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Yang-Mills solutions on Euclidean Schwarzschild space

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We show that the apparently periodic Charap-Duff Yang-Mills "instantons" in time-compactified Euclidean Schwarzschild space are actually time independent. For these solutions, the Yang-Mills potential is constant along the time direction (no barrier) and therefore, there is no tunneling. We also demonstrate that the solutions found to date are three-dimensional monopoles and dyons. We conjecture that there are no time-dependent solutions in the Euclidean Schwarzschild background.

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Studies [1,2] of Yang-Mills (YM) instantons coupled to Einstein's gravity started not long after their discovery in flat space [3], but this subject has not received proper interest since. The Lorentzian Einstein-YM system holds quite a number of surprises—one of which is a soliton solution [4] which is absent both in pure gravity and in pure YM theory [5]. In Euclidean space, the energy momentum tensor of self-dual YM solutions vanishes and therefore gravity is not disturbed by the presence of instantons but does affect them[1] in a number of different ways: in particular it can change the space-time topology and it can bring a scale (or multiple scales). If space is conformally flat, the self-duality equations of flat-space YM theory are intact and the flat space Belavin-Polyakov-Schwarz-Tyupkin (BPST) instantons are formally solutions. But this is too naive: one has to take into account the existence of the horizons and the global topology of the space-time. Euclidean de Sitter space is a good example in this context: there is a horizon and the time is compactified. In the literature [7] one can find *static* three-dimensional solutions but apparently no fully four-dimensional instanton solutions.

Here we shall take the background to be Euclidean Schwarzschild space. YM theory in this background was studied long ago by Charap and Duff (CD) [1,2], who found self-dual solutions. However, the physical meaning of the solutions of CD has not hitherto been resolved. CD acknowledge that their "instanton" solutions, being periodic in Euclidean time, do not allow a tunneling interpretation between vacua. This work started with the intention to show that periodic instanton solutions in a curved background can be given a normal tunneling interpretation in the same spirit as the caloron solutions [8] of YM theory on $S^1\times R^3$. As demonstrated in [9], the caloron gauge field, initially constructed to be explicitly periodic, is no longer periodic in the Weyl gauge ($A_0^a=0$), which is more suitable for the Hamiltonian processes, such as tunneling. But, unlike the calorons, the apparently time-periodic YM solutions of CD in Euclidean Schwarzschild turned out to be actually *time independent*, when looked at the proper gauge. As we shall show, these solutions are more like BPS monopoles than instantons. In fact, the YM potential is constant along the time direction for these solutions and there is no barrier to tunnel.

The Euclidean Schwarzschild space, in Schwarzschild and Kruskal coordinates, respectively, is

$$ds^2=H(r)dt^2+\frac{1}{H(r)}dr^2+r^2(d\theta^2+\sin^2\theta d\phi^2),$$

$$H(r)=1-\frac{2M}{r}, \tag{1}$$

$$ds^2=\frac{32M^3}{r}\exp\left(-\frac{r}{2M}\right)(dz^2+dy^2)$$
$$+r^2(d\theta^2+\sin^2\theta d\phi^2), \tag{2}$$

where

$$z^2+y^2=\left(\frac{r}{2M}-1\right)\exp\left(\frac{r}{2M}\right),\quad \frac{y-iz}{y+iz}=\exp\frac{-it}{2M}. \tag{3}$$

The Euclidean time can be compactified to remove the singularity at the origin [we set the gravitational constant G and the Yang-Mills coupling constant g_{YM} to unity: $G=g_{YM}=1$]:

$$t\to t+\beta,\quad \beta=8\pi M,\quad r\geqslant 2M. \tag{4}$$

However, once compactified, the $M\to 0$ limit is tricky, and formally it yields a three- instead of four-dimensional Euclidean space. (This suggests that YM instantons in this background, if they existed at all, would not limit to the flat space 4D BPST instanton for $M\to 0$.)

We take $SU(2)$ YM theory and adopt the spherically symmetric instanton ansatz for the YM gauge connection,

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[1]A similar situation arises in three dimensions for a different reason, the imaginary nature of the Chern-Simons action in Euclidean space: there are Chern-Simons gravity-YM-Higgs "gravitating monopole-instanton" solutions which also do not curve space [6].

$$A=\frac{1}{2}\tau^{a}A_{\mu}^{a}dx^{\mu}$$
$$=\frac{\tau^{a}}{2}\left\{A_{0}\frac{x_{a}}{r}dt+A_{1}\frac{x_{a}x_{k}}{r^{2}}dx_{k}+\frac{\phi_{1}}{r}\left(\delta_{ak}-\frac{x_{a}x_{k}}{r^{2}}\right)dx_{k}+\epsilon_{akl}\frac{\phi_{2}-1}{r^{2}}x_{k}dx_{l}\right\},\qquad(5)$$

where τ^{a} are Pauli matrices. We should mention that Charap-Duff [2] have only three unknown functions in their ansatz, as they work with an already gauge-fixed instanton ansatz ($x^{j}A_{j}^{a}=A_{1}=0$) from the start. On the other hand, we have not fixed the gauge yet. This will be a crucial point for the physical interpretation of the solutions.

Our task is to solve the self-duality equations, $F=\pm *F$, but there are at least one apparent and one serious obstruction to the existence of non-trivial instanton solutions. The apparent one is that naive topological considerations seem to imply that there will be no non-trivial solutions of equations of motion. The topology of the Euclidean Schwarzschild space is $R^{2}\times S^{2}$. At spatial infinity ($r\rightarrow\infty$), the space has the topology $S_{\beta}^{1}\times S_{\infty}^{2}$ (we add the point $t=0$ to the time direction). The relevant homotopy groups vanish $[\Pi_{2}(SU(2)]=0]$ and $\Pi_{1}[SU(2)]=0$; considered as a map, $A:S^{1}\times S^{2}\rightarrow SU(2)$ cannot have a winding number. But this argument is not sufficient to prove that there are no non-trivial solutions, the existence of calorons in the same (asymptotic) topology being a counterexample. There *can* indeed be non-trivial maps between $S^{1}\times S^{2}$ and $SU(2)$ [10]. For explicit construction of these maps in the finite temperature gauge theory context, we refer the reader to [11] and [12].

Another way to avoid this topological "simplicity" is to consider solutions that have non-trivial holonomy for the Polyakov loop at spatial infinity: $\mathcal{P}\exp\int_{0}^{\beta}A_{0}dt\neq\pm 1$. This will effectively break the gauge group to $U(1)$ and non-trivial homotopy will guarantee the solutions. But more generally we can rely on the cohomology arguments for the existence of non-trivial solutions. Namely we need the topological charge to be non-vanishing, expressed as an integral over the boundary : $Q\sim\int_{M}trF\wedge F=\int_{M}dK=\int_{S^{2}\times S^{1}}K$. A non-exact but closed 3-form K can be constructed since the relevant cohomology groups are non-trivial: $H^{1}(S^{1},R)=R$ and $H^{2}(S^{2},R)=R$. This argument alone does not imply the quantization of the topological charge. This is obtained by the requirement of the single valuedness of the path-integral $\langle\exp(iQ)\rangle$.

The more serious obstruction for the existence of arbitrary size instanton solutions is the existence of a scale, $2M$. It is clear that YM theory in the Schwarzschild background is not scale invariant and the best one can do is to look for "constrained instantons" as in [13]. These are approximate solutions with scales restricted to a domain, usually given by the scale in the theory. At the end of this paper, we will argue that Euclidean Schwarzschild space does not allow such solutions. We next show that the solutions reported in the literature are 3D monopoles rather than 4D instantons.

The gauge field strength , $F=dA-iA\wedge A$, is computed to be [14,15]

$$F=\frac{1}{2}\{(\dot{A}_{1}-A_{0}')\tau_{3}dt\wedge dr+[(\dot{\phi}_{2}-A_{0}\phi_{1})\tau_{1}+(\dot{\phi}_{1}+A_{0}\phi_{2})\tau_{2}]dt\wedge d\theta-[(\dot{\phi}_{1}+A_{0}\phi_{2})\tau_{1}+(-\dot{\phi}_{2}+A_{0}\phi_{1})\tau_{2}]dt\wedge\sin\theta d\phi$$
$$+[(\phi_{2}'-A_{1}\phi_{1})\tau_{1}+(\phi_{1}'+A_{1}\phi_{2})\tau_{2}]dr\wedge d\theta+[(-\phi_{1}'-A_{1}\phi_{2})\tau_{1}+(\phi_{2}'-A_{1}\phi_{1})\tau_{2}]dr\wedge\sin\theta d\phi$$
$$-(1-\phi_{1}^{2}-\phi_{2}^{2})\tau_{3}d\theta\wedge\sin\theta d\phi\}.\qquad(6)$$

Here ϕ' and $\dot{\phi}$ denote derivative with respect to r and t, respectively. The dual field strength, in the Euclidean Schwarzschild background, is

$$*F=\frac{1}{2}\left\{-\frac{1}{r^{2}}(1-\phi_{1}^{2}-\phi_{2}^{2})\tau_{3}dt\wedge dr+H(r)[(\phi_{1}'+A_{1}\phi_{2})\tau_{1}+(-\phi_{2}'+A_{1}\phi_{1})\tau_{2}]dt\wedge d\theta+H(r)[(\phi_{2}'-A_{1}\phi_{1})\tau_{1}\right.$$
$$+(\phi_{1}'+A_{1}\phi_{2})\tau_{2}]dt\wedge\sin\theta d\phi-\frac{1}{H(r)}[(\dot{\phi}_{1}+A_{0}\phi_{2})\tau_{1}+(-\dot{\phi}_{2}+A_{0}\phi_{1})\tau_{2}]dr\wedge d\theta+-\frac{1}{H(r)}[(\dot{\phi}_{2}-A_{0}\phi_{1})\tau_{1}$$
$$\left.+(\dot{\phi}_{1}+A_{0}\phi_{2})\tau_{2}]dr\wedge\sin\theta d\phi+r^{2}(\dot{A}_{1}-A_{0}')\tau_{3}d\theta\wedge\sin\theta d\phi\right\}.\qquad(7)$$

Before we look at the equations of motion, let us show that there is a nice dimensional reduction of the four-dimensional YM action to a 2D Abelian Higgs model in a curved background:

$$S_{YM}=\int_M \mathrm{tr} F\wedge {}^*F$$
$$=8\pi\int_\Sigma d^2x\sqrt{h}\left\{\frac{1}{2}h^{\mu\nu}D_\mu\varphi_i D_\nu\varphi_i\right.$$
$$\left.+\frac{1}{8}h^{\mu\alpha}h^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}+\frac{1}{4}(1-\varphi_i^2)^2\right\}. \quad (8)$$

Space-time indices refer to (t,r) only and $i,j=(1,2)$, where $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ and $D_\mu\varphi_i=\partial_\mu\varphi_i+\epsilon_{ij}A_\mu\varphi_j$ are the two-dimensional Abelian field strength and covariant derivative, respectively. Σ is a semi-infinite strip in the upper-half plane with the following metric:

$$ds^2=h_{\mu\nu}dx^\mu dx^\nu=\frac{H(r)}{r^2}dt^2+\frac{1}{r^2H(r)}dr^2. \quad (9)$$

This reduction of course follows from the conformal invariance of the Yang-Mills action. One can simply pull out a factor of r^2 from the metric (1). The result (8) merely generalizes that of the flat space case [14]: four-dimensional Yang-Mills theory on R^4, for spherically symmetric solutions, reduces to the two-dimensional Abelian-Higgs model on the upper half-plane with the Poincaré metric $ds^2=r^{-2}(dt^2+dr^2)$.

Now let us study the self-duality equations $F=\pm{}^*F$, expressed as

$$\dot{A}_1-A_0'=\mp\frac{1}{r^2}(1-\phi_1^2-\phi_2^2),$$
$$\dot{\phi}_2-A_0\phi_1=\pm H(r)(\phi_1'+A_1\phi_2), \quad (10)$$
$$\dot{\phi}_1+A_0\phi_2=\mp H(r)(\phi_2'-A_1\phi_1).$$

Both the equations of motion and the action are invariant under the non-Abelian gauge transformations $(U(\vec{x},t)=\exp -i[f(r,t)/2]\hat{x}\cdot\vec{\tau})$ which transform the ansatz functions in the following way:

$$\tilde{A}_0=A_0+\dot{f},$$
$$\tilde{A}_1=A_1+f',$$
$$\tilde{\phi}_2=+\phi_2\cos f+\phi_1\sin f,$$
$$\tilde{\phi}_1=-\phi_2\sin f+\phi_1\cos f. \quad (11)$$

In flat space, $M=0$ and $H(r)=1$, Eqs. (10), augmented with a gauge condition, are integrable and were solved in [14] to obtain n-instanton solutions located on the t axis at arbitrary locations and with arbitrary sizes. (Another interesting fact about these equations was noted in [19] and [20]: Eq. (10) corresponds to extremal surfaces in the $R^{(2,1)}$, for $H(r)=1$. For generic $H(r)$, Eqs. (10) can be shown to reduce to a problem of finding a (non-extremal) surface whose Gaussian curvature is a function of H(r) [21].)

Now let us look at non-zero M. The charge 1 solution given by [2] is

$$A_1=0,\quad A_0=\lambda-\frac{M}{r^2},$$
$$\phi_1=\left(1-\frac{2M}{r}\right)^{1/2}\cos(\lambda t+\omega_0),$$
$$\phi_2=\left(1-\frac{2M}{r}\right)^{1/2}\sin(\lambda t+\omega_0). \quad (12)$$

This solution will be smooth $r\in[2M,\infty)$ and $t\in[0,8\pi M]$, if one sets $\lambda=1/4M$. The non-Abelian gauge field is periodic $[A_\mu^a(r,t=0)=A_u^a(r,t=8\pi M)]$. ω_0 is an integration constant which denotes the angular location of the instanton in the time interval and without loss of generality we choose $\omega_0=\pi/2$. Observe that there is no (arbitrary) size for the solution. In this periodic gauge, the flat space limit $(M\rightarrow 0)$ does not seem to be well defined. But it is clear that we can gauge transform this solution to a time-independent solution using Eq. (11), with a gauge transformation function $f(r,t)=-\lambda t$. Then the solution will be

$$A_1=0,\quad A_0=-\frac{M}{r^2},$$
$$\phi_1=0,\quad \phi_2=\left(1-\frac{2M}{r}\right)^{1/2}. \quad (13)$$

This solution has an artificial (gauge) singularity at $r=2M$ and this is the same solution as the one CD obtained from the spin connection in their first paper on the subject [1]. We can show that it is a charge 1 monopole, by associating an Abelian field strength through 't Hooft's [16] definition (and also see Abbott-Deser [17])

$$f_{\mu\nu}=\partial_\mu(\hat{A}_0^aA_\nu^a)-\partial_\nu(\hat{A}_0^aA_\mu^a)+\epsilon^{abc}\hat{A}_0^a\partial_\mu\hat{A}_0^b\partial_\nu\hat{A}_0^c \quad (14)$$

where $\hat{A}_0^a=A_0^a/\sqrt{(A_0^aA_0^a)}$,

$$f_{0i}=\frac{x_iA_0'}{r},\quad f_{ij}=\frac{\epsilon_{ija}x^a}{r^2}. \quad (15)$$

The electric field $E_i=f_{0i}$ decays rapidly and so the electric charge of the solution is zero but the magnetic charge is 1. Thus, as promised, we have demonstrated that the solution of [2] is time independent and corresponds to a monopole. Another way to see this is to look at the gauge-invariant Yang-Mills potential. It reads

$$V(t)=2\pi\int_{2M}^\infty dr\left\{2H(r)(\phi_1'+A_1\phi_2)^2+2H(r)\right.$$
$$\left.\times(\phi_2'-A_1\phi_1)^2+\frac{1}{r^2}(1-\phi_1^2-\phi_2^2)^2\right\}. \quad (16)$$

For Eq. (12), we have

$$V = \frac{\pi}{2M}. \tag{17}$$

It is time independent and so there is no barrier to tunnel unlike the flat-space BPST instanton or the time-periodic caloron [9]. Its kinetic energy is

$$E_{kinetic}(t) = 2\pi \int_{2M}^{\infty} dr \left\{ \frac{2}{H(r)} (\dot{\phi}_1 + A_0\phi_2)^2 + \frac{2}{H(r)} (\dot{\phi}_2 - A_0\phi_1)^2 + r^2(\dot{A}_1 - A_0')^2 \right\}, \tag{18}$$

which is computed to be

$$E_{kinetic}(t) = \frac{\pi}{2M}. \tag{19}$$

We have seen that, even though the monopole has an associated electric field, its flux at infinity is zero. The "mass" of the monopole can be defined as the sum of the kinetic and the potential energies: namely π/M. In flat space the monopole action ($\int$mass dt) is divergent, but here, because of the compactness of the time dimension, its action is $8\pi^2$. This is numerically the same as the topological charge 1 instanton action, but as argued above, the solution at hand is a BPS monopole.

We can also give a suggestive argument of how to interpret the mass of the monopole. Needless to say, A_0^a plays the role of an adjoint Higgs field, as is already clear from the definition of the Abelian field strength (14). The flat space BPS 't Hooft-Polyakov monopole, obtained from YM fields has a mass $4\pi v$, where v is the expectation value of $|A_0^a|$ at spatial infinity. Even though the explicit solutions of the flat space and the curved-space BPS monopole are quite different, they nevertheless allow similar interpretations in the non-singular gauge (12). We have $A_0(r\rightarrow\infty) = 1/4M$: the inverse of the Schwarzschild radius determines the symmetry breaking scale and so the mass of the curved BPS monopole is

$$M_{monopole} = \frac{\pi}{M} = 4\pi A_0(r\rightarrow\infty). \tag{20}$$

The second, dyon, solution reported in [2] is

$$A_1 = \phi_1 = \phi_2 = 0, \quad A_0 = \pm\left(c - \frac{1}{r}\right). \tag{21}$$

Computing Eq. (15) for this solution one observes that this solution has both electric and magnetic charges of unity. Its Abelian nature was also explored in [18] from a different perspective. The YM potential for this BPS dyon is twice that of the monopole (17).

Departing from CD, new solutions can be obtained, one of which is the following[2]

$$\tilde{A}_0 = 0, \quad \tilde{A}_1 = -\frac{2Mt}{r^3},$$
$$\tilde{\phi}_1 = \left(1 - \frac{2M}{r}\right)^{1/2} \cos\left(\frac{Mt}{r^2} + \omega_0\right),$$
$$\tilde{\phi}_2 = \left(1 - \frac{2M}{r}\right)^{1/2} \sin\left(\frac{Mt}{r^2} + \omega_0\right). \tag{22}$$

This is again a topological charge 1 solution but since $\tilde{A}_0^a = 0$, it does not seem to allow a monopole interpretation. But one can easily show that this solution is related to the CD solution (12) by a large gauge transformation $W(\vec{x},t) = \exp[-if(r,t)\hat{x}\cdot\sigma/2]$, where $f(r,t) = t[(M/r^2) - \lambda]$. Therefore Eq. (22) is simply in "bad," Weyl, gauge as far as the monopole structure is concerned. The Yang-Mills potential and the rest of the gauge invariant objects are the same as the CD solution of course. But the decomposition of the topological charge differs in these two gauges.

Following the discussion of [9], let us now see how the topological charge can be decomposed into the radial part and the Chern-Simons parts,

$$\begin{aligned} Q &= \frac{1}{8\pi^2}\int_M \mathrm{tr} F\wedge F \\ &= 4\pi \int_\Sigma d^2x \left\{ \epsilon_{\mu\nu}\epsilon_{ij} D_\mu\phi_i D_\nu\phi_j - \frac{1}{2}\epsilon_{\mu\nu}F_{\mu\nu}(1-\phi_i\phi_i) \right\} \\ &= \int_0^{8\pi M} dt \frac{d}{dt} CS(t) + \int_{2M}^{\infty} dr \frac{d}{dr} K(r). \end{aligned} \tag{23}$$

In terms of the ansatz fields, the Chern-Simons term[3] $CS \equiv \int d^3x K_0$, is

$$CS(t) = \frac{1}{2\pi}\int_{2M}^{\infty} dr [\phi_1'\phi_2 - \phi_1\phi_2' - A_1(1-\phi_1^2-\phi_2^2) + \phi_1'] \tag{24}$$

while the radial part is

$$K(r) = \frac{1}{2\pi}\int_0^{8\pi M} dt [\phi_1\dot{\phi}_2 - \phi_2\dot{\phi}_1 + A_0(1-\phi_1^2-\phi_2^2) - \dot{\phi}_1]. \tag{25}$$

[2]Dyon solution is $A_1 = \pm(c - t/r^2)$ and the rest of the fields vanish.

[3]Our signs are different from those of [9].

For CD gauge (12), we have

$$CS(t)=0, \quad K(r)=1-\frac{8M^3}{r^3}, \tag{26}$$

for which all the contributions to the topological charge come from the radial part. In the Weyl gauge (22),

$$CS(t)=\frac{t}{8\pi M}, \quad K(r)=\frac{4M^2}{r^2}\left(1-\frac{2M}{r}\right) \tag{27}$$

all the contributions to the topological charge come from the Chern-Simons part.

In conclusion we have demonstrated that the apparently time periodic instanton solutions in Euclidean Schwarzschild background are actually time independent and do not describe tunneling. To get this result, it was important to work in the proper gauge. All the solutions reported in the literature (including the ones we proposed here) are monopoles in the sense of 't Hooft and Polyakov: they are non-singular three-dimensional solutions with an associated Abelian field strength. This clarifies the physical interpretation of these solutions.

We have also argued that even though the topological obstruction brought by gravity can be circumvented, the dimensionful scale (Schwarzschild radius) brought by it eliminates four-dimensional (arbitrary size) instantons.

How about ''constrained instantons'' [13]? Looking at the equations of motion (10) we see that, for $r \gg 2M$, the equations reduce to those of flat space with a compact time. Therefore the usual caloron solution is [8] indeed a solution for $r \gg 2M$:

$$A_0=-\partial_r \ln\rho, \quad A_1=\partial_0 \ln\rho$$
$$\phi_1=r\partial_0 \ln\rho, \quad \phi_2=1-r\partial_r \ln\rho \tag{28}$$

where

$$\rho(r,t)=1+\frac{\lambda^2}{4Mr}\frac{\sinh\left(\frac{r}{4M}\right)}{\cosh\left(\frac{r}{4M}\right)-\cos\left(\frac{t}{4M}\right)}. \tag{29}$$

But clearly, for distances $r \gg \beta = 8\pi M$, time dependence drops out and the caloron looks exactly like a three-dimensional (time-independent) dipole [22]. Therefore because of the proximity of the two scales, β and $2M$, there is practically no room for time-dependent ''constrained instanton'' solutions.[4] Numerical study of the self-duality equations would be necessary to decide this issue.

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[4]Here we exclusively deal with periodic solutions. On the other hand, ''quasi-periodic'' solutions can be constructed for multi-instantons. (A quasi-periodic solution broadly means that the ratio of the periods of two constituent instantons is irrational and one cannot define a ''universal'' period). Quasi-periodic solutions in *flat* space were found by Chakrabarti in [23]. It remains to be seen if Euclidean Schwarzschild space allows these types of solutions.

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