the total momentum. Now, disentangling the T product and making the external operators $e^{\pm i\hat{P}x}$ and $e^{\pm i\hat{P}y}$ act on the vacuum states to the left or to the right, one can arrive at the expression

$$\langle \widetilde{0} | T J_\mu(x) J_\nu(y) | \widetilde{1} \rangle$$
$$= \begin{cases} e^{-i\Delta P y} \langle \widetilde{0} | J_\mu(x-y) J_\nu(0) | \widetilde{1} \rangle, & x_0 > y_0, \\ e^{-i\Delta P x} \langle \widetilde{0} | J_\mu(y-x) J_\nu(0) | \widetilde{1} \rangle, & y_0 > x_0, \end{cases} \qquad (78)$$

where ΔP is the difference of the four-momenta of the two vacua, which, actually, has only the zeroth component—the difference of the vacuum energies, ΔE. If the Fourier transform is now applied to Eq. (78),

$$\mathcal{P}_{\mu\nu}(p,q) = \int d^3x \, d^3y \, e^{ipx} e^{-iqy} \langle \widetilde{0} | T J_\mu(x) J_\nu(y) | \widetilde{1} \rangle, \qquad (79)$$

then

$$\mathcal{P}_{\mu\nu}(p,q) \sim \delta(p_0 - q_0 - \Delta E) \delta^{(3)}(\vec{p} - \vec{q}), \qquad (80)$$

that is, one has an effective source pumping energy into the system. This energy can be spent to produce photons on mass shell, each with an energy $\omega = \Delta E/2 \simeq 10$ MeV for the ra-

dius of the excited-vacuum bubble, which is of the order of the interaction radius, that is, of order about 0.5 fm.

Notice that, if the BCS vacuum structure is ignored and the trivial vacuum $|0\rangle$ is used instead of $|\widetilde{0}\rangle$ and $|\widetilde{1}\rangle$ in Eq. (77) (this corresponds to the chiral angles φ and $\widetilde{\varphi}$ being identically put to zero), then the latter expression coincides with the standard perturbative vacuum polarization operator supplied by an external energy source. In order to perform calculations taking into account the nontrivial functions φ and $\widetilde{\varphi}$, one should express all quantities in terms of the quark creation and annihilation operators inherent to one and the same vacuum, and then apply the standard Wick contractions. For example, relating everything to the trivial-vacuum operators, one can find

$$|\widetilde{0}\rangle = S_0|0\rangle, \quad |\widetilde{1}\rangle = S_1|0\rangle \qquad (81)$$

with the operator S_0 defined in Eq. (46) and a similar definition of S_1 with the obvious change $\varphi \to \widetilde{\varphi}$. Then the correlator (78) reads

$$\langle \widetilde{0} | T J_\mu(x) J_\nu(y) | \widetilde{1} \rangle = \frac{\langle 0 | S_0^+ T J_\mu(x) J_\nu(y) S_1 | 0 \rangle}{\langle 0 | S_0^+ S_1 | 0 \rangle}, \qquad (82)$$

where we used the correct normalization which removes the disconnected diagrams. Note that in Eq. (82) the currents are also expected to be expressed in terms of the trivial-vacuum quark creation/annihilation operators, so that, starting from the quark field (43), one must use transformations (50) (and similar ones for the excited vacuum replica) to have everything self-consistent.

An important comment is in order here. In the above analysis we did not consider the mechanism of the vacuum excitation from the ground state to the replica. Indeed, to excite the vacuum inside some region in space, one should have a trigger—a localized operator with the vacuum quantum numbers. Such an operator could be constructed out of the background gluonic fields [see Eq. (38) above]—the only bricks at our disposal. If this is done, the outgoing photons or pions, produced as a result of the excited vacuum decay have to carry information about the vacuum–gluonic-field correlations and, if measured experimentally, offer the possibility of direct probes of the QCD vacuum structure.

V. CONCLUSIONS AND OUTLOOK

In the present paper we studied the properties of the vacuum states in potential models for QCD. We found that there is only one vacuum in the two-dimensional case, and it provides spontaneous breaking of the chiral symmetry. Lack of extra dimensions prevents this simple model from having vacuum replicas. On the contrary, the four-dimensional theory reveals a richer vacuum structure, not only containing the trivial solution with unbroken chiral symmetry, but also possessing excited vacuum states, which we found numerically for the oscillator and linear confining forces. In the

latter case the Coulomb and a constant potential were also added to have a better correspondence with real QCD. Thus we draw the following picture. Starting from the theory with bare quarks and performing the Bogoliubov-Valatin transformation to "dress" them, we find several different types of this "dressing," which correspond to different vacuum states and differently broken chiral symmetry. The mostly broken phase, that is, the one with the maximal (in the absolute value) chiral condensate and the solution for the chiral (Bogoliubov) angle without knots, possesses the minimal vacuum energy and thus defines the true vacuum of the theory. In addition, there are other solutions of the mass-gap equation with one, two, and more knots, defining other phases of the theory, which are less energetically favorable than the ground state, but still possess lower vacuum energy than the trivial vacuum. All these excited solutions lead to spontaneous breaking of the chiral symmetry, although the chiral condensate decreases (in absolute value) for each next replica of the vacuum. The number of such replicas depends on the parameters of the interaction, and we found only one of them to exist for a combination of the linearly rising, Coulomb, and constant potential with a realistic set of parameters, listed in Table I. We tried several contributions of the parameters σ_0, α_s, and U consistent with the phenomenology and numerically found it to be impossible to get rid of this replica.

Thus, starting from the system in its true vacuum state and exciting it, e.g., by means of heating or adding a strong field, one will finally arrive at the totally disordered chirally symmetric phase, although at an intermediate stage the system will quasistabilize (one or several times), which corresponds to its reordering over a new, excited, vacuum state. It is no surprise that all excited vacuum replicas are metastable since the hadronic spectra built over them contains tachyons.

Still, they may exist for a finite period of time comparable with the characteristic time of hadronic processes. Then the latter may go through the intermediate stage with formation of a local bubble of the excited vacuum, which decays then to the ground-state true vacuum by emitting pairs of photons and/or pions. The latter could be detected experimentally and thus serve as a signal of the existence of the excited vacuum state. Moreover, such measurements can serve as a direct probe of the QCD vacuum.

Of course, the question may be raised as to what extent we rely on potential models of QCD, and if the statement concerning the excited vacuum replicas is, indeed, model independent. However, potential models have been shown to be very efficient in studies of chiral symmetry breaking, hadronic properties and decays, and so on. Thus we do believe that, at least qualitatively, their predictions, including the existence of excited vacuum replicas, will persist in more realistic models for QCD, as well as in the true theory, if it is ever solved.

ACKNOWLEDGMENTS

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