Global strings in high density QCD

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We show that several types of global strings occur in color superconducting quark matter due to the spontaneous violation of relevant $U(1)$ symmetries. These include the baryon $U(1)_B$, and approximate axial $U(1)_A$ symmetries as well as an approximate $U(1)_S$ arising from kaon condensation. We discuss some general properties of these strings and their interactions. In particular, we demonstrate that the $U(1)_A$ strings behave as superconducting strings. We draw some parallels between these strings and global cosmological strings and discuss some possible implications of these strings to the physics in neutron star cores.

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I. INTRODUCTION

Domain walls and strings are common examples of topological defects which are present in various field theories [1,2]. Domain walls are configurations of fields related to the nontrivial mapping $\pi_0(M)$, while topologically stable strings are due to the mapping $\pi_1(M)$ where M is the manifold of degenerate vacuum states. While it is generally believed that there are no domain walls or strings in the standard model due to the triviality of the corresponding mappings, such objects may exist in extensions to the standard model, or in phases where the symmetries of the standard model are broken.

Topological defects that occur in extensions to the standard model may play an important role in cosmology as described in Ref. [2]; however, the focus of this paper will be strings and domain walls that exist in high density matter where the symmetries of the standard model are broken. As was recently demonstrated [3], in the regime of high baryon densities when the chemical potential μ is much larger than the QCD scale $\mu \gg \Lambda_{\rm QCD}$, QCD supports domain walls. The existence and long lifetime of these domain walls is based on the following facts: (1) the instanton density is small at large chemical potential, suppressing the effect of the chiral anomaly and giving rise to the approximate $U(1)_A$ symmetry; (2) this $U(1)_A$ symmetry is spontaneously broken, and (3) the decay constant of the pseudoscalar singlet boson (η') is large and its mass small at large μ.

In this paper we show that, along with these domain walls, there exist topological string configurations with interesting physical properties. Specifically, we discuss the three flavor $N_f=3$ color-flavor locking (CFL) phase, and the two flavor $N_f=2$ superconducting (2SC) phase. In the $N_f=3$ CFL phase both the $U(1)_B$ and the approximate $U(1)_A$ symmetries of QCD are spontaneously broken. In addition, a third approximate $U(1)_S$ symmetry, related to the formation of a kaon condensate, is broken (see Sec. II B). Thus, at least three types of strings are possible in the presence of a kaon condensate and two types of strings are possible if no kaon condensate forms. In the $N_f=2$ 2SC phase, the $U(1)_B$ symmetry is not broken and we only expect one type of string from the breaking of the approximate $U(1)_A$ symmetry. The important point is that the asymptotic freedom of QCD allows us to assert the existence of these strings in the high baryon density regime, and the properties of these strings can be determined by controllable weak-coupling calculations.

Naïvely, these strings are neutral objects with respect to electromagnetic interactions, and therefore, one might think that they play no role in electromagnetic dynamics. However, for the $U(1)_A$ strings, this naïve expectation is incorrect due to the anomalous coupling of the singlet Goldstone field with a massless "photon"—actually a mixture of the bare electromagnetic field and one component of the color gluon fields. We demonstrate that such strings behave as superconducting strings, and we estimate the maximum current that they can carry. Thus, QCD strings at high density may affect the electromagnetic properties of high density matter, influencing, for example, the magnetic fields in the cores of neutron stars.

Regarding the baryonic $U(1)_B$ strings present in the CFL phase: it has been known for quite a while [4] that, although superfluid vortices (in liquid helium, for example) and static global string solutions (related to the spontaneous breaking of a global symmetry, $U(1)_B$ in this case) are closely related, there are some differences. In particular, fluid vortices carries angular momentum while a static global strings have no angular momentum. What was demonstrated in [4] was that the two can be identified if the static strings are immersed in a uniform background with nonzero density. In high density QCD, such a background is naturally present, thus we demonstrate that the $U(1)_B$ global strings can be identified with the superfluid vortices likely to form in a rotating CFL phase with nonzero angular momentum as has been conjecture to exist in the cores of neutron stars (see, for example, [5,6]). A similar mechanism leads to the formation of $U(1)_S$ strings.

The mechanism that forms the $U(1)_A$ strings in the 2SC phase, which do not carry angular momentum, is less obvious to us and, in this paper, we simply assume that some mechanism exists to form these types of objects. In general, there will be complicated dynamical interactions between the strings, which may lead to the formation of more complicated stable objects like rings, springs, and vortons. In this paper we limit ourselves to describing the properties of individual strings and their dominant pairwise interactions.

This paper is organized as follows: In Sec. II we describe the string solutions and calculate the relevant parameters of the string in terms of the fundamental parameters of the theory. In Sec. II B 2 we discuss how the $U(1)_B$ strings present in the CFL phase are not static, but rather spin with a time dependent phase. These spinning strings can then be identified with the superfluid vortices. In Sec. III we discuss the anomalous electromagnetic properties of the $U(1)_A$ strings. We solve the relevant Maxwell equations in the string background and demonstrate that the $U(1)_A$ strings behave as if they are superconducting. Finally, in Sec. IV, we discuss the interactions between strings and speculate on their effects on the physics of high density quark matter and neutron stars.

II. STRINGS IN HIGH DENSITY QCD

When one considers QCD at high density, the relevant excitations are due to quarks with momenta close to the Fermi surface. At high densities, these low-energy excitations have high momenta and, because of asymptotic freedom, one might hope to regain theoretical control in a weak coupling regime. Although the situation is not quite as simple as it appears, we do gain theoretical control. In particular, the color anti-triplet $\overline{\mathbf{3}}$ single gluon interaction is attractive, and, for low energies, this leads to a preferred Bardeen-Cooper-Schrieffer (BCS) pairing of quarks with opposite momenta that reduces the energy of the vacuum state. This is the phenomenon of color superconductivity. (For a review, see [7].)

To be specific, we consider the simplest model where high-density $U(1)_A$ type strings appear: QCD with $N_f=2$ massless quark flavors (u and d) and $N_c=3$ colors. This model is a rather good approximation to realistic quark matter at moderate densities. At higher densities, the approximation of $N_f=3$ massless quarks becomes quite good. This will be discussed later in Sec. II B. The most important qualitative difference between the $N_f=2$ and the $N_f=3$ phase is the emergence of new spontaneously broken symmetries: the $U(1)_B$ in the three flavor case and the $U(1)_S$ symmetry if a kaon condensate forms in the CFL phase. As a consequence, if a kaon condensate forms in the CFL phase, then there emerge two new types of global strings related to the $U(1)_B$ and $U(1)_S$ symmetries. If a kaon condensate does not form, then only one new type of string related to the $U(1)_B$ symmetry emerges. In what follows we use the same normalization factors as the paper [3] on domain walls in dense QCD.

A. $N_f=2$ superconducting phase (2SC)

We recall that the ground state at high baryon densities is a superconducting state [8–13], characterized by the condensation of diquark Cooper pairs. The superconducting ground state spontaneously breaks the symmetry of the bare QCD Lagrangian through the non-zero diquark condensates $\langle\Psi^{ia}_{\alpha}\Psi^{jb}_{\beta}\rangle$ which represent the Cooper pairs. Here we explicitly show the flavor (α, β, etc.), color (a, b, etc.) and spinor (i, j, etc.) indices. In this section we consider the $N_f=2$ case where the strange quark is treated as heavy. In this 2SC phase, the diquark condensates have the form

$$\langle\Psi^{i\alpha}_{La}\Psi^{j\beta}_{Lb}\rangle^{*}=\epsilon^{ij}\epsilon^{\alpha\beta}\epsilon_{abc}X^{c}, \tag{1a}$$

$$\langle\Psi^{i\alpha}_{Ra}\Psi^{j\beta}_{Rb}\rangle^{*}=\epsilon^{ij}\epsilon^{\alpha\beta}\epsilon_{abc}Y^{c}. \tag{1b}$$

The condensates X^c and Y^c are complex color 3-vectors which are aligned along the same direction in the ground state. They spontaneously break the color $SU(3)_c$ group down to $SU(2)_c$. The lengths of these vectors are equal $|X|=|Y|$ and have been computed perturbatively [3,14]:

$$|X|=|Y|=\frac{3}{2\sqrt{2}\pi}\frac{\mu^2\Delta}{g}. \tag{2}$$

In perturbation theory, there is an approximate degeneracy of the ground state with respect to the relative $U(1)_A$ phase between X^c and Y^c which is a symmetry of the QCD Lagrangian at the classical level. A nonzero value (1) for the vacuum condensate implies that the $U(1)_A$ symmetry is spontaneously broken and, thus, the corresponding pseudo-Goldstone boson—the η'—enters into the theory. The $U(1)_B$ symmetry of the QCD Lagrangian also appears to be broken, but is in fact restored by a simultaneous $SU(3)_c$ rotation. Thus, only the $U(1)_A$ symmetry is spontaneously broken in the $N_f=2$ 2SC phase.

Following [3], to describe the η' physics in an explicit way, we construct the gauge-invariant order parameter

$$\Sigma=Y^{\dagger}X\equiv Y^{*}_{c}X^{c}, \tag{3}$$

where the phase φ_A is to be identified with the dynamical η' field,

$$\Sigma=|\Sigma|e^{-i\varphi_A}=|\Sigma|e^{-i\eta'/f}. \tag{4}$$

It is evident that Σ carries a non zero $U(1)_A$ charge:

$$\Psi\rightarrow e^{i\gamma_5\alpha/2}\Psi\Rightarrow X\rightarrow e^{-i\alpha}\ \text{ and }\ Y\rightarrow e^{i\alpha} \tag{5a}$$

$$\Rightarrow\Sigma\rightarrow e^{-2i\alpha} \tag{5b}$$

$$\Rightarrow\varphi_A\rightarrow\varphi_A+2\alpha. \tag{5c}$$

At low energies, the dynamics of the Goldstone mode φ_A are described by an effective Lagrangian, which, to leading order in derivatives, must take the following form [3]:

$$\mathcal{L}=f^2[(\partial_0\varphi_A)^2-u^2(\partial_i\varphi_A)^2]-V_{\text{inst}}(\varphi_A). \tag{6}$$

For large chemical potentials $\mu\gg\Lambda_{\text{QCD}}$, the leading perturbative values for the decay constant f and velocity u [15], and the instanton contribution describing the explicit anomalous breaking of the $U(1)_{\text{A}}$ symmetry [3] have been calculated:

$$V_{\text{inst}}(\varphi_A)=-a\mu^2\Delta^2\cos\varphi_A, \tag{7a}$$

$$f^2=\frac{\mu^2}{8\pi^2}, \quad u^2=\frac{1}{3}. \tag{7b}$$

In this formula Δ is the BCS gap, and a is a dimensionless parameter [3]

$$a\sim\left(\frac{\Lambda_{\rm QCD}}{\mu}\right)^{29/3} \tag{8}$$

that goes to zero at large μ. In what follows it will be important that the η' mass is asymptotically small, as can be seen from Eq. (6):

$$m_{\eta'}=\frac{\mu}{f}\Delta\sqrt{\frac{a}{2}}=2\pi\Delta\sqrt{a}. \tag{9}$$

The effective Lagrangian (6) is justified for describing the light η' degree of freedom, but to describe global strings we must formulate an effective theory for fluctuations in the magnitude of the condensate $|\Sigma|$. From Eq. (2) we have that $|\langle\Sigma\rangle|=9\mu^4\Delta^2/(8\pi^2g^2)$. Thus, we introduce the dynamical field $\Phi(x)$ of dimension one and expectation value $|\langle\Phi\rangle|=\langle\rho\rangle=\Delta$:

$$\Sigma(x)=\Delta\left(\frac{3\mu^2}{2\sqrt{2}\pi g}\right)^2\Phi(x). \tag{10}$$

With this definition, the η' dynamical field is merely the phase of a complex field

$$\Phi=|\Phi|\exp(-i\varphi_A)=\rho\exp(-i\varphi_A) \tag{11}$$

as in the Abelian Higgs model.

The effective potential for this condensate has been calculated [16] for asymptotically large μ. In terms of the field $\rho(x)=|\Phi(x)|$, an approximate expression[1] for the potential [16] can be used which is a good description for ρ close to its vacuum expectation value $\rho\approx\langle\rho\rangle=\Delta$

$$V_\Phi(\rho)=-\frac{\mu^2\Delta}{\pi^2}\rho\left[1-\ln\left(\frac{\rho}{\Delta}\right)\right]. \tag{12}$$

Finally, in terms of a single complex field $\Phi=\rho\exp(-i\varphi_A)$ the effective Lagrangian describing the η' phase φ_A and the absolute value for the condensate ρ can be represented in the following simple way:

$$\mathcal{L}_{\rm eff}=\frac{f^2}{\Delta^2}(|\partial_0\Phi|^2-u^2|\partial_i\Phi|^2)+\frac{\mu^2\Delta}{\pi^2}\rho\left[1-\ln\left(\frac{\rho}{\Delta}\right)\right]+a\mu^2\Delta\rho\cos\varphi_A \tag{13a}$$

$$=\frac{f^2}{\Delta^2}[(\partial_0\rho)^2-u^2(\partial_i\rho)^2]+\frac{f^2\rho^2}{\Delta^2}[(\partial_0\varphi_A)^2-u^2(\partial_i\varphi_A)^2]+\frac{\mu^2\Delta}{\pi^2}\rho\left[1-\ln\left(\frac{\rho}{\Delta}\right)\right]+a\mu^2\Delta\rho\cos\varphi_A \tag{13b}$$

where the normalization factor f^2/Δ^2 for the kinetic term has been chosen to reproduce Eq. (6) where $\rho\sim\langle\rho\rangle$ which correctly describes the dynamics of the light pseudo Goldstone η' meson.

We should comment here that the potential presented in Eq. (12) was derived in [16] for very large μ and when the field Φ is close to its vacuum expectation value $\langle\rho\rangle\simeq\Delta$. It deviates considerably from this form when ρ is far from its vacuum expectation value $\rho\neq\Delta$. Besides that, there is no justification to keep only the lowest derivative term for the massive mode ρ in the expression (13). Finally, there is an ambiguity in the definition of the dynamical field ρ describing heavy $\sim\Delta$ degrees of freedom: any smooth function of $F(\rho)$ [for example, $\exp(\rho)$] is appropriate for the description of the dynamics of the heavy degrees of freedom. This is a marked contrast with the description of the light Goldstone fields where the physics does not depend on specific parametrization of the light fields. There are many other deficiencies of the Lagrangian (13) describing massive field ρ which we shall not comment about.

We are not pretending to have derived a Lagrangian describing a heavy (order Δ) degree of freedom. Rather, we want to demonstrate the qualitative features of the effective potential: that it has a Mexican hat shape (as it should) and that the internal relevant scales are of order Δ and not μ. Indeed, the general scale $\sim\mu^2$ factors from the expression (13). Therefore, the description of the strings which follows serves mainly to illustrate the qualitative features of the strings. In particular, the details of the strings core are not well founded; however, we shall see that the behavior of the strings far from the core is governed by the form of the potential where $\rho\sim\langle\rho\rangle=\Delta$ where the effective potential is valid. This region gives a logarithmic contribution to the string tension which dominates the energy of the strings. Thus, despite the fact that the effective theory (13) breaks down for small ρ, the description of the strings far from the core is well justified.

1. Global strings

First we consider the properties of an isolated global string which is symmetric about the z axis. The term $V_{\rm inst}(\varphi_A)$, which explicitly breaks the $U(1)_A$ symmetry, is

[1]Note, the normalization for the condensate in [16] differs from our normalization for $|X|$ by a factor of 16.

small, and vanishes in the high density limit. Thus, our approximation of the global string is a good description out to lengths scales of order $m_{\eta'}^{-1}$.

With this simplification, we are looking for a static, classical field configuration $\Phi(r,\phi)=\rho(r,\phi)\exp[-i\varphi_A(r,\phi)]$ which minimizes the energy density or string tension α:

$$\alpha=\int\int[\mathcal{H}(\rho,\varphi_A)-\mathcal{H}_{\text{vac}}]\,\mathrm{d}x\,\mathrm{d}y \tag{14}$$

where $\mathcal{H}$ is the energy density of the field configuration:

$$\mathcal{H}=\frac{f^2u^2}{\Delta^2}\left\{(\partial_i\rho)^2+\rho^2(\partial_i\varphi_A)^2 - \frac{\mu^2\Delta^3}{\pi^2 f^2u^2}\rho\left[1-\ln\left(\frac{\rho}{\Delta}\right)\right]\right\},$$

and $\mathcal{H}_{\text{vac}}=\mathcal{H}(\langle\rho\rangle,\langle\varphi_A\rangle)=-\mu^2\Delta^2/\pi^2$ is a trivial shift of the background vacuum energy introduced so that $\mathcal{H}\to 0$ far from the core of the string.

To simplify the equations, we introduce the dimensionless field $\tilde{\rho}$ and the dimensionless coordinates $\tilde{x}_i$:

$$\tilde{\rho}=\frac{\rho}{\Delta},\quad \langle\tilde{\rho}\rangle=1,\quad \tilde{x}_i=mx_i=2\sqrt{3}\Delta x_i. \tag{15}$$

The natural length scale is thus set by the parameter $m=2\sqrt{3}\Delta$: the mass of the excitations about the condensate $\rho=\langle\rho\rangle+\delta\rho$ in our model (12). In terms of these dimensionless parameters, the energy density becomes

$$\mathcal{H}=24f^2u^2\Delta^2\{\tfrac{1}{2}(\partial_i\tilde{\rho})^2+\tfrac{1}{2}\tilde{\rho}^2(\partial_i\varphi_A)^2-\tilde{\rho}[1-\ln\tilde{\rho}]\}$$

where all the derivatives are with respect to the dimensionless coordinates $\tilde{x}_i$. To minimize the string tension, we can drop the overall factor and then determine the equations of motion. To achieve the appropriate boundary conditions, φ_A will wind uniformly n times as a function of ϕ

$$\varphi_A(r,\phi)=\varphi_A(\phi)=n\phi=n\tan^{-1}\left(\frac{y}{x}\right). \tag{16}$$

Converting to polar coordinates we have

$$\alpha=4\pi f^2u^2\int_0^{\tilde{R}}\left(\frac{\tilde{\rho}'^2}{2}+\frac{\tilde{\rho}^2n^2}{2\tilde{r}^2}+V(\tilde{\rho})\right)\tilde{r}\,\mathrm{d}\tilde{r} \tag{17}$$

where the prime signifies differentiation with respect to $\tilde{r}$,

$$V(\tilde{\rho})=1-\tilde{\rho}[1-\ln\tilde{\rho}], \tag{18}$$

and we have introduced an outer limit $\tilde{R}$ to the string's size to make the tension finite. The equations of motion follow from an application of a variational principle:

$$\tilde{\rho}''+\frac{\tilde{\rho}'}{\tilde{r}}=\frac{\tilde{\rho}n^2}{\tilde{r}^2}+\frac{\mathrm{d}V(\tilde{\rho})}{\mathrm{d}\tilde{\rho}} \tag{19}$$

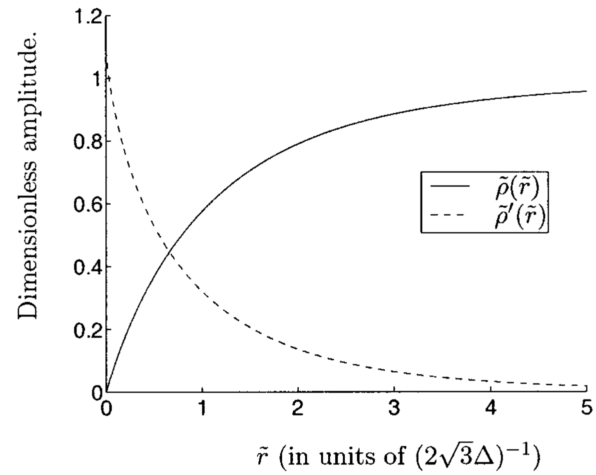

FIG. 1. Radial $\tilde{\rho}(\tilde{r})$ dependence of the $n=1$ string in dimensionless units: the vertical scale is set by Δ and the horizontal scale is set by $(2\sqrt{3}\Delta)^{-1}$.

with boundary conditions $\tilde{\rho}(0)=0$ and $\tilde{\rho}(\infty)=1$. The solution for the lowest energy string with winding $n=1$ is presented in Fig. 1.

The string is governed by two relevant parameters: its core size and its tension. As can be seen in Fig. 1, the core size is of order $\tilde{r}_c\sim 1$ while the tension of a global string diverges as $\log(\tilde{R})$ where $\tilde{R}$ is an upper cutoff determined by the environment of the string. For large distances the second term in Eq. (17) dominates and we have a logarithmic divergence. We plot the cumulative energy as a function of the upper cutoff $\tilde{R}$ in Fig. 2.

Physically, the effective potential (12) is only valid for $\rho\sim\langle\rho\rangle$. Thus, the details of the string core must be interpreted with caution. The large distance behavior and the logarithmic divergence in the string tension, however, are well justified.

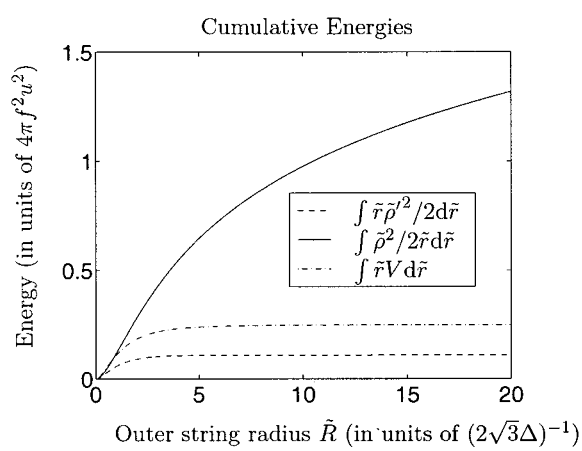

FIG. 2. Cumulative contributions to the string tension of an $n=1$ string from the core out to the cutoff radius $\tilde{R}$ (all integrals from 0 to $\tilde{R}$: $\int_0^{\tilde{R}}$) in dimensionless units: the vertical scale is set by $4\pi f^2u^2$ and the horizontal scale is set by $(2\sqrt{3}\Delta)^{-1}$. The three curves correspond to the three terms in Eq. (17). The first kinetic term approaches 0.11 while the potential term approaches 0.25. Notice that the second term gives a logarithmic divergence in $\tilde{R}$.

2. Domain walls

The formation of global strings discussed in the previous section neglected the instanton contribution $V_{\text{inst}}(\varphi_A)$ which explicitly breaks the $U(1)_A$ symmetry responsible for the global strings. Thus, the preceding analysis and picture is really only justified on distance scales small compared to those set by the anisotropy $V_{\text{inst}}(\varphi_A)$, i.e. for $r \ll R \sim m_{\eta'}^{-1}$, which, in the high density limit, is much larger than the core size Δ^{-1} as can be seen from Eqs. (8) and (9). For distances larger than $r \gg R \sim m_{\eta'}^{-1}$, the appropriate description is no longer one of global strings, but one of QCD domain walls bounded by strings: the situation is similar to the so-called $N=1$ axion model [17].

Here we summarize a few results regarding QCD domain walls that will be relevant for our discussions later on. As classically stable objects, QCD domain walls were first analyzed in [18] for $\mu=0$ in the large N_c limit. Similar objects were later shown to be stable for $N_c=3$ at high densities [3]. We shall not repeat the analysis presented in [3] but quote some important results that we will make use of later.

The thickness of the domain walls is set by the mass $m_{\eta'}$ (9) of the excitations in the φ_A field about the true vacuum $\varphi_A=0$. In addition, the energy density per unit area of the domain walls (the wall tension) is

$$\sigma = 8\sqrt{2a}\, u f \mu \Delta \sim \sqrt{a}\,\mu^2 \Delta. \tag{20}$$

Thus, on scales much smaller than $R \sim m_{\eta'}^{-1}$, the description of the global strings is valid; however, when one looks at scales larger than this, then one sees that these strings are really attached to domain walls.

B. $N_f=3$ CFL phase

For very high densities, it can be a good approximation to neglect the strange quark mass. In this case there will be three flavors $N_f=3$ in our model. Again, the ground state is a superconducting state, characterized by the condensation of diquark Cooper pairs [19,20]. The superconducting ground state spontaneously breaks the symmetry of the bare QCD Lagrangian through the non-zero diquark condensates $\langle \Psi_\alpha^{ia} \Psi_\beta^{jb} \rangle$ which represent the Cooper pairs. [Again, we explicitly show the flavor (α, β, etc.), color (a, b, etc.) and spinor (i, j, etc.) indices.] This condensate has the form

$$\langle \Psi_{La}^{i\alpha} \Psi_{Lb}^{j\beta} \rangle^* \sim \epsilon^{ij} \epsilon^{\alpha\beta\gamma} \epsilon_{abc} X_\gamma^c, \tag{21a}$$

$$\langle \Psi_{Ra}^{i\alpha} \Psi_{Rb}^{j\beta} \rangle^* \sim \epsilon^{ij} \epsilon^{\alpha\beta\gamma} \epsilon_{abc} Y_\gamma^c. \tag{21b}$$

Now the condensates X_γ^c and Y_γ^c are complex color-flavor matrices. Following [21,22], we introduce

$$\Sigma_\beta^\alpha = Y^\dagger X = (Y^*)_c^\alpha X_\beta^c. \tag{22}$$

The matrix Σ is a color singlet and describes the meson octet as well as the φ_A axial singlet:

$$\Sigma = |\Sigma| e^{i\vec{\pi}\cdot\vec{\lambda}/f_\pi - i\varphi_A} \tag{23}$$

where $\vec{\pi}$ is the octet of Goldstone bosons, $\vec{\lambda}$ are the generators of $SU(3)$ (the Gell-Mann matrices) normalized as $\text{Tr}(\lambda^a \lambda^b) = 2\delta^{ab}$ and f_π is the appropriate decay constant computed to leading order in α_s [21–23],

$$f_\pi^2 = \frac{21 - 8\ln 2}{18}\,\frac{\mu^2}{2\pi^2}. \tag{24}$$

As we shall discuss shortly, the non-trivial structure of $SU(3)$ admits the possibility of further condensations of these mesons: in particular, of the kaon K^0 and thus suggests the presence of additional $U(1)$ symmetries and strings. We shall defer discussion of these strings until Sec. II B 1. For the rest of this section, we consider only the "symmetric CFL" phase where

$$\langle \Sigma_\beta^\alpha \rangle \sim \delta_\beta^\alpha \tag{25}$$

with no further meson condensation.

The $U(1)_B$ symmetry is not reflected in Σ; however, unlike in the $2SC$ phase where it was restored via a simultaneous color rotation, here the $U(1)_B$ symmetry is also spontaneously broken giving rise to both axial and baryonic strings.

To be precise, we define the singlet phases φ_A and φ_B describing the Goldstone bosons related to the spontaneously broken symmetries $U(1)_A$ and $U(1)_B$ through the following structure:

$$\langle \Psi_{La}^{i\alpha} \Psi_{Lb}^{j\beta} \rangle^* \sim \epsilon^{ij} \epsilon^{\alpha\beta c} \epsilon_{abc} e^{-i\varphi_A - i\varphi_B}, \tag{26a}$$

$$\langle \Psi_{Ra}^{i\alpha} \Psi_{Rb}^{j\beta} \rangle^* \sim \epsilon^{ij} \epsilon^{\alpha\beta c} \epsilon_{abc} e^{i\varphi_A - i\varphi_B}. \tag{26b}$$

In terms of the φ_A and φ_B fields, the corresponding strings can be described as was done above for the 2SC phase: the only difference is that the $U(1)_B$ strings are not attached to domain walls. This is because the $U(1)_B$ symmetry is not explicitly broken: i.e. the equations describing the φ_B string contain no terms analogous to those proportional to a in Eq. (13).

The effective Lagrangian in the CFL phase for the axial Goldstone field φ_A can be derived in the same manner as before (6):

$$\mathcal{L}_A = f_A^2[(\partial_0 \varphi_A)^2 - u_A^2(\partial_i \varphi_A)^2] - V_{\text{inst}}(\varphi_A). \tag{27}$$

For large chemical potentials, $\mu \gg \Lambda_{\text{QCD}}$, the leading perturbative values for f_A and u_A have been calculated [15,21,22,24]:

$$V_{\text{inst}}(\phi_A) = -a' \mu^2 \Delta^2 \cos(\phi_A) \tag{28a}$$

$$f_A^2 = \frac{9\mu^2}{\pi^2}, \quad u_A^2 = \frac{1}{3}. \tag{28b}$$

The instanton contribution that explicitly violates the $U(1)_A$ symmetry has also been calculated [25]

$$a' \sim \left(\frac{m_s}{\mu}\right)\left(\frac{\Lambda_{\rm QCD}}{\mu}\right)^9 \tag{29}$$

which again vanishes for large μ.

To leading order in perturbation theory, the parameters f_B and u_B for the Goldstone boson associated with the spontaneous breaking of $U(1)_B$ symmetry are identical to the ones presented in Eq. (28b) [15,21,22]. However, the structure of the effective Lagrangian for the baryon Goldstone field φ_B is a little bit different than for the φ_A field due to the explicit presence of the chemical potential μ in the original QCD Lagrangian. This gives rise to a time-dependent global string solution as we discuss in Sec. II B 2. First, however, we discuss a slightly more complicated CFL phase which may admit an additional type of global string.

1. $N_f=3$ CFL+K^0 phase

It was recently argued [23,26,27] that the so-called "symmetric CFL phase" discussed above where $\langle \Sigma^\alpha_\beta\rangle \sim \delta^\alpha_\beta$ is unlikely to occur in nature due to the sizable presence of the strange quark mass m_s. Instead, it was argued that, in the CFL phase at high baryon density, a kaon K^0 condensate is likely to form [23,26,27]. If these arguments are correct and a kaon condensate forms, then the kaon K^0 condensation spontaneously breaks the global $U(1)_S$ symmetry associated with the kaon phase φ_S in addition to the $U(1)_A$ and $U(1)_B$ symmetries discussed above.[2] Thus, one expects the appearance in the spectrum of a new Goldstone boson associated with the phase φ_S.

One can present essentially the same argument as before to conclude that, if this condensate forms, a new type of the global string related to the φ_S phase exists which we shall refer to as the $U(1)_S$. The internal structure of the $U(1)_S$ string has similar properties to the $U(1)_A$ and $U(1)_B$ strings discussed previously, namely that at the center of the core, the relevant condensate $|K^0|$ vanishes while far from the core it approaches the vacuum expectation value.

We should note at this point that the structure of the condensate $\langle\Sigma\rangle$ describing the CFL+K^0 phase [23,26,27] is somewhat more complicated than $\langle \Sigma^\alpha_\beta\rangle \sim \delta^\alpha_\beta$ which described the symmetric CFL phase. For example, in the simplest case of pure K^0 condensation (see footnote 2), the fields have the form

$$\Sigma = |\Sigma| \exp\{i\theta[\sin(\phi_S)\lambda_6 + \cos(\varphi_S)\lambda_7]\} \tag{30a}$$

$$= |\Sigma| \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & (\sin\theta)e^{i\varphi_S} \\ 0 & -(\sin\theta)e^{-i\varphi_S} & \cos\theta \end{pmatrix}. \tag{30b}$$

[2]If the isospin symmetry were exact and the electromagnetic interactions turned off, then there would be two Goldstone bosons [28,29] associated with the condensation of the degenerate K^0 and K^+ bosons. In this paper we consider the realistic case when the isospin symmetry is not exact and only the single K^0 field—the lightest field—condenses.

Here, $\sin(\theta)$ is the parameter characterizing the strength of the kaon condensate. Notice that, although the form is somewhat more complex due to the fact that the fields are now phases, this has essentially the same form as Eq. (11) with the magnitude of the condensate being played by $|\Sigma \sin(\theta)|$ and the phase φ_S. As before, the relevant magnitude of the fluctuations of the condensate Σ is set by the scale Δ of the diquark gap and the relative magnitude of the kaon condensation in the proposed effective theories [23,26,27] is fixed by the ratio of the induced strangeness chemical potential $\mu_{eff} \simeq m_s^2/2p_F$ to the K^0 mass M_{K^0} such that

$$\cos\theta = \frac{M^2_{K^0}}{\mu^2_{eff}}. \tag{31}$$

Though the internal structure of this string could be complicated, the string tension α (to logarithmic accuracy) is determined by the corresponding well-known decay constant $f_\pi \sim \mu$ and, therefore, has the same order of magnitude as the $U(1)_A$ and $U(1)_B$ strings discussed above. Indeed, with a logarithmic accuracy the string tension is determined by the magnitude of the K^0 condensate and therefore

$$\alpha \simeq 2\pi u^2 f_\pi^2 \sin^2\theta \ln R. \tag{32}$$

The case of the $U(1)_S$ strings is a little bit more complicated because of the two factors in the condensate (30) in the CFL+K^0 phase. In order for the condensate to vanish in the core of the $U(1)_S$ strings, the following term from Eq. (30) must vanish:

$$|\Sigma|\sin(\theta). \tag{33}$$

This can happen in two ways: either the diquark condensate can vanish in the core so that $|\Sigma|\to 0$ or the kaon condensate can vanish $|\sin(\theta)|\to 0$. The scale of the field fluctuations that could cause the diquark condensate to vanish are of the same order of Δ as before with the $U(1)_A$ and $U(1)_B$ strings. The typical scale of fluctuations required to suppress the K^0 condensation (30) is m_s^2/μ. For our qualitative discussions, we limit ourselves to the case of maximal K^0 condensation: $\sin(\theta)\simeq 1$. This could be achieved for large m_s and reasonably small μ and might be physically relevant in the cores of neutron stars for example. In this case, we can neglect the relatively heavy fluctuations of order $m_s^2/\mu > \Delta$ and instead consider the core of the $U(1)_S$ strings to be governed by the lighter diquark fluctuations of order Δ. In this case, the core size will be about the same as for the $U(1)_A$ and $U(1)_B$ strings and set by the scale Δ^{-1} as deduced from our analysis of Eq. (19). In the case of less than maximal kaon condensation, then the softer fluctuations of order $m_s^2/\mu < \Delta$ will start to play a role, aiding in the relaxation of the core by reducing the $|\sin(\theta)|$ factor and allowing the core size to expand. To fully analyze the structure of $U(1)_S$ strings in this the case of slight kaon condensation requires a more complicated analysis.

If the $U(1)_S$ symmetry were exact, then the string structure would persist as with the $U(1)_B$ strings. As with the $U(1)_A$ symmetry, however, the $U(1)_S$ symmetry is only approximate and is explicitly violated by strangeness-violating weak interactions. This gives rise (to lowest order) to an additional contribution to the potential of the forms (7a), (28a). As with the $U(1)_A$ symmetry, these effects give rise to a mass for the pseudo Goldstone boson associated with $U(1)_S$ and lead to the formation of domain walls [30]. Thus, the equation describing the core of the $U(1)_S$ string is the same as Eq. (13) with the parameters f and coefficient a in Eq. (13) being replaced by f_π and a_S.

What is important is that, if the decay constant $f_\pi \sim \mu$ is roughly the same as before and proportional to μ, then the coefficient a_S is extremely small (but not zero) as was argued recently [30],

$$a_S \approx \frac{18\sqrt{2}}{g^2} G_F m_u m_s \cos\theta_C \sin\theta_C, \quad m^2_{\varphi_S} \sim a_S \Delta^2. \quad (34)$$

In this formula θ_C is the Cabibbo angle and G_F is the Fermi constant. Numerically, the scale corresponding to the pseudo Goldstone boson mass m_{φ_S} is of order $m_{\varphi_S} \sim 50$ keV.

Thus, the CFL phase contains two modifications over the 2SC phase: First, the $U(1)_B$ baryon symmetry is spontaneously broken which gives rise to $U(1)_B$ strings—these will be identified with superfluid vortices shortly. Second, the mesons resulting from the symmetric CFL condensation may further condense giving rise to addition symmetries which may be spontaneously broken. In particular, the inclusion of a moderate strange quark mass may, while preserving most of the CFL features, induce the formation of a K^0 kaon condensate. If this occurs, we have shown that $U(1)_S$ strings can form. Furthermore, due to the weak interactions which explicitly violate $U(1)_S$, these strings will actually bound domain walls.

A further modification comes from the presence of a nonzero chemical potential. Whereas the strings we have described so far have been static configurations, when one introduces a finite chemical potential (resulting from a finite density), these strings acquire a time dependence. Thus, a finite baryon and kaon (strange quark) chemical potential will induce time dependence into the string configurations for the $U(1)_S$ and $U(1)_B$ strings. In contrast, the static $U(1)_A$ configuration remains valid (until one considers interactions and dynamics). We discuss the effect of finite chemical potentials in the next section.

2. Spinning strings

The goal of this section is to relate the description of global $U(1)$ strings in terms of a complex scalar field to the more intuitive notion of rotation vortices. In particular, we discuss the $U(1)_B$ strings associated with the superfluid vortices in the CFL state to show that, as expected, these carry angular momentum, and thus may be copiously formed in rotating neutron stars.

The two descriptions of global strings and of superfluid vortices should be equivalent. Naïvely, however, there appear to be some discrepancies. For example, near a straight superfluid vortex at rest there is a velocity field moving circularly around it, carrying momentum and kinetic energy. But a simple global string [for example, defined by Lagrangian (6)] is a time-independent solution of the equations of motion. The momentum density away from the string core is $\sim f^2 \partial_0 \varphi_A \partial_i \varphi_A$ which is zero in the rest frame of a static global string.

The precise relation between the global string formulation (which we discuss in this paper) and superfluid vortices was given in [4]. There it was demonstrated that, if a global string is immersed in a uniform, Lorentz non-invariant background, then the static solution (with φ_B to be identified with the azimuthal angle) is replaced by a time-dependent solution $\varphi_B \to \varphi_B + \omega t$, where the coefficient ω is determined by the density of the background and the magnitude of the condensate. In the quark superconducting phase, such a background is naturally present and the constant ω is uniquely determined by the chemical potential μ.

Intuitively,[3] the $U(1)_B$ strings are fluid vortices with a baryon current carried around the string by the motion of quarks with both left- and right-handed quarks moving together and with both left and right anti-quarks moving in the opposite direction. In the $U(1)_A$ strings, the left-handed quarks and right-handed anti-quarks circulate in one direction and the right-handed quarks and left-handed anti-quarks circulate in the other direction. The $U(1)_S$ strings are described by a similar picture except with strange quarks. A static global $U(1)_B$ string, thus, consists of an equal number of quarks and anti-quarks such that the net angular momentum is zero. This is what a static relativistic global $U(1)_B$ string would describe in the CFL phase; however, in the CFL phase, the baryon number is violated and there is a net overdensity of baryons (i.e., there is a finite chemical potential) and thus, there are more quarks than anti-quarks giving a net angular momentum to the string, and—as we shall show in the rest of this section—that the description as a string must be time dependent: a "spinning global string."

We now derive the effective theory for φ_B. In order to restore the dependence on μ, one can use the following trick which, in the present context, was originally suggested in [31,32] and consequently has been used in a number of papers [15,21,22,33,34]. The idea is to make use of the fact that μ enters the QCD Lagrangian in the same way as the zeroth component of a gauge potential:

$$\mathcal{L}_\Psi = \bar{\Psi}(i\gamma^\nu \partial_\nu - m)\Psi + \mu \Psi^\dagger \Psi \quad (35a)$$

$$= \bar{\Psi}(i\gamma^\nu \partial_\nu + \mu\gamma^0 - m)\Psi. \quad (35b)$$

Therefore, one can formally promote the global $U(1)_B$ baryon symmetry to a local one by introducing a gauge field $B_\nu = (B_0, \vec{0})$ coupled to the baryon current with coupling constant μ:

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\Psi}(i\gamma^\nu \nabla_\nu - m)\Psi \quad (36)$$

where $\nabla_\nu = \partial_\nu + i\mu B_\nu$.

Indeed, under the $U(1)_B$ rotations we have

[3] We thank Krishna Rajagopal for this simple picture.

$$\Psi \rightarrow e^{i\alpha}\Psi, \tag{37a}$$

$$\varphi_B \rightarrow \varphi_B + 2\alpha, \tag{37b}$$

$$B_\nu \rightarrow B_\nu - \mu^{-1}\partial_\nu \alpha, \tag{37c}$$

which leave the microscopical QCD Lagrangian unchanged. An effective description must respect this symmetry, and therefore, in the effective Lagrangian description of the baryon Goldstone mode φ_B (26), one must replace the derivative $\partial_\nu \varphi_a$ in Eq. (27) by the covariant derivative $D_\nu \varphi_B = \partial_\nu \varphi_B + 2\mu B_\nu$. In matter with uniform density, we fix $B_0 = 1$ and $B_i = 0$ so that

$$D_0\varphi_B \equiv (\partial_0 \varphi_B + 2\mu), \quad D_i \varphi_B \equiv \partial_i \varphi_B \tag{38}$$

and thus

$$\mathcal{L}_B = f_B^2[(D_0\varphi_B)^2 - u_B^2(D_i\varphi_B)^2]. \tag{39}$$

From this equation we see that we can restore the original form (without covariant derivatives) if the baryonic Goldstone mode φ_B—the phase of the condensate (26)—receives a time dependence in the ground state

$$e^{-i\varphi_B} \sim e^{i2\mu t}. \tag{40}$$

Note, in the previous literature concerning the CFL phase [15,21,22], this time dependence was irrelevant. However, for the discussions of strings as emphasized in [4] this dependence is essential. In particular, following [4], to analyze QCD strings and their interactions in a nontrivial environment, one can introduce an equivalent description in terms of a two-index antisymmetric tensor field $B_{\mu\nu}$ where

$$f\partial_\mu \varphi_B \sim \epsilon_{\mu\nu\lambda\sigma}\partial^\nu B^{\lambda\sigma}. \tag{41}$$

Using this formalism, one can calculate the Lorentz force between strings, similar to the Magnus force in the nonrelativistic limit. As is known [4], this Magnus force can stabilize a moving vorton: the configurations where a string forms a closed loop. See also the textbook [2] for a review. One can also introduce a quantity similar to the nonrelativistic vorticity and demonstrate that it is quantized (in our notations) in units of μ^{-1}. We shall not discuss these interesting topics in this paper: once the appropriate correspondence is made, the techniques of [4] can be applied and used to address questions such as the formation, stability and radiation of structures like helices, rings, vortons, etc. The styles of analyses are reviewed in the textbook [2] as applied to cosmic strings. The point we are making is that similar techniques can be used in the contexts of dense quark matter.

The important point that we make is that, if the CFL phase with the global $U(1)_B$ symmetry is realized in the interior of a rotating neutron star where there is a non-zero chemical potential μ, then the global strings that form will be spinning and will carry angular momentum. Thus, drawing upon the analogy with liquid helium, we expect that, if the CFL phase rotates, then spinning global $U(1)_B$ strings will form.

One consequence of this connection between angular momentum and the global string is that, if global strings are formed through the transfer of angular momentum, then there will be a correlation between the direction of the angular momentum and the sign of the winding of the string. To see this, note that the angular momentum of the field φ_B is proportional to

$$M_{ij} \sim \int x_j \partial_0 \varphi_B \partial_i \varphi_B - x_i \partial_0 \varphi_B \partial_j \varphi_B. \tag{42}$$

The terms $\partial_0\varphi_B \sim \mu$ pickup the sign of the chemical potential and the terms $\partial_i \varphi_B \sim n$ pickup the sign of the winding or topological "charge" n (16) of the string. In the core of a neutron star, for example, the angular momentum has a definite sign (as set by the rotation of the star) and the chemical potential has a definite sign (the core is composed of baryons, not antibaryons). Thus, the sign of the topological charge is correlated with the sign of the angular momentum. If the formation of these strings is related to the rotation of the bulk phase, then there will be an excess of one type of string (either positive or negative winding).

Exactly the same arguments can be made for the $U(1)_S$ strings which originate from kaon condensation. The only modification that must be made to Eq. (39) is to replace the chemical potential μ by an induced chemical potential $\mu_{\rm eff} \simeq m_s^2/2p_F$, see [23,26,27]. The argument based on Eq. (42) about the correlation between the sign of the topological charge of the string with the sign of the angular momentum also holds.

The situation with $U(1)_A$ strings is expected to be quite different: conservation of P parity implies that the number of strings and anti-strings must be the same. This is due to the fact that $U(1)_B$ strings do not transform into anti-strings under the exchange $R \leftrightarrow L$ while $U(1)_A$ strings do, as can be seen[4] from the definition (26).

III. ELECTROMAGNETIC PROPERTIES

Up until this point, we have discussed the existence of global $U(1)$ strings and their correspondence with superfluid vortices. However, since they only involve excitations close to the Fermi surface, there will not be enough to affect the thermodynamics of the superconducting phases. In addition, the fields φ_A and φ_B are neutral, so one might naïvely expect that they have little effect on electromagnetic physics either. It turns out, however, that the axial strings φ_A have nontrivial electromagnetic properties.

A. Anomalous electromagnetism of the η'

In this section we are mainly concerned with electromagnetic interaction of the neutral η' meson. The simplest way to derive the corresponding low-energy effective Lagrangian, which includes the massless electromagnetic $F_{\mu\nu}$ field and the light η' field (6), is to follow the standard procedure and consider the transformation properties of the path integral under the $U(1)_A$ chiral transformation (5). As is known, the measure is not invariant under these transformations due to

[4]We thank Misha Stephanov for presenting this argument to us.

the chiral anomaly: it receives an additional contribution $\delta\mathcal{L}=(\alpha/2)\partial^\nu J_\nu^A$. The expression for the anomaly $\partial^\nu J_\nu^A$ is well known and takes the form

$$\partial^\nu J_\nu^A=\frac{g^2}{16\pi^2}N_f G^a\tilde{G}^a+\frac{e^2}{8\pi^2}N_c F\tilde{F}\sum_{f=1}^{N_f} Q_f^2, \tag{43}$$

where we have included the electromagnetic fields $F_{\mu\nu}$ along with the gluon fields $G^a_{\mu\nu}$ and their duals:

$$\tilde{F}_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\lambda\sigma}F^{\lambda\sigma}, \quad \tilde{G}^a_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\lambda\sigma}G^{a\lambda\sigma}. \tag{44}$$

One should note that the expression for the anomaly is an operator relation which is valid for any finite chemical potential μ. Indeed, the anomaly arises from the ultraviolet properties of the theory and is not sensitive to the finite chemical potential as long as the regulator fields are heavier than μ.

The second step is an identification of the parameters from the Lagrangian δL with the physical fields of the theory. For example, in QCD with $\mu=0$ the electromagnetic field $F_{\mu\nu}$ (43) from the original theory is the observable physical field (in contrast with superconducting phases when $\mu\neq 0$, see below). The parameter α of $U(1)_{\mathrm{A}}$ chiral transformation is identified with the physical η' field which is nothing but the singlet phase of the chiral condensate,

$$\langle\bar{\Psi}_R\Psi_L\rangle\sim e^{-i\alpha}\sim\exp\left(-i\frac{2\eta'}{f_\pi\sqrt{N_f}}\right). \tag{45}$$

Therefore, the anomalous effective Lagrangian of η' coupled to photons takes a familiar form:

$$\delta\mathcal{L}_{\eta'\gamma\gamma}=\frac{e^2}{8\pi^2}N_c F\tilde{F}\sum_{f=1}^{N_f} Q_f^2\frac{\eta'}{f_\pi\sqrt{N_f}}. \tag{46}$$

In a similar manner, one can derive the well-known effective Lagrangian describing the famous $\pi^0\to\gamma\gamma$ decay. In this case, one should consider the expressions for the third component of the axial current,

$$\partial^\mu(\bar{u}\gamma_\mu\gamma_5 u-\bar{d}\gamma_\mu\gamma_5 d)=\frac{e^2}{8\pi^2}F\tilde{F}, \tag{47}$$

and identify the corresponding transformation parameter α_3 with Goldstone mode π^0 such that the effective Lagrangian takes the familiar form

$$\delta\mathcal{L}_{\pi^0\gamma\gamma}=\frac{e^2}{8\pi^2}F\tilde{F}\frac{\pi^0}{f_\pi\sqrt{2}}. \tag{48}$$

Now, we want to derive a similar expression for the effective Lagrangian describing interaction of the light η' field with electromagnetism in CFL and 2SC phases.[5] As we mentioned above, the operator expression for the anomaly (43) remains unchanged: the singlet phase α defined in Eq. (5) is identified with the physical Goldstone mode η', which is now the phase of the diquark condensate instead of the chiral condensate

$$\Sigma\sim e^{-i\varphi_A}|\Sigma|, \quad \alpha=\frac{\varphi_A}{2}=\frac{\eta'}{2f_A}. \tag{49}$$

This is not the end of the story, however, because, as it has been known since [36], in dense QCD matter the electromagnetic field strength $F_{\mu\nu}$ and the electric charge e are not the appropriate physical quantities. Rather, a combination of the electromagnetic field A_μ with the eighth component of the gluon field A^8_μ acts as a physical massless photon $\mathcal{A}$ field [36]:

$$\mathcal{A}_\mu=A_\mu\cos\theta+A^8_\mu\sin\theta \tag{50a}$$

$$\mathcal{A}^8_\mu=-A_\mu\sin\theta+A^8_\mu\cos\theta \tag{50b}$$

$$\cos\theta=\frac{g}{\sqrt{g^2+4\eta^2e^2}} \tag{50c}$$

$$\sin\theta=\frac{2e\eta}{\sqrt{g^2+4\eta^2e^2}} \tag{50d}$$

$$\mathfrak{e}=\frac{eg}{\sqrt{g^2+4\eta^2e^2}}, \tag{50e}$$

where the $\mathcal{A}$ is the physical "photon" field and $\mathfrak{e}$ is the physical charge. The values for the parameter η entering expressions (50) are given by [36]

$$\eta_{CFL}=\frac{1}{\sqrt{3}}, \quad \eta_{2SC}=-\frac{1}{2\sqrt{3}}. \tag{51}$$

We note that our expression for the angle θ in terms of e and g is obtained from the one presented in [36] by changing $g\to g/2$. The difference is due to the non-standard definition of the strong coupling constant g in [36] (the absence of the factor 1/2 in front of g in the covariant derivative in [36]). We use the standard definition for the strong coupling constant such that the chiral anomaly is given by Eq. (43). Using our normalization for the η' field and expressing the anomaly in terms of the physical electromagnetic field $\mathcal{F}_{\mu\nu}$ we arrive at the following effective Lagrangian describing the interaction of the η' with the electromagnetic fields in the CFL phase:

[5]Anomalous electromagnetic interaction for the $SU(3)$ Goldstone modes in the CFL phase was discussed previously using a different approach [35]. As far as we know, the anomalous electromagnetic interaction with the singlet η' phase has not been previously discussed in the literature.

$$\delta\mathcal{L}^{(\mathrm{CFL})}_{\eta'\mathcal{F}\tilde{\mathcal{F}}}=\frac{\mathfrak{e}^2}{4\pi^2}\mathcal{F}\tilde{\mathcal{F}}\frac{\varphi_A}{2}. \tag{52}$$

It is interesting to note that half of this result is due to the original electromagnetic interaction, $\sim F\tilde{F}$, while the other half is due to the gluonic part of the anomaly (43). One can repeat the same calculations for the 2SC phase with the result similar to Eq. (52):

$$\delta\mathcal{L}^{(2\mathrm{SC})}_{\eta'\mathcal{F}\tilde{\mathcal{F}}}=\frac{\mathfrak{e}^2}{8\pi^2}\mathcal{F}\tilde{\mathcal{F}}\,\frac{\varphi_A}{2}. \tag{53}$$

However, for the 2SC phase, only one-sixth of this contribution is due to the original gluon term in the anomaly. One should also note that the same procedure determines the anomalous coupling constants $\sim g_{\eta'\mathcal{F}\mathcal{G}}$ describing the decay of a heavy gluon to an η' and a photon—the decay similar to $\rho\to\pi\gamma$ in usual QCD. In the present paper we shall not discuss further this physics involving the heavy particle $(m_G\sim\Delta)$.

B. Superconducting strings

In this section we wish to analyze the system of light particles η' (the phase φ_A) and photon $\mathcal{F}_{\mu\nu}$. Combining Eqs. (27) and (52) we have the following effective Lagrangian:[6]

$$\mathcal{L}_{\mathrm{eff}}=f^2[(\partial_0\varphi_A)^2-u^2(\partial_i\varphi_A)^2]-V_{\mathrm{inst}}(\varphi_A)$$
$$-\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}+\frac{\mathfrak{e}^2}{8\pi^2}\left(\frac{\varphi_A}{2}\right)\epsilon_{\mu\nu\lambda\sigma}\mathcal{F}^{\mu\nu}\mathcal{F}^{\lambda\sigma}. \tag{54}$$

We approach this problem for large μ. In this case, as a first approximation, we can omit the instanton contribution V_{inst} as we did for the discussion of a single string. This allows us to treat the problem of string electromagnetism exactly at large distances. If we turn on the small instanton term and explicitly break the $U(1)_A$ symmetry, then domain walls are also allowed and the problem becomes much more complicated. We speculate on possible effects of these physically relevant domain walls in the conclusion.

The effective theory (54) is almost identical to that studied in [39]. We begin our analysis by considering a single spinning global axial string lying along the z axis and use cylindrical coordinates (r,ϕ):

$$\Phi(t,r,\phi)=\rho(r)e^{i\varphi_A(t,\phi)}. \tag{55}$$

[6]To be more precise, one should take into account the deviation of the dielectric constant ϵ from 1. This deviation is discussed in [37,38] where it is shown that $\epsilon=1+(8\alpha/9\pi)(\mu^2/\Delta^2)$, where $\alpha=\mathfrak{e}^2/4\pi$ is the modified fine structure constant [see Eq. (50)]. We do not attempt to make any numerical estimates in this paper; therefore, we ignore this numerical correction in what follows. We thank Krishna Rajagopal for bringing to our attention the possible numerical importance of this correction.

The radial solution $\rho(r)$ was presented in Fig. 1, but we will only be concerned with distances large with respect to the core size of the string $r\gg r_c\sim\Delta$. In this regime, the effective theory (54) becomes valid and we have the following equations of motion for the "electromagnetic" field:

$$\partial_\mu\mathcal{F}^{\mu\nu}=\frac{\mathfrak{e}^2}{2\pi^2}(\partial_\mu\varphi_A)\tilde{\mathcal{F}}^{\mu\nu}=\frac{2\alpha}{\pi}(\partial_\mu\varphi_A)\tilde{\mathcal{F}}^{\mu\nu} \tag{56}$$

where $\alpha=\mathfrak{e}^2/4\pi$ is the modified fine structure constant and the background field φ_A has the solution

$$\varphi_A(t,\phi)=\dot{\varphi}_A t+\phi \quad \text{where} \quad \phi=\tan^{-1}\frac{y}{x}. \tag{57}$$

We assume that the rate at which the string is spinning is small:

$$\dot{\varphi}_A\equiv\frac{\partial\varphi_A}{\partial t}\ll\Delta^{-1}. \tag{58}$$

The origin for this assumption is that, while φ_B strings have very high frequencies of order of μ, Eq. (40), the φ_A strings do not spin on their own. They can, however, receive some angular momentum through the interaction with the surrounding medium and/or the φ_B strings. The interaction of the Goldstone fields with other particles is expected to be suppressed by some power of μ^{-1}. Therefore, we expect $\dot{\varphi}_A$ to be small with respect to all relevant scales.

The anomalous Maxwell's equations (54) in the presence of a time-dependent global string background have the following form far from the core of the string $(r\gg\Delta^{-1})$:

$$-\frac{\partial\mathcal{E}_r}{\partial t}+\left(\frac{1}{r}\frac{\partial\mathcal{B}_z}{\partial\phi}-\frac{\partial\mathcal{B}_\phi}{\partial z}\right)=-\frac{2\alpha}{\pi r}\mathcal{E}_z-\frac{2\alpha\dot{\varphi}_A}{\pi}\mathcal{B}_r, \tag{59a}$$

$$-\frac{\partial\mathcal{E}_z}{\partial t}+\left(\frac{1}{r}\frac{\partial(r\mathcal{B}_\phi)}{\partial r}-\frac{1}{r}\frac{\partial\mathcal{B}_z}{\partial\phi}\right)=\frac{2\alpha}{\pi r}\mathcal{E}_r-\frac{2\alpha\dot{\varphi}_A}{\pi}\mathcal{B}_z, \tag{59b}$$

$$-\frac{\partial\mathcal{E}_\phi}{\partial t}-\frac{\partial\mathcal{B}_z}{\partial r}+\frac{\partial\mathcal{B}_r}{\partial z}=-\frac{2\alpha\dot{\varphi}_A}{\pi}\mathcal{B}_\phi, \tag{59c}$$

$$\frac{1}{r}\frac{\partial(r\mathcal{E}_r)}{\partial r}+\frac{\partial\mathcal{E}_z}{\partial z}+\frac{1}{r}\frac{\partial\mathcal{E}_\phi}{\partial\phi}=\frac{2\alpha}{\pi r}\mathcal{B}_\phi. \tag{59d}$$

In the limit of $\dot{\varphi}_A\to0$, these results reduce to those presented in [39] with the replacement $2\alpha\to\alpha$ which is due to the extra factor 2 in our expression (52) for the anomaly as compared with the axion model considered in [39].

We were not able to find a complete solution of these linear equations over all of space; however, a z-independent solution for radii larger than the core size but not too large $\Delta^{-1}\sim r_c\ll r\ll\dot{\varphi}_A^{-1}$ can be easily found:

$$\mathcal{E}_r = C_+ r^{-1+2\alpha/\pi} + C_- r^{-1-2\alpha/\pi}, \tag{60a}$$

$$\mathcal{B}_\phi = C_+ r^{-1+2\alpha/\pi} - C_- r^{-1-2\alpha/\pi}, \tag{60b}$$

$$\mathcal{B}_z = \dot{\varphi}_A (C_+ r^{2\alpha/\pi} + C_- r^{-2\alpha/\pi}), \tag{60c}$$

with the other field components vanishing. Coefficients $C_\pm$ need to be determined by matching the solution (60) with the behavior of fields in the core region $r \simeq \Delta$ where our effective Lagrangian approach breaks down. In what follows, we estimate these coefficients using dimensional arguments.

The solution (60) is reduced to the corresponding expression [39] in the limit $\dot{\varphi}_A = 0$. In this case, the solution becomes valid for arbitrarily large r. Our approximate solution (60) is limited to the range $\Delta^{-1} \ll r \ll \dot{\varphi}_A^{-1}$. At shorter ranges we must further develop our microscopic model.

The most important feature of the solution (60) is that, for a given $C_+ \neq 0$ or $C_- \neq 0$, the solution has the Lorentz property $\mathcal{B}_\phi = \pm \mathcal{E}_r$—as if the string carries a light-like current four-vector with current density along the z axis $\mathrm{j}_\mu = (\mathrm{j},0,0,\pm \mathrm{j})$ with the whole system spinning with small frequency $\dot{\varphi}_A \ll 1$. The presence of a small $\mathcal{B}_z$ field in Eq. (60c) which is due to the spinning of the static solution can be easily understood by boosting to the local frame moving with the speed $v_\phi = -\dot{\varphi}_A r$ at the point r.

We should note here that the presence of a current with the property $\mathrm{j}_\mu = (\mathrm{j},0,0,\pm \mathrm{j})$ in the system defined by Eq. (54) was established by analyzing the only large distance physics without formulating a microscopic model of the core. A similar result was obtained in [39]. There, however, the effective theory was taken to be exact, allowing for a detailed analysis of the core physics. In that case, it can be seen that such a current results from a charged Dirac fermionic zero mode present in the string background (see [40–43] for more details). Unfortunately, in our case, the effective theory (54) breaks down at short distance scales: we only have theoretical control over the large distance physics; however, we believe that the microscopical explanation of the property $\mathrm{j}_\mu = (\mathrm{j},0,0,\pm \mathrm{j})$ in our case is very similar to the microscopical explanation given in [39]: namely, that it is due to zero modes of the charged fermion field which travels inside the core of the string.

The presence of zero modes [localized in the (xy) direction] in the string background is a very general property of such a background and is a trivial consequence of an index theorem [40–43]. Although this topic is beyond the scope of the present paper, we would like to note that, in the color superconducting CFL or 2SC phases, the interaction between the gapped fermions close to the Fermi surface and the diquark condensate has a more complicated algebraic structure than the simple Yukawa coupling of a single fermion considered previously [39,41,42]. Nevertheless, one can explicitly demonstrate the presence of the fermionic zero modes which would have been the regular gapped excitations in the absence of the string background. Therefore, we believe that the microscopical explanation of the result $\mathrm{j}_\mu = (\mathrm{j},0,0,\pm \mathrm{j})$ in our case is analogous to the explanation given in [39]. However, unlike the case in [39], we expect that both the coefficients $C_\pm$ are nonzero due to the lack of Lorentz invariance in our system. (The dispersion laws for in-medium fermions do not have a standard Lorentz-invariant form.)

One of the consequences of localized charged fermionic zero modes in a string background is superconductivity. Indeed, as was demonstrated in [41], the problem can be reduced to a two-dimensional effective theory which describes massless charged fermions (our original zero modes in four dimensions), and photons (our physical "photon" fields $\mathcal{A}_\mu$) which are coupled to the massless fermions through the physical charge $\mathfrak{e}$. It was also demonstrated in [41] that such a system describes a superconducting string in the sense that, if external electric field is applied along the z direction, it results in persistent current along the string. We shall not repeat all these well known arguments, which are based on the bosonization technique of the (effectively two-dimensional) localized zero modes. Instead, we refer the reader to the original paper [41].

Thus, we can only present a dimensional estimate at this time: For a given orientation of the string solution $\varphi_A(r,\phi) = +\phi$ we expect that both $C_\pm$ of Eq. (60) are of the same order and can be estimated dimensionally to be

$$C_- \sim \mathfrak{e}\Delta(\Delta)^{-2\alpha/\pi}, \quad C_+ \sim \mathfrak{e}\Delta(\Delta)^{+2\alpha/\pi}. \tag{61}$$

This result behaves in the same way as if fermions trapped in transverse zero modes travel in opposite directions (without cancellation, $C_+ \neq C_-$) at the speed of light. We should remark that, due to the linearity of Maxwell's equations, a linear superposition of two sources $c_+(\mathrm{j},0,0,+\mathrm{j}) + c_-(\mathrm{j},0,0,-\mathrm{j})$ produces a desirable superposition of the corresponding solutions (60). Such electromagnetic behavior of the string is reminiscent of the behavior of free superconducting strings [41,44].

The question of how the external electric field may be generated is a different question which is not addressed in the present paper. However, we should remark here that, in the core of a neutron star, the current could be generated by the motion of the strings through external magnetic fields already present in neutron star [41].

To conclude this section, we estimate the maximum current which can be achieved in a color superconductor. Under an external electric field applied parallel to the axis of the string, the fermion current will grow. Eventually, however, it will saturate: If the mass of the fermions away from the string is Δ, then once the Fermi momentum of the charge carriers rises above $p_F > \Delta$, it will be energetically favorable for the fermion to leave the string. Therefore, one expects that the maximum current supported by a single string would be of the order

$$\mathrm{j}_{\max} \sim \frac{\mathfrak{e}\Delta}{2\pi} \tag{62}$$

which cannot exceed 2×10^3 A.

We do not expect, however, that a single, infinitely long string is realized in nature. Rather, we expect that the strings will take the shape of a ring or organize into more complicated objects. We presently cannot say more about the results of the rather complicated dynamics of strings in dense quark matter in the presence of strong external magnetic fields. However, we refer to some of the results discussed in [41] and the textbook [2]. As a first step to understanding the dynamics of this complicated system, one has to understand the forces which act between the strings. We discuss this subject in the Conclusion.

IV. CONCLUSION

In this paper we have considered the existence of classically stable global string configurations in high density QCD. We have shown that, in the $N_f=3$ color flavor locking (CFL) phase, baryonic strings resulting from the $U(1)_B$ symmetry of the condensate can carry angular momentum. A similar situation arises for $U(1)_S$ strings if a kaon condensate forms. In addition, we have shown that the axial strings resulting from the approximate $U(1)_A$ symmetry may exist in both the $N_f=3$ CFL phase and the $N_f=2$ (2SC) phase.

Even though the fields that form these global strings are neutral, we show that the $U(1)_A$ strings have non-trivial electromagnetic properties due to the anomalous coupling between the η' field and a massless "photon" that is a mixture of the eighth gluon and photon fields. Due to this anomalous interaction, axial strings behave as superconducting string.

As we have already mentioned, however, in general, strings will not be formed as infinitely long strings in isolation. The details of the interaction between strings is quite varied and complicated: stable structures like helices, rings and vortons may form, or the strings may form networks or tangle. In addition to that, $U(1)_A$ strings as well as $U(1)_S$ strings will be bounded by domain walls which themselves can decay into string-antistring configurations. In order to understand the possible dynamics of the system one must know different forces which determine the dynamics.

In this Conclusion we shall only comment on a few simple possibilities here and refer the reader to the textbook [2] for a discussion about general string dynamics. We shall consider the following interactions between strings:

(a) Inter-vortex force: F_{VV}.
(b) Domain wall force: F_{DW}.
(c) Anomalous electromagnetic force: F_{EM}.

We shall briefly summarize these forces and provide qualitative estimates in terms of the parameters of the QCD phase. In the following we assume that two infinitely long global strings lie parallel to the z axis with separation d. We label the strings by their topological "charge" n which corresponds to their winding number (16) with respect to the positive z axis. Strings of the same charge have the same orientation: String of opposite charge can annihilate. All forces are presented as a force per unit length of the string.

(a) Inter-vortex force. The most important and well-known interaction is due to the exchange of massless Goldstone particles. The force is estimated by considering the energy of a configuration of two strings. This approximation is valid for distances d much larger than the core size $d \gg \Delta^{-1}$ [45,46]

$$F_{VV} \sim \mp \frac{4\pi f^2}{d}. \tag{63}$$

This force is repulsive for strings with the same charge, and attractive for strings of opposite charge. Physically, strings of opposite charge tend to attract (energetically they try to annihilate and restore the vacuum everywhere) while strings of the same charge repel: strings of higher charge $n>1$ will split into several strings of single winding which will tend to move away from each other.

For baryonic $U(1)_B$ strings this result extends to arbitrarily large distances d because the Goldstone bosons are massless. For $U(1)_S$ strings, this description is only valid for distances $d<m^{-1} \simeq (50\ \text{keV})^{-1}$ because the Goldstone bosons are in fact massive due to weak interaction [30]. A similar situation occurs for the axial $U(1)_A$ strings where this description is only valid for distances $d<m_{\eta'}^{-1}$ because the Goldstone bosons is massive due to instanton effects (9). Beyond this range, this inter-vortex force falls away and the domain wall force starts to dominate.

(b) Domain wall force. On distance scales $d>m^{-1}$, where $m=m_{\eta'}$ for $U(1)_A$ strings and $m=m_{\varphi_S} \sim \sqrt{a_S}\Delta$ for $U(1)_S$ strings, Eq. (34), one will see that the $U(1)_A$ and $U(1)_S$ strings are really embedded in domain walls of thickness m^{-1}. The scale for this interaction is set by the corresponding domain wall tension σ calculated for φ_A field in [3] and for φ_S field in [30]:

$$F_{DW} \sim \sigma \sim f^2 m. \tag{64}$$

This force can be attractive for opposite charge strings if a domain wall connects them. Domain walls are not formed between strings of the same charge if we neglect the process of nucleation which is equivalent to the creation of the string-antistring pairs.

(c) Anomalous electromagnetic force. For distances between $\Delta^{-1} \ll d \ll \dot{\varphi}_A^{-1}$ the axial string behaves as current carrying wires. Through the exchange of "photons," these strings thus interact with force

$$F_{EM} \sim \frac{\mathrm{jj}}{d} \sim \pm \frac{\alpha}{\pi} \frac{\Delta^2}{d}. \tag{65}$$

This force is attractive for strings of the same charge and repulsive for strings of opposite charge. Physically, strings carrying current in the same direction want to bunch together into a "wire." In comparison with the other two forces, however, the anomalous electromagnetic force is suppressed by factors of α and Δ/μ and can be neglected in comparison with these other forces.

Besides that, when a $U(1)_A$ string moves at velocity v across an external magnetic field, the effective superconducting current should build at a very high rate (see [42]). In such a situation, an electric field is also generated leading to the creation of particles, a process which we do not discuss here.

One more effect which we would like to mention here and which deserves further study is the binding strings by the domain wall force (64). In a situation similar to the $N=1$

axion model [17] such a configuration decays very quickly. In our case when $U(1)_A$ string is superconducting and can form a ring with a dragged magnetic flux crossing the ring, the situation may not be so obvious. One should better understand the system of a superconducting string with an attached domain wall in the dense background in the presence of a nonzero magnetic field before one can make any conclusions regarding the system. We suspect that due to the interactions which were not present in the axion case, some stable configurations (such as vortons with attached domain walls) will be possible.

The most likely place to encounter bulk high-density superconducting quark matter is in the core of neutron stars. Typically, neutron stars are rapidly spinning, and we suspect that during the formation of the core, this rotation would impart some angular momentum to the superconducting core. As we showed in Sec. II B 2, spinning global $U(1)_B$ and $U(1)_S$ strings carry angular momentum that is correlated with the string's charge.

It remains to be seen whether these strings can have any impact on observable effects (they might, for example, affect glitches: sudden increases in the rotation frequency ω of neutron stars by as much as $\Delta\omega/\omega \sim 10^{-6}$, or the magnetic field structure and evolution).

There will likely be many other effects due to the interactions and electromagnetic properties of global $U(1)$ strings that will play a role in the physics of superconducting phases of high density quark matter. These effects might be closely related to analogous effects studied for cosmic strings (we refer the reader to the textbook [2]), and studied in condensed matter physics. We hope that the present paper will initiate some activity in this direction.

Note added in proof. Shortly after the appearance of this paper, D. Kaplan and S. Reddy introduced a paper, see Ref. [47], with some overlap. In particular, they consider $U(1)_S$ strings from a different perspective where the kaon condensate vanishes rather than the diquark condensate. To fully describe these $U(1)_S$ vortices, one should include both this effect and the effects discussed in this paper.

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