Shortest cut in brane cosmology

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We consider brane cosmology, studying the shortest null path on the brane for photons, and in the bulk for gravitons. We derive the differential equation for the shortest path in the bulk for a $1+4$ cosmological metric. The time cost and the redshifts for photons and gravitons after traveling their respective paths are compared. We consider some numerical solutions of the shortest path equation, and show that there is no shortest path in the bulk for the Randall-Sundrum vacuum brane solution, for the linear cosmological solution of Binétruy *et al.* for $\omega=-1,-2/3$, and for some expanding brane universes.

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I. INTRODUCTION

The possibility of using extra dimensions in order to explain features related to unified field theories was advocated several decades ago by Kaluza and Klein. After dying out for many years the idea was reestablished in the context of supergravity and string theory, especially in the latter, where extra dimensions are required in order that the theory is rendered well defined. Meanwhile, other problems have been posed in the framework of unified theories. One of them is the huge hierarchy between the electroweak scale ($\sim$100 GeV) and the Planck scale ($\sim 10^{19}$ TeV). One possibility to explain that difference is based on the dynamics of supersymmetry, a very beautiful idea that has not, unfortunately, given solutions to these issues. In the usual Kaluza-Klein theory, and also in the modern proposals to deal with extra dimensions, while the $1+3$ (physical) dimensions open up to infinity, the extra dimensions are confined in a region of the size of the Planck length, namely, $\sim 10^{-33}$ cm, staying beyond experimental verification, today or in the near future.

However, it has been shown recently that it is possible to explain the hierarchy between the electroweak and the Planck scales by dimensional reduction without compactifying the extra dimensions. Moreover, the usual $(1+3)$-dimensional Einstein theory of gravity can be reproduced on the macroscopic distance scale [1–5]. This is quite different from the standard approach, in which extra dimensions open up at short distances only, whereas above a certain length scale, physics is effectively described by $1+3$-dimensional theories. Our $1+3$ dimensional Universe would be a three-dimensional brane existing in a higher dimensional theory, thus displaying a certain number of additional dimensions. A further proposal to deal with the additional dimensions is to have them compactified on a submillimeter scale, unifying in a natural way the electroweak and Planck scales [6,7].

The possibility of relaxing the constraints on the size of the extra dimensions is very appealing. Such is the case of the Randall-Sundrum (RS) model [1,2], where the Universe is $(1+4)$-dimensional and the standard model fields are localized on a three-brane embedded in the four-dimensional space. Only gravitational fields can propagate in all four space directions. At the phenomenological length scale the Kaluza-Klein zero modes are responsible for the well-posed Einstein $(1+3)$-dimensional theory of gravity and the excitations provide a correction. Because of the "warp factor" of the brane, a mass scale around that of the Planck mass corresponds to a TeV mass scale in the *visible* brane. This explains the hierarchy problem. The cosmological consequence of this model is also under active investigation [8–20], as well as alternative "asymmetrically warped" [21] and non-Z_2 symmetric cosmological models [22]. All these models lead to new perspectives on many interesting aspects such as the question of the cosmological constant.

The construction of the brane universe can be traced to the study of $E_8\times E_8$ string theory, presumably 11 dimensional, with the field theory limit studied in [15], and where matter fields exist in ten-dimensional branes at the edge of the space-time. The issue of higher dimensionality and its consequences for the early universe have often been discussed in the recent literature [16]. Problems related to higher derivative gravity [17] and the cosmological constant [18] have also been studied, in addition to the AdS conformal field theory correspondence and Cardy formula [19].

In spite of the attractive aspects of the model, causality can be violated, as first noticed in [23] and [24]. We have two choices facing this situation. Either we accept the viewpoint that true causality should be defined by the null geodesics in the $1+4$ universe instead of in the $1+3$ brane spacetime, or we find some mechanism to avoid such a violation on the brane. In the first case, the violation must be negligible in low energy experiments, otherwise, it would have already been found. The question is whether it could be substantial in cosmology. If the answer is positive, it might help in solving the well known horizon problem as discussed in [23] and [24]. In this paper, we consider the following prob-

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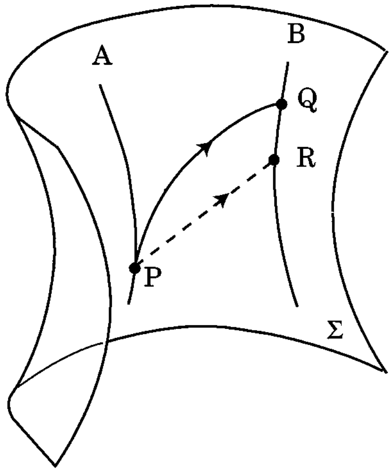

FIG. 1. Two possible paths for massless signal propagation. Solid curve PQ is a null geodesic on the brane Σ and broken line PR is a null geodesic in the bulk (modified from [23]).

lem. Suppose there are two observers A and B on the brane. A can send a series of photons or gravitons to B in order to establish communication (see Fig. 1). According to the brane cosmology, photons travel on the brane while gravitons may travel in the bulk. We consider three questions: (i) what is the shortest path for gravitons, and is it on the brane or in the bulk; (ii) how much earlier can the gravitons arrive at B; and (iii) what is the difference in the redshift for photons and gravitons after they arrive at B.

II. PRELIMINARIES

We shall consider a five-dimensional metric describing brane cosmology. We thus set up a five-dimensional action of the form [9]

$$S^{(5)}=-\frac{1}{2\kappa_5^2}\int d^5x\sqrt{-\tilde{g}}\tilde{R}+\int d^5x\sqrt{-\tilde{g}}\mathcal{L}_m. \tag{1}$$

The constant κ_5 is related to the Planck mass by $\kappa_5^2=M_{Pl}^{-3}$. The five-dimensional metric is

$$ds_5^2=-n^2(t,y)dt^2+a^2(t,y)\gamma_{kj}dx^kdx^j+b^2(\tau,y)dy^2 \tag{2}$$

where γ_{kj} represents a maximally symmetric three-metric. The energy-momentum appearing in the Einstein equation $G_{AB}=\kappa_5^2\mathcal{T}_{AB}$ is decomposed as

$$\mathcal{T}_{AB}=\hat{T}_{AB}+T_{AB} \tag{3}$$

where $\hat{T}_{AB}$ is the energy-momentum tensor of the bulk matter (in the RS scenario it comes from the bulk cosmological constant Λ, that is, $\hat{T}_{AB}=-\Lambda\delta_B^A$) and T_{AB} corresponds to the matter content on the brane located at $y=0$. We are interested in the case where the energy-momentum tensor of the brane matter can be expressed as

$$T_B^A=\frac{\delta(y)}{b}\mathrm{diag}(-\rho-\sigma,p-\sigma,p-\sigma,p-\sigma,0). \tag{4}$$

Here, σ is the brane tension in the RS scenario. The energy density ρ and the pressure p come from the ordinary matter on the brane and are independent of the position. Assuming $\mathbf{Z}_2$ symmetry and $\sigma=0$, the Einstein equation permits the following exact cosmological brane solution [9] (corresponding to $\Lambda=0$, $\sigma=0$, $\gamma_{jk}=\delta_{jk}$):

$$\begin{aligned}a&=a_0(t)(1+\lambda|y|),\\ n&=n_0(t)(1+\mu|y|),\\ b&=b_0,\end{aligned} \tag{5}$$

where b_0 is constant in time (a redefinition of y makes it 1) and $n_0(t)$ is an arbitrary function (a suitable redefinition of t fixes it to be 1). In the above,

$$\lambda=-\frac{\kappa_5^2}{6}b_0\rho, \tag{6}$$

$$\mu=\frac{\kappa_5^2}{2}\left(\omega+\frac{2}{3}\right)b_0\rho, \tag{7}$$

where κ_5^2 is related to the five-dimensional Newton constant G_5 by $\kappa_5^2=8\pi G_5$, and the matter equation of state is $p=\omega\rho$ as usual.

For $\omega=-1$, we have the inflationary case

$$a_0(t)=e^{Ht},\quad H=\frac{\kappa_5^2}{6}\rho=\text{const}, \tag{8}$$

while for $\omega\neq-1$, the known solution for a tensionless brane is recovered;

$$a_0=t^q,\quad \kappa_5^2\rho=\frac{6q}{t},\quad q=\frac{1}{3(1+\omega)}. \tag{9}$$

Remarkably, the exact solution in the RS model can also be obtained [25]. Note that the parameters ρ_b and p_b in [25] are related to the corresponding ones here in this paper by the relations $\rho_b=\rho+\sigma$, $p_b=p-\sigma$. The solution can be written in terms of the function

$$\begin{aligned}a(t,y)=\Bigg\{&\frac{1}{2}\left(1+\frac{\kappa_5^2(\sigma+\rho)^2}{6\Lambda}\right)a_0^2+\frac{3\mathcal{C}}{\kappa_5^2\Lambda a_0^2}\\ &+\left[\frac{1}{2}\left(1-\frac{\kappa_5^2(\sigma+\rho)^2}{6\Lambda}\right)a_0^2-\frac{3\mathcal{C}}{\kappa_5^2\Lambda a_0^2}\right]\\ &\times\cosh(\mu y)-\frac{\kappa_5(\sigma+\rho)}{\sqrt{-6\Lambda}}a_0^2\sinh(\mu|y|)\Bigg\}^{1/2}.\end{aligned} \tag{10}$$

We now construct the remaining function

$$n(t,y)=\frac{\dot{a}(t,y)}{\dot{a}_0(t)}. \tag{11}$$

As for Eq. (33) in [25], we also have

$$\dot{\rho}+3\frac{\dot{a}_0}{a_0}(\rho+p)=0. \tag{12}$$

Defining

$$\lambda=\sqrt{\frac{\Lambda}{6\kappa_5^2}+\frac{\sigma^2}{36}}, \tag{13}$$

and assuming $\lambda\geqslant 0$ and $p=\omega\rho$, the Friedmann equation can be solved in the case $\mathcal{C}=0$, $k=0$. For $\lambda>0$,

$$a_0(t)=a_\star\rho_\star^q\left\{\frac{\sigma}{36\lambda^2}[\cosh(\kappa_5^2\lambda t/q)-1]+\frac{1}{6\lambda}\sinh(\kappa_5^2\lambda t/q)\right\}^q. \tag{14}$$

For $\lambda=0$, which is the case of the RS model,

$$a_0(t)=a_\star(\kappa_5^2\rho_\star)^q\left(\frac{1}{72q^2}\kappa_5^2\sigma t^2+\frac{1}{6q}t\right)^q \tag{15}$$

where $a_\star,\rho_\star$ are constant [the origin of time being chosen so that $a_0(0)=0$].

III. THE SHORTEST CUT AND THE REDSHIFT

Equation for the shortest cut

We consider the generic metric (2) for $b=1$. Consider two points r_A and r_B on the brane. In general, there is more than one null geodesic connecting r_A to r_B in the $1+4$ space-time. The trajectories of photons must be on the brane and those of gravitons may be outside as assumed here. We consider the shortest path for both photons and gravitons. Since the three-metric is spherically symmetric, we can omit the angular part and just consider the problem for

$$ds_3^2=-n^2(t,y)dt^2+a^2(t,y)f^2(r)dr^2+dy^2. \tag{16}$$

The photon path is on the brane $[n(t,0)=1]$; therefore,

$$-dt^2+a_0^2(t)f^2(r)dr^2=0, \tag{17}$$

which can be immediately integrated as

$$\int_{r_A}^{r}f(r')dr'=\int_{t_A}^{t}\frac{dt'}{a_0(t')}. \tag{18}$$

The graviton path is defined in terms of the geodesic equation

$$-n^2(t,y)dt^2+a^2(t,y)f^2(r)dr^2+dy^2=0. \tag{19}$$

We suppose that the path is parametrized by $y=y(t)$. Thus the relation $r=r(t)$ is obtained by

$$\int_{r_A}^{r}f(r')dr'=\int_{t_A}^{t}\frac{\sqrt{n^2(t,y)-\dot{y}^2(t)}}{a(t,y)}dt. \tag{20}$$

We are looking for the path for which t_B reaches its minimum when $r=r_B$. For this purpose, we consider the general case

$$\int_{r_A}^{r_B}f(r')dr'=\int_{t_A}^{t_B}\mathcal{L}[y(t),\dot{y}(t);t]dt. \tag{21}$$

For an adjacent path $y=y(t)+\delta y(t)$, we have

$$\int_{r_A}^{r_B}f(r')dr'=\int_{t_A}^{t_B+\delta t_B}\mathcal{L}[y(t)+\delta y(t),\dot{y}(t)+\delta\dot{y}(t);t]dt; \tag{22}$$

therefore we find the usual condition

$$-\delta t_B\mathcal{L}[y(t_B),\dot{y}(t_B);t_B]=\delta\int_{t_A}^{t_B}\mathcal{L}[y(t),\dot{y}(t);t]dt. \tag{23}$$

The problem is transformed into the Euler-Lagrange problem

$$\delta\int_{t_A}^{t_B}\mathcal{L}[y(t),\dot{y}(t);t]dt=0. \tag{24}$$

In our case,

$$\mathcal{L}[y(t),\dot{y}(t);t]=\frac{\sqrt{n^2(t,y)-\dot{y}^2(t)}}{a(t,y)}, \tag{25}$$

and we have

$$\frac{\partial\mathcal{L}}{\partial y}=-a^{-2}a'(n^2-\dot{y}^2)^{1/2}+a^{-1}(n^2-\dot{y}^2)^{-1/2}nn',$$

$$\frac{\partial\mathcal{L}}{\partial\dot{y}}=-a^{-1}(n^2-\dot{y}^2)^{-1/2}\dot{y}. \tag{26}$$

The Euler-Lagrange equation thus reads

$$-\ddot{y}+\left(\frac{\dot{a}}{a}+\frac{\dot{n}}{n}\right)\dot{y}+\left(\frac{2n'}{n}-\frac{a'}{a}\right)\dot{y}^2-\frac{\dot{a}}{an^2}\dot{y}^3+\left(\frac{a'}{a}n^2-nn'\right)=0. \tag{27}$$

From this equation we can see that the shortest path is on the brane only when

$$\frac{a'}{a}n^2-nn'=0, \tag{28}$$

i.e.,

$$\partial_y\left(\frac{a}{n}\right)=0. \tag{29}$$

Further, if there exists a solution, when y reaches its maximum, where $\dot{y}=0$ and $\ddot{y}<0$, we have

$$-\ddot{y}+\left(\frac{a'}{a}n^2-nn'\right)=0. \tag{30}$$

Thus, $(a'/a)n^2-nn'$, i.e., $\partial_y(an^{-1})$ must be negative at this point.

The equation is a very difficult nonlinear ordinary differential equation. There is no guarantee of the existence of the required solutions. In order to obtain a solution with both ends on the brane, we can make the Fourier expansion

$$y(t)=\sum_{l=1}^{+\infty} y_l \sin\frac{l\pi}{t_{gB}-t_A}(t-t_A), \tag{31}$$

$$a(t,y)=A(y)+\sum_{l=1}^{+\infty}\left[a_l^s(y)\sin\frac{l\pi}{t_{gB}-t_A}(t-t_A)+a_l^c(y)\cos\frac{l\pi}{t_{gB}-t_A}(t-t_A)\right], \tag{32}$$

$$n(t,y)=N(y)+\sum_{l=1}^{+\infty}\left[n_l^s(y)\sin\frac{l\pi}{t_{gB}-t_A}(t-t_A)+n_l^c(y)\cos\frac{l\pi}{t_{gB}-t_A}(t-t_A)\right], \tag{33}$$

and then substitute back into the differential equation to obtain the coefficients y_l. Here t_{gB} is the time when the graviton arrives at r_B, which is different from the time $t_{\gamma B}$ when the photon arrives at r_B. It should be determined self-consistently by the equation

$$\int_{r_A}^{r_B} f(r')dr'=\int_{t_A}^{t_{gB}}\frac{\sqrt{n^2(t,y)-\dot{y}^2(t)}}{a(t,y)}dt \tag{34}$$

once the solution is obtained.

If we want to find the path for a graviton so that it can reach the farthest distance within a given time interval $[t_A,t_B]$, we can also use the Euler-Lagrange equation. Then the length difference between geodesics for photons and gravitons within a given time interval can be evaluated:

$$\int_{r_A}^{r_g} f(r')dr'=\int_{t_A}^{t_B}\frac{\sqrt{n^2(t,y)-\dot{y}^2(t)}}{a(t,y)}dt, \tag{35}$$

$$\int_{r_A}^{r_\gamma} f(r')dr'=\int_{t_A}^{t_B}\frac{dt'}{a_0(t')}. \tag{36}$$

Photon and graviton redshift

In general, if A sends out massless signals at x_A^μ and $x_A^\mu+dx_A^\mu$, these signals will reach B at x_B^μ and $x_B^\mu+dx_B^\mu$. The relation of $x_A^\mu, x_A^\mu+dx_A^\mu$ and $x_B^\mu, x_B^\mu+dx_B^\mu$ can be obtained by solving the geodesic equation. Then the redshift of the signal is [26]

$$\frac{\nu_B}{\nu_A}=\sqrt{\frac{g_{00}(x_B)}{g_{00}(x_A)}}\frac{g_{0\mu}(x_A)dx_A^\mu}{g_{0\nu}(x_B)dx_B^\nu}=\sqrt{\frac{g_{00}(x_A)}{g_{00}(x_B)}}\frac{dx_A^0}{dx_B^0}. \tag{37}$$

For a static metric such as the Schwarzschild case, it can be shown that $dx_A^0=dx_B^0$; therefore,

$$\frac{\nu_B}{\nu_A}=\sqrt{\frac{g_{00}(x_A)}{g_{00}(x_B)}}. \tag{38}$$

For the time-dependent RW metric we have

$$\frac{dx_A^0}{dx_B^0}=\frac{R(x_A^0)}{R(x_B^0)}, \tag{39}$$

in which case the redshift is given by

$$\frac{\nu_B}{\nu_A}=\frac{R(x_A^0)}{R(x_B^0)}. \tag{40}$$

Thus, in the geometric-optics limit, the redshifts in the two cases can be systematically discussed.

Here, we consider that another graviton starts traveling from r_A at a later time $t_A+\delta t_A$. Its shortest path is in general different from the previous one. Let us denote it as $y_*=y_*(t)$. Then the time when it arrives at r_B will be a later time $t_{gB}+\delta t_{gB}$:

$$\int_{r_A}^{r_B} f(r')dr'=\int_{t_A+\delta t_A}^{t_{gB}+\delta t_{gB}}\frac{\sqrt{n^2(t,y_*)-\dot{y}_*^{\,2}(t)}}{a(t,y_*)}dt. \tag{41}$$

Therefore we have the equality

$$\int_{t_A}^{t_{gB}}\frac{\sqrt{n^2(t,y)-\dot{y}^2(t)}}{a(t,y)}dt=\int_{t_A+\delta t_A}^{t_{gB}+\delta t_{gB}}\frac{\sqrt{n^2(t,y_*)-\dot{y}_*^2(t)}}{a(t,y_*)}dt. \tag{42}$$

For infinitesimal dt_A and dt_B, we have

$$dt_B\left(\frac{\sqrt{n^2(t,y)-\dot{y}^2(t)}}{a(t,y)}\right)\Bigg|_B=dt_A\left(\frac{\sqrt{n^2(t,y)-\dot{y}^2(t)}}{a(t,y)}\right)\Bigg|_A. \tag{43}$$

Thus, the graviton redshift is given by

$$\frac{\nu_{gB}}{\nu_{gA}}=\frac{a_0(t_A)}{a_0(t_B)}\sqrt{\frac{1-\dot{y}^2(t_B)}{1-\dot{y}^2(t_A)}}, \tag{44}$$

while for the photon we have

$$\frac{\nu_{gB}}{\nu_{gA}}=\frac{a_0(t_A)}{a_0(t_B)}. \tag{45}$$

IV. EXAMPLES

RS vacuum solution

In this case [1,2]

$$n(y,t)=a(y,t)=e^{-k|y|}. \tag{46}$$

Equation (27) turns out to be

$$\ddot{y}+k\dot{y}^2=0. \tag{47}$$

It has two possible solutions; one is $y=y_A=0$, and the other is $y=y_0+k^{-1}\ln(t-t_0)$. The second solution does not meet our requirement because it will not end on the brane. So the shortest path must be on the brane. This agrees with the conclusion in [23].

The linear cosmological solution

We first consider the case $\omega=-\frac{2}{3}$ so that from Eq. (7) $\mu=0$, $a(t,y)=t-y$, $\lambda=-1/t$. The equation is

$$-(t-y)\ddot{y}+\dot{y}+\dot{y}^2-\dot{y}^3-1=0. \tag{48}$$

Let $t-y=u$; then

$$u\ddot{u}+\dot{u}^3-2\dot{u}^2=0, \tag{49}$$

or

$$\frac{1}{2\dot{u}^2-\dot{u}^3}\frac{d}{dt}\dot{u}^2=\frac{2\dot{u}}{u}. \tag{50}$$

Therefore,

$$\int\frac{d\dot{u}}{2\dot{u}-\dot{u}^2}=\int\frac{du}{u}, \tag{51}$$

$$\frac{\dot{u}}{2-\dot{u}}=cu^2. \tag{52}$$

We can obtain the solution (t_0 and c are two integration constants)

$$y=t_0\pm\sqrt{(t-t_0)^2+\frac{1}{c}}. \tag{53}$$

It is obvious that this path cannot end on the brane either. Furthermore, we consider the case $\omega=-1$, $\lambda=\mu=\text{const}\times a_0(t)=e^{Ht}$. So $\partial_y(a/n)=0$. Therefore the shortest path is on the brane.

The general linear cosmological solution

Consider the case $\omega\neq-1$ [9]:

$$a_0(t)=t^q,\quad \lambda=-\frac{q}{t},\quad \mu=w\frac{q}{t},\quad w=2+3\omega, \tag{54}$$

$$a(t,y)=t^q-qt^{q-1}y,\quad n(t,y)=1+\frac{q\omega}{t}y, \tag{55}$$

$$\dot{a}(t,y)=qt^{q-1}-q(q-1)t^{q-2}y,\quad a'(t,y)=-qt^{q-1}, \tag{56}$$

$$\dot{n}(t,y)=-q\omega t^{-2}y,\quad n'(t,y)=q\omega t^{-1}. \tag{57}$$

Letting $y=tf(t)$ in Eq. (27), we get a nonlinear differential equation,

$$\begin{aligned}&-[1+(2q\omega-q)f+(q^2\omega^2-2q^2\omega^2)f^2-q^3\omega^2f^3](t^2\ddot{f}+2t\dot{f})+[q+(2\omega q^2-q^2+q-q\omega)f+(q^2\omega-q^2\omega^2-2\omega q^3\\&+2\omega q^2)f^2+(2q^3\omega^2-q^4\omega^2)f^3](t\dot{f}+f)+[2q\omega-q)+q^3\omega^2f^2](t\dot{f}+f)^2-[q-q(q-1)f](t\dot{f}+f)^3+[(-q-q\omega)\\&+(q^2-4q^2-3q^2\omega^2)f+(3q^3\omega-6q^3\omega^2-3q^3\omega^3)f^2+(-2q^4\omega^3+3q^4-q^4\omega^4)f^3+(q^5\omega^3-q^5\omega^4)f^4]=0.\end{aligned} \tag{58}$$

The analysis of this differential equation is beyond our capability. We leave it as it stands and pass to a discussion of some simple cases where numerical analysis can be performed.

The case considered by Binétruy *et al.* [25] is that of a three-brane universe in the five-dimensional space-time with a cosmological constant. For an equation of state $p=\omega\rho$ they found explicit solutions which we use in order to study the question of the existence of shortcuts. The solutions are very involved, and we first disentangle the equations using a MAPLE program, and further on numerically solve the differential equations. We shall consider the matter dominated ($\omega=0$) and radiation dominated ($\omega=1/3$) cases.

The solution of the gravity equations reads [25]

$$a(t,y)=\left\{\frac{1}{2}\left(1+\frac{\kappa^2\rho_b^2}{6\rho_B}\right)+\frac{1}{2}\left(1-\frac{\kappa^2\rho_b^2}{6\rho_B}\right)\cosh(\mu y)-\frac{\kappa\rho_b}{\sqrt{-6\rho_B}}\sinh(\mu|y|)\right\}^{1/2}a_0(t),$$

$$n(t,y)=\frac{\dot{a}(t,y)}{\dot{a}_0(t)}, \tag{59}$$

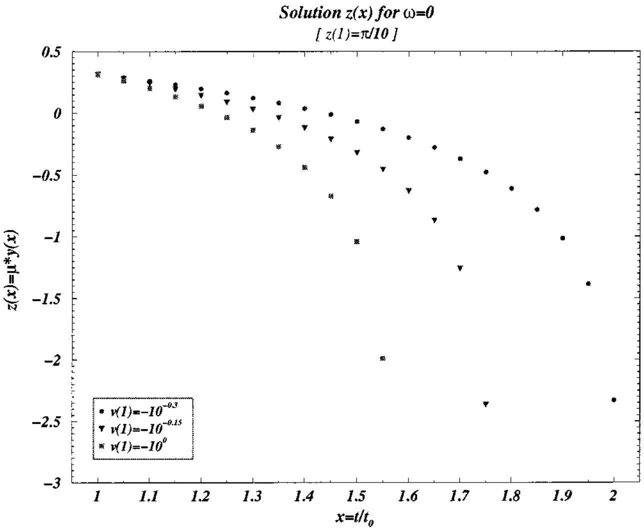

FIG. 2. Diagram for $y(1) \sim 0.3 l_P$.

where

$$a_0(t) = a_\star (\kappa^2 \rho_\star)^{1/q} \left(\frac{q^2}{72} \kappa^2 \rho_\Lambda t^2 + \frac{q}{6} t \right)^{1/q},$$

$$\mu = \sqrt{-\frac{2\kappa^2}{3} \rho_B}, \tag{60}$$

with $a_\star$ and $\rho_\star$ constants.

In addition, ρ_b and ρ_B are the matter densities on the brane and on the bulk, respectively. We have to choose these constants, which we do according to the method we are using to discuss the possibilities of shortcuts. We choose the parameters according to the discussion in Binétruy *et al.* [9]:

$$\rho_b = \rho_\Lambda + \rho, \tag{61}$$

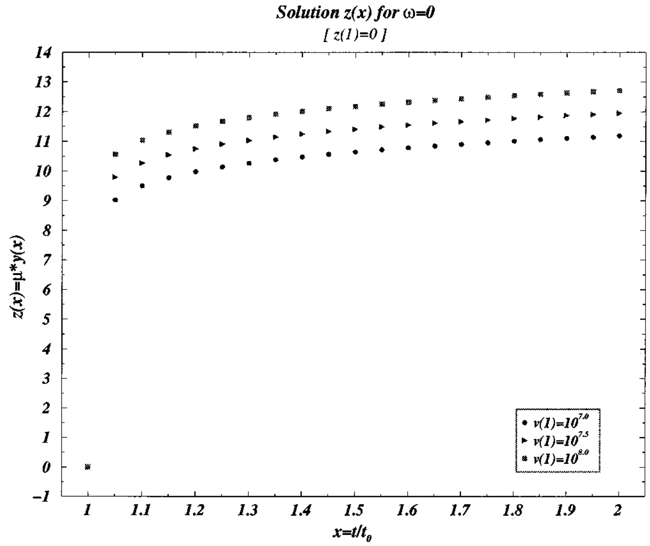

FIG. 4. The same as before, with positive initial velocity.

where ρ stands for the ordinary energy density in cosmology given by

$$\rho = \rho_\star (a_0 / a_\star)^{-q}, \quad q = 3(1+\omega). \tag{62}$$

The intrinsic tension of the brane, ρ_Λ, has to be identified with Newton's constant in order to recover the standard cosmology, that is,

$$8\pi G = \frac{\kappa^4 \rho_\Lambda}{6}, \tag{63}$$

when $\rho \ll \rho_\Lambda$. Moreover, the five-dimensional coupling constant κ, the five-dimensional Newton constant $G_{(5)}$, and the Planck mass $M_{(5)}$ are related by

$$\kappa^2 = 8\pi G_{(5)} = M_{(5)}^{-3}. \tag{64}$$

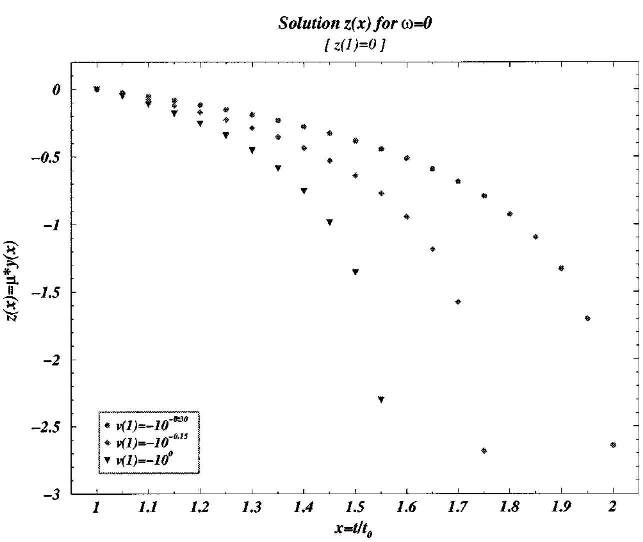

FIG. 3. The same diagram as before, with y beginning at the brane.

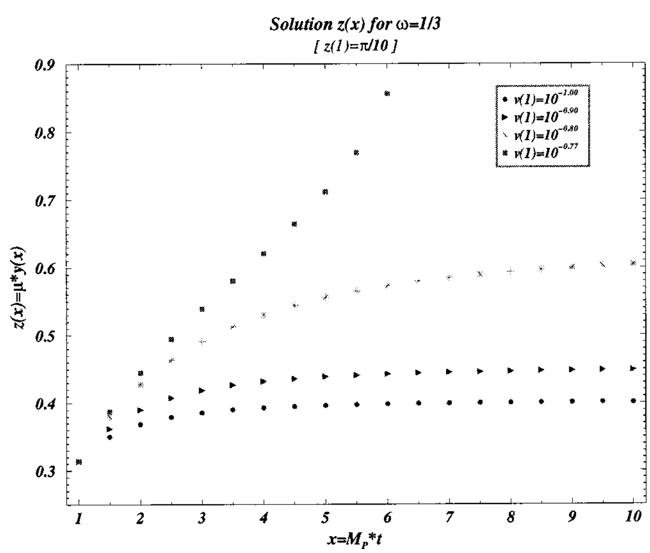

FIG. 5. Diagram for $y(1) \sim 0.3 l_P$ in the radiation dominated case. Notice the plateau followed by the case of lowest initial velocity.

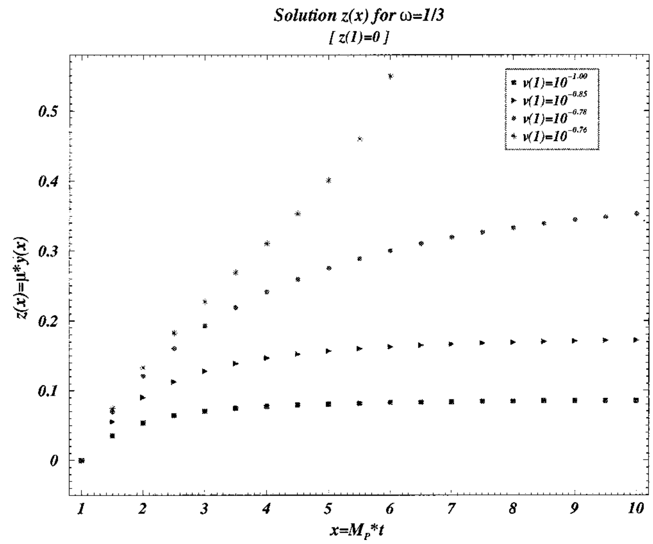

FIG. 6. The same as before, with vanishing initial position with respect to the brane.

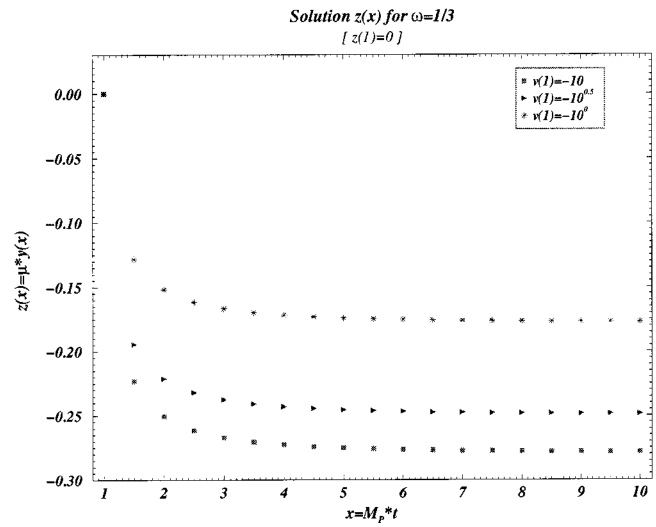

FIG. 7. The same as before, with negative initial velocity.

Furthermore, we follow Randall and Sundrum and relate the bulk energy density ρ_B and the cosmological constant density ρ_Λ by

$$\rho_B = -\frac{\kappa^2 \rho_\Lambda^2}{6}. \tag{65}$$

At this point all constants are defined in terms of the Planck mass, and our discussion of the evolution of gravity signs can be established.

For the matter dominated case, $\omega = 0$, we experimented using different initial conditions. In general, we prefer to start with $y \neq 0$ in order to avoid any spurious solution in the differential equation, which is rather singular. We thus suppose that y starts at the order of the Planck length. Figures 2–4 show some results. We have chosen to plot the adimensional function $z(x) = \mu y(x)$, where μ corresponds to two Planck mass units M_P and $x = t/t_0$, t_0 being the present age of the universe.

Each graph contains a set of curves corresponding to three typical velocities, whose values are shown in the legend of each graph, producing similar behaviors. In Figs. 2 and 3 we use negative initial velocities and, independently of the chosen initial point y, the curve decays and escapes, never returning to the same brane. In the case of positive initial velocities, Fig. 4 shows three curves from which we can see that the greater the initial velocity is, the further away from the brane the object will travel.

Summarizing, these graphs show that no path comes back to the brane after splitting off further inside the bulk. After the split, some paths go off quickly, while others remain almost parallel to the brane for an indefinite time.

We thus believe, based on these results, that the shortest path is inside the brane, and is the one followed by light. However, there is certainly room for more paths due to the extremely complicated character of the differential equation involved in the problem. We tried to further investigate whether new structures could arise in such differential equations, but we failed in finding them. The fact is that no shortcut has been found.

In the radiation dominated era the equations are further complicated. In this case solutions are shown in Figs. 5–7. Again, we have plotted the adimensional function $z(x)$, where $x = M_P t$ in this case.

Figures 5 and 6 show a plateau behavior for low positive initial velocities; however, there is a threshold velocity for which the curve decouples and escapes to infinity. Figure 7 shows curves for three negative initial velocities. Again, the wave tries to follow the brane from a distance depending on the initial velocity value, as we saw in the matter dominated case.

Thus, in the radiation dominated era, $\omega = \frac{1}{3}$, many solutions tend to remain not far from the original bulk, but still, never returning. Their meaning is not known, and again no shortcut has been found.

We thus arrive at the conclusion that the present cosmology is still simple, and is included in the large class of cosmologies with no shortcuts. Our investigation must now head toward cosmologies displaying genuinely new structures, such as black holes in the bulk [21,20].

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[1] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).
[2] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).
[3] J. Garriga and T. Tanaka, Phys. Rev. Lett. **84**, 2778 (2000).
[4] R. Gregory, V. A. Rubakov, and S. M. Sibiryakov, Phys. Rev. Lett. **84**, 5928 (2000).
[5] C. Csáki, J. Erlich, and T. J. Hollowood, Phys. Rev. Lett. **84**, 5932 (2000).
[6] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998).
[7] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **436**, 257 (1998).
[8] E. Witten, in *Marina del Rey 2000*, *Sources and detection of dark matter and dark energy in the universe*, 27-36, hep-th/0002297.
[9] P. Binétruy, C. Deffayet, and D. Langlois, Nucl. Phys. **B565**, 269 (2000).
[10] J. M. Cline, C. Grojean, and G. Servant, Phys. Rev. Lett. **83**, 4245 (1999).
[11] C. Csáki, M. Graesser, C. Kolda, and J. Terning, Phys. Lett. B **462**, 34 (1999).
[12] S.-H. Henry Tye and I. Wasserman, Phys. Rev. Lett. **86**, 1682 (2001).
[13] D. Langlois, Phys. Rev. Lett. **86**, 2212 (2001).
[14] V. Barger, T. Han, T. Li, J. D. Lykken, and D. Marfatia, Phys. Lett. B **488**, 97 (2000).
[15] P. Horava and E. Witten, Nucl. Phys. **B460**, 506 (1996); **B475**, 94 (1996).
[16] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper, and J. March-Russell, Nucl. Phys. **B567**, 189 (2000); N. Kaloper and A. Linde, Phys. Rev. D **60**, 103509 (1999); E. Abdalla and L. Alejandro Correa-Borbonet, Phys. Lett. B **489**, 383 (2000).
[17] S. Nojiri and S. D. Odintsov, J. High Energy Phys. **07**, 049 (2000).
[18] S. Nojiri, O. Obregon, and S. D. Odintsov, hep-th/0105300.
[19] Bin Wang, Elcio Abdalla, and Ru-Keng Su, hep-th/0106086.
[20] C. Csáki, M. Graesser, L. Randall, and J. Terning, Phys. Rev. D **62**, 045015 (2000).
[21] C. Csáki, J. Erlich, and C. Grojean, Nucl. Phys. **B604**, 312 (2001).
[22] A. Davis, C. Rhodes, and I. Vernon, J. High Energy Phys. **11**, 015 (2001).
[23] H. Ishihara, Phys. Rev. Lett. **86**, 381 (2001).
[24] R. Caldwell and D. Langlois, Phys. Lett. B **511**, 129 (2001).
[25] P. Binétruy, C. Deffayet, U. Ellwanger, and D. Langlois, Phys. Lett. B **477**, 285 (2000).
[26] S. S. Feng and C. G. Huang, Int. J. Theor. Phys. **36**, 1179 (1997).