Brane world cosmology with Gauss-Bonnet interaction

B. Abdesselam* and N. Mohammedi†

Laboratoire de Mathématiques et Physique Théorique, Université François Rabelais, Faculté des Sciences et Techniques, Parc de Grandmont, F-37200 Tours, France

(Received 18 October 2001; published 29 March 2002)

We study a Randall-Sundrum model modified by a Gauss-Bonnet interaction term. We consider, in particular, a Friedmann-Robertson-Walker metric on the brane and analyze the resulting cosmological scenario. It is shown that the usual Friedmann equations are recovered on the brane. The equation of state relating the energy density and the pressure is uniquely determined by the matching conditions. A cosmological solution with negative pressure is found.

DOI: 10.1103/PhysRevD.65.084018 PACS number(s): 04.50.+h, 98.80.−k

I. INTRODUCTION

The possibility that our universe is a four-dimensional brane embedded in a higher-dimensional spacetime has been extensively discussed recently. The most popular model in this context is the one proposed by Randall and Sundrum [1]. This scenario is based on the metric

$$ds^2 = A^2(y)\,\eta_{ij}dx^i dx^j + dy^2, \tag{1.1}$$

where η_{ij} is a flat Minkowski four-dimensional metric on the brane and $A^2(y)$ is the warp factor depending only on $|y|$. Perturbations of this metric reproduce the expected $1/r$ Newtonian potential on the brane (the observed universe). This is due to the fact that the zero modes of the perturbation propagate on the brane only (they tend rapidly to zero in the fifth dimension). The other modes (the massive Kaluza-Klein modes) merely give a correction in $1/r^3$ to this potential.

One of the first developments of this model was the generalization of the Randall-Sundrum ansatz to include a wider class of metrics [2–15]. Different geometries were treated by considering solutions to the Randall-Sundrum model with metrics which, up to a nonconstant conformal factor, can be written as

$$ds^2 = A^2(y) g_{ij}(x,y) dx^i dx^j + dy^2. \tag{1.2}$$

The requirement that the zero modes of the perturbations around these metrics be localized on the brane imposes further constraints on this class of geometries [3].

Since the Randall-Sundrum model is a string inspired picture [16], one would like to understand the implications of higher curvature terms in such a brane world universe. These terms naturally arise in the string effective action beyond first order in the string tension α'. The inclusion of these terms is also of relevance to cosmology and inflation. We should stress that in the setting of Ref. [16], α' corrections (higher curvature terms) arise on the ten-dimensional brane and lead to Gauss-Bonnet combinations after compactification down to five dimensions. In the eleven-dimensional bulk, on the other hand, the corrections are quartic in the curvature and might not yield Gauss-Bonnet terms after compactification. This is why we would like to characterize our model as a string inspired theory and not a string derived one.

It turns out that generic higher curvature terms lead, in general, to a delocalization of gravity from the brane [17,18]. A combination of these terms in a Gauss-Bonnet form yields, however, the desired Randall-Sundrum behavior of the zero modes of the perturbations. We should mention that the analyses of Refs. [17,18] is carried out with a brane possessing a flat metric of the form given in Eq. (1.1). In this context, various other issues were also treated using higher curvature terms [19–28]. Furthermore, the Gauss-Bonnet combination in five dimensions is the only nonlinear term in the curvature which yields second order field equations. It is essentially this feature which favors the inclusion of such terms.

In this paper, we deal with a Randall-Sundrum model complemented by a Gauss-Bonnet density where the five-dimensional metric is of the form in Eq. (1.2). We start by considering a metric on the brane with spherical symmetry. It is shown that the only possible solution in this case is a de Sitter or anti–de Sitter spacetime for the brane. This is in contrast to the case without a Gauss-Bonnet term where black hole geometries are allowed [3]. Our bulk metric involves a warp factor that presents an oscillatory regime among other possibilities.

A second study consists in taking a Friedmann-Robertson-Walker metric on the brane. We recover the equations of ordinary cosmology on the brane. This is to be compared to previous brane world cosmology models [29–51] where, among other things, the square of the Hubble parameter is found to be proportional to the square of the energy density. Here, the matching conditions are so restrictive that they determine the equation of state relating the energy density to the pressure. Various inflationary solutions with a cosmological constant on the brane are determined. Another solution with a time-dependent energy density and pressure is also found. However, the pressure for this matter is negative and cannot describe ordinary dust. On the other hand, a scalar field is found whose energy-momentum tensor could describe this behavior.

*Permanent address: Laboratoire de Physique Théorique, Centre Universitaire Mustapha Stambouli, 29000 Mascara, Algeria. Email address: boucif@celfi.phys.univ-tours.fr

†Email address: nouri@celfi.phys.univ-tours.fr

II. THE MODEL

We consider a five-dimensional spacetime with coordinates $(x^0 \equiv t, x^1, x^2, x^3, x^4 \equiv y)$, where (t, x^1, x^2, x^3) denotes the usual four-dimensional spacetime and $x^4 \equiv y$ is the coordinate of the fifth dimension, which is an orbifold S^1/Z_2 where the Z_2 action identifies y and $-y$. The five-dimensional indices are denoted by $M,N,\ldots=0,\ldots,4$ and the four-dimensional brane world indices are $i,j,\ldots=0,\ldots,3$. We will neglect the matter interaction and consider the five-dimensional gravitational action

$$S=\int d^5x\sqrt{-G}(\alpha R+\Lambda+\beta L_{\mathrm{GB}}), \tag{2.1}$$

where α and β are two coupling constants and Λ is the five-dimensional cosmological constant. The Gauss-Bonnet Lagrangian density is

$$L_{\mathrm{GB}}=R_{MNPQ}R^{MNPQ}-4R_{MN}R^{MN}+R^2. \tag{2.2}$$

The equations of motion corresponding to our action are[1]

$$\begin{aligned}\mathcal{E}_{MN}\equiv{}&\alpha\left(R_{MN}-\frac{1}{2}g_{MN}R\right)-\frac{1}{2}\Lambda g_{MN}+2\beta\left(R_M^{PQS}R_{NPQS}\right.\\&\left.+2R^{PQ}R_{MPQN}-2R_{MP}R_N^P+RR_{MN}-\frac{1}{4}g_{MN}L_{\mathrm{GB}}\right)\\={}&0.\end{aligned} \tag{2.3}$$

The most important property of these equations is that they reproduce Einstein's equations in four dimensions and contain, in five dimensions, derivatives of the metric of order no higher than 2. This latter fact is crucial in the context of the brane world scenario as it avoids the problem of encountering powers of a delta function.

In contrast to the case of Einstein's equations, the set of equations (2.3) possesses, for a given β and a given Λ, two (anti–)de Sitter solutions [52,53]. This can be seen by considering the simple case of a maximally symmetric spacetime as expressed by

$$R_{MNPQ}=-\sigma(G_{MP}G_{NQ}-G_{MQ}G_{NP}). \tag{2.4}$$

The various curvature terms in Eq. (2.3) are then easily computed in terms of σ and G_{MN} and one obtains the equation

$$[\Lambda-12\sigma(\alpha-2\beta\sigma)]G_{MN}=0. \tag{2.5}$$

Therefore σ takes two possible values as given by

$$\sigma=\frac{1}{4\beta}\left(\alpha\pm\sqrt{\alpha^2-\frac{2}{3}\beta\Lambda}\right). \tag{2.6}$$

This last equation can be inverted to get an expression for Λ

$$\Lambda=12\sigma(\alpha-2\beta\sigma). \tag{2.7}$$

This value of Λ will be needed in the rest of the paper.

Since the aim of this paper is the study of the Randall-Sundrum model supplemented by a Gauss-Bonnet interaction term, it is natural to ask whether interesting solutions, such as Eq. (2.4), still exist in the context of the brane world scenario. As a first step, a class of rich structures can be explored by considering spherically symmetric geometries on the brane. In particular, one would like to know if black hole geometries are present. Furthermore, most of the equations encountered in the spherically symmetric case will be of use when investigating the cosmological solutions. We start, therefore, with a spherically symmetric line element as given by

$$\begin{aligned}ds^2={}&A^2(y)[-N(r)dt^2+M(r)dr^2+r^2(d\theta^2+\sin^2\theta d\phi^2)]\\&+dy^2\end{aligned} \tag{2.8}$$

and examine the equations of motion

$$\mathcal{H}_N^M\equiv\mathcal{E}_N^M+T_N^M=0, \tag{2.9}$$

where the nonzero components of the energy-momentum tensor T_N^M are

$$T_j^i=\delta_j^i\lambda\,\delta(y) \tag{2.10}$$

with λ denoting the cosmological constant on the brane. We obtain four different equations corresponding to the components $\mathcal{H}_0^0$, $\mathcal{H}_1^1$, $\mathcal{H}_2^2=\mathcal{H}_3^3$, and $\mathcal{H}_4^4$. Subtracting $\mathcal{H}_1^1$ from $\mathcal{H}_0^0$ leads to

$$M(r)=\frac{1}{N(r)}. \tag{2.11}$$

Substituting for $M(r)$ and subtracting $\mathcal{H}_2^2$ from $\mathcal{H}_0^0$ yields

$$N(r)=1+\mu r^2+\frac{\nu}{r}, \tag{2.12}$$

where μ and ν are two constants of integration.

After substituting for $N(r)$, the equation corresponding to $\mathcal{H}_4^4=0$ can be cast in the form

$$12(\mu+A'^2)[\alpha A^2-2\beta(\mu+A'^2)]-\Lambda A^4=\frac{12\nu^2\beta}{r^6}. \tag{2.13}$$

Our notation is explained in the footnote below.[2] Since the left-hand side of this equation is independent of r, it is clear that one must have $\nu=0$. This condition, however, is not needed if $\beta=0$ (see, for example, Ref. [3]). With $\nu=0$, the last equation takes then the simple form

[1] Our conventions are such that $R_{NPQ}^M=\partial_P\Gamma_{NQ}^M+\Gamma_{PR}^M\Gamma_{NQ}^R-(P\leftrightarrow Q)$ and $R_{MN}=R_{MQN}^Q$.

[2] Since A is a function of $|y|$ we have $dA/dy=A'(d|y|/dy)$ where A' denotes the derivative of A with respect to its argument $|y|$ and $d|y|/dy=2\Theta(y)-1$ where $\Theta(y)$ is the Heaviside function. Notice that $(d|y|/dy)^2=1$ and we have $(dA/dy)^2=A'^2$. On the other hand $d^2A/dy^2=A''+2A'\delta(y)$ where A'' denotes the second derivative of A with respect to its argument $|y|$.

$$\mu + A'^2 = \sigma A^2, \tag{2.14}$$

where σ is as previously defined. The solution to our last differential equation is

$$A(y) = \frac{\mu}{4\gamma\sigma}\exp(\pm\sqrt{\sigma}|y|) + \gamma\exp(\mp\sqrt{\sigma}|y|). \tag{2.15}$$

Here γ is an integration constant. It is worth recalling that μ is the curvature on the four-dimensional brane.

We are left with one equation to solve, namely, $\mathcal{H}_0^0$

$$6\alpha A(\mu + A'^2) + 6[\alpha A^2 - 4\beta(\mu + A'^2)]\frac{d^2A}{dy^2} - \Lambda A^3 + 2\lambda A^3\delta(y) = 0. \tag{2.16}$$

This equation involves second derivatives of $A(y)$ which generate delta functions as explained in the footnote. Using Eq. (2.14) in $\mathcal{H}_0^0$ and matching the delta functions yields the fine tuning conditions

$$\lambda = -6(\alpha - 4\beta\sigma)\frac{A'(0)}{A(0)}. \tag{2.17}$$

Substituting then for $A(y)$ in Eq. (2.16), fixes the five-dimensional cosmological constant Λ to its original value $\Lambda = 12\sigma(\alpha - 2\beta\sigma)$.

Notice that the warp factor $A(y)$ can present various behaviors depending on the values of σ and μ. Let us start by discussing the case when the curvature on the brane μ vanishes and σ is positive. The warp factor is then given by $A(y) = \gamma\exp(\mp\sqrt{\sigma}|y|)$. If we require now that $A(y)$ decays to zero away from the position of the brane then only the negative sign in the exponential is retained. This requirement is essential for the stability of the solution when the Gauss-Bonnet term is not present [3]. It is possible that the same requirement holds here too. However, a complete proof is much more involved and beyond the scope of the present study. The second case in this discussion corresponds to a nonvanishing μ and a positive σ. Here both factors in the expression of $A(y)$ are present and one has to choose the two constants μ and γ in such a way that $A(y)$ tends to zero as y approaches infinity. Finally, if σ is negative then an oscillatory regime is obtained. It would be interesting to investigate the stability of this last case.

III. COSMOLOGICAL SOLUTIONS

One of the first questions that have been asked in the context of the Randall-Sundrum model is whether significant modifications are brought to the standard cosmological scenario. In the search for interesting brane world cosmological solutions, most of the studies have so far been based on a line element of the form (see, for example, Refs. [30,31])

$$ds^2 = -n^2(t,y)dt^2 + S^2(t,y)\gamma_{ab}(x)dx^a dx^b + b^2(t,y)dy^2, \tag{3.1}$$

where γ_{ab} is a maximally symmetric three-dimensional metric. Notice that this metric, in contrast to the Randall-Sundrum solution, contains three warp factors (though b can be consistently set to 1). Furthermore, the brane metric (found by setting $y=0$) is not explicitly in a standard Friedmann-Robertson-Walker form. It is only after a change of the time coordinate t that it can be brought to the standard form. It is also worth mentioning that by choosing a metric having one warp factor [this can be achieved, for example, by choosing $n(t,y) = A(y)$, $S(t,y) = A(y)h(t)$ and $b(t,y) = 1$] then the only possible solution is a cosmological solution having constant energy density and constant pressure on the brane. The cosmological implication of the above metric will be highlighted below.

It is one of the aims of the present study to show that cosmological solutions with one warp factor are possible. This situation can be realized after a modification of the Randall-Sundrum model by the addition of a Gauss-Bonnet term in the bulk. The metric for this analysis is taken to have the form

$$ds^2 = A^2(y)\left\{-dt^2 + a(t)^2\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]\right\} + dy^2. \tag{3.2}$$

The nonvanishing components of the energy-momentum tensor T_N^M are now given by

$$T_0^0 = -\rho(t)\delta(y),$$
$$T_1^1 = T_2^2 = T_3^3 = p(t)\delta(y). \tag{3.3}$$

By choosing an energy-momentum tensor in this form, we have already assumed that no matter escapes through the fifth dimension.

The equations of motion we would like to solve are still $\mathcal{H}_N^M = \mathcal{E}_N^M + T_N^M = 0$. There are three different types of equations $\mathcal{H}_0^0$, $\mathcal{H}_1^1 = \mathcal{H}_2^2 = \mathcal{H}_3^3$, and $\mathcal{H}_4^4$. The first of these $\mathcal{H}_0^0$ is

$$6[a^2(\alpha A^2 - 4\beta A'^2) + 4\beta(k + \dot{a}^2)]\frac{d^2A}{dy^2} - 6\alpha A(k + \dot{a}^2) + 6\alpha a^2 AA'^2 - \Lambda A^3 a^2 - 2\rho a^2 A^3\delta(y) = 0, \tag{3.4}$$

where $\dot{a}$ is the derivative of a with respect to t. Matching the delta functions in this last equation yields

$$\rho = \frac{6A'(0)}{a^2[A(0)]^3}(a^2\{\alpha[A(0)]^2 - 4\beta[A'(0)]^2\} + 4\beta(k + \dot{a}^2)). \tag{3.5}$$

The second equation $\mathcal{H}_1^1$ is given by the expression

$$2[3a^2(\alpha A^2 - 4\beta A'^2) + 8\beta a\ddot{a} + 4\beta(k + \dot{a}^2)]\frac{d^2A}{dy^2} - 2\alpha A(k + \dot{a}^2) + 6\alpha a^2 AA'^2 - \Lambda a^2 A^3 - 4\alpha a\ddot{a}A + 2pa^2A^3\delta(y) = 0. \tag{3.6}$$

Again, matching the delta functions coming from d^2A/dy^2 and those coming from the energy-momentum tensor gives

$$p=-\frac{2A'(0)}{a^2[A(0)]^3}(3a^2\{\alpha[A(0)]^2-4\beta[A'(0)]^2\}+8\beta a\ddot{a}+4\beta(k+\dot{a}^2)). \tag{3.7}$$

Using the expression of ρ we deduce the following expression for the Hubble parameter:

$$H^2\equiv\left(\frac{\dot{a}}{a}\right)^2=\frac{[A(0)]^3}{24\beta A'(0)}\rho-\frac{k}{a^2}+\left(-\frac{\alpha[A(0)]^2}{4\beta}+[A'(0)]^2\right). \tag{3.8}$$

This is the first Friedmann relation of ordinary cosmology. Similarly, combining the expression of ρ with that corresponding to p, yields the second Friedmann equation

$$\ddot{a}=-\frac{[A(0)]^3}{48\beta A'(0)}(3p+\rho)a+\left(-\frac{\alpha[A(0)]^2}{4\beta}+[A'(0)]^2\right)a. \tag{3.9}$$

Differentiating H^2 and using the expression of $\ddot{a}$ results in the usual conservation equations

$$\dot{\rho}a+3(p+\rho)\dot{a}=0. \tag{3.10}$$

This conservation equation can be easily understood in terms of Bianchi identities. Indeed, a little algebra involving the use of the Bianchi identities of the Riemann tensor shows that $\nabla^M\mathcal{E}_{MN}=0$. Since we are dealing with the equation $\mathcal{E}_{MN}+T_{MN}=0$, we deduce that $\nabla_M T^M_N=0$. Due to the fact that our energy-momentum tensor is such that $T^4_4=0$, the above conservation relation on the brane is reached. This means that there is no flow of matter along the fifth dimension. In conclusion, our gravitational theory with the Gauss-Bonnet term leads to ordinary Friedmann equations.

At this point let us summarize the main result obtained with the metric (3.1) and without a Gauss-Bonnet term [30,31]. In this case, the Hubble parameter takes the form

$$\left(\frac{\dot{a}}{a}\right)^2=\mathcal{C}_1\rho^2+\frac{\mathcal{C}_2}{a^4}+\frac{\mathcal{C}_3}{a^2}+\mathcal{C}_4\Lambda, \tag{3.11}$$

where $\mathcal{C}_1$ and $\mathcal{C}_4$ are related to the square of the five-dimensional gravitational constant, $\mathcal{C}_3$ is proportional to the curvature of the three-dimensional metric γ_{ab}, while $\mathcal{C}_2$ is an integration constant. The radius of the universe $a(t)$ is defined by $a(t)=S(t,y=0)$. Furthermore, one has the usual conservation equation $\dot{\rho}a+3(p+\rho)\dot{a}=0$ which is a consequence of the Bianchi identities.

The most noticible feature of Eq. (3.11) is that the energy density of the brane enters quadratically and the time evolution of the Hubble parameter depends on an effective radiation term $\mathcal{C}_2/a^4$. An attempt to obtain ordinary cosmology is made by choosing $\mathcal{C}_2=0$ and assuming that $\rho=\rho_1+\rho_2$ with ρ_2 much smaller than the constant energy density ρ_1 [30,31]. By neglecting terms involving ρ_2^2 and carrying out some fine-tuning procedures, one can obtain a standard Friedmann-type equation. However, neither the artificial splitting of ρ nor the fine-tuning is justified.

After this digression, let us return now to our set of equations. Once the matching is carried out, we can deal with the two equations $\mathcal{H}^1_1$ and $\mathcal{H}^0_0$ away from $y=0$. Subtracting $\mathcal{H}^1_1$ from $\mathcal{H}^0_0$ then gives

$$(k+\dot{a}^2-a\ddot{a})(\alpha A-4\beta A'')=0. \tag{3.12}$$

If the first factor $(k+\dot{a}^2-a\ddot{a})$ vanishes then an interesting solution is given by

$$a(t)=\frac{k}{4\kappa\tau^2}\exp(\pm\tau t)+\kappa\exp(\mp\tau t), \tag{3.13}$$

where τ and κ are two integration constants.

Upon substituting for $a(t)$ in $\mathcal{H}^4_4$, we obtain

$$12(A'^2-\tau^2)[\alpha A^2-2\beta(A'^2-\tau^2)]-\Lambda A^4=0. \tag{3.14}$$

This equation is exactly the one found in Eq. (2.13) with $\nu=0$ and where μ is replaced by $-\tau^2$. The solution $A(y)$ to this equation is therefore as given in Eq. (2.15) upon replacing μ by $-\tau^2$.

Putting the expression of $a(t)$ in $\mathcal{H}^0_0$ leads to the differential equation

$$6\alpha A(A'^2-\tau^2)+6[\alpha A^2-4\beta(A'^2-\tau^2)]A''-\Lambda A^3=0. \tag{3.15}$$

Again, this equation is that found in Eq. (2.16) away from the position of the brane and where μ is replaced by $-\tau^2$. Therefore, replacing for $A(y)$ in this last differential equation fixes the bulk cosmological constant to be $\Lambda=12\sigma(\alpha-2\beta\sigma)$. We should mention that this solution leads to the following relation between the energy density ρ and the pressure p:

$$p=-\rho=\lambda, \tag{3.16}$$

where λ is as given in Eq. (2.17). The Hubble parameter for this solution is given by

$$H^2=-\frac{k}{a^2}+\tau^2, \tag{3.17}$$

with $a(t)$ as given in Eq. (3.13). Similarly, we find that

$$\frac{\ddot{a}}{a}=\tau^2. \tag{3.18}$$

We distinguish, therefore, two cases. The first corresponds to $\tau^2>0$ and leads to an inflationary regime whenever one of the two exponentials in $a(t)$ dominates. The second situation arises when $\tau^2<0$. In this case the scale factor is given by

$$a(t)=\varepsilon\cos\left(\frac{\sqrt{-k}}{\varepsilon}t+\varphi\right), \tag{3.19}$$

where ε and φ are two real integration constants. The corresponding $A(y)$ is given by Eq. (2.15) where μ is replaced by $-k/\varepsilon^2$. Of course, this solution is valid only when k is negative and describes a repeatedly collapsing universe. The Hubble parameter in this case is $H^2=-k/a^2+k/\varepsilon^2$.

The other solution to $(k+\dot{a}^2-a\ddot{a})=0$ is given by

$$a(t)=\pm\sqrt{-k}t+\delta \tag{3.20}$$

with δ an integration constant. This solution is physical only for k negative. Substituting this expression of $a(t)$ in $\mathcal{H}_4^4$ leads to the differential equation

$$A'^2=\sigma A^2, \tag{3.21}$$

where σ is as defined in Eq. (2.6). Therefore $A(y)$ is given by

$$A(y)=\psi\exp(\sqrt{\sigma}|y|)\quad\text{or}\quad A(y)=\omega\exp(-\sqrt{\sigma}|y|) \tag{3.22}$$

for two integration constants ψ and ω. The solution in Eqs. (3.20) and (3.22) automatically satisfies $\mathcal{H}_0^0$. Furthermore, we have $p=-\rho=\lambda$ and the Hubble parameter is $H^2=-k/a^2$.

Let us now return to the second possibility as allowed by Eq. (3.12), namely, when $(\alpha A-4\beta A'')=0$. This case is of course present only when β is different from zero and we have the solution

$$A(y)=\xi\exp\left(\frac{1}{2}\sqrt{\frac{\alpha}{\beta}}|y|\right)+\theta\exp\left(-\frac{1}{2}\sqrt{\frac{\alpha}{\beta}}|y|\right), \tag{3.23}$$

where ξ and θ are two integration constants. Substituting this solution in $\mathcal{H}_4^4$ leads to

$$\begin{aligned}a^3(3\alpha^2-2\beta\Lambda)\Bigg\{&\Bigg[\xi\exp\left(\frac{1}{2}\sqrt{\frac{\alpha}{\beta}}|y|\right)+\theta\exp\Bigg(-\frac{1}{2}\\&\times\sqrt{\frac{\alpha}{\beta}}|y|\Bigg)\Bigg]^4-6\xi^2\theta^2\Bigg\}-48\ddot{a}[\beta^2(k+\dot{a}^2)+\alpha\beta\xi\theta a^2]\\&-6\xi\theta a[8\alpha\beta(k+\dot{a}^2)+\xi\theta a^2(5\alpha^2+2\beta\Lambda)]\\&=0.\end{aligned} \tag{3.24}$$

It is clear, from the separation of variables, that one must have

$$\Lambda=\frac{3\alpha^2}{2\beta} \tag{3.25}$$

and the above differential equation reduces then to

$$[\beta(k+\dot{a}^2)+\alpha\xi\theta a^2](\beta\ddot{a}+\alpha\xi\theta a)=0. \tag{3.26}$$

Regardless of which factor vanishes first, the solution to this last differential equation takes the form

$$a(t)=\zeta\exp\left(\sqrt{-\frac{\alpha\xi\theta}{\beta}}t\right)+\chi\exp\left(-\sqrt{-\frac{\alpha\xi\theta}{\beta}}t\right), \tag{3.27}$$

where ζ and χ are two integration constants and if the first factor of the differential equation vanishes then

$$\beta k+4\alpha\xi\theta\zeta\chi=0. \tag{3.28}$$

With $\Lambda=(3\alpha^2)/(2\beta)$, the equation corresponding to $\mathcal{H}_0^0$ yields

$$(\alpha A-4\beta A'')[a^2(\alpha A^2-4\beta A'^2)+4\beta(k+\dot{a}^2)]=0 \tag{3.29}$$

and is therefore automatically satisfied (since we are examining the case when $\alpha A-4\beta A''=0$).

If the first factor in Eq. (3.26) vanishes then we have

$$p=-\rho=0. \tag{3.30}$$

The Hubble parameter for this case is

$$H^2=-\frac{k}{a^2}-\frac{\alpha\xi\theta}{\beta}. \tag{3.31}$$

On the other hand if the second factor in Eq. (3.26) vanishes then we have the equation of state

$$p=-\frac{1}{3}\rho=-8\frac{A'(0)}{[A(0)]^3}\frac{1}{a^2}(\beta k+4\alpha\xi\theta\zeta\chi), \tag{3.32}$$

where $A(y)$ is given by Eq. (3.23). The corresponding expression for the Hubble parameter is

$$H^2=-\frac{\alpha\xi\theta}{\beta}\left(1-\frac{4\zeta\chi}{a^2}\right) \tag{3.33}$$

with $a(t)$ as given in Eq. (3.27). In both cases we have

$$\frac{\ddot{a}}{a}=-\frac{\alpha\xi\theta}{\beta}. \tag{3.34}$$

We notice that if $-\alpha\xi\theta/\beta$ is positive then the universe is accelerating; otherwise $a(t)$ oscillates in time.

Let us examine in more detail the solution found here and in particular the case when $-\alpha\xi\theta/\beta$ is positive. If the first factor in Eq. (3.27) vanishes, which requires $\beta k+4\alpha\xi\theta\zeta\chi=0$, then no matter is present on the brane (since $p=-\rho=0$). Nevetheless, the universe is accelerating as $\ddot{a}/a=-\alpha\xi\theta/\beta$. The question to be asked now is what causes this acceleration. The only source of acceleration is a combined effect due to the bulk cosmological constant and the Gauss-Bonnet correction term. This is because neither Λ nor β can be set to zero in our solution. The case corresponding to the vanishing of the second factor in Eq. (3.27) will be investigated below.

IV. DISCUSSION

Among the solutions found in this analysis, special attention should be paid to that described by a time-dependent energy density and pressure as in Eq. (3.32). However, the equations of states $p=-\rho/3$ cannot correspond to an ordinary dust as the pressure p is negative. Let us suppose that the only matter present on the brane is a self-interacting scalar field with energy-momentum tensor

$$T_{ij}=\left[\partial_i\phi\partial_j\phi-\frac{1}{2}g_{ij}[\partial^k\phi\partial_k\phi+V(\phi)]\right]\delta(y), \tag{4.1}$$

where g_{ij} is a Friedmann-Robertson-Walker metric on the brane. If the field ϕ depends only on time then one has

$$\rho=-T_0^0=\frac{1}{2}[\dot{\phi}^2+V]\delta(y),$$

$$p=T_1^1=T_2^2=T_3^3=\frac{1}{2}[\dot{\phi}^2-V]\delta(y). \tag{4.2}$$

The field ϕ is subject to the equations of motion

$$\frac{1}{\sqrt{-g}}\partial_i(\sqrt{-g}\partial^i\phi)-\frac{1}{2}V'=0 \tag{4.3}$$

with V' being the derivative of the potential V with respect to ϕ.

The equation of state $p=-\rho/3$ leads to

$$\dot{\phi}^2=\frac{1}{2}V \quad\Rightarrow\quad V'=4\ddot{\phi} \tag{4.4}$$

while the equation of motion of the scalar field yields

$$\ddot{\phi}+3\frac{\dot{a}}{a}\dot{\phi}=-\frac{1}{2}V'. \tag{4.5}$$

Combining these last two equations results in

$$\frac{\ddot{\phi}}{\dot{\phi}}+\frac{\dot{a}}{a}=0. \tag{4.6}$$

Therefore $\dot{\phi}=\upsilon/a$, for some integration constant υ. Using the expression of $a(t)$ as given in Eq. (3.27), we find that

$$\phi(t)=\frac{\upsilon}{\sqrt{-\dfrac{\alpha\xi\theta\zeta\chi}{\beta}}}\arctan\left[\sqrt{\frac{\zeta}{\chi}}\exp\left(\sqrt{-\frac{\alpha\xi\theta}{\beta}}t\right)\right]+\varpi, \tag{4.7}$$

where ϖ is an integration constant. In order to be able to express the potential V as a function of ϕ, it is convenient to rewrite this last equation in the form

$$\exp\left(\sqrt{-\frac{\alpha\xi\theta}{\beta}}t\right)=\sqrt{\frac{\chi}{\zeta}}\tan\left[\frac{1}{\upsilon}\sqrt{-\frac{\alpha\xi\theta\zeta\chi}{\beta}}(\phi-\varpi)\right]. \tag{4.8}$$

Since the potential is given by $V=2\dot{\phi}^2=2\upsilon^2/a^2$, we find that

$$V(\phi)=\frac{\upsilon^2}{2\zeta\chi}\sin^2\left[\frac{2}{\upsilon}\sqrt{-\frac{\alpha\xi\theta\zeta\chi}{\beta}}(\phi-\varpi)\right]. \tag{4.9}$$

This potential has an infinite number of minima.

As it is known, cosmological scenarios with negative pressure are used in explaining the current acceleration of our universe (see, for example, Ref. [54]). The scalar field providing this pressure is known as quintessence [55]. Similar models are also constructed in the context of brane world cosmology [56–62]. It seems that we have found here another model for quintessence where the scalar field is governed by a very simple potential. However, a word of caution is needed here. Let us recall that in standard cosmology, the equation which determines the acceleration of the universe at later times is given by $\ddot{a}/a=-(4\pi G/3)(\rho+3p)+\lambda/6$, where λ is a cosmological constant. If λ is set to zero (or equivalently absorbed into the definition of ρ and p) then the universe is accelerating for an equation of state $p=s\rho$ with $-1\leqslant s<\frac{1}{3}$. On the other hand, if the equation of state is such that $p=-\frac{1}{3}\rho$ and $\lambda=0$ then $\ddot{a}/a=0$ and the universe is uniformally evolving.

In our case, we have $p=-\frac{1}{3}\rho$, however, $\ddot{a}/a=-\alpha\xi\theta/\beta$. This situation can only be explained by a nonvanishing cosmological constant λ. Indeed, if $\lambda\neq 0$ then the requirement that the universe is accelerating amounts to demanding that $p<\lambda/24\pi G-\rho/3$. When the equation of state $p=-\frac{1}{3}\rho$ is obeyed then the last inequality requires the cosmological constant λ to be positive, and we have $\ddot{a}/a=\lambda/6$. In our case, we have an equation of state for a perfect fluid, with $p=-\frac{1}{3}\rho$, however with a nonvanishing cosmological equal to $\lambda=-6\alpha\xi\theta/\beta$.

Finally we would like to mention that if the cosmological constant λ is equal to zero, then current data seem to favor an equation of state $p=s\rho$ with $-1\leqslant s\leqslant -0.7$. However, no limits on the possible values of s are known to us in the case when the contribution of the cosmological constant is taken into account. It remains as a challenge to decide whether the model presented here could be taken as a serious proposal for explaining the current acceleration of the universe.

ACKNOWLEDGMENTS

One of us (B.A.) would like to thank Professor Peter Forgács for his kind invitation to the University of Tours where this work was carried out. He is also grateful to the members of the group for their kind hospitality. N.M. would like to thank Claude Barrabes, Bruno Boisseau, Bernard Linet and Micheal Volkov for very useful discussions.

[1] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999); **83**, 3370 (1999).
[2] A. Chamblin, S.W. Hawking, and H.S. Reall, Phys. Rev. D **61**, 065007 (2000).
[3] T. Padmanabhan and S. Shankaranarayanan, Phys. Rev. D **63**, 105021 (2001).
[4] I. Giannakis and Hai-cang Ren, Phys. Rev. D **63**, 125017 (2001).
[5] R. Dick, Class. Quantum Grav. **18**, R1 (2001).
[6] D. Birmingham and M. Rinaldi, Mod. Phys. Lett. A **16**, 1887 (2001).
[7] P. Kraus, J. High Energy Phys. **12**, 011 (1999).
[8] C. Gómez, B. Janssen, and P.J. Silva, J. High Energy Phys. **04**, 027 (2000).
[9] M. Singh Modgil, Sukanta Panda, and G. Sengupta, ''Rotating Brane World Black Holes,'' hep-th/0104122.
[10] Parampreet Singh and Naresh Dadhich, Phys. Lett. B **511**, 291 (2001).
[11] I.I. Kogan, S. Mouslopoulos, and A. Papazoglou, Phys. Lett. B **501**, 140 (2001).
[12] N. Kaloper, Phys. Rev. D **60**, 123506 (1999).
[13] S. Nojiri and S.D. Odintsov, Phys. Lett. B **493**, 153 (2000).
[14] N. Deruelle and J. Katz, Phys. Rev. D **64**, 083515 (2001).
[15] S. Kobayashi, K. Koyama, and J. Soda, Phys. Rev. D **65**, 064014 (2002).
[16] L.P. Hořava and E. Witten, Nucl. Phys. **B460**, 506 (1996); **B475**, 94 (1996).
[17] J.E. Kim, B. Kyae, and H.M. Lee, Phys. Rev. D **62**, 045013 (2000); Nucl. Phys. **B582**, 296 (2000); **B591**, 587(E) (2000).
[18] I.P. Neupane, Phys. Lett. B **512**, 137 (2001).
[19] N. Deruelle and T. Doležel, Phys. Rev. D **62**, 103502 (2000).
[20] K.A. Meissner and M. Olechowski, Phys. Rev. D **65**, 064017 (2002).
[21] I. Low and A. Zee, Nucl. Phys. **B585**, 395 (2000).
[22] O. Corradini and Z. Kakushadze, Phys. Lett. B **494**, 302 (2000).
[23] M. Giovannini, Phys. Rev. D **63**, 064011 (2001).
[24] N.E. Mavromatos and J. Rizos, Phys. Rev. D **62**, 124004 (2000).
[25] S. Nojiri and S.D. Odintsov, Phys. Lett. B **484**, 119 (2000); J. High Energy Phys. **07**, 049 (2000).
[26] Y.M. Cho, I.P. Neupane, and P.S. Wesson, Nucl. Phys. **B621**, 388 (2002).
[27] K. Kashima, Prog. Theor. Phys. **105**, 301 (2001).
[28] S. Nojiri, S.D. Odintsov, and S. Ogushi, Phys. Rev. D **65**, 023521 (2002).
[29] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper, and J. March-Russell, Nucl. Phys. **B567**, 189 (2000).
[30] P. Binétruy, C. Deffayet, U. Ellwanger, and D. Langlois, Phys. Lett. B **477**, 285 (2000).
[31] P. Binétruy, C. Deffayet, and D. Langlois, Nucl. Phys. **B565**, 269 (2000).
[32] T. Nihei, Phys. Lett. B **465**, 81 (1999).
[33] C. Csáki, M. Graesser, C. Kolda, and J. Terning, Phys. Lett. B **462**, 34 (1999).
[34] J.M. Cline, C. Grojean, and G. Servant, Phys. Rev. Lett. **83**, 4245 (1999); Phys. Lett. B **472**, 302 (2000).
[35] D.J.H. Chung and K. Freese, Phys. Rev. D **61**, 023511 (2000).
[36] P. Kanti, I.I. Kogan, K.A. Olive, and M. Pospelov, Phys. Lett. B **468**, 31 (1999); Phys. Rev. D **61**, 106004 (2000).
[37] H.B. Kim and H.D. Kim, Phys. Rev. D **61**, 064003 (2000).
[38] E.E. Flanagan, S.-H. Henry Tye, and I. Wasserman, Phys. Rev. D **62**, 024011 (2000).
[39] C. Csáki, M. Graesser, Lisa Randall, and J. Terning, Phys. Rev. D **62**, 045015 (2000).
[40] D. Ida, J. High Energy Phys. **09**, 014 (2000).
[41] M. Cvetič and J. Wang, Phys. Rev. D **61**, 124020 (2000).
[42] S. Mukohyama, T. Shiromizu, and K. Maeda, Phys. Rev. D **62**, 024028 (2000); **63**, 029901(E) (2001).
[43] R. Maartens, D. Wands, B. Bassett, and I. Heard, Phys. Rev. D **62**, 041301(R) (2000).
[44] R. Maartens, V. Sahni, and T.D. Saini, Phys. Rev. D **63**, 063509 (2001).
[45] C. Csáki, J. Erlich, T.J. Hollowood, and J. Terning, Phys. Rev. D **63**, 065019 (2001).
[46] H. Collins and B. Holdom, Phys. Rev. D **62**, 105009 (2000).
[47] C. Barceló and M. Visser, Phys. Lett. B **482**, 183 (2000).
[48] P. Bowcock, C. Charmousis, and R. Gregory, Class. Quantum Grav. **17**, 4745 (2000).
[49] L. Anchordoqui, C. Nuñez, and K. Olsen, J. High Energy Phys. **10**, 050 (2000).
[50] L. Anchordoqui, J. Edelstein, C. Nuñez, S.P. Bergliaffa, M. Schvellinger, M. Trobo, and F. Zyserman, Phys. Rev. D **64**, 084027 (2001).
[51] J. Lesgourgues, S. Pastor, M. Peloso, and L. Sorbo, Phys. Lett. B **489**, 411 (2000).
[52] N. Deruelle and J. Madore, Mod. Phys. Lett. A **1**, 237 (1986).
[53] N. Deruelle and L. Farina-Busto, Phys. Rev. D **41**, 3696 (1990).
[54] P.M. Garnavich *et al.*, Astrophys. J. **509**, 74 (1998).
[55] B. Ratra and P.J.E. Peebles, Phys. Rev. D **37**, 3406 (1988).
[56] B. Chen and F.-L. Lin, Phys. Rev. D **65**, 044007 (2002).
[57] Y.S. Myung, Mod. Phys. Lett. A **16**, 1963 (2001); **16**, 2187 (2001).
[58] G. Huey and J.E. Lidsey, Phys. Lett. B **514**, 217 (2001).
[59] S. Mizuno and K. Maeda, Phys. Rev. D **64**, 123521 (2001).
[60] V. Sahni, M. Sami, and T. Souradeep, Phys. Rev. D **65**, 023518 (2002).
[61] M. Sami, Class. Quantum Grav. **27**, 228 (2001).
[62] A. Feinstein, K.E. Kunze, and M.A. Vázquez-Mozo, Phys. Rev. D **64**, 084015 (2001).