

Synergistic warm inflation

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We consider an alternative warm inflationary scenario in which n scalar fields coupled to a dissipative matter fluid cooperate to produce power-law inflation. The scalar fields are driven by an exponential potential and the bulk dissipative pressure coefficient is linear in the expansion rate. We find that the entropy of the fluid attains its asymptotic value in a characteristic time proportional to the square of the number of fields. This scenario remains nearly isothermal along the inflationary stage. The perturbations in energy density and entropy are studied in the long-wavelength regime and seen to grow roughly as the square of the scale factor. They are shown to be compatible with COBE measurements of the fluctuations in temperature of the CMB.

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I. INTRODUCTION

The very early Universe was supposedly populated by a host of scalar fields but soon only one came to dominate the dynamics; the others settled in the minimum of its potential. Of particular interest from the point of view of cosmological inflation are scalar fields with exponential potentials since these are natural candidates to drive power-law inflation in Friedmann-Lemaître-Robertson-Walker (FLRW) universes, i.e., $a(t) \propto t^\alpha$ where $a(t)$ is the scale factor and $\alpha = \text{const} > 1$ [1].

As is well known, scalar fields possessing exponential potentials appear naturally in different theories of fundamental physics as superstrings and higher dimensional theories [2], in $N=2$ supergravity [3], as well as in theories undergoing dimensional reduction to an effective four-dimensional theory [4]. However, in many cases they happen to be too steep and fail to do the job as $\alpha \leq 1$. Nevertheless, it has been shown that if one considers n noninteracting scalar fields with exponential potentials they can cooperate to achieve power-law inflation in spite of the fact that no single field can achieve it by itself [5]. The rationale behind it is that while each field descends toward the minimum of its potential the cosmic expansion rate (to which all the fields cooperate) acts as a friction force upon each of them. This result has been extended to Bianchi type I and VII₀ cosmologies as well [6]. A further interesting feature is that the larger n is, the closer the resulting spectrum of initial cosmic perturbations is to that of Harrison and Zeldovich. The proponents of this scenario termed it “assisted inflation;” however we find it more fitting to call it “synergistic inflation,” and we shall do so henceforth.

A more realistic problem arises when in addition to the scalar fields one considers a matter fluid. The price to be paid is a rather involved set of field equations even in the simplest case when the fluid interacts with the scalar fields only gravi-

tationally. In a recent paper Coley and van den Hoogen qualitatively analyzed the autonomous system of two scalar fields of the kind discussed above in a curved FLRW universe and showed that the system has an equilibrium point compatible with a stable phase of power-law inflation. This feature persists even if a perfect fluid with baryotropic equation of state is allowed in the picture [7]. Again the authors assumed that the fields do not interact among themselves nor with the matter fluid.

At first sight the presence of a matter fluid (such as a sea of relativistic particles) may be seen as rather irrelevant because the fast inflationary expansion will very soon dilute away these particles. However, as shown by Berera [8] if a coupling between the inflaton field and the matter fluid is assumed, things change drastically to the point that when inflation ends the Universe is far from being as cool as predicted by ordinary inflation models. In fact it can be hot enough to resume the radiation dominated phase of standard big-bang cosmology; thus the reheating phase is no longer necessary.

The main target of this paper is to study the scenario of warm inflation with n scalar fields having exponential potentials and interacting with the matter fluid. Section II considers first the more simple case of a single scalar field plus a dissipative matter fluid. Then the analysis is extended to n scalar fields. In Sec. III the scalar fields are allowed to decay into the matter fluid thereby the temperature of the latter does not fall drastically and so the warm inflationary phase can be followed by the conventional radiation dominated period without any intermediate reheating. Section IV studies the perturbations in energy density and entropy brought about by warm inflation; they do not conflict with the observed temperature anisotropies of the cosmic microwave background radiation. Finally, Sec. V summarizes our main findings. Units have been chosen so that $c = 8\pi G = 1$.

II. THE SYNERGISTIC MECHANISM

Let us assume a FLRW universe filled with a self-interacting scalar field plus a dissipative matter fluid. The

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stress-energy tensor of this mixture is

$$T_{ab} = (\rho + p + \pi)u_a u_b + (p + \pi)g_{ab}, \quad (1)$$

where $\rho = \rho_m + \rho_\phi$ and $p = p_m + p_\phi$. Here ρ_m and p_m are the energy density and pressure of the matter fluid with equation of state given by $p_m = (\gamma_m - 1)\rho_m$ and with its baryotropic index in the interval $1 \leq \gamma_m \leq 2$. Likewise ρ_ϕ and p_ϕ , the energy density and pressure of the minimally coupled self-interacting field ϕ , i.e.,

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi), \quad (2)$$

are related by an equation of state similar to that of the matter, namely $p_\phi = (\gamma_\phi - 1)\rho_\phi$, so that its baryotropic index is given by

$$\gamma_\phi = \frac{\dot{\phi}^2}{(\dot{\phi}^2/2) + V(\phi)}, \quad (3)$$

where for non-negative potentials $V(\phi)$ one has $0 \leq \gamma_\phi \leq 2$, and an overdot means derivative with respect to cosmic time. In general γ_ϕ varies as the Universe expands, and the same is true for γ_m since the massive and massless components of the matter fluid redshift at different rates.

The Friedmann equation together with the energy conservation of the normal matter fluid with bulk dissipative pressure and Klein-Gordon equation can be written as

$$\Omega_m + \Omega_\phi + \Omega_K = 1, \quad (K = 1, 0, -1), \quad (4)$$

$$\dot{\rho}_m + 3H \left(\gamma_m + \frac{\pi}{\rho_m} \right) \rho_m = 0, \quad (5)$$

$$\dot{\rho}_\phi + 3H \gamma_\phi \rho_\phi = 0, \quad (6)$$

where $H \equiv \dot{a}/a$ denotes the Hubble factor. We have further introduced the density parameters $\Omega_m \equiv \rho_m/\rho_c$, $\Omega_\phi \equiv \rho_\phi/\rho_c$, with $\rho_c \equiv 3H^2$ the critical density, and $\Omega_K \equiv -K/(aH)^2$.

In terms of these quantities we can introduce an overall baryotropic index γ :

$$\gamma\Omega = \gamma_m\Omega_m + \gamma_\phi\Omega_\phi, \quad (7)$$

where we have made use of the definition $\Omega \equiv \Omega_m + \Omega_\phi$. The flatness problem is solved by the attractor solution $\Omega = 1$ of Eq. (4). In addition, the ratio Ω_ϕ/Ω_m becomes asymptotically a constant, meaning that the matter content of the universe does not dilute altogether as inflation proceeds. In [9] it was shown that these constant solutions are stable in the $(\Omega, \Omega_m, \Omega_\phi)$ space. The fixed point solution $\Omega = 1, \Omega_m = \Omega_{m0}$ and $\Omega_\phi = \Omega_{\phi0}$, respectively, of Eq. (4)–(6) is obtained when the partial adiabatic indices and the dissipative pressure are related by

$$\gamma_m + \frac{\pi}{\rho_m} = \gamma_\phi = -\frac{2\dot{H}}{3H^2}, \quad (8)$$

accordingly the smaller γ_ϕ , the larger the dissipative effects.

Typically bulk viscosity arises in mixtures either of different particles species, as in a radiative fluid, or of the same species but with different energies, as in a Maxwell-Boltzmann gas. Physically, we can think of π as the internal “friction” that sets in due to the different cooling rates in the expanding mixture. Any dissipation in exact FLRW universes has to be scalar in nature, and in principle it may be modeled as a bulk viscosity effect within a nonequilibrium thermodynamic theory such as the Israel-Stewart’s [10]. In that formulation and under certain general circumstances, the evolution equation for the bulk dissipative pressure takes the form

$$\pi + \tau \dot{\pi} = -3\xi H, \quad (9)$$

where the positive-definite quantity ξ stands for the phenomenological coefficient of bulk viscosity, T the temperature of the fluid, and τ the relaxation time associated to the dissipative pressure—i.e., the time the system would take to reach the thermodynamic equilibrium state if the velocity divergence were suddenly turned off [11]. Usually ξ is given by the kinetic theory of gases or a fluctuation-dissipation theorem or both [12]. Expression (9) has been widely used in the literature [13] and it meets the requirements of causality and stability to be demanded to any physically acceptable transport equation [14].

Combining Eqs. (8) and (9) we obtain the equation of motion of the attractor solutions satisfying flatness, accelerated expansion and the nondilution condition

$$\nu^{-1} \left(\frac{\ddot{H}}{H} + 3\gamma_m \dot{H} \right) + \dot{H} + \frac{3\gamma_m}{2} H^2 - \frac{3\xi}{2\Omega_{ma}} H = 0. \quad (10)$$

Here $\nu = (\tau H)^{-1}$ is the number of relaxation times in a Hubble time—for a quasistatic expansion ν is proportional to the number of particle interactions in a Hubble time. Perfect fluid behavior occurs in the limit $\nu \rightarrow \infty$, and a consistent hydrodynamical description of the fluids requires $\nu > 1$.

The problem of a homogeneous scalar field driven by a exponential potential

$$V = V_0 e^{-A\phi}, \quad (11)$$

minimally coupled to gravity in a flat FLRW spacetime with a linear viscosity coefficient

$$\xi = \xi_0 H, \quad (12)$$

where V_0 and ξ_0 are constants, has the solution

$$a = a_0 (t/t_0)^\alpha, \quad (13)$$

$$\phi = \phi_0 \ln(t/t_0). \quad (14)$$

The quantities

$$\alpha = \frac{2}{(1 - \Omega_{ma})A^2}, \quad (15)$$

$$\xi_0 = \Omega_{ma}(\gamma_m - \gamma_{\phi a})[1 - 3\gamma_{\phi a}\nu^{-1}], \quad (16)$$

and

$$\gamma_{\phi a} = \frac{2}{3\alpha} \quad (17)$$

are obtained by solving the system (4)–(6), (8), (10) and (11); the subindex a stands for asymptotic value of the corresponding quantity. The power-law expansion (13) will be inflationary for $A^2 < 2(1 - \Omega_{ma})^{-1}$.

Rewriting Eq. (10) in terms of the field baryotropic index γ_ϕ , we get

$$\gamma'_\phi = 3\gamma_\phi^2 - (\nu + 3\gamma_m)\gamma_\phi + \nu\left(\gamma_m - \frac{\xi}{\Omega_{ma}H}\right), \quad (18)$$

where a prime indicates derivative with respect to $\eta = \ln a$. When the phenomenological coefficient of bulk viscosity is given by Eqs. (12) and (16), that is ξ_a , Eq. (18) admits the constant solution $\gamma_\phi = \gamma_{\phi a}$. It gives an accelerated expansion in the late time regime when $\gamma_{\phi a} < 2/3$. As $\xi > 0$ and $\gamma_m \geq 1$, the hydrodynamical parameter ν is restricted to $\nu > 3\gamma_{\phi a}$. The case of constant ξ_0 arises for instance in a radiating fluid, and the nearly linear regime, with slowly varying ν and γ_m , was already investigated in the quasiperfect limit, corresponding to $\nu^{-1} \rightarrow 0$ [9].

To analyze the stability of the solution $\gamma_\phi = \gamma_{\phi a}$ we insert Eq. (12) in Eq. (18) to obtain

$$\gamma'_\phi = 3(\gamma_\phi^2 - \gamma_{\phi a}^2) - (\nu + 3\gamma_m)(\gamma_\phi - \gamma_{\phi a}). \quad (19)$$

As $\gamma_{\phi a} < 2/3, \nu > \max(3\gamma_{\phi a}, 1)$ and $\gamma_m \geq 1$, Eq. (19) shows that $\partial\gamma'_\phi/\partial\gamma_\phi < 0$ in a neighborhood of $\gamma_{\phi a}$. Hence this constant solution is asymptotically stable, showing that all solutions of Eq. (10), that is all the accelerated attractors of the system (4), (5), (6), (9), are themselves attracted towards the constant solution $\gamma_\phi = \gamma_{\phi a}$ provided that they satisfy $\xi \sim \xi_a$ at late time.

Let us now assume that instead of having just one scalar field, we have n homogeneous noninteracting scalar fields, ϕ_i with exponential potentials $V_i = V_{0i} e^{-A_i\phi_i}$. In that case the Einstein-Klein-Gordon equations can be recast as

$$\Omega_m + \sum_{i=1}^n \Omega_{\phi_i} + \Omega_K = 1 \quad (K=1,0,-1), \quad (20)$$

$$\dot{\rho}_m + 3H\left(\gamma_m + \frac{\pi}{\rho_m}\right)\rho_m = 0, \quad (21)$$

$$\dot{\rho}_{\phi_i} + 3H\gamma_{\phi_i}\rho_{\phi_i} = 0 \quad (i=1,2,\dots,n). \quad (22)$$

As it stands the problem in its full generality is rather involved. Therefore to bring it to a form amenable to analytical treatment we shall consider henceforth a simplified version

characterized by $A_1 = A_2 = \dots = A_n \equiv A$ and $V_{01} = V_{02} = \dots = V_{0n} \equiv V_0$. As we shall see in the next section, we can expect in this particular case that all scalar fields share the same asymptotic limit. So, in the remaining of this section we will assume $\phi_1 = \phi_2 = \dots = \phi_n \equiv \phi$, so that $V_1 = V_2 = \dots = V_n \equiv V = V_0 e^{-A\phi}$ and Eq. (20) becomes

$$\Omega_m + n\Omega_{\phi_i} + \Omega_K = 1 \quad (K=1,0,-1), \quad (23)$$

while Eq. (22) simplifies to Eq. (6). Following parallel steps to that leading to Eqs. (15)–(17) it can be seen that the Einstein-Klein-Gordon system has the spatially flat power-law attractor solution (13) but now with

$$\alpha = \frac{2n}{(1 - \Omega_{ma})A^2}, \quad (24)$$

showing that the n scalar fields ϕ cooperate to a stronger inflation.

Let us now assume that the n homogeneous scalar fields, ϕ_i are driven by a general potential $V = V(\phi_i)$. In that case the Einstein-Klein-Gordon equations are

$$3H^2 = \frac{1}{2} \sum_{i=1}^n \dot{\phi}_i^2 + V + \rho_m - \frac{3K}{a^2}, \quad (25)$$

$$\ddot{\phi}_i + 3H\dot{\phi}_i + V_{,\phi_i} = 0, \quad (26)$$

along with Eq. (21) ($V_{,\phi_i}$ stand for $\partial V/\partial\phi_i$). From these equations we get

$$\dot{H} = -\frac{1}{2} \sum_{i=1}^n \dot{\phi}_i^2 - \frac{1}{2}(\gamma_m \rho_m + \pi) + \frac{K}{a^2}. \quad (27)$$

In order to investigate the stable scalar field configurations it is expedient to introduce the ancillary quantity

$$\omega = \frac{\sum_{i=1}^n \dot{\phi}_i^2}{n\dot{\phi}_\alpha^2}, \quad (28)$$

which reduces to $\omega = 1$ for the completely symmetric configuration $\phi_1 = \phi_2 = \dots = \phi_n$. Using Eqs. (25)–(28) we find the differential equation for ω :

$$\dot{\omega} = 2 \frac{nV_{,\phi_\alpha}\dot{\phi}_\alpha\omega - \dot{V}}{n\dot{\phi}_\alpha^2}. \quad (29)$$

(In this section no summation convention applies to repeated Greek indices.) If we further assume that the potential satisfies the condition

$$\dot{V} = nV_{,\phi_\alpha}\dot{\phi}_\alpha, \quad (30)$$

then Eq. (29) becomes

$$\dot{\omega} = 2 \frac{V_{,\phi_\alpha}}{\dot{\phi}_\alpha} (\omega - 1). \quad (31)$$

This has a fixed point solution, namely $\omega = 1$. Further, with the aid of Eqs. (27), (28) the general solution of Eq. (31) can be found in terms of the scale factor, the matter fluid and the bulk viscous pressure

$$\omega = \left(1 + \frac{nc}{2a^6 \left[\dot{H} + \frac{1}{2}(\gamma_m \rho_m + \pi) - \frac{K}{a^2} \right]} \right)^{-1}, \quad (32)$$

where c is an arbitrary integration constant. Evaluating Eq. (32) in the asymptotic regime of the attractor solution $\gamma_\phi = \gamma_{\phi a}$ and the potentials $V_i = V_{0i} e^{-A_i \phi_i}$, which asymptotically satisfies the condition (30), it can be easily shown that the particular solution $\omega = 1$ is an attractor for evolutions that behave asymptotically as $a \propto t^\alpha$ with $\alpha > 1/3$. On the other hand, this result strongly suggests that the special case in which all scalar fields are equal may be the late-time attractor of more general scenarios.

III. WARM INFLATION

Warm inflation arises when a strong enough coupling between the scalar field and matter fluid (which we shall assume perfect) exists. The former decays into the latter (which acts as a thermal bath) while the inflaton slowly rolls down the potential [8]. The decay is phenomenologically implemented by inserting a (usually constant) friction term Γ in the equation of evolution for ϕ

$$\ddot{\phi} + 3(H + \Gamma)\dot{\phi} + V'(\phi) = 0. \quad (33)$$

We adopt this picture except that (i) we consider no slow-roll (although it can be straightforwardly incorporated), and (ii) rather than a single field we have n scalar fields all of them with identical exponential potential. Here ϕ stands for any of these fields. Accordingly Eq. (33) is not just a single equation but n identical equations; besides for mathematical simplicity we assume the same friction term for each field.

Obviously the coupling between the n scalar fields and the matter fluid introduces a source term in the energy balance equation for the latter

$$\dot{\rho}_m + 3\gamma_m H \rho_m = 3\Gamma \dot{\phi}^2. \quad (34)$$

So there is a continuous transfer of energy from the scalar fields to the matter adjusted in such a way that the former experience a damped evolution and give rise to a nearly isothermal expansion. Accordingly, like in standard warm inflation, no reheating mechanism is needed at the end of inflation. Moreover, thermal rather than quantum fluctuations produce the primordial spectrum of density perturbations [15–19].

We can identify the phenomenological coupling with an effective dissipative pressure π^* along the evolution on the attractor. Then comparing Eq. (5) with Eq. (34) we get

$$\pi^* = -\frac{\Gamma \dot{\phi}^2}{H}. \quad (35)$$

In the case at hand the attractor condition (8) becomes dynamic because the starred magnitudes include the interaction between the scalar field and radiation

$$\gamma_m + \frac{\pi^*}{\rho_m} = \gamma_\phi^* = -\frac{2\dot{H}}{3H^2}, \quad (36)$$

where $\gamma_\phi^* \equiv \gamma_\phi - (\pi^*/\rho_\phi)$. Using this together with Eqs. (2) and (3) it follows that

$$\Gamma = \left(\frac{\gamma_m}{\gamma_\phi^*} - 1 \right) \frac{\rho_m}{\rho_\phi} H \equiv RH \quad (37)$$

(bear in mind that $\gamma_m > \gamma_\phi^*$). This implies that the scalar field evolves with an “effective” expansion rate $\tilde{H} = (1+R)H$. Once converged to the attractor solution, R becomes a constant and the effective power-law exponent results larger by a factor $(1+R)$:

$$\alpha = \frac{2n(1+R)}{(1-\Omega_{ma})A^2}. \quad (38)$$

This means that this interaction between the scalar fields and radiation further assist to inflation.

Let us investigate the entropy production \dot{S} along this era. Using Eqs. (36) and (5) we get that matter redshifts as $\rho_m = \rho_{me}(a/a_e)^{-3\gamma_\phi^*}$ (subindex e means evaluation at inflation exit). As it is customary in warm inflationary scenarios we assume that this matter behaves as radiation $\rho_m = c_1 T^4$, where $c_1 = \pi^2/30g$ and g is the effective number of relativistic degrees of freedom, so we have

$$T = T_e (a/a_e)^{-3\gamma_\phi^*/4}. \quad (39)$$

Then, using Eqs. (8), (12), (39), (13) and $\dot{S} = \pi^{*2}/\xi T$, we get

$$S(t) = S_e \left[1 - \left(\frac{\tau_1}{t} \right)^{3/2} \right], \quad (40)$$

where

$$\tau_1 = \left[\frac{16(\gamma_\phi^* - 4/3)^2 \Omega_{ma}^2}{9\gamma_\phi^{*3} \xi_0 T_e S_e t_e^{1/2}} \right]^{2/3} \quad (41)$$

is the characteristic time for the entropy density to attain a constant value S_e and is directly related to the time for the start of warm inflation.

We see from Eq. (39) that the requirement of a nearly isothermal expansion along the inflationary era imposes a

constraint on the value of γ_ϕ^* . Namely, if T_i is the temperature of the radiation fluid at the beginning of inflation, T_e at the end, and N is the number of e-folds, we have

$$\frac{T_i}{T_e} = e^{-3\gamma_\phi^* N/4}. \quad (42)$$

Assuming $T_i/T_e = \mathcal{O}(1)$ we find that $\gamma_\phi^* \approx 10^{-2}$. From Eqs. (17) and (24) we see that this can be easily achieved, with n in the range of 10–100, without fine-tuning the potential slope.

Inflationary scenarios need to achieve a graceful exit from inflation. In our case this is not a problem since the ratio ρ_ϕ/ρ_m is not exactly constant, as can be seen from Eqs. (33) and (34). Therefore, the continuous transfer of energy from the n scalar fields to the matter fluid slowly increases the energy of the latter and decreases that of the former. Thus the acceleration equation $\ddot{a}/a = -\frac{1}{6}(\rho + 3p)$ implies that the universe ceases to accelerate when both energy densities equalize. This criterion for ending inflation coincides with Taylor and Berera's [16].

IV. EVOLUTION OF PERTURBATIONS

This section considers the evolution of energy density, entropy and curvature fluctuations in the perturbative long-wavelength regime during the attractor era. Scalar perturbations are covariantly and gauge-invariantly characterized by the spatial gradients of scalars. Energy density inhomogeneities are described by the comoving fractional density gradient [20]

$$\delta_i = \frac{aD_i\rho}{\rho}, \quad (43)$$

where D_i stands for the covariant spatial derivative $D_j A_i \dots = h_j^k h_i^l \dots \nabla_k A_l \dots$. The scalar part $\delta \equiv aD^i \delta_i = (aD)^2 \rho/\rho$ encodes the total scalar contribution to energy density inhomogeneities. It relates to the usual gauge-invariant density perturbation scalar ε_m through $\delta = \nabla^2 \varepsilon_m$, where ∇^2 is the Laplacian for the metric of the 3-surfaces of constant curvature [21,22]. Also the comoving expansion gradient, the normalized pressure gradient, and normalized entropy gradient are defined by [20,23]

$$\theta_i = aD_i\theta, \quad p_i = \frac{aD_i p}{\rho}, \quad e_i = \frac{anTD_i s}{\rho}, \quad (44)$$

n being the particle number density, T the temperature, and s the specific entropy per particle. The evolution equation for scalar density perturbations reads [23]

$$\begin{aligned} & \delta + H(8 - 6\gamma + 3c_s^2)\delta - \frac{3}{2}H^2\{-10 + 14\gamma - 3\gamma^2 - 6c_s^2 \\ & \quad + [1 - 3(\gamma - 1)^2 + 2c_s^2]k\}\delta - c_s^2 D^2 \delta \\ & = \mathbf{S}[e] + \mathbf{S}[\pi^*] + \mathbf{S}[q] + \mathbf{S}[\sigma], \end{aligned} \quad (45)$$

where

$$c_s^2 = \left(\frac{\partial p}{\partial \rho}\right)_s, \quad r = \frac{1}{nT} \left(\frac{\partial p}{\partial s}\right)_\rho, \quad (46)$$

are, respectively, the adiabatic speed of sound and a non-baryotropic index. The sources in the right-hand side of Eq. (45) arising, respectively, from entropy perturbations, bulk viscous stress, energy flux, and shear viscous stress are given in [23]. Since in our case there are no shear viscous stress ($\sigma_{ij}=0$) and $\mathbf{S}[q]$ vanishes by choosing the energy frame ($q_i=0$), we reproduce here only the expressions for $\mathbf{S}[e]$ and $\mathbf{S}[\pi^*]$:

$$\mathbf{S}[e] = r(3KH^2 + D^2)e, \quad (47)$$

$$\mathbf{S}[\pi^*] = -(3KH^2 + D^2)\mathcal{B}, \quad (48)$$

where the scalar entropy perturbation

$$e = aD^i e_i = \frac{a^2 n T}{\rho} D^2 s \quad (49)$$

and the dimensionless perturbation scalar

$$\mathcal{B} = \frac{a^2 D^2 \pi^*}{\rho}, \quad (50)$$

related to the inhomogeneous part of the bulk viscous stress, were defined. Also, the entropy perturbation equation in the energy frame is

$$\dot{e} + 3H(c_s^2 - \gamma + 1 + r)e = -3H\mathcal{B}. \quad (51)$$

The coupled system that governs scalar dissipative perturbations in the general case is given by the energy density perturbation equation (45), the entropy perturbation equation (51), the equation for the scalar bulk viscosity (9), and the equation for temperature perturbations.

When only bulk viscous stress dissipation is present, the coupled system can be reduced to a pair of coupled equations in δ (third order in time) and e (second order in time). For a flat background, the equations are [23]

$$\begin{aligned}
& \tau \ddot{\delta} + [1 + 3(2 - \gamma + c_s^2)\tau H] \dot{\delta} + H\{8 - 6\gamma + 3c_s^2 + 3\tau(c_s^2) \cdot - \frac{1}{2}[-14 + 75\gamma - 48\gamma^2 + (21\gamma - 30)c_s^2]\tau H\} \delta - \frac{3}{2}H^2\{6 - 10\gamma \\
& + 5\gamma^2 - 6c_s^2 - 4\tau(c_s^2) \cdot - 2[-6 + 18\gamma - 15\gamma^2 + 5\gamma^3 + (6 - 28\gamma + 10\gamma^2)c_s^2]\tau H\} \delta \\
& = \frac{a^2 \xi}{\rho \gamma} D^2(D^2 \delta) + \frac{3a^2(\gamma - 1)H}{\rho \gamma} D^2(D^2 \delta) + \tau c_s^2 D^2 \delta + \tau r D^2 \dot{e} + \left[(1 - 3\gamma\tau H)c_s^2 + \tau(c_s^2) \cdot + 3\left(\frac{\partial \xi}{\partial \rho}\right)_s \right] D^2 \delta \\
& + \left[(1 - 3\gamma\tau H)r + \tau \dot{r} + 3\left(\frac{\partial \xi}{\partial s}\right)_\rho \right] D^2 e, \tag{52}
\end{aligned}$$

and

$$\begin{aligned}
& \tau \ddot{e} + [1 - \frac{3}{2}(-2 + 3\gamma - 2c_s^2 - 2r)\tau H] \dot{e} - 3H \left[\gamma - 1 - c_s^2 - r + 3\gamma(\gamma - c_s^2)\tau H + \tau(c_s^2 + r) \cdot - \frac{\rho}{\gamma} \left(\frac{\partial \xi}{\partial s}\right)_\rho \right] e \\
& = -\frac{\xi}{\gamma} \dot{\delta} + \frac{3H}{\gamma} \left[(\gamma - 1)\xi + \rho \left(\frac{\partial \xi}{\partial \rho}\right)_s \right] \delta. \tag{53}
\end{aligned}$$

We shall consider here the evolution of the energy density and entropy perturbations in the attractor stage with the conditions $r=0$ and $\partial\nu/\partial s=0$. Together with Eq. (12) they imply

$$\left(\frac{\partial \xi}{\partial s}\right)_\rho = 0, \tag{54}$$

$$\left(\frac{\partial \xi}{\partial \rho}\right)_s = \left(\frac{\partial \xi}{\partial \rho_m}\right)_s = \frac{\xi_0}{6\Omega_{ma}H}. \tag{55}$$

In this case Eq. (52) decouples to give

$$\begin{aligned}
\ddot{\delta} + \frac{c_1}{t} \dot{\delta} + \frac{c_2}{t^2} \dot{\delta} + \frac{c_3}{t^3} \delta = c_4 t^{2\alpha} D^4 \delta + c_5 t^{2\alpha} D^4 \delta + c_s^2 D^2 \delta \\
+ \left(\frac{c_6}{t} + c_7\right) D^2 \delta, \tag{56}
\end{aligned}$$

and Eq. (53) becomes

$$\ddot{e} + \frac{c_8}{t} \dot{e} + \frac{c_9}{t^2} e = \frac{c_{10}}{t^2} \dot{\delta} + \frac{c_{11}}{t^2} \delta \tag{57}$$

where the constant coefficients $c_1 \dots c_{11}$ depend upon the parameters of the model: $\nu, \kappa_1, \kappa_2, \Omega_m, \alpha, \gamma, c_s^2$, and the value of the scale factor a_e at the exit from inflation. For our purposes, only the explicit expression for c_1, c_8 , and c_9 are relevant, being

$$c_1 = \alpha\nu + 3\alpha(2 - \gamma - c_s^2),$$

$$c_8 = \alpha \left[\nu - \frac{3}{2}(3\gamma - 2 - 2c_s^2) \right],$$

$$c_9 = -3\alpha^2 [\nu(\gamma - 1 - c_s^2) + 3\gamma(\gamma - c_s^2)]. \tag{58}$$

We deal with the system (56), (57) by performing separation of variables in the form $\delta = \delta_x \delta_t$ and $e = e_x e_t$, where δ_x and e_x depend upon the spatial variables while δ_t and e_t are functions of the coordinate time t . Then, Eq. (56) can be recast as

$$\begin{aligned}
\frac{\ddot{\delta}_t}{\delta_t} + \frac{c_1}{t} \frac{\dot{\delta}_t}{\delta_t} + \frac{c_2}{t^2} \frac{\dot{\delta}_t}{\delta_t} + \frac{c_3}{t^3} = t^{2\alpha} \left(c_4 \frac{\delta_t}{\delta_t} + c_5 \right) \frac{D^4 \delta_x}{\delta_x} \\
+ \left(c_s^2 \frac{\delta_t}{\delta_t} + \frac{c_6}{t} + c_7 \right) \frac{D^2 \delta_x}{\delta_x}, \tag{59}
\end{aligned}$$

which can only hold if

$$(D^2 - \mu) \delta_x = 0, \tag{60}$$

μ being an arbitrary constant. In Fourier transformed space it becomes an identity that holds for an arbitrary amplitude δ_k with $\mu = -k^2/a^2$, where k/a is the physical wavenumber. Also, Eq. (57) leads to

$$\frac{t^2(\ddot{e}_t/e_t) + c_8 t(\dot{e}_t/e_t) + c_9}{c_{10}(\dot{\delta}_t/e_t) + c_{11}(\delta_t/e_t)} = \frac{\delta_x}{e_x}, \tag{61}$$

which requires $e_x = A \delta_x$, with A a constant. Then the evolution equation for mode μ becomes

$$\ddot{\delta}_i + \frac{c_1}{t} \dot{\delta}_i + \left(\frac{c_2}{t^2} - \mu^2 - t^{2\alpha} c_4 - \mu c_s^2 \right) \delta_i + \left[\frac{c_3}{t^3} - \mu^2 t^{2\alpha} c_5 - \mu \left(\frac{c_6}{t} + c_7 \right) \right] \delta_i = 0. \quad (62)$$

As we are interested in the asymptotic behavior of the perturbations along the attractor regime, it suffices to consider the dominant terms in Eq. (62) for large time. As $\alpha > 0$ we have

$$\ddot{\delta}_i + \frac{c_1}{t} \dot{\delta}_i - \mu^2 t^{2\alpha} c_4 \delta_i - \mu^2 t^{2\alpha} c_5 \delta_i \cong 0. \quad (63)$$

To study the asymptotic evolution of the long-wavelength modes much larger than the Hubble scale ($k/aH \ll 1$), we expand δ_i in powers of μ^2 as $\delta_i \cong \delta_i^{(0)} + \mu^2 \delta_i^{(1)}$. Then, replacing this expression in Eq. (63) and retaining terms up to first order in μ^2 , we obtain

$$\left[\ddot{\delta}_i^{(1)} + \frac{c_1}{t} \dot{\delta}_i^{(1)} - t^{2\alpha} c_4 \delta_i^{(0)} - t^{2\alpha} c_5 \delta_i^{(0)} \right] \mu^2 + \ddot{\delta}_i^{(0)} + \frac{c_1}{t} \dot{\delta}_i^{(0)} = 0. \quad (64)$$

Its zeroth-order solution is

$$\delta_i^{(0)}(t) = \frac{A_1}{(1-c_1)(2-c_1)} t^{2-c_1} + A_2 t + A_3, \quad (65)$$

for $c_1 \in \{1, 2\}$ and being $A_i, i=1, 2, 3$ arbitrary integration constants. Then, $\delta_i^{(1)}$ satisfies the inhomogeneous equation

$$\ddot{\delta}_i^{(1)} + \frac{c_1}{t} \dot{\delta}_i^{(1)} - t^{2\alpha} c_4 \delta_i^{(0)} - t^{2\alpha} c_5 \delta_i^{(0)} = 0, \quad (66)$$

whose general solution has the form

$$\delta_i^{(1)} = \frac{B_1}{(1-c_1)(2-c_1)} t^{2-c_1} + B_2 t + B_3 + a_1 t^{2\alpha+4-c_1} + a_2 t^{2\alpha+3} + a_3 t^{2\alpha+5-c_1} + a_4 t^{2\alpha+4}, \quad (67)$$

where the coefficients $B_i, i=1, 2, 3$ are integration constants and $a_i, i=1, \dots, 4$ depend on α, c_1 , and the B_i . Bearing in mind that $\alpha \gg 1$, we find that the dominant mode of the energy density perturbations in the super-Hubble regime grows asymptotically like $t^{2\alpha+4} \propto a^{2+(4/\alpha)} \simeq a^2$, independently of the wavenumber, until inflation exit.

On the other hand, Eq. (53) becomes, in the leading regime,

$$\ddot{e}_i + \frac{c_8}{t} \dot{e}_i + \frac{c_9}{t^2} e_i = c_{12} t^{2\alpha+1} \left[1 + \frac{3}{\nu} \left(\gamma - 1 + \frac{1}{2\Omega_{ma}} \right) t \right], \quad (68)$$

whose solution is

$$e_i(t) = B_4 t^{\lambda_1} + B_5 t^{\lambda_2} + c_{13} t^{2\alpha+3} \left[1 + \frac{3}{\nu} \left(\gamma - 1 + \frac{1}{2\Omega_{ma}} \right) t \right], \quad (69)$$

where $\lambda_{1,2}$ are the roots of the equation $\lambda^2 + (c_8 - 1)\lambda + c_9 = 0$, B_4, B_5 are arbitrary integration constants, and c_{12}, c_{13} are functions of the parameters and the previously defined integration constants. It follows that the dominant mode of the entropy perturbations also grows as $a^{2+(4/\alpha)} \simeq a^2$ for $\nu = \mathcal{O}(1)$.

To deal with the evolution of the curvature perturbations generated by these energy density and entropy fluctuations we will turn to the standard metric-based gauge-invariant approach. In a particular choice for slicing of space-time named the longitudinal gauge, the metric describing the inhomogeneous perturbations of the spatially flat FLRW background takes the simple form

$$ds^2 = (1 + 2\Phi) dt^2 - a^2(t) (1 - 2\Psi) \delta_{ij} dx^i dx^j, \quad (70)$$

in terms of the gauge invariant Bardeen potentials Φ and Ψ [21,24]. Besides, when the shear stress vanishes, it follows from the equations of motion for the gauge invariant variables that $\Phi = \Psi$. So, just a single scalar degree of freedom, say Φ , is required to describe linear perturbations of the metric. We get two second order equations, namely

$$\nabla^2 \Phi - 3aH\Phi' - 3a^2 H^2 \Phi = \frac{1}{2} a^2 \delta\rho, \quad (71)$$

$$\begin{aligned} \Phi'' + 3aH(1 + c_s^2)\Phi' - c_s^2 \nabla^2 \Phi + [2a(aH)' \\ + (1 + 3c_s^2)a^2 H^2] \Phi \\ = \frac{1}{2} a^2 (\delta p + \delta\pi^* - c_s^2 \delta\rho), \end{aligned} \quad (72)$$

where the prime denotes derivative with respect to conformal time, and the source terms contain the energy density perturbation $\delta\rho$, the equilibrium pressure perturbation δp and the dissipative pressure perturbation $\delta\pi^*$.

When the source term of Eq. (72) vanishes, and the scales are larger than the Hubble radius such that the spatial gradients can be neglected, Eq. (72) can be recast in terms of the curvature perturbation on uniform-density hypersurfaces [25–27]

$$\zeta \equiv \Phi + H \frac{\delta\rho}{\dot{\rho}} = \Phi + \frac{2}{3} \frac{H^{-1} \dot{\Phi} + \Phi}{\gamma + \pi^*/\rho} \quad (73)$$

as a conservation law $\dot{\zeta} = 0$ [28,29]. We consider that the equilibrium pressure perturbation is isentropic during warm inflationary and radiation-dominated eras, and that the dissipative pressure perturbation switches off during the transition between both eras with a relaxation time that is a fraction of a Hubble time. Hence, one should expect that $\dot{\zeta}$ soon vanishes along this transition so that the value ζ_e at inflation exit on super-Hubble scales may be equated to that at reentry of long wavelength modes to Hubble radius during the

radiation- or matter-dominated eras. To find this value we need the evolution of Φ along the attractor era for long-wavelength modes. Again, neglecting the spatial derivative term and using the relationship $\epsilon_{mk} = -\delta_t \delta_k / k^2 \approx -a^2 \delta_k / k^2$, the Fourier transform of Eq. (71) becomes

$$a \frac{d\Phi_k}{da} + \Phi_k = \frac{\delta_k}{2k^2} a^2. \quad (74)$$

Its solution

$$\Phi_k = \frac{C_1}{a} + \frac{\delta_k}{6k^2} a^2 \quad (75)$$

shows that Φ grows asymptotically as a^2 during the attractor regime. Inserting this result back in Eq. (73) we find that ζ also grows as a^2 during this regime. This nonconservation of the super-Hubble curvature perturbations is a consequence of the growth of the entropy perturbations, which, in its turn, is due to dissipation effects as Eq. (51) shows. Then, taking into account that $\gamma_\phi^* \ll 1$, we find that $\zeta \approx 3\alpha\Phi \approx -\alpha\epsilon_m/2$ at inflation exit. Hence the power spectra of ζ and Φ at that moment are proportional to the power spectrum of the primordial energy density perturbations.

For a mode that crosses outside the Hubble radius at scale factor a_A during inflation and reenters to the Hubble radius at scale factor a_B during the radiation-dominated era, we find the number of e -foldings before the end of inflation $N_A = \alpha/(\alpha-1)\ln(a_B/a_e)$ by using the continuity of the energy density at the turnover between the warm inflation and the radiation dominated eras. Then, for this mode of perturbations, the regime of growth as a^2 starts at scale factor $a_1 \gg a_A$, when both $aH \gg k$ and the evolution is close to the attractor, and it continues until inflation exit. Then we obtain

$$\frac{a_e}{a_1} = \frac{1}{\sigma} \left(\frac{a_B}{\beta a_N} \right)^{\alpha/(\alpha-1)} \approx \frac{a_B}{\beta \sigma a_N}, \quad (76)$$

where $\sigma = a_1/a_A$, a_N is the scale factor at the start of nucleosynthesis era, corresponding to a temperature $T_N \approx 1$ MeV, $\beta = a_e/a_N$ and we are taking $\alpha \gg 1$. As in this warm inflationary scenario there is no need to accommodate a reheating stage, inflation may end shortly before nucleosynthesis and we may take safely $\beta \approx 10^{-2}$. Then we obtain an upper bound on the growth of perturbation modes crossing inside the Hubble radius during the radiation-dominated era by considering the matter-radiation equality scale $k_{eq}^{-1} \approx 100$ Mpc, corresponding to a cluster of galaxies. As $a_{eq}/a_N \approx 10^6$, taking $\sigma \gtrsim 10^3$ we find that $\epsilon_{me}/\epsilon_{m1} = \zeta_e/\zeta_1 \lesssim 10^{10}$. Similarly, for a mode crossing inside the Hubble radius during the matter-dominated era, we obtain

$$\frac{a_e}{a_1} \approx \frac{3}{4\beta\sigma} \left(\frac{a_{eq}}{a_N} \right) \left(\frac{a_B}{a_{eq}} \right)^{1/2}. \quad (77)$$

Thus, scales $k_{hor}^{-1} \approx 10^4$ Mpc, corresponding to the observable universe, give the upper bound on the growth of perturbation modes $\zeta_e/\zeta_1 \lesssim 10^{13}$.

After the inflationary stage, we recover the standard picture of conserved isentropic curvature perturbations and well known calculations show that the density contrast at Hubble scale entry $(\delta\rho/\rho)_{k=aH}$ is proportional to ζ , or equivalently to the comoving curvature perturbation \mathcal{R} , with a proportionality factor of $2/5$ during pressureless matter dominated era, or $4/9$ during the radiation-dominated era [30]. So, using COBE normalization for the power spectrum of curvature $(2/5)\mathcal{P}_{\mathcal{R}}^{1/2} = 1.91 \times 10^{-5}$ at the scale $k^{-1} \approx 10^3$ Mpc, we find that at inflation exit $\epsilon_{mk} = 9.55 \times 10^{-5}/\alpha \approx 10^{-6}$ for $\alpha \approx 100$. Besides, due to the proportionality of power spectra, the observed bound on the spectral slope of curvature perturbations $n = 1.0 \pm 0.2$ implies the same bound on the spectral slope of the energy density perturbations at inflation exit. Recalling that the amplitude δ_k is an arbitrary function, these observational bounds impose no fine tuning constraints whatsoever on the parameters of the scalar field potential.

V. DISCUSSION

We have proposed a new inflationary scenario whose main ingredients are n scalar fields and a dissipative matter fluid. The former decay into the latter at a high rate Γ . While no single scalar field can achieve inflation by its own they cooperate synergistically to produce it. We have derived the attractor condition Eq. (8) and shown that the presences of dissipation does not spoil the linear relationship between the power-law exponent α and the number of fields, preserving the stability of the symmetric configuration of n identical fields.

We have described the interaction between the scalar fields and the radiation fluid in the warm inflationary scenario by means of an effective bulk dissipative pressure and generalized the attractor condition. Likewise, we have resorted to the synergistic mechanism to calculate the production of entropy and the evolution of temperature. The exit temperature results lower but approximately of the same order than the initial temperature. This renders the reheating phase redundant.

Further, we have found that the combination of the synergistic mechanism and the decay of the scalar fields into the matter fluid produces significant entropy perturbations with proportional spectral amplitude and dominant mode evolution to that of energy density perturbations on large wavelength scales until inflation exit. This steep growth contrasts with other models of inflation where long wavelength curvature modes evolve isentropically (see, e.g., [29], [31]); however in our case there is a continuous transfer of energy from the scalar fields to matter. Moreover, observational bounds on the curvature perturbations at Hubble scale entry do not force on our model any slow-roll constraints on the scalar field.

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