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Big bang nucleosynthesis constraints on bulk neutrinos

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We examine the constraints imposed by the requirement of successful nucleosynthesis on models with one large extra hidden space dimension and a single bulk neutrino residing in this dimension. We solve the Boltzmann kinetic equation for the thermal distribution of the Kaluza-Klein modes and evaluate their contribution to the energy density at the big bang nucleosynthesis epoch to constrain the size of the extra dimension $R^{-1}\equiv\mu$ and the parameter $\sin^2 2\theta$ which characterizes the mixing between the active and bulk neutrinos.

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I. INTRODUCTION

There has been a great deal of interest and activity in the last two years concerning the possibility that there may be one or more extra space dimensions in nature which have sizes of the order of a millimeter [1]. This has been driven by the realization that string theories provide a completely new way to view a multidimensional space-time in terms of a brane-bulk picture, where a brane is a lower dimensional space-time manifold that contains known matter and forces and the bulk consists of the brane plus the rest of the space dimensions where gravity and perhaps other particles are present. The resulting picture replaces the Planck scale with the string scale as the new fundamental scale beyond the standard model. The relation between the familiar Planck scale and the string scale M_* is given by the formula [1]

$$M_{Pl}^2 = M_*^{2+n} R_1 R_2 R_3 \cdots R_n . \tag{1}$$

For $R_1 \simeq R_2 = R$ and $R_3 \simeq R_4 \simeq \cdots = M_*^{-1}$, this relation leads to $R \simeq$ millimeter for $M_* \simeq$ TeV. The fact that the familiar inverse square law of gravity allows for the existence of such submillimeter size extra dimensions made these models interesting for phenomenology [2]. An added attraction was the fact that a whole new set of particles is present at the TeV scale, making such theories accessible to collider tests. Furthermore, since there are no high scales in the theory, there is no hierarchy problem between the weak and the Planck scale; this provided an alternative resolution to the familiar gauge hierarchy problem. Obviously, the picture would become much more interesting if collider experiments such as those planned at Large Hadron Collider (LHC) or Fermilab Tevatron fail to reveal any evidence for supersymmetry.

Even though these models present an attractive alternative to the standard grand unification scenarios, there are two arenas where the simplest TeV string scale, large extra dimension models lead to problems: (i) one has to do with understanding neutrino masses and (ii) the second is in the domain of cosmology and astrophysics.

The reason for the first is that the smallness of neutrino masses is generally thought to be understood via the seesaw mechanism [3], the fundamental requirement of which is the existence of a scale $\geqslant 10^{11}$ or 10^{12} GeV, if the neutrino masses are in the sub-eV range. Clearly this is a much higher scale than M_* of the TeV scale models. A second problem is that if one considers only the standard model group on the brane, operators such as $LHLH/M_*$ could be induced by string theory in the low energy effective Lagrangian. For TeV scale strings this would obviously lead to unacceptable neutrino masses.

In the domain of cosmology, the problems are related to the existence of the Kaluza-Klein (KK) tower of gravitons generally, which lead to overclosure of the Universe unless the highest temperature of the universe is about 1 MeV [4]. This can cause potential problems not only with big bang nucleosynthesis but also with understanding of the origin of matter, inflation, etc. There are also arguments based on SN1987A observations diffuse and gamma ray background that require that $M_* \geqslant 50$–100 TeV [5].

The neutrino mass problem was realized early on and a simple solution was proposed in Ref. [6]. The suggestion was to postulate the existence of one or more gauge singlet neutrinos ν_B in the bulk which couple to the lepton doublets in the brane. We will call this the bulk neutrino. After electroweak symmetry breaking, this coupling can lead to neutrino Dirac masses, which are of order $h v_{wk} M_* / M_{Pl}$, where v_{wk} is the $\mathrm{SU}(2)_L$ breaking scale of the standard model and h is a typical leptonic Yukawa coupling between the brane and the bulk neutrinos. This leads to $m_\nu \simeq h \times 10^{-4}$ eV. The dominant nonrenormalizable terms have to be forbidden in this model. The simple way to accomplish this is to assume the existence of a global $B-L$ symmetry in the theory. The only difficulty with this assumption is that string theories are not supposed to have any global symmetries and one has to find a way to generate an effective $B-L$ symmetry at low energies without putting it in at the beginning.

There is an alternative scenario for neutrino masses [7,8], where one abandons the TeV string scale but maintains one large extra dimension and avoids the problem associated with nonrenormalizable operators. The relation in Eq. (1) then gets modified to the form

$$M_{Pl}^2 = M_*^3 R. \tag{2}$$

Since the string scale in these models is in the intermediate range, i.e., 10^9 GeV or so, the cosmological overclosure

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problem is avoided. The active neutrino masses in such models could arise from seesaw mechanism or from the presence of bulk neutrinos. The inclusion of the bulk neutrino, however, brings new neutrinos into the theory which can be ultralight (i.e., $R^{-1}\equiv\mu$, where R is the size of the large extra dimension) and can play the role of the sterile neutrino, which may be required, e.g., if the Liquid Scintillation Neutirno Detector (LSND) results are confirmed.

Both the above approaches have the common feature that they introduce a bulk neutrino into the theory, which is equivalent in the brane to an infinite tower of sterile neutrinos. All of these neutrino modes mix with the active neutrinos in the process of mass generation [6,7]. For extra dimension size of order $\sim$mm, the KK modes have masses typically of order $nR^{-1}\sim n\times10^{-3}$ eV or so, where $n=0,1,2,3,\ldots$. The presence of this dense tower of extra sterile states coupled to the known neutrinos leads to a variety of new effects in the domain of particle physics [9] and cosmology [10–12], which in turn impose constraints on the allowed size of the extra dimensions. In this paper, we focus on the cosmological constraints that may arise from the contribution of the neutrino states to big bang nucleosynthesis.

The constraints from big bang nucleosynthesis on bulk neutrinos were considered in Refs. [10,11], where the cases of $\geqslant 2$ extra dimensions were discussed. As was noted there, light KK modes of the bulk neutrinos could easily be produced in the early Universe, when the temperature is of order a MeV or more. Thus, there is the danger that they could make large contribution to the energy density of the Universe at the epoch of big bang nucleosynthesis (BBN) and completely destroy our current, successful understanding of the primordial abundance of He^4, D, and Li^7 [13]. In particular, present abundance data from metal poor stars are well understood provided we do not have more than one extra active neutrino in the theory in addition to the three known ones, i.e., (ν_e,ν_μ,ν_τ). By the same token, if there are extra species of neutrinos that do not have conventional weak interactions, their masses and mixings to known neutrinos must obey severe constraints [14].

Our goal in this paper is to reconsider this issue in the context of models with only one large extra dimension. The first reason for undertaking this analysis is that the class of models with intermediate string scale $\sim 10^9$ GeV and one large extra dimension [7,8] have certain theoretical advantages and they are also free of the cosmological and astrophysical problems that seem to plague the TeV scale models. Secondly, the number of KK modes in this case is much fewer than in models with larger numbers of large extra dimensions and therefore one would expect the constraints to be somewhat less restrictive.

We also wish to emphasize that one large extra dimension could also occur in models with string scale in the 100 TeV range, where one can satisfy the Planck-scale–string-scale relation in Eq. (1), if the compactification is not isotropic, e.g., for a string scale of 100 TeV, if two extra dimensions have sizes $r\sim\mathrm{GeV}^{-1}$ and one has $R\sim$millimeter, i.e., $M_{Pl}^2=M_*^5Rr^2$.

We organize this paper as follows. In Sec. II, we solve the Boltzmann equation to study the generation of the bulk neutrinos from active neutrino interactions in the early Universe. The numerical calculations leading to our final results are given in Sec. III.

II. BOLTZMANN EQUATION AND BBN CONSTRAINTS

The class of models we will be interested in are assumed to have one large extra dimension with a single bulk neutrino ν_B, which means that masses of the KK modes of ν_B are integer multiples of the basic scale $\mu\equiv R^{-1}\sim10^{-3}$ eV. This is one of the parameters that we expect the BBN discussion to constrain. The second parameter is the mixing of the KK modes with the active brane neutrinos, e.g., ν_e. It is true in both classes of models, i.e., both TeV and intermediate scale types [6,8], that the typical mixing parameter of the active neutrino to the nth KK mode scales like $\theta_{en}\simeq\theta/n$, in the range of interest for phenomenology. The parameter θ depends on the size of the extra dimension and other parameters of the theory such as the weak scale, Yukawa coupling h, etc. For instance, in TeV scale models [6], one has $\theta\sim\sqrt{2}hv_{wk}M_*R/M_{Pl}$, whereas in the local $B-L$ models, the relation is $\theta\simeq hv_{wk}v_RR/M_{Pl}$, where we have chosen $M_*\simeq v_R$. In general, therefore, BBN discussion will give a correlated constraint between μ and θ. Obviously, for $\theta=0$, there is no BBN constraint on μ or the size of the extra dimension.

We will also assume that the Universe starts its ‘‘big bang journey’’ somewhere around a GeV or so and when it starts, the Universe is essentially swept clean of the sterile neutrino modes. This can happen in inflation models with a low reheat temperature. We choose such a low reheat temperature essentially for reasons that in models with large extra dimensions higher temperatures would lead to closure due to production of graviton KK modes.

To see the origin of constraints, let us note that at high temperatures (i.e., $T\gg$MeV), there are two ways the KK modes of the sterile bulk neutrino can be created: (i) neutrino scattering and annihilations and (ii) the oscillation of the active neutrinos into the sterile KK modes. It is important to stress that in building up the oscillation the scattering process is important, since otherwise there will be back-and-forth oscillation and no buildup of the sterile modes.

Since there is an infinite KK tower of these neutrino modes, the higher the temperature the larger the number of modes that can get created. Once these modes are created, they may decay or annihilate to produce the lighter particles (lighter neutrino modes or KK modes of the graviton, etc.). In general, it is reasonable to expect that this process of decay or annihilation will not be efficient enough [11] to eliminate all the KK modes. As a result, many of them will stay around at the BBN temperature and contribute to the energy density. The present understanding of big bang nucleosynthesis [13] relies on the assumption that the total energy density at the BBN era is $\rho_{BBN}=(\pi^2/30)g^*T^4$ with $g^*=10.75$ coming from the contribution of photons, e^+e^-, and the three species of neutrinos. The uncertainties in our knowledge of the He^4, D_2, and Li^7 content of the Universe allow that one could have $g^*\sim12.5$ (or one extra species of neutrino). We will require that any additional contribution to

ρ_{BBN} coming from the bulk neutrinos generated at higher temperature be less than the contribution to ρ_{BBN} equivalent to one extra species of neutrino.

We now employ the Boltzmann equations to get the constraints on μ and θ. Our procedure is to calculate the distribution function for the sterile KK modes produced in the matter oscillation of ν_e, including any possible depletion of their density due to decays all the way down to the BBN temperature. We then calculate their cumulative contribution to the energy density ρ at the BBN epoch and demand that this be less than the corresponding contribution of one extra species of neutrino. This is the procedure followed in [16,11].

To estimate this new contribution to energy density ρ_{BBN}, we have to calculate the number of KK states produced at a given temperature. Let us denote by $f_k = f_k(p,t)$, the distribution of the kth mode of the neutrino at the epoch t. The KK modes are not in equilibrium at any epoch. The time evolution of the nonequilibrium density is governed by the Boltzmann equation given below [11,16]:

$$\left(\frac{\partial}{\partial t} - Hp\frac{\partial}{\partial p}\right) f_k = \Gamma(\nu_a \to \nu_k) f_{\nu_a} - \frac{m_k}{E_k}\frac{1}{\tau_k} f_k + \sum_{l>k} C_{k,l}[f_l], \tag{3}$$

where we have neglected the contribution of the pair annihilation of the KK modes in the right hand side, since it is a very small effect. In Eq. (3), t (time), and p (momentum) are the two independent variables with k, μ, and $\sin^2 2\theta$ fixed. $E_k(p) = \sqrt{p^2 + m_k^2}$. H is the instantaneous Hubble expansion rate and is clearly a function of time $H(t)$. $\Gamma(\nu_a \to \nu_k) = \Gamma_k$, the production rate of bulk neutrino, is given by $(\Gamma/2)\langle P(\nu_a \to \nu_k)\rangle$ where $(\Gamma/2) = 2G_F^2 T^5$ is half the interaction rate of the active neutrino in the thermal bath. Taking matter effects [15] into account, the probability P is given by

$$\langle P(\nu_a \to \nu_k)\rangle \simeq \frac{1}{2}\frac{\sin^2 2\theta_k}{1 - 2z\cos 2\theta_k + z^2}. \tag{4}$$

Note that we have used the averaged probability P to get rid of the momentum dependence of the production rate. We will only use this p-averaged probability in our numerical calculation. The averaging is not necessary for the general solution we are going to find later in this section. In the above equation, f_{ν_a} is thermal distribution of active neutrino ν_a. τ_k is the lifetime of the kth mode in its rest frame, and $1/\tau_k$ is the total decay width. For small k, the dominant contributions come from the partial width of $\nu_k \to 3\nu$ decay which is given by $\sin^2\theta_k G_f^2 m_k^5/192\pi^3 = \sin^2 2\theta_k G_f^2 \mu^5 k^5/(4\times 192\pi^3)$ [11]. For big k, it is from $\Sigma_{k'=1}^{k-1} \nu_k \to \nu_{k'} h_{k-k'} \sim m_k^3(k-1)/12\pi M_{Pl}^2 \sim k^4\mu^3/12\pi M_{Pl}^2$. We included both contributions in our total decay rate. To make the solution more general, we now let all of the functions in Eq. (3) except $H(t)$ depend on both p and t.

The general solution of Eq. (3) without knowing the form of functions introduced in the equation and $C_{k,l} = 0$ is found to be

$$f_k(p,t) = \int_{t_i}^{t} \Gamma_k(p'(x,t),x) f_{\nu_a}(p'(x,t),x) e^{-m_k \alpha(x,t)} dx \tag{5}$$

with the initial condition $f_k(p,t_i) = 0$. We have defined

$$p'(x,t) = p e^{\int_x^t H(x')dx'} \tag{6}$$

and

$$\alpha(x,t) = \int_x^t \frac{1}{\tau_k(p'(x',t),x')\sqrt{p'^2(x',t) + m_k^2}} dx'. \tag{7}$$

Note from Eq. (6) that $p'(t,t) = p$. The above solution can be checked easily by the observation that

$$\left(\frac{\partial}{\partial t} - Hp\frac{\partial}{\partial p}\right) p'(x,t) = 0 \tag{8}$$

and

$$\left(\frac{\partial}{\partial t} - Hp\frac{\partial}{\partial p}\right) F(p'(x,t)) = 0 \tag{9}$$

for any function $F(p'(x,t))$ with no explicit dependence on t and p. The time derivative on the integration upper limit gives the first term on the right hand side of Eq. (3). While acting on the upper limit of $\alpha(x,t)$, it gives the second term on the right hand side of Eq. (3). The total energy density of bulk neutrino is then given by

$$\rho(t) = \int_0^\infty \int_1^\infty \frac{E_k p^2}{2\pi^2} f_k(p,t) dk\, dp \tag{10}$$

in the continuous k approximation. In the next section, we discuss the numerical results that follow from the above discussion.

III. NUMERICAL RESULTS

In this section, we calculate the total energy density of the universe at $T = 1$ MeV from Eq. (10). We use temperature (T) instead of time in the integration and proceed in two different ways.

In the first method, we make some approximations to simplify the integral in Eq. (10) and calculate analytically as far as possible and then numerically estimate the final integrals that follow from it. In the second method, we evaluate the entire integral in Eq. (10) numerically. The two results approximately agree with each other.

In the calculation, we use $H(T) = 1.66\sqrt{g_*}T^2/M_{Pl}$ and $t = 0.301 g_*^{-1/2} M_{Pl}/T^2$ as given by standard cosmology. After introducing dimensionless variables $\overline{p} = p/k\mu$, $x = T/T_f$, and $\overline{k} = k\mu/T_f$, and using the notation p and k instead of $\overline{p}$ and $\overline{k}$ in the final form Eq. (10) reduces to

$$\rho(T_f)=\mathcal{A}\int_1^{T_i/T_f}\int_\epsilon^\infty\int_0^\infty \frac{k^2x^2p^2\sqrt{1+p^2}\{1+(x/4)[d\ln g_*(x)/dx]\}}{\sqrt{g_*}(1+z^2-2z\sqrt{1-\sin^2 2\theta\mu^2/k^2T_f^2})}e^D\mathcal{F}\,dp\,dk\,dx, \tag{11}$$

where

$$\mathcal{A}=\frac{0.301}{\pi^2}\mu\sin^2 2\theta G_F^2 M_{pl}T_f^6, \tag{12}$$

$$D=-\frac{0.301M_{pl}}{\sqrt{10.75}}\left(\frac{\mu^2\sin^2 2\theta G_F^2T_f}{4\times192\pi^2}k^3+\frac{T_f^2}{12\pi M_{Pl}^2\mu}k^4\right)K(x,p), \tag{13}$$

$$K(x,p)=\left(\sqrt{1+p^2}-\frac{c^{-1/4}}{x}\sqrt{p^2+\frac{c^{-1/2}}{x^2}}-p^2\ln\frac{1+\sqrt{1+p^2}}{c^{-1/4}/x+\sqrt{p^2+c^{-1/2}/x^2}}\right),$$

$$\mathcal{F}=\frac{1}{e^{kpc^{1/4}}+1}, \tag{14}$$

and $c=g_*(x)/g_*(T_f)$, $\epsilon=\mu/T_f$. In order to simplify the calculation, we also make the following approximations.

We assume that the effective degree of freedom g_* is not affected by the density of the bulk neutrino during the production of bulk neutrinos. g_* is given by the standard model of cosmology and approximated by a step function as follows: $g_*=61.75$ from $T=1$ GeV to 200 MeV, $g_*=17.5$ from $T=200$ MeV to 100 MeV, and $g_*=10.75$ from $T=100$ MeV to 1 MeV. Here we actually approximate the degree of freedom as a step function of temperature.

We use $\delta m^2\equiv m_k^2-m_\nu^2\approx k^2\mu^2$. Our δm^2 are always positive and so the lepton number generated from neutrino oscillations with nonzero initial lepton number is small [17,18]. We can ignore the contribution of lepton number to the ν_e potential in matter. This gives

$$z=-0.215589\times10^{-18}\frac{x^6}{k^2}\equiv -z_0\frac{x^6}{k^2}. \tag{15}$$

c is a constant of $O(1)$ in the step function approximation of g_*.

In order to use the analytical method we make some further approximations. Note that the integration in Eq. (11) is suppressed by two exponential functions: one from the decay term (D term) e^D and a second from the $\mathcal{F}$ term. We can therefore cut off the integration when either one of them gets smaller than e^{-100}. We want to extract some of the regions of k and p space in which the $\mathcal{F}$ term decays much faster than the D term within the parameter space of interest and so the latter can be ignored. However, it is easier to do it another way. As the contribution from the region where $\mathcal{F}<e^{-100}$ can be neglected, we have to examine only the region where $pk<100$ to make sure the D term can be approximated to 1. This is the same as finding the region where $pk<100$ and both $4k^4/\mu\times10^{-19}$ and $64k^3\mu^2\sin^2 2\theta\times10^{-19}$ are less than 0.1. It is easy to see that the only region that may not satisfy the above condition is

$$k>\min\left[\left(\frac{\mu\times10^{18}}{4}\right)^{1/4},\left(\frac{10^{18}}{64\mu^2\sin^2 2\theta}\right)^{1/3}\right]\equiv k_{min} \tag{16}$$

and

$$p<\frac{1}{k_{min}}<1. \tag{17}$$

Now we can calculate the total energy density by splitting the integration into three parts: (i) $p>1$, (ii) $p<1$ and $k<k_{min}$, and (iii) $p<1$ and $k>k_{min}$. We will include the decay term only for (iii). We also simplify our analysis by substituting $\mathcal{F}=\frac{1}{2}e^{-kp}$ with a possible error of factor 2. For simplicity, we will now treat g_* as a constant and so $c=1$. Setting $\cos 2\theta=1$ affects only the term due to the matter effect. This is a very good approximation as long as z is negative. For cases (i) and (ii), Eq. (11) now becomes

$$\rho=\frac{\mathcal{A}}{2\sqrt{g_*}}\int_1^{10^3}\int_\epsilon^\infty\int_0^\infty\frac{k^2x^2p^2\sqrt{1+p^2}}{(1-z)^2}e^{-kp}dp\,dk\,dx. \tag{18}$$

The x integration can now be done analytically. we approximate $\sqrt{1+p^2}$ with p for case (i) where $p>1$ and with 1 for case (ii) where $p<1$. With this simplification, the p integration can also be done analytically. For k, we can always divide it into three parts: $k>10$, $0.01<k<10$, and $k<0.01$ since $k_{min}>10^3$. We will leave the factor $\mathcal{A}/\sqrt{g_*}$ to the end of the discussion. Now we look only at the integration. The results are summarized below

(i) $p>1$. For $k>10$, we have

$$\frac{10^9}{6}\int_{10}^\infty e^{-k}k\,dk=83233. \tag{19}$$

For $0.01<k<10$, we have

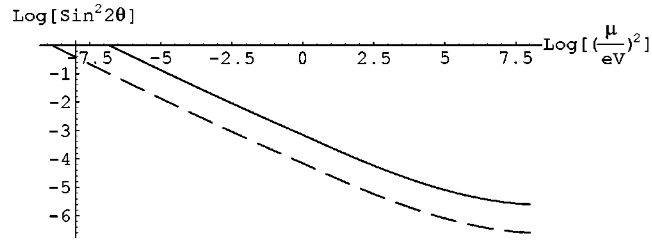

FIG. 1. The solid and the long dashed lines separate the allowed and forbidden regions of R^{-1} and mixing, with the space above the lines forbidden. The solid and dashed lines correspond, respectively, to the extra allowed neutrino degree of freedom ΔN_ν equal to 1 and 0.1.

$$10^9\int_{0.01}^{10} e^{-k}\left(\frac{1}{2k^2}+\frac{1}{2k}+\frac{1}{4}+\frac{k}{12}\right)\times\left(\frac{1}{1+0.216/k^2}+\frac{\tan^{-1}(0.465/k)}{0.465/k}\right)dk = 7.85\times 10^9. \tag{20}$$

For $k<0.01$,

$$\frac{1}{2}\int_\epsilon^{0.01}\left(\frac{10^9}{0.216}+\frac{\pi}{2k\sqrt{z_0}}-\frac{1}{k^2+z_0}-\frac{\tan^{-1}(\sqrt{z_0}/k)}{k\sqrt{z_0}}\right)dk = \frac{10^7}{0.432}+\frac{\pi}{4\sqrt{z_0}}\ln\frac{0.01}{\epsilon}-\frac{\pi/2-\tan^{-1}(\epsilon/\sqrt{z_0})}{2\sqrt{z_0}} - \frac{1}{2\sqrt{z_0}}\int_\epsilon^{0.01}\frac{\tan^{-1}(\sqrt{z_0}/k)}{k}dk. \tag{21}$$

(ii) $p<1$ and $k<k_{min}$. For $k>10$, we have

$$\frac{10^9}{3}\int_{10}^{k_{min}}\frac{1}{k}dk=\frac{10^9}{3}\ln\frac{k_{min}}{10}. \tag{22}$$

For $0.01<k<10$, we have

$$10^9\int_{0.01}^{10}\left[\frac{1}{6k}-e^{-k}\left(\frac{1}{6k}+\frac{1}{6}+\frac{k}{12}\right)\right]\times\left(\frac{1}{1+0.216/k^2}+\frac{\tan^{-1}(0.465/k)}{0.465/k}\right)dk = 4.52\times 10^8. \tag{23}$$

For $k<0.01$, we have

$$\frac{1}{36}\int_\epsilon^{0.01}\left(\frac{10^9k^4}{0.216}+\frac{\pi k^3}{2\sqrt{z_0}}-\frac{k^4}{k^2+z_0}-\frac{k^3\tan^{-1}(\sqrt{z_0}/k)}{\sqrt{z_0}}\right)dk. \tag{24}$$

This is a very small number which can be neglected.

(iii) In this region, the decay term haves to be included. However, we can remove the p dependence by setting $p=1$ in the function $K(x,p)$, defined above. The matter effect can also be neglected as $k_{min}>500$. After the p integration, with the fact that $e^{-k_{min}}\leqslant 1$ we have the upper bound

$$\int_1^{10^3}\int_1^\infty x^2 e^{-(c_1k^4+c_2k^3)K(x,1)}dk\,dx. \tag{25}$$

In the above equation, c_1 and c_2 are coefficients dependent on k_{min}. One of them is 0.1 and the other $\leqslant 0.1$ by the definition of k_{min}. We also use $k=k_{min}$ in the result of the p integration. For the upper bound, we can just take either the c_1 or the c_2 term whichever has the value of 0.1. This gives

$$\int_1^{10^3}x^2\frac{\Gamma(\frac{1}{4},0.1K(x,1))}{4[0.1K(x,1)]^{1/4}}dx \tag{26}$$

or

$$\int_1^{10^3}x^2\frac{\Gamma(\frac{1}{3},0.1K(x,1))}{3[0.1K(x,1)]^{1/3}}dx. \tag{27}$$

Both are numerically of the order of 10^8. Our results from other regions give about 10^{10}. This part contributes only a few percent of the total. We can use the upper bound 4.6×10^8.

The total energy density of bulk neutrinos is the sum of all the above, which is found to be

$$\rho=\frac{\mathcal{A}}{\sqrt{g_*}}\left(8.785\times 10^9+\frac{\pi}{4\sqrt{z_0}}\ln\frac{0.01}{\epsilon}-\frac{\pi/2-\tan^{-1}(\epsilon/\sqrt{z_0})}{2\sqrt{z_0}}-\frac{1}{2\sqrt{z_0}}\times\int_\epsilon^{0.01}\frac{\tan^{-1}(\sqrt{z_0}/k)}{k}dk+\frac{10^9}{3}\ln\frac{k_{min}}{10}\right). \tag{28}$$

Compared to the ν_e equilibrium energy density at $T=1$ MeV, which is 2.824×10^{-13} (GeV)^4, the constraint that effective neutrino degrees of freedom should be less than 1, gives the constraint on the parameter space

$$\left(\frac{\mu}{eV}\right)^{0.936}\sin^2 2\theta\leqslant 2.5\times 10^{-4}. \tag{29}$$

We also numerically integrate Eq. (10) without making any simplification except the step function g_* and the positive δm^2. The numerical result obtained from the Boltzmann equation are shown in Fig. 1. The extra effective degree of freedom equal to 1 is shown in Fig. 1 as a solid line. The numerical fit is obtained as

$$\left(\frac{\mu}{\mathrm{eV}}\right)^{0.92}\sin^2 2\theta\leqslant 7.06\times 10^{-4} \tag{30}$$

for $\mu\leqslant 1$ eV.

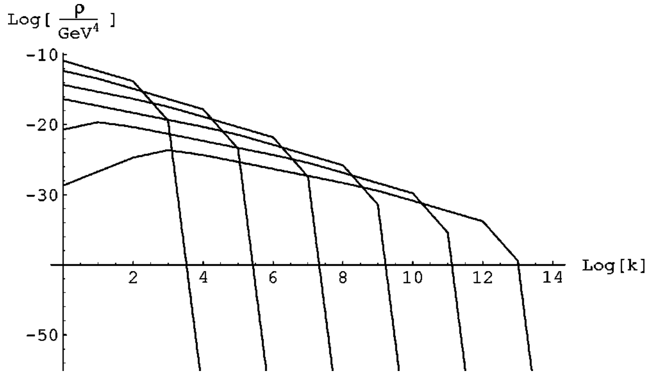

FIG. 2. The various lines denote the contributions of the individual modes to the total energy density as a function of the KK mode number for various values of inverse size (in eV) of the extra dimension. The rightmost curve corresponds to $\mu = 10^{-6}$ eV and for each successive curve μ goes up by a factor of 100.

In Fig. 2, we give the contributions of individual modes to the total energy density as a function of the mode number for $\sin^2 2\theta = 10^{-4}$ and for a range of values for $R^{-1} \equiv \mu$ from 10^{-6} eV to 10^4 eV in increasing steps of 10^2 eV. The total contribution is the integral over each line for a given μ.

IV. CONCLUSION

In this paper, we have studied the constraints of big bang nucleosynthesis on models with one large extra dimension and a single bulk neutrino by solving the Boltzmann equation for the production of the sterile KK modes at the epoch of big bang nucleosynthesis. We isolate the allowed and forbidden domains in the R^{-1} and mixing space and find that this allows a range of radius of the extra dimension and mixing of bulk and active neutrinos that is of interest for studying solar neutrino oscillations [8]. Our results complement the work of Abazajian, Fuller, and Patel [11] who have derived bounds for the cases of four, five, and six extra dimensions.

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