Parity violating bosonic loops at finite temperature

V. S. Alves,[1,2] Ashok Das,[2] Gerald V. Dunne,[3] and Silvana Perez[1,2]
[1]Departamento de Física, Universidade Federal do Pará, 66075-110 Belém, Brazil
[2]Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627-0171
[3]Department of Physics, University of Connecticut, Storrs, Connecticut 06269-3046
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The finite temperature parity-violating contributions to the polarization tensor are computed at one loop in a system without fermions. The system studied is a Maxwell-Chern-Simons-Higgs system in the broken phase, for which the parity-violating terms are well known at zero temperature. At nonzero temperature the static and long-wavelength limits of the parity violating terms have very different structure, and involve nonanalytic log terms depending on the various mass scales. At high temperature the boson loop contribution to the Chern-Simons term goes like T in the static limit and like $T \log T$ in the long-wavelength limit, in contrast with the fermion loop contribution which behaves like $1/T$ in the static limit and like $\log T/T$ in the long wavelength limit.

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I. INTRODUCTION

The study of induced Chern-Simons terms in $2+1$ dimensional field theory at finite temperature has produced some interesting new insights into large gauge invariance and parity-violating effective actions at finite temperature [1–5]. Almost all previous studies (with the exception of [6]) have concentrated on the induced Chern-Simons term arising from fermion loop contributions to the gauge field self-energy in $2+1$ dimensions. In this paper we study finite temperature induced Chern-Simons terms in a bosonic theory. The induced Chern-Simons terms are generated in radiative loop corrections, due to the presence of a bare Chern-Simons term. Specifically, we consider a Maxwell-Chern-Simons-Higgs (MCSH) model in the spontaneously broken phase. At zero temperature, induced Chern-Simons terms in such models have been studied in great detail, revealing an intricate relation between spontaneous parity violation and spontaneous symmetry breaking [7–10]. One motivation for this present paper is to understand how this generalizes to finite temperature.

The induced Chern-Simons coefficient is extracted from the zero-momentum limit of the parity violating part of this self-energy [11]. At finite T, this procedure is not unique [12] since Feynman diagrams are not analytic in external momenta at finite temperature [13], because the thermal heat bath breaks Lorentz invariance. In a static limit, with $q^0=0$ and $|\vec{q}|\rightarrow 0$, an induced Chern-Simons term is found with a temperature dependent coefficient [14]. As first pointed out in [6], this result appears (when carried over to a non-Abelian theory) to violate large gauge invariance since the coefficient of the induced Chern-Simons term in a non-Abelian theory should take discrete values [15]. This puzzle has been resolved for the fermion loop when the background has the character of a static Abelian magnetic field with integer flux Φ, because in this case the problem factorizes into Φ copies of an exactly solvable $(0+1)$-dimensional model [1–4]. Then one finds that the finite temperature effective action has an infinite series of parity-violating terms (of which the Chern-Simons term is only the first), each of which has a T dependent coefficient at finite T. Nevertheless, the series is such that the full effective action changes under a large gauge transformation in a manner that is independent of T. These new parity-violating terms are non-extensive (i.e., they are not integrals of a density) and they explicitly vanish at zero temperature (as they must since the zero T effective action should be extensive). This issue of large gauge invariance of the finite temperature effective action is considerably more difficult to resolve in genuinely time-dependent and genuinely non-Abelian backgrounds, although much recent progress has been made in understanding the parity-violating parts of multi-leg amplitudes at finite temperature [5].

Another motivation for our study is the question of the analytic structure of the bosonic self-energy at finite temperature. This issue has been analyzed previously [16] for massive gauge bosons in four dimensional space-time, where the Chern-Simons parity-violating issues are not relevant. In the four dimensional case it was found that in the broken phase the different bosonic masses appearing in the bosonic loop meant that the zero energy-momentum limit was actually analytic, despite the well-known physical difference between the Debye and plasmon masses, which can be defined through the static and long wavelength limits, respectively [16]. In this current paper, we find that in three dimensional space-time, for a model with parity violation, the zero energy-momentum limit is not unique, even though the bosonic masses entering the one-loop calculation are different.

In Sec. II we define the bosonic model to be studied, and present the finite temperature propagators necessary for a perturbative analysis. In Sec. III we present the one loop results for the parity violating part of the finite temperature self energy in both the static and long wavelength limits. Section IV contains our conclusions.

II. MAXWELL-CHERN-SIMONS-HIGGS MODEL

We consider an Abelian gauge field A_μ in $2+1$ dimensions with both a Maxwell and a Chern-Simons term in the

Lagrangian, interacting with a charged scalar field Φ which has a symmetry breaking quartic potential:

$$\mathcal{L}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{\kappa}{2}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda+|D_\mu\Phi|^2 -\frac{\lambda}{4}(|\Phi|^2-v^2)^2. \tag{1}$$

In the spontaneously broken phase, where Φ has a nonzero vacuum expectation value $\langle\Phi\rangle=v$, we expand the scalar field about this vacuum expectation value (VEV) as $\Phi=v+(1/\sqrt{2})(\sigma+i\chi)$ and obtain the following Lagrangian in the R_ξ gauge (ignoring the ghost Lagrangian which is not relevant to our calculations):

$$\begin{aligned}\mathcal{L}=&-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{\kappa}{2}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda+\frac{m^2}{2}A_\mu A^\mu-\frac{1}{2\xi}(\partial_\mu A^\mu)^2\\&+\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma-\frac{1}{2}m_\sigma^2\sigma^2+\frac{1}{2}\partial_\mu\chi\partial^\mu\chi-\frac{1}{2}m_\chi^2\chi^2\\&-e\sigma\overleftrightarrow{\partial}^\mu\chi A_\mu+\frac{e^2}{2}(\sigma^2+\chi^2+2\sqrt{2}v\sigma)A_\mu A^\mu\\&-\frac{\lambda}{2\sqrt{2}}v\sigma(\sigma^2+\chi^2)-\frac{\lambda}{16}(\sigma^2+\chi^2)^2.\end{aligned} \tag{2}$$

Here the various mass parameters are

$$\begin{aligned}m^2&=2e^2v^2\\m_\sigma^2&=\lambda v^2\\m_\chi^2&=\xi m^2.\end{aligned} \tag{3}$$

As mentioned above in the Introduction, for the corresponding system *without* the Chern-Simons term (i.e. for $\kappa=0$), the finite temperature behavior of the polarization tensor was studied in [16]. There, one of the key features was the difference between the bosonic masses appearing in the one-loop calculation. The model *with* a Chern-Simons term is more interesting for two reasons. First, the presence of the Chern-Simons term leads to a different mass generation mechanism for the gauge field, with it acquiring two (rather than one) massive modes in the broken phase [17]. Second, the Chern-Simons coupling leads to parity-violating contributions to the polarization tensor, whose finite temperature behavior is the subject of this paper. Both these differences can be seen clearly in the propagator structure of the model.

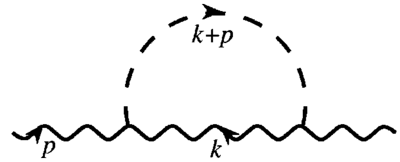

FIG. 1. One loop self-energy diagram for computing the induced parity violating Chern-Simons coefficient. The wavy line represents the gauge field and the dashed line represents the scalar particle.

A. Zero temperature propagators

At zero temperature, the gauge field propagator is

$$\begin{aligned}D_{\mu\nu}(p)=&\frac{-1}{(p^2-m_+^2+i\epsilon)(p^2-m_-^2+i\epsilon)}\Bigg[\eta_{\mu\nu}(p^2-m^2)\\&-p_\mu p_\nu\frac{(1-\xi)(p^2-m^2)+\xi\kappa^2}{p^2-\xi m^2}+i\kappa\epsilon_{\mu\nu\lambda}p^\lambda\Bigg]\end{aligned} \tag{4}$$

where the two massive modes are identified by the poles at

$$m_\pm^2=\frac{\kappa^2+2m^2\pm|\kappa|(\kappa^2+4m^2)^{1/2}}{2} \tag{5}$$

from which we deduce the (positive) masses

$$m_\pm=\frac{|\kappa|}{2}\left(\sqrt{1+\frac{4m^2}{\kappa^2}}\pm 1\right). \tag{6}$$

Note also the presence in $D_{\mu\nu}(p)$ of the term proportional to $\epsilon_{\mu\nu\lambda}p^\lambda$, which manifestly breaks parity. The scalar field σ has the standard bosonic propagator $D_\sigma(p)=1/(p^2-m_\sigma^2)$.

B. Finite temperature propagators

At finite *T*, propagators can be presented either in the imaginary-time or real-time formalism [18–20]. Here we record the propagators in both forms for the model in Eq. (2).

1. Imaginary time

In the imaginary-time formalism, the gauge field propagator is ($\kappa\rightarrow i\kappa$ in the Euclidean space)

$$\begin{aligned}D_{\mu\nu}^{(\beta)}(p)=&\frac{1}{(w_n^2+\vec{p}^2+m_+^2)(w_n^2+\vec{p}^2+m_-^2)}\\&\times\Bigg[\delta_{\mu\nu}(w_n^2+\vec{p}^2+m^2)-p_\mu p_\nu\\&\times\frac{(1-\xi)(w_n^2+\vec{p}^2+m^2)-\xi\kappa^2}{w_n^2+\vec{p}^2+\xi m^2}-\kappa\epsilon_{\mu\nu\lambda}p_\lambda\Bigg]\end{aligned} \tag{7}$$

and the scalar σ field propagator is

$$D_\sigma^{(\beta)}(p)=\frac{1}{w_n^2+\vec{p}^2+m_\sigma^2}. \tag{8}$$

Here, $-ip^0\to\omega_n=2\pi nT$ are the usual bosonic Matsubara modes.

2. Real time

In the real-time formalism, the degrees of freedom are doubled in the standard way [20] in order to account for the transfer of energy into and out of the thermal heat bath. The propagators thus acquire a 2×2 matrix structure, the components of which are listed below, in the closed time path formalism, for the MCSH system in the broken phase. For the gauge field,

$$D_{\mu\nu}^{(\beta)++}(p)=-\left[\eta_{\mu\nu}(p^2-m^2)-p_\mu p_\nu\frac{(1-\xi)(p^2-m^2)+\xi\kappa^2}{p^2-\xi m^2}+i\kappa\epsilon_{\mu\nu\lambda}p^\lambda\right]\times\left[\frac{1}{(p^2-m_+^2+i\epsilon)(p^2-m_-^2+i\epsilon)}-2i\pi n_B(|p^0|)\delta((p^2-m_+^2)(p^2-m_-^2))\right]$$

$$D_{\mu\nu}^{(\beta)+-}(p)=2i\pi\left[\eta_{\mu\nu}(p^2-m^2)-p_\mu p_\nu\frac{(1-\xi)(p^2-m^2)+\xi\kappa^2}{p^2-\xi m^2}+i\kappa\epsilon_{\mu\nu\lambda}p^\lambda\right]\times[\theta(-p^0)+n_B(|p^0|)]\delta((p^2-m_+^2)(p^2-m_-^2))$$

$$D_{\mu\nu}^{(\beta)-+}(p)=2i\pi\left[\eta_{\mu\nu}(p^2-m^2)-p_\mu p_\nu\frac{(1-\xi)(p^2-m^2)+\xi\kappa^2}{p^2-\xi m^2}+i\kappa\epsilon_{\mu\nu\lambda}p^\lambda\right]\times[\theta(p^0)+n_B(|p^0|)]\delta((p^2-m_+^2)(p^2-m_-^2))$$

$$D_{\mu\nu}^{(\beta)--}(p)=-\left[\eta_{\mu\nu}(p^2-m^2)-p_\mu p_\nu\frac{(1-\xi)(p^2-m^2)+\xi\kappa^2}{p^2-\xi m^2}+i\kappa\epsilon_{\mu\nu\lambda}p^\lambda\right]\times\left[\frac{-1}{(p^2-m_+^2-i\epsilon)(p^2-m_-^2-i\epsilon)}-2i\pi n_B(|p^0|)\delta((p^2-m_+^2)(p^2-m_-^2))\right].$$

For the scalar σ field,

$$D_\sigma^{(\beta)++}(p)=\frac{1}{p^2-m_\sigma^2+i\epsilon}-2i\pi n_B(|p^0|)\delta(p^2-m_\sigma^2)$$

$$D_\sigma^{(\beta)+-}(p)=-2i\pi[\theta(-p^0)+n_B(|p^0|)]\delta(p^2-m_\sigma^2)$$

$$D_\sigma^{(\beta)-+}(p)=-2i\pi[\theta(p^0)+n_B(|p^0|)]\delta(p^2-m_\sigma^2)$$

$$D_\sigma^{(\beta)--}(p)=\frac{-1}{p^2-m_\sigma^2-i\epsilon}-2i\pi n_B(|p^0|)\delta(p^2-m_\sigma^2).$$

III. ONE-LOOP RESULTS

In this section we compute the parity-violating part, $\Pi_{\mu\nu}^{(PV)}$, of the polarization tensor $\Pi_{\mu\nu}$, as represented by the one-loop Feynman diagram in Fig. 1. The parity-violating contribution arises from the $\epsilon_{\mu\nu\lambda}k^\lambda$ part of the gauge propagator. We first review briefly the zero temperature result.

A. Zero temperature parity-violating part

The parity-violating part of the diagram in Fig. 1 is

$$\Pi^{\mu\nu(PV)}=8i\kappa e^4v^2\epsilon^{\mu\nu\lambda}\int\frac{d^3k}{(2\pi)^3}\frac{1}{[(k+p)^2-m_\sigma^2]}\times\frac{k_\lambda}{(k^2-m_+^2)(k^2-m_-^2)}. \tag{9}$$

A straightforward use of Feynman parameters shows that this can be expressed as

$$\Pi^{\mu\nu(PV)}=-\epsilon^{\mu\nu\lambda}p_\lambda\Pi(p^2) \tag{10}$$

where

$$\Pi(p^2)=16i\kappa e^4v^2\int\frac{d^3k}{(2\pi)^3}\int_0^1 d\alpha\int_0^{1-\alpha}d\beta\frac{\alpha}{[k^2+\alpha(1-\alpha)p^2-\alpha m_\sigma^2-\beta m_+^2-(1-\alpha-\beta)m_-^2]^3}. \tag{11}$$

The induced Chern-Simons coefficient is deduced from the value of $\Pi(p^2=0)$:

$$\begin{aligned}\Pi(p^2=0)&=16i\kappa e^4v^2\int_0^1 d\alpha\int_0^{1-\alpha}d\beta\frac{1}{32\pi}\frac{\alpha}{[\alpha(m_\sigma^2-m_-^2)+\beta(m_+^2-m_-^2)+m_-^2]^{3/2}}\\&=\frac{-2i\kappa e^4v^2}{3\pi(m_\sigma^2-m_+^2)^2(m_\sigma^2-m_-^2)^2(m_+^2-m_-^2)}\{-m_\sigma^5(m_+^2-m_-^2)+2m_\sigma^4(m_+^3-m_-^3)\\&\quad-m_\sigma^3(m_+^4-m_-^4)-4m_\sigma^2(m_+^3m_-^2-m_+^2m_-^3)+3m_\sigma(m_+^4m_-^2-m_+^2m_-^4)-2(m_+^4m_-^3-m_+^3m_-^4)\}.\end{aligned} \tag{12}$$

Notice that the dependence on the three different masses, m_σ, m_+ and m_-, is quite involved.

At this stage we pause to compare with some previous results corresponding to special cases of this result. In the pure Chern-Simons limit, in which the Maxwell term is removed from the Lagrangian, the corresponding result was computed in [7]. This limit can be obtained from our result by sending $e^2\to\infty$ and $\kappa\to\infty$, in such a way that the ratio e^2/κ is kept finite. In terms of the masses, in this limit $m_+\to\infty$, $m_-\to m^2/|\kappa|=2e^2v^2/|\kappa|$ (finite), and m_σ is unaffected. In this limit, our result reduces to

$$\Pi(p^2=0)=\frac{2ie^2}{3\pi}\frac{\kappa}{|\kappa|}\frac{\left(1+\frac{1}{2}\frac{m_\sigma}{m_-}\right)}{\left(1+\frac{m_\sigma}{m_-}\right)^2} \tag{13}$$

which is in agreement with [7]. Furthermore, when the remaining masses, m_σ and m_-, are equal, this gives

$$\Pi(p^2=0)=\frac{ie^2}{4\pi}\frac{\kappa}{|\kappa|}. \tag{14}$$

This is exactly the mass relationship ($m_+\to\infty$ and $m_\sigma=m_-$) that arises in the bosonic sector of the N=2 supersymmetric Chern-Simons-Higgs system [21], and this result (14) agrees with the known result for this SUSY model [22].

B. Finite temperature parity-violating part

We now consider the calculation of the finite temperature one-loop parity-violating part of the polarization tensor. Such a calculation can be performed either using the imaginary time or the real time formalism of finite temperature field theory. In this paper we record the imaginary time calculation; we have also done the calculation using the real time formalism (the appropriate amplitudes to compare are the retarded ones), and obtain exactly the same results.

In the imaginary-time formalism, the parity-violating part of $\Pi_{\mu\nu}$ is

$$\Pi_{\mu\nu(PV)}=8\kappa e^4v^2\epsilon_{\mu\nu\lambda}\frac{1}{\beta}\sum_{n=-\infty}^{\infty}\int\frac{d^2k}{(2\pi)^2}\frac{k_\lambda}{[(k^0+p^0)^2+(\vec{k}+\vec{p})^2+m_\sigma^2](k_0^2+\vec{k}^2+m_+^2)(k_0^2+\vec{k}^2+m_-^2)} \tag{15}$$

where the Matsubara energies are $k^0=2\pi n/\beta$ and $p^0=2\pi l/\beta$, with n and l being integers.

1. Static limit

At finite temperature there are different physical limits for the external energy-momentum p, due to the preferred Lorentz frame of the heat bath. These different limits reflect different physical processes. The static and long-wavelength limits correspond to the Debye and plasmon masses, respectively. The physical origin of the usual nonanalyticity in these two different limits is due to the fact that a virtual particle can be absorbed by real particles in the medium, thus opening new channels that are not present at zero temperature in the absence of the thermal heat bath [20]. We first consider the *static limit* in which we first set $p^0=0$, and then take the limit $|\vec{p}|\to 0$. First, observe that, in this static limit,

$$\Pi_{ij}^{\text{static }(PV)}=0, \tag{16}$$

since the k^0 sum (i.e. the sum over n) clearly vanishes when the index $\lambda=0$. The remaining parity-violating components are

$$\Pi_{0i}^{\mathrm{static}(PV)} = 8\kappa e^4 v^2 \epsilon_{ij} \frac{1}{\beta} \sum_{n=-\infty}^{\infty} \int \frac{d^2k}{(2\pi)^2} \frac{k_j}{[(k^0)^2+(\vec{k}+\vec{p})^2+m_\sigma^2](k_0^2+\vec{k}^2+m_+^2)(k_0^2+\vec{k}^2+m_-^2)}. \tag{17}$$

The induced Chern-Simons coefficient is determined by the coefficient of $\epsilon_{ij}p_j$ in the limit $|\vec{p}|\to 0$, so we look for the term linear in the spatial momentum $\vec{p}$. Thus, we expand

$$\frac{1}{[(k^0)^2+(\vec{k}+\vec{p})^2+m_\sigma^2]} = \frac{1}{[(k^0)^2+\vec{k}^2+m_\sigma^2]} - \frac{2\vec{p}\cdot\vec{k}}{[(k^0)^2+\vec{k}^2+m_\sigma^2]^2} + O(\vec{p}^2). \tag{18}$$

The first term in this expansion contributes 0 when the spatial $\vec{k}$ momentum integral is done in Eq. (17). However, the second term produces a term linear in $\vec{p}$. Using symmetric integration, we replace $k_i k_j \to \frac{1}{2}\vec{k}^2\delta_{ij}$, and obtain

$$\Pi_{0i}^{\mathrm{static}(PV)} = \epsilon_{ij}p_j \Pi_{\mathrm{static}}(\vec{p}^2) \tag{19}$$

where

$$\Pi_{\mathrm{static}}(\vec{p}^2=0) = -8\kappa e^4 v^2 \frac{1}{\beta} \sum_{n=-\infty}^{\infty} \int \frac{d^2k}{(2\pi)^2} \frac{\vec{k}^2}{[(k^0)^2+\vec{k}^2+m_\sigma^2]^2(k_0^2+\vec{k}^2+m_+^2)(k_0^2+\vec{k}^2+m_-^2)}$$

$$= 8\kappa e^4 v^2 \frac{\partial}{\partial m_\sigma^2} \frac{1}{\beta} \sum_{n=-\infty}^{\infty} \int \frac{d^2k}{(2\pi)^2} \frac{\vec{k}^2}{[(k^0)^2+\vec{k}^2+m_\sigma^2](k_0^2+\vec{k}^2+m_+^2)(k_0^2+\vec{k}^2+m_-^2)}. \tag{20}$$

It is convenient to perform the sum over Matsubara modes using the Sommerfeld-Watson transformation [23,20] of the sum into a contour integral:

$$\sum_{n=-\infty}^{\infty} f(n) = -\pi \sum_{\mathrm{residues}} [f(z)\cot(\pi z)] \tag{21}$$

where the sum is over the residues at the poles of $f(z)$. Thus, defining

$$\omega_\sigma = \sqrt{\vec{k}^2+m_\sigma^2}, \quad \omega_+ = \sqrt{\vec{k}^2+m_+^2}, \quad \omega_- = \sqrt{\vec{k}^2+m_-^2} \tag{22}$$

we find that

$$\frac{1}{\beta}\sum_{n=-\infty}^{\infty} \frac{1}{\left[\left(\frac{2\pi n}{\beta}\right)^2+\omega_\sigma^2\right]\left[\left(\frac{2\pi n}{\beta}\right)^2+\omega_+^2\right]\left[\left(\frac{2\pi n}{\beta}\right)^2+\omega_-^2\right]}$$

$$= \frac{1}{2}\left[\frac{\frac{1}{\omega_\sigma}\coth\left(\frac{\beta\omega_\sigma}{2}\right)}{(m_\sigma^2-m_+^2)(m_\sigma^2-m_-^2)} - \frac{\frac{1}{\omega_+}\coth\left(\frac{\beta\omega_+}{2}\right)}{(m_\sigma^2-m_+^2)(m_+^2-m_-^2)} + \frac{\frac{1}{\omega_-}\coth\left(\frac{\beta\omega_-}{2}\right)}{(m_\sigma^2-m_-^2)(m_+^2-m_-^2)}\right]. \tag{23}$$

We can separate the zero temperature contribution from the finite temperature correction by using the simple identity

$$\coth\left(\frac{x}{2}\right) = 1 + \frac{2}{e^x-1} \tag{24}$$

in which we recognize the Bose-Einstein distribution function $n(x)=1/(e^x-1)$. Then the zero temperature contribution can be expressed as

$$\Pi_{\mathrm{static}}^{(T=0)} = 4\kappa e^4 v^2 \frac{\partial}{\partial m_\sigma^2} \int \frac{d^2k}{(2\pi)^2} \vec{k}^2 \left[\frac{1/\omega_\sigma}{(m_\sigma^2-m_+^2)(m_\sigma^2-m_-^2)} - \frac{1/\omega_+}{(m_\sigma^2-m_+^2)(m_+^2-m_-^2)} + \frac{1/\omega_-}{(m_\sigma^2-m_-^2)(m_+^2-m_-^2)}\right]. \tag{25}$$

These integrals may be performed with a consistent UV regulator, yielding a finite result that agrees precisely with the zero temperature result quoted in Eq. (12) (it is worth noting here that the Chern-Simons coefficient in the Euclidean space is *i* times that of the Minkowski space).

The finite temperature correction to this zero temperature result is given by

$$\Pi^{(\beta)}_{\rm static}=8\kappa e^4 v^2\frac{\partial}{\partial m_\sigma^2}\int\frac{d^2k}{(2\pi)^2}\vec{k}^2\Bigg[\frac{1}{(m_\sigma^2-m_+^2)(m_\sigma^2-m_-^2)}\frac{1}{\omega_\sigma}\frac{1}{e^{\beta\omega_\sigma}-1}-\frac{1}{(m_\sigma^2-m_+^2)(m_+^2-m_-^2)}\frac{1}{\omega_+}\frac{1}{e^{\beta\omega_+}-1}+\frac{1}{(m_\sigma^2-m_-^2)(m_+^2-m_-^2)}\frac{1}{\omega_-}\frac{1}{e^{\beta\omega_-}-1}\Bigg]. \tag{26}$$

Thus, we need to evaluate an integral of the form

$$I=\int\frac{d^2k}{(2\pi)^2}\frac{\vec{k}^2}{\omega}\frac{1}{e^{\beta\omega}-1}=\frac{1}{2\pi\beta^3}\int_{m\beta}^{\infty}dx\frac{[x^2-(m\beta)^2]}{e^x-1} \tag{27}$$

where $\omega=\sqrt{\vec{k}^2+m^2}$. We note that

$$\int_y^\infty dx\frac{1}{e^x-1}=-\log(1-e^{-y}) \tag{28}$$

$$\int_y^\infty dx\frac{x^2}{e^x-1}=\int_0^\infty dx\frac{x^2}{e^x-1}-\int_0^y dx\frac{x^2}{e^x-1}$$
$$=2\zeta(3)-\sum_{n=0}^{\infty}\frac{\mathcal{B}_n}{(n+2)n!}y^{n+2} \tag{29}$$

where the $\mathcal{B}_n$ are the Bernoulli numbers. Therefore, the high temperature expansion of I is

$$I=\frac{1}{2\pi\beta^3}\Bigg[(m\beta)^2\log(1-e^{-m\beta})+2\zeta(3)-\sum_{n=0}^{\infty}\frac{\mathcal{B}_n}{(n+2)n!}(m\beta)^{n+2}\Bigg]$$
$$=\frac{1}{2\pi\beta^3}\Bigg[2\zeta(3)+(m\beta)^2\left(\log(m\beta)-\frac{1}{2}\right)+\dots\Bigg]. \tag{30}$$

Thus, in the static limit, at high temperature, the leading behavior is

$$\Pi^{(\beta)}_{\rm static}=\frac{4\kappa e^4 v^2}{\pi\beta}F(m_+,m_-,m_\sigma) \tag{31}$$

where

$$F(m_+,m_-,m_\sigma)=\frac{m_+^2\log(m_+/m_-)}{(m_+^2-m_-^2)(m_\sigma^2-m_+^2)^2}+\frac{(m_+^2m_-^2-m_\sigma^4)\log(m_\sigma/m_-)}{(m_\sigma^2-m_+^2)^2(m_\sigma^2-m_-^2)^2}+\frac{1}{2(m_\sigma^2-m_+^2)(m_\sigma^2-m_-^2)}$$
$$=\frac{m_-^2\log(m_+/m_-)}{(m_+^2-m_-^2)(m_\sigma^2-m_-^2)^2}+\frac{(m_+^2m_-^2-m_\sigma^4)\log(m_\sigma/m_+)}{(m_\sigma^2-m_+^2)^2(m_\sigma^2-m_-^2)^2}+\frac{1}{2(m_\sigma^2-m_+^2)(m_\sigma^2-m_-^2)}$$
$$=\frac{m_+^2\log(m_+/m_\sigma)}{(m_+^2-m_-^2)(m_\sigma^2-m_+^2)^2}-\frac{m_-^2\log(m_-/m_\sigma)}{(m_+^2-m_-^2)(m_\sigma^2-m_-^2)^2}+\frac{1}{2(m_\sigma^2-m_+^2)(m_\sigma^2-m_-^2)}. \tag{32}$$

It is very interesting to notice that the temperature dependence inside the logarithmic terms cancels out, leaving logarithms only involving ratios of the masses m_+, m_- and m_σ.

Note that to lowest order in perturbation theory we can neglect the possible T dependent mass shift that arises from a calculation of the finite temperature effective potential. Also, it is possible to study the high temperature limit where T is sufficiently high compared to the relevant mass scales ($T\gg m\sim ev$, and $T\gg m_\sigma\sim\sqrt{\lambda}v$) but still below any symmetry restoration temperature scale T_c whose magnitude is determined by $T_c\ln T_c\sim m^2/\lambda$. For supersymmetric Chern-Simons-Higgs systems, a detailed analysis of symmetry breaking considerations, at both one and two loop, has been peformed in [24].

We now consider this result in the mass limits considered in Sec. III A. First, we take the pure Chern-Simons limit in which $m_+\sim|\kappa|\to\infty$, and $m_-\to m^2/|\kappa|=$finite. From the above, in this limit the leading high temperature behavior is

$$\Pi^{(\beta)}_{\rm static}\to-\frac{\kappa}{|\kappa|}\frac{e^2m_-}{\pi\beta}\left[\frac{2m_-^2\log\left(\frac{m_-}{m_\sigma}\right)+m_\sigma^2-m_-^2}{(m_\sigma^2-m_-^2)^2}\right]. \tag{33}$$

When the two remaining masses are equal (i.e., $m_-=m_\sigma$), we find

$$\Pi^{(\beta)}_{\rm static}\to-\frac{\kappa}{|\kappa|}\frac{e^2}{2\pi\beta m_-}. \tag{34}$$

Indeed, in this case we can keep the *full* temperature dependence. Returning to the expressions (20) and (23) which have the full temperature dependence, we can take the limit $m_+\to\infty$, and $m_\sigma\to m_-$, to obtain a remarkable simplification:

$$\Pi_{\rm static}\to-\frac{\kappa}{|\kappa|}\frac{e^2}{4\pi}\coth\left(\frac{\beta m_-}{2}\right). \tag{35}$$

This should be compared with the the corresponding fermion loop result for a fermion of mass M:

$$\Pi_{\rm static}=\frac{\kappa}{|\kappa|}\frac{e^2}{4\pi}\tanh\left(\frac{\beta M}{2}\right). \tag{36}$$

2. Long wavelength limit

In this section we consider the long wavelength limit at finite temperature. In this limit we set $\vec{p}=0$, and consider $p^0\to0$. This must be done with care in the imaginary time formalism, because p^0 is discrete and must be analytically continued back to real time where it is a continuous variable. In the long wavelength limit, the parity violating part of the polarization tensor is

$$\Pi^{\rm LW(\it PV)}_{\mu\nu}=8\kappa e^4v^2\epsilon_{\mu\nu\lambda}\frac{1}{\beta}\sum_{n=-\infty}^{\infty}\int\frac{d^2k}{(2\pi)^2}\frac{k_\lambda}{[(k^0+p^0)^2+\vec{k}^2+m_\sigma^2][(k^0)^2+\vec{k}^2+m_+^2][(k^0)^2+\vec{k}^2+m_-^2]}. \tag{37}$$

By symmetry it is clear that

$$\Pi^{\rm LW(\it PV)}_{0i}=0 \tag{38}$$

while $\Pi^{\rm LW(\it PV)}_{ij}$ is nonzero. This is the opposite of what was found above in the static limit, where $\Pi^{\rm static(\it PV)}_{ij}=0$ and $\Pi^{\rm static(\it PV)}_{0i}\neq0$. In fact,

$$\Pi^{\rm LW(\it PV)}_{ij}=8\kappa e^4v^2\epsilon^{ij}\frac{1}{\beta}\sum_{n=-\infty}^{\infty}\int\frac{d^2k}{(2\pi)^2}\frac{k_0}{[(k^0+p^0)^2+\vec{k}^2+m_\sigma^2][(k^0)^2+\vec{k}^2+m_+^2][(k^0)^2+\vec{k}^2+m_-^2]}. \tag{39}$$

The sum over Matsubara modes can be done, as before, using a contour integral representation

$$\begin{aligned}
&\frac{1}{\beta}\sum_{n=-\infty}^{\infty}\frac{k_0}{[(k^0+p^0)^2+\vec{k}^2+m_\sigma^2][(k^0)^2+\vec{k}^2+m_+^2][(k^0)^2+\vec{k}^2+m_-^2]}\\
&=-\frac{\pi}{\beta}\left(\frac{\beta}{2\pi}\right)^5\sum_{\rm residues}\left[\frac{z\cot(\pi z)}{\left[\left(z+\frac{\beta p_0}{2\pi}\right)^2+\left(\frac{\beta\omega_\sigma}{2\pi}\right)^2\right]\left[\left(z+\frac{\beta p_0}{2\pi}\right)^2+\left(\frac{\beta\omega_+}{2\pi}\right)^2\right]\left[\left(z+\frac{\beta p_0}{2\pi}\right)^2+\left(\frac{\beta\omega_-}{2\pi}\right)^2\right]}\right]\\
&=\frac{1}{4}\left[\frac{\coth\left(\frac{\beta\omega_\sigma}{2}\right)}{\omega_\sigma}\frac{(-p^0+i\omega_\sigma)}{[(-p^0+i\omega_\sigma)^2+\omega_+^2][(-p^0+i\omega_\sigma)^2+\omega_-^2]}-\frac{\coth\left(\frac{\beta\omega_\sigma}{2}\right)}{\omega_\sigma}\frac{(p^0+i\omega_\sigma)}{[(p^0+i\omega_\sigma)^2+\omega_+^2][(p^0+i\omega_\sigma)^2+\omega_-^2]}\right.\\
&\quad-\frac{i\coth\left(\frac{\beta\omega_+}{2}\right)}{[(p^0+i\omega_+)^2+\omega_\sigma^2][\omega_+^2-\omega_-^2]}+\frac{i\coth\left(\frac{\beta\omega_+}{2}\right)}{[(p^0-i\omega_+)^2+\omega_\sigma^2][\omega_+^2-\omega_-^2]}-\frac{i\coth\left(\frac{\beta\omega_-}{2}\right)}{[(p^0+i\omega_-)^2+\omega_\sigma^2][\omega_+^2-\omega_-^2]}\\
&\quad\left.+\frac{i\coth\left(\frac{\beta\omega_-}{2}\right)}{[(p^0-i\omega_-)^2+\omega_\sigma^2][\omega_+^2-\omega_-^2]}\right].
\end{aligned} \tag{40}$$

This expression can now be analytically continued in p^0, and then Taylor expanded to linear order in p^0, in order to determine the Chern-Simons coefficient. We write

$$\Pi_{ij}^{\mathrm{LW}(PV)}=\epsilon_{ij}p^0\Pi_{\mathrm{LW}}(p^0) \tag{41}$$

where

$$\Pi_{\mathrm{LW}}(p^0=0)=-4\kappa e^4v^2\int\frac{d^2k}{(2\pi)^2}\left[\left\{\frac{1}{\omega_\sigma(m_\sigma^2-m_+^2)(m_\sigma^2-m_-^2)}-\frac{2\omega_\sigma}{(m_\sigma^2-m_+^2)^2(m_\sigma^2-m_-^2)}-\frac{2\omega_\sigma}{(m_\sigma^2-m_+^2)(m_\sigma^2-m_-^2)^2}\right\}\right.$$
$$\left.\times\coth\left(\frac{\beta\omega_\sigma}{2}\right)+\frac{2\omega_+\coth\left(\frac{\beta\omega_+}{2}\right)}{(m_\sigma^2-m_+^2)^2(m_+^2-m_-^2)}-\frac{2\omega_-\coth\left(\frac{\beta\omega_-}{2}\right)}{(m_\sigma^2-m_-^2)^2(m_+^2-m_-^2)}\right]. \tag{42}$$

Once again, we separate the zero temperature piece from the finite temperature correction using the simple identity (24). Then the zero temperature part of Eq. (42) is finite, with consistent UV regulators for the momentum integrals, and agrees precisely with the direct zero temperature result in Eq. (12).

The nonzero temperature contribution can be expressed as

$$\Pi_{LW}^{(\beta)}=-\frac{4\kappa e^4v^2}{\pi\beta^3}\left[\int_{\beta m_\sigma}^{\infty}\frac{dx}{e^x-1}\left\{\frac{\beta^2}{(m_\sigma^2-m_+^2)(m_\sigma^2-m_-^2)}-\frac{2x^2}{(m_\sigma^2-m_+^2)^2(m_\sigma^2-m_-^2)}-\frac{2x^2}{(m_\sigma^2-m_+^2)(m_\sigma^2-m_-^2)^2}\right\}\right.$$
$$\left.+\int_{\beta m_+}^{\infty}\frac{dx}{e^x-1}\frac{2x^2}{(m_\sigma^2-m_+^2)^2(m_+^2-m_-^2)}-\int_{\beta m_-}^{\infty}\frac{dx}{e^x-1}\frac{2x^2}{(m_\sigma^2-m_-^2)^2(m_+^2-m_-^2)}\right]. \tag{43}$$

The dominant contribution at high temperature is easily computed using the integrals listed earlier in Eqs. (28),(29). This dominant contribution in the long wave limit, at high temperature, gives

$$\Pi_{ij}^{\mathrm{LW}(PV)(\beta)}=\frac{4\kappa e^4v^2\epsilon^{ij}p_0}{\beta\pi}\frac{\log(\beta m_\sigma)}{(m_\sigma^2-m_+^2)(m_\sigma^2-m_-^2)}. \tag{44}$$

We note several things about this result. First, there is still a logarithmic dependence on the temperature. Second, the long wavelength limit gives a completely different result for the parity violating part of the self energy, as compared to the static limit. This is true even though the two masses in the bosonic loop are quite different. So, there is still a nonanalyticity in the self-energy, contrary to what had been found earlier in a simpler model [16] without parity violation. For completeness, we note here that, at high temperature, the contribution due to a fermion loop to the Chern-Simons term goes as $\sim\beta$ in the static limit and as $\sim\beta\ln\beta$ in the long wave limit [25].

IV. CONCLUSIONS

To conclude, we emphasize that the induced Chern-Simons terms that appear from bosonic loops have a completely different temperature dependence from those induced by a fermion loop. In fact, the contribution from the bosonic loop grows at high temperature both in the static as well as the long wavelength limits, as opposed to the contribution from the fermionic loop which decreases at high temperature in both these limits [25]. This behavior is most clearly seen by comparing the static limit results (35) and (36) in the pure Chern-Simons limit. The bosonic loop contribution goes like $-\coth(\beta m/2)$, while the familiar fermion loop static limit contribution goes like $\tanh(\beta m/2)$. In the bosonic theory, in the static limit, there will be a sequence of new T-dependent parity violating terms, beyond the Chern-Simons term, just like in the fermionic theory in the static limit [1–4]. This is one step towards answering the question of large gauge invariance at finite temperature in non-fermionic theories. However, this is still an Abelian theory. To resolve this question in a *non-Abelian* Chern-Simons Higgs system (such as those studied at zero temperature in [7–10]) one needs also to analyze the gluon and ghost loop terms at finite temperature. This is a much more difficult problem, and is beyond the scope of this paper.

In the model studied in this paper, the induced Chern-Simons terms arise in loop corrections because of the presence of a parity violating bare Chern-Simons term in the bare Lagrangian. This bare Chern-Simons term has a number of consequences. First, it gives the gauge field two massive modes in the spontaneously broken phase. Second, it introduces parity violating interactions. We have shown that the induced parity violating contributions to the self energy behave, at finite temperature, in a very different way from the parity preserving contributions studied previously in [16]. Specifically, the parity preserving terms have a unique zero momentum limit, even at finite temperature, while the parity violating terms have a non-unique limit at finite temperature. We have demonstrated this by computing the parity violating terms in both the static and long wavelength limit. The leading high temperature parts for these parity violating terms are given in Eqs. (31), (32) and (44), and they are clearly different. In the long wavelength limit there is a logarithmic dependence on the temperature, while in the static limit this cancels out leaving logarithmic dependence on mass ratios.

In addition, we have analyzed the limits in which the Maxwell term is removed, leaving a pure Chern-Simons theory, and also in which the remaining scalar masses are taken to be equal. In these cases the temperature dependence simplifies considerably. We understand [26] that Gomes and collaborators are analyzing a related model involving a pure Chern-Simons gauge Lagrangian coupled to a Higgs field with a sextic potential. It would be interesting to compare their results to ours in the appropriate limit.

Finally, the temperature dependent parity violating contributions to the self-energy mix with the temperature dependent parity conserving contributions in order to determine the physical masses in thermal equilibrium. This issue has been analyzed in [6] for the Chern-Simons-Yang-Mills system at finite temperature. A similar analysis for the Maxwell-Chern-Simons-Higgs system with symmetry breaking would be an interesting application of the results in this current paper.

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