

Renormalization-group running of the cosmological constant and its implication for the Higgs boson mass in the standard model

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The renormalization-group equation for the zero-point energies associated with vacuum fluctuations of massive fields from the standard model is examined. Our main observation is that at *any* scale the running is necessarily dominated by the heaviest degrees of freedom, in clear contradistinction with the Appelquist-Carazzone decoupling theorem. Such an enhanced running would represent a disaster for cosmology, unless a fine-tuned relation among the masses of heavy particles is imposed. In this way, we obtain $m_H \approx 550$ GeV for the Higgs boson mass, a value safely within the unitarity bound, but far above the more stringent triviality bound for the case when the validity of the standard model is pushed up to the grand unification (or Planck) scale.

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There are now increasing indications, based on observations on rich clusters of galaxies [1], searches for type Ia Supernovae [2] and measurements of the cosmic microwave background anisotropy [3], that today's universe is undergoing a phase of accelerated expansion. This is usually attributed to the presence of a cosmological constant. Although the simplest explanation is a time-independent (i.e. "true") cosmological constant Λ , many scenarios have also been discussed involving a dynamical cosmological constant $\Lambda(t)$. There have recently been a number of suggestions regarding the nature of the latter, the most popular candidate being known under the name of "quintessence" [4] (a classically unstable field that is rolling towards its true minimum which is presumed to vanish).

The problem of the cosmological constant—how to reconcile its value from cosmological observations, $\Lambda \sim 10^{-47} \text{ GeV}^4$ (to be of the same order as the critical energy density) with particle physics scales describing all known and unknown phase transitions in the early universe or with $\Lambda_{Pl} \sim 10^{72} \text{ GeV}^4$ in the case of vacuum fluctuations with the Planck scale cutoff—arises when an ordinary field theory is discussed in relation to gravity. It is therefore adequate to formulate the theory on the classical curved background [5,6]. However, it is true that the net cosmological constant, being the sum of a certain number of essentially disparate contributions, may classically always be set to zero by applying some fine-tuning. It is to our current understanding that the problem is intimately related to quantum gravity, leaving thereby string theory as the only framework for properly addressing it [7].

In two recent papers [5,8] Shapiro and Sola found that even by taking the quantum effects of the standard model, one could not fix the value of the cosmological constant to any definite constant (including zero), because any such a

constraint would necessarily be invalidated at a different scale (the energy scale changes in the course of the universe's evolution) owing to renormalization group (RG) running effects. If the nature of the RG behavior were such that near the scale $\mu=0$ one is allowed to set $\Lambda(\mu=0)=0$ (a relation suggesting some unknown symmetry principle), then the above scenario could mimic quintessence models, but without invoking a rolling scalar field. On the other hand, if one could not set $\Lambda(\mu=0)=0$, then the usage of the anthropic principle would probably be the only alternative. It was argued in [5,8] that the scaling dependence of the cosmological constant should be consistent with the standard cosmological model. That means that, given a value for Λ at far infrared, the running should reproduce the value for the cosmological constant inferred from present observations at the present-day scale ($\Omega_\Lambda^0 \approx 0.6-0.7, \mu_0 \approx 2 \times 10^{-3} \text{ eV}$), and, also, should not spoil the success of nucleosynthesis ($\Lambda \lesssim \rho_R$) at the much higher scale $\mu \sim m_e$. Although the aim of Shapiro and Sola in [5,8] was not to explain fine-tuning to 55 decimals required to explain the present value of Λ , careful examination of the running of Λ could prove useful as it could reveal a close relation between the SM parameters (particle masses and couplings) and the parameters of observational cosmology.

The main result of [5,8] contains two nice features: (i) the running near the present-day scale involves only light neutrino masses, and by taking them to correspond to the large-mixing-angle MSW solution of the solar neutrino problem, we immediately arrive at the right value $|\Lambda| \approx 10^{-47} \text{ GeV}^4$; (ii) although the net value of the cosmological constant requires fine-tuning to 55 decimals, the running of the same quantity requires no fine-tuning at all, thereby making its scaling dependence trivially consistent with the standard cosmological scenario. There is, however, a bad feature too: in order to set the condition $\Lambda(\mu=0)=0$ (as to avoid the anthropic principle for explanation of the observed values of cosmological parameters), one should inevitably accept the existence of some light scalar with a mass a few times the neutrino mass, which apparently leads us beyond the standard model. All the above features stem from the fact that the

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authors of [5,8] explicitly assumed the validity of the Appelquist and Carazzone decoupling theorem [9]. In particular, this means that the quantum effects of some particle are taken into consideration only at scales higher than the mass of this particle ($\mu > m$). As a consequence of this decoupling of heavy particles, only light neutrinos contribute to the running at present scales $\sim 10^{-3}$ eV. In the present paper, we scrutinize the decoupling theorem and its role in the running of Λ in the standard model, and find that although the contribution of a particle having a mass m is suppressed at $\mu < m$, it is still much larger than the contributions from lighter particles with $\mu > m_i$. Thus, the heaviest particles do dominate the running at any scale, and in order to have the RG behavior in accordance with standard cosmology a fine-tuned relation connecting the heaviest masses should exist. In this way, we obtain an interesting prediction for the Higgs boson mass in terms of other particle masses in the standard model. Although the amount of fine-tuning in this relation is significantly reduced in comparison with the original problem (55 decimals), it is still considerable (28 decimals).

Let us start with the discussion of the cosmological constant Λ which enters the Einstein equation in the following way:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + 8\pi Gg_{\mu\nu}\Lambda = -8\pi GT_{\mu\nu}, \quad (1)$$

where Λ is a dimensionful parameter with the dimension $(mass)^4$. The classical general relativity does not bring any specific preference for the value of Λ . Its value is therefore arbitrary.

With the advent of particle physics and quantum field theory it became clear that Λ can be interpreted as the vacuum energy density. In fact, there are additional sources of the cosmological constant coming from particle physics. Field condensates at the classical level, and zero-point energies at the quantum level, are two well-known generators of the vacuum energy. Therefore, we have at least three sources of the cosmological constant: (1) the original Einstein constant; (2) field condensate contributions at the classical level; (3) particle zero-point energies at the quantum level.

The formulation of the theory [5,6,10] is rather simple— one constructs a renormalizable gauge theory (the gauged Higgs Lagrangian, for example) in an external gravitational field.¹ In fact, one starts with the usual matter action in flat space-time, and replaces the partial derivatives by the covariant ones, the Minkowski metric by the general one, and d^4x by $d^4x\sqrt{-g}$. The cosmological constant Λ that enters the Einstein Lagrangian may be regarded as a bare parameter, and used to absorb eventual divergences coming from the quantum fluctuation in the particle Lagrangian. In such a way, the divergences of particle field theory are absorbed into the bare Λ , and are therefore reduced to the geometry. It turns out that, for example, the vacuum action necessary to

insure the renormalizability of the gauged scalar Lagrangian should contain the terms $R^2_{\mu\nu\rho\sigma}$, $R^2_{\mu\nu}$, R^2 , and $\square R$, with the corresponding coefficients a_i which are the bare parameters. In this way, all divergences in the vacuum action can be removed by the appropriate renormalization of the matter fields, their masses and couplings, bare parameters a_i , G_{bare} , Λ_{bare} , and the nonminimal parameter ξ_{bare} which enters the action via a term of the form $\xi\phi^\dagger\phi R$.

Generally, the scalar field ϕ with the potential energy $V(\phi)$ has the following contribution to the action:

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2}g^{\mu\nu}(\partial_\mu\phi\partial_\nu\phi) - V(\phi) \right]. \quad (2)$$

If ϕ_{vac} is the value of the field $\phi(x)$ which minimizes the potential $V(\phi)$, then the lowest state has $T_{\mu\nu} = g_{\mu\nu}V(\phi_{vac})$, which is the classical scalar field contribution to the vacuum energy. As an example let us take the Higgs scalar field with the potential $V(\phi) = -m^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$. Then the Higgs condensate contribution (at the classical level) to the cosmological constant is

$$\Lambda^{cond} = -\frac{m^4}{4\lambda}. \quad (3)$$

We shall turn to the discussion of the above expression later.

The second source of the contributions to the cosmological constant are quantum fluctuations (zero-point energy) of the free fields. Each free quantum field (in the case of bosonic fields being basically a collection of an infinite number of harmonic oscillators) contributes an infinite amount of the vacuum energy to the cosmological constant.

In the following we calculate and discuss the running cosmological constant $\Lambda(\mu)$ for the case of a scalar field. It will turn out that the decoupling theorem, although perfectly valid for Green functions in the field theory, fails in the case of the calculation of the cosmological constant.

Using the dimensional regularization in $d = 4 + 2\epsilon$ dimensions and the minimal subtraction (MS) renormalization scheme, one gets for the quantum fluctuations (zero-point energy) of the scalar field

$$ZPE = \frac{M^4}{64\pi^2} \left(\frac{1}{\epsilon} + \gamma - \ln 4\pi + \ln \frac{M^2}{\mu^2} - \frac{3}{2} \right). \quad (4)$$

Defining the relation between the bare (Λ_{bare}) and renormalized (Λ) quantities as

$$\Lambda_{bare} = \mu^{d-4}(\Lambda + z_\Lambda M^4), \quad (5)$$

one gets for the counterterm z_Λ

$$z_\Lambda = -\frac{1}{64\pi^2} \frac{1}{\epsilon}. \quad (6)$$

It is straightforward to write down the renormalization group equation (RGE) for Λ , which reads

¹A rather successful attempt towards quantum gravity has recently been traced using the effective average action and nonperturbative renormalization with infrared cutoff [11].

$$(4\pi)^2 \mu \frac{\partial}{\partial \mu} \Lambda(\mu) = \frac{1}{2} M^4. \quad (7)$$

Once derived, Eq. (7) should be valid for any value of μ . However, the relation (7) has been derived using the MS renormalization scheme, which is a mass-independent renormalization scheme. It is well known that such a scheme does not give the correct mass behavior of the β functions. Therefore, the expression (7) gives the correct behavior of $\beta_\Lambda \equiv \mu(\partial\Lambda/\partial\mu)$ only for $\mu \gg M$. For $\mu \ll M$, we would expect the decoupling of the heavy particle with mass M , i.e. β_Λ is expected to be approximately zero.

However, it would be premature to claim the validity of the decoupling theorem [9], because on purely dimensional grounds, one expects the corrections of the type μ^2/M^2 [12,13] to be insufficient to suppress the quartic power of the mass M in Eq. (7). To be more precise, let us assume that there are two scalar particles, one with a heavy mass M , and the other with a light mass m . Then, the RGE becomes

$$(4\pi)^2 \mu \frac{\partial}{\partial \mu} \Lambda(\mu) = \frac{1}{2} M^4 + \frac{1}{2} m^4 \quad (8)$$

at the scale μ , $\mu \gg M, m$. However, for $m \ll \mu \ll M$, one would expect the heavy scalar to decouple with the suppression factor μ^2/M^2 and Eq. (8) would have the form

$$(4\pi)^2 \mu \frac{\partial}{\partial \mu} \Lambda(\mu) = \frac{1}{2} a \frac{\mu^2}{M^2} M^4 + \frac{1}{2} m^4, \quad (9)$$

where a is the number of order $\mathcal{O}(1)$. Obviously, the suppression factor μ^2/M^2 is not sufficient to suppress the contribution of the heavy scalar, since

$$\mu^2 M^2 \gg m^4 \quad (10)$$

and the heavy scalar does not decouple. The reason for such a peculiar behavior of the cosmological constant is its high dimensionality ($mass^4$).

The calculation of zero-mode contributions for a given massive field can be related to the evaluation of the vacuum bubble diagrams (diagrams without external legs). The aforementioned calculation results in a divergent quantity which must be properly regularized. We shall consider ‘‘cutoff’’ regularization procedure for a bosonic degree of freedom (e.g. a real scalar field) which is more suitable for our purposes since it displays the structure of divergences more clearly. Other regularization schemes (e.g. dimensional regularization) yield equivalent results. The treatment of fermionic degrees of freedom is completely analogous to the treatment of bosonic degrees of freedom up to the opposite sign. The zero-point energy of a real scalar field is [14]

$$ZPE = \frac{1}{(4\pi)^2} A_0^4 + \frac{1}{2} \frac{1}{(4\pi)^2} \left[A^2 m^2 - \frac{m^4}{4} - \frac{m^4}{2} \ln \frac{A^2 + m^2}{m^2} \right], \quad (11)$$

plus additional terms which vanish as $A \rightarrow \infty$, A being the four dimensional cutoff. The term A_0^4 corresponds to the

zero-point energy in the massless limit. Since we are dealing with the divergent quantity, a consistent procedure of renormalization must be invoked. Various divergent contributions have to be renormalized by adding appropriate counterterms. Quartically and quadratically divergent terms have to be subtracted completely (i.e. the choice of counterterms is unique) while in the case of a logarithmically divergent term, the most general counterterm includes some scale dependence. In order to examine the effects of mass thresholds, it is necessary to apply a renormalization scheme in which the counterterms are scale and mass dependent [12,13]. This requirement clearly disqualifies the most widely applied renormalization schemes, such as the MS or $\overline{\text{MS}}$ schemes. There exists a version of the MS scheme [15] which incorporates the effects of mass thresholds (named by its authors as decoupling subtraction). This scheme keeps the contributions of massive particles at scales above the mass, while it excludes them completely at the scales below the mass and therefore implies a ‘‘sharp cutoff’’ approximation. However, it is also based on the assumption of validity of decoupling of the massive field at low scales. This last feature is yet to be tested in the case of the cosmological constant. The subtraction scheme, on the other hand, meets the aforementioned demand. The counterterm in this scheme is obtained by setting some exterior scale (like the momentum squared) in the divergent Green function to a predetermined value (usually referred to as a renormalization point). From Eq. (11) it is clear that in our case there is no exterior scale (we treat the mass m as a parameter), so even the subtraction scheme cannot be applied directly. One possible way out of this predicament is to use a very general form of the counterterm and then limit its form by imposing some reasonable conditions on the running of relevant quantities (contribution to the zero-point energy part of the cosmological constant). This approach leads to the following relation between unrenormalized and renormalized zero-point energy parts of the cosmological constant:

$$\Lambda_{bare} = \Lambda - \frac{1}{(4\pi)^2} A_0^4 - \frac{1}{2} \frac{1}{(4\pi)^2} A^2 m^2 + \frac{1}{2} \frac{1}{(4\pi)^2} m^4 \ln \frac{A^2 + m^2}{\mu^2 g\left(\frac{m}{\mu}\right)}, \quad (12)$$

where the function $\mu^2 g(m/\mu)$ represents the general scale and mass dependence of the counterterm. The results of renormalization in the subtraction scheme [12,13] strongly suggest the form of the counterterm determined by the function

$$\mu^2 g\left(\frac{m}{\mu}\right) = \mu^2 + m^2. \quad (13)$$

We consider this choice the most natural and consequently use it in the concrete calculations in the rest of the paper. Nevertheless, one can perform a more general analysis starting from the rather general form of the counterterm. By

introducing $x = m/\mu$, the running of the vacuum part of the cosmological constant becomes

$$\mu \frac{\partial \Lambda}{\partial \mu} = \frac{1}{(4\pi)^2} \frac{m^4}{2} \left[1 - \frac{1}{2} \frac{x g'(x)}{g(x)} \right], \quad (14)$$

where the prime denotes the derivative with respect to x . Let us start from the general form of the counterterm determined by the function

$$g(x) = \sum_{l=-n}^m c_l x^l. \quad (15)$$

Valuable information can be gained by considering the following interesting limits of the expression governing the running of Eq. (14):

$$\lim_{x \rightarrow \infty} \left[1 - \frac{1}{2} \frac{x g'(x)}{g(x)} \right] = 1 - \frac{m}{2} + \frac{1}{2} \frac{c_{m-1}}{c_m} \frac{1}{x} + \mathcal{O}\left(\frac{1}{x^2}\right),$$

$$\lim_{x \rightarrow 0} \left[1 - \frac{1}{2} \frac{x g'(x)}{g(x)} \right] = 1 + \frac{n}{2} + \frac{1}{2} \frac{c_{-n+1}}{c_{-n}} x + \mathcal{O}(x^2). \quad (16)$$

The first limit covers the behavior for large x , i.e. at scales μ much smaller than the mass m . At low scales one expects suppressed contributions of very massive fields. If we formulate this expectation as a requirement, serious constraints on the index m can be obtained. For $m \geq 3$, the running is unsuppressed and negative. The negative running at low scales, together with the positive running at higher scales, implies a change of sign at some intermediate scale which is clearly an undesirable property. For $m = 0, 1$, the running is positive, but unsuppressed. Only for $m = 2$, we obtain the suppressed behavior as required. In the opposite limit of small x , i.e. large scales μ compared with the mass m , we demand to recover the behavior displayed by the $\overline{\text{MS}}$ and $\overline{\text{MS}}$ schemes. Namely, in this limit, the effect of mass threshold can be completely neglected, which is exactly the property of the $\overline{\text{MS}}$ and $\overline{\text{MS}}$ schemes. Therefore, the condition $n = 0$ follows directly. Taking into account the considerations given above, the most general form of the counterterm (15) is given by $\mu^2 g(m/\mu) = \mu^2 + c_1 m \mu + c_2 m^2$ (the coefficient in front of μ^2 can be absorbed by the redefinition of the μ scale). Since terms linear in the mass m are nonspecific for relativistic calculations, it is evident that our choice (13) fits nicely into the allowed form of the counterterm.

Now when the question of the renormalization scheme is settled, we can look at the running of the vacuum part of the cosmological constant in some particle physics model with its own spectrum of massive bosonic and fermionic degrees of freedom (relevant in our case). The common property of the running in all models is the nonexistence of decoupling at low scales. Namely, for the contribution of the real scalar field to the running of the zero-point energy part of the cosmological constant we obtain

$$\mu \frac{\partial \Lambda}{\partial \mu} = \frac{1}{(4\pi)^2} \frac{1}{2} m^4 \quad (17)$$

in the $\mu \gg m$ limit. In the opposite $\mu \ll m$ limit, the running becomes

$$\mu \frac{\partial \Lambda}{\partial \mu} = \frac{1}{(4\pi)^2} \frac{1}{2} m^2 \mu^2 \quad (18)$$

as anticipated in the relation (9). One can clearly see that the suppression of very massive fields is present, but insufficient to insure their decoupling.

In the case of the standard model, the running acquires the form

$$(4\pi)^2 \mu \frac{\partial \Lambda}{\partial \mu} = -2 \sum_i N_i m_i^4 \frac{\mu^2}{\mu^2 + m_i^2} + 3 m_W^4 \frac{\mu^2}{\mu^2 + m_W^2} + \frac{3}{2} m_Z^4 \frac{\mu^2}{\mu^2 + m_Z^2} + \frac{1}{2} m_H^4 \frac{\mu^2}{\mu^2 + m_H^2}, \quad (19)$$

where the index i denotes fermions, N_i being 3 for quarks and 1 for leptons. Direct integration of Eq. (19) gives

$$(4\pi)^2 (\Lambda(\mu) - \Lambda(0)) = - \sum_i N_i m_i^4 \ln \frac{\mu^2 + m_i^2}{m_i^2} + \frac{3}{2} m_W^4 \ln \frac{\mu^2 + m_W^2}{m_W^2} + \frac{3}{4} m_Z^4 \ln \frac{\mu^2 + m_Z^2}{m_Z^2} + \frac{1}{4} m_H^4 \ln \frac{\mu^2 + m_H^2}{m_H^2}. \quad (20)$$

The expression given above indicates that the contribution of very massive fields is nonnegligible at all scales. As for neutrinos, recent experiments indicate that neutrinos do have nonzero masses. The question of these masses is still unsettled, but it is general agreement that they are in the region below $\mathcal{O}(1 \text{ eV})$. As these masses are far below all the other masses in play, we shall put them all to zero as a starting approximation. The investigation of possible subtle effects due to nonzero neutrino masses will be discussed elsewhere.

In this framework we can focus our attention to the effects of running at scales rather below the mass of the electron, the lightest particle in our approach. Since all the masses are large compared with the scale μ , it is convenient to expand the logarithms in the relation (20). This procedure yields

$$\Lambda(\mu) - \Lambda(0) = \frac{1}{(4\pi)^2} \frac{1}{4} \mu^2 \left[m_H^2 + 3m_Z^2 + 6m_W^2 - 4 \sum_i N_i m_i^2 \right] + \frac{1}{(4\pi)^2} \mu^4 \left[\frac{1}{2} \sum_i N_i - \frac{5}{4} \right] + \mathcal{O}\left(\frac{\mu^6}{m_{large}^2}\right). \quad (21)$$

The analysis of the relation given above tells us instantly that large masses in the μ^2 term drive the numerical value of Λ far out of the range consistent with observation, unless the expression in the brackets of the same term vanishes. Therefore, to avoid inconsistency with observation, we obtain a stringent condition on the Higgs boson mass, i.e. an explicit expression for m_H in terms of masses of other particles in the standard model:

$$m_H^2 = 4 \sum_i N_i m_i^2 - 3m_Z^2 - 6m_W^2. \quad (22)$$

Using the numerical values from [16] we obtain $m_H \approx 550$ GeV. It is clear that the relation (22) implies the relation between the squares of masses ranging from ~ 1 MeV to ~ 100 GeV and, accordingly, introduces a certain fine-tuning of masses of the standard model.

If the Higgs boson mass is fixed by the requirement (22), the running of the zero-point energy part of the cosmological constant is given by the μ^4 term of the expression (21). The running below the electron mass should not be too intensive in order to preserve the conditions for nucleosynthesis. Using the expression for the energy density of radiation during nucleosynthesis $\rho_R = (\pi^2/30)g_* T^4$, as well as making a natural choice for the scale $\mu = T$, one obtains the relation

$$\frac{\Lambda(\mu) - \Lambda(0)}{\rho_R} = \frac{555}{32\pi^4 g_*}. \quad (23)$$

With the numerical value $g_* = 3.36$, the expression given above acquires the numerical value 0.053, a value within the range that does not disturb [17] nucleosynthesis (note that even more stringent constraints obtained very recently in [18] are obeyed). It is interesting to notice that in the radiation dominated universe the quantity on the left-hand side of Eq. (23) is constant for $\mu \ll 1$ MeV. This phenomenon of ‘‘scaling’’ has already been met in the investigations of scalar field cosmologies with potentials having attractor solutions and its appearance here represents a very interesting and potentially important similarity.

Finally, the relation (21) together with the constraint (22) enables us to calculate the value of Λ at present scale of the evolution of the universe. If we take the value $\mu_0 = 0.002$ eV suggested in [5,8], we obtain $\Lambda(\mu) - \Lambda(0) \approx 10^{-48} \text{ GeV}^4$, a value reasonably close to the observed value of dark energy density of order 10^{-47} GeV^4 . From Eq. (21) the amount of fine-tuning at present is estimated to be 1 in $(100 \text{ GeV})^2/\mu_0^2 \approx 10^{28}$.

At this point a remark is in order. All our preceding results have been obtained using a specific form of the function

determining the counterterm $\mu^2 g(m/\mu) = \mu^2 + m^2$. However, if we use a more general form $\mu^2 g(m/\mu) = \mu^2 + c_2 m^2$, where c_2 is naturally expected to be of order 1, the fine-tuning expression for the Higgs mass (22) remains completely the same, while the μ^4 term stays of the same order of magnitude and the conclusions of comparisons of our results with observations remain unchanged.

Let us now discuss our value (≈ 550 GeV) in view of experimental and theoretical constraints on the mass of the standard model Higgs boson. The experimental lower limit is 114 GeV [19] at 95% confidence level, a value somewhat higher than the ‘‘best fit’’ value obtained from electroweak precision data [20] (106 GeV). At the same time, $m_H < 220$ GeV at 95% confidence level. Thus, these data suggest that the Higgs boson mass should not be too much larger than the present limit from direct searches. On theoretical grounds, a well-known upper limit on the Higgs boson mass comes from the unitarity of the scattering matrix. Even the most restrictive bound (~ 800 GeV) obtained from the scattering process, $Z_L W_L \rightarrow Z_L W_L$, is considerably higher than our value. On the other hand, the triviality of the standard model admits it only as an effective theory, valid below some energy scale Λ . If the validity of the standard model is pushed up to extremely high scales (grand unification or Planck), the triviality bound is more stringent than the unitarity bound, being $\lesssim 200$ GeV for the quartic coupling taken to reside in the perturbative domain, $1 \leq \lambda \leq 10$. Thus, even on purely theoretical grounds, one can see (upon including the stability lower bound) that m_H in the 100–200 GeV range is preferred.

Since we take the heaviest masses from the standard model, our model implicitly assumes the validity of the standard model up to the highest scales, thereby violating the triviality bound. One can therefore claim, using arguments based solely on the running of the cosmological constant, the existence of some intermediate energy scale at which we should expect nonstandard phenomena to take place. Models in which nonstandard physics compensates the effect of a heavy Higgs boson (at the same time fitting precision data as good as the standard model) can be found, for example, in a recent review [21]. The above conclusion may however not be definite as, for the full treatment, one also needs to include scaling effects from the vacuum energy induced by a scalar Higgs potential of the standard model, which we consider next.

The Higgs field will contribute to Λ via the vacuum condensate. The contribution is given by Eq. (3) and the renormalization group equation reads

$$\frac{d}{dt} \Lambda^{cond}(t) = -\frac{m^2}{2\lambda} \frac{dm^2}{dt} + \frac{m^4}{4\lambda^2} \frac{d\lambda}{dt}, \quad (24)$$

where $t = \ln(\mu/\mu_0)$ and $m_H^2 = 2m^2$.

Again, one would expect the Higgs contribution at the scale $\mu \ll m_H$ to diminish owing to the decoupling. Unfortunately, it is not very difficult to convince oneself that the suppression factors are of the form μ^2/m_H^2 , and cannot compete with the overall m_H^4 factor. To see this, one inspects Eq.

(24). It is clear that one has to calculate dm^2/dt and $d\lambda/dt$ in, for example, the momentum subtraction scheme. Let us check, for instance, the contribution of the self-energy diagram (the Higgs loop with two external Higgs legs) which contributes to the running of $m^2(t)$. The renormalized contribution is proportional to the m_H^2 multiplied by the following integral:

$$j(m_H, \mu) = \int_0^1 dx \frac{\mu^2 x(1-x)}{m_H^2 + \mu^2 x(1-x)}. \quad (25)$$

For $\mu \gg m_H$, the integral gives

$$j(m_H, \mu) = 1 + O(m_H^2/\mu^2) \quad (26)$$

and in the limit $\mu \ll m_H$, one gets

$$j(m_H, \mu) = \frac{1}{6} \frac{\mu^2}{m_H^2} + O(\mu^4/m_H^4). \quad (27)$$

Since dm^2/dt in Eq. (24) multiplies m^2 , the overall mass factor is m_H^4 for $\mu \gg m_H$, whereas the suppression in Eq. (27) is not enough to suppress it for $\mu \ll m_H$. We have checked that the same is true for the one-loop, four-point function (a fermion box with four external Higgs legs), which contributes to the running of $\lambda(t)$, and, again, the decoupling theorem fails in the calculation of the running of $\Lambda^{cond}(t)$. The examples discussed above indicate that the same behavior as in the running of the zero-point energy part of $\Lambda(\mu)$, is expected for Λ^{cond} as well.

In conclusion, we made a study to demonstrate a scaling dependence of the cosmological constant by showing that its

observational value is not preserved at different energy scales. The running due to one-loop vacuum bubble graphs associated with massive fields from the standard model is obtained in a closed analytic form. We have noted that the quantum theory of gravity plus matter truncated at the one-loop level is an adequate description because of the non-renormalizability of gravity. Contrary to the expectation from the Appelquist-Carazzone decoupling theorem, we have found that more massive fields do play a dominant role in the running at any scale. We have also indicated that the same behavior should persist in the running of the cosmological constant induced by spontaneous symmetry breaking through the Higgs mechanism. Owing to heavy masses involved in the running, the standard cosmological scenario may be found in jeopardy unless some fine-tuning is applied. As a result, we get the prediction $m_H \simeq 550$ GeV for the Higgs boson mass. Since this mass is not favored by the present constraints, one may consider our results obtained from the running of the cosmological constant as an independent indication that the standard model cannot be the full theory at all scales. Finally, the effects described in this paper are a feature of any quantum field theory comprising massive fields. Consequently, one expects the same type of relations between masses of that theory (stemming from the consistency with observation at low scales) in any extension (e.g. SUSY, GUTs) of the standard model.

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