Steady-state eternal inflation

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Since the advent of inflation, several theorems have been proven suggesting that although inflation can (and generically does) continue eternally into the future, it cannot be extended eternally into the past to create a ''steady-state'' model with no initial time. Here we provide a construction that circumvents these theorems and allows a self-consistent, geodesically complete, and physically sensible steady-state eternally inflating universe, based on the flat slicing of de Sitter space. This construction could be used as the background spacetime for creation events that form big-bang-like regions, and hence could form the basis for a cosmology that is compatible with observations and yet which avoids an initial singularity or beginning of time.

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I. INTRODUCTION

Following the discovery of the cosmic expansion, cosmology became dominated by two alternative paradigms. The first, the big bang (BB), is based on general relativity applied to a physical system obeying the cosmological principle (CP) of spatial homogeneity and isotropy. In the BB, the observed universe evolved in a finite time from a dense singular state before which classical space and time did not exist. The second, the steady state (SS), is based on the *perfect* cosmological principle (PCP) that the statistical properties of the universe are independent also of time. In the SS, the universe always has and always will exist in a state statistically like its current one, and time has no beginning [1,2].

Observations of the thermal microwave background and evolution in quasars and galaxies turned most astronomers away from the SS, and its proponents were forced to make their models only ''quasisteady,'' with expansion and contraction cycles explaining the observed evolution [3]. But while the SS has approached the BB, the BB has also approached the SS, in the form of ''eternal inflation:'' there is a broad consensus among its architects that inflation—now considered by many to be an indispensable part of the BB cosmology—never ends once it begins [4,5]. Rather, inflation always continues somewhere and continually spawns new thermalized regions, creating a mixture of inflating and noninflating areas that approaches some quasisteady-state distribution eternally into the future [6].

The SS cosmology is appealing because it avoids an initial singularity, has no beginning of time, and does not require an initial condition for the universe. This led some to hope that inflation could be extended eternally into the past to likewise avoid these unpalatable necessities [7]. Attempts to do this failed, however, and these failures have motivated the formulation of several singularity theorems attempting to show that under very general assumptions inflating spaces must contain singularities, so that inflation can be at best ''semi-eternal'' into the future [8,9]. The most recent such theorem, for example, attempts to show that an observer following ''almost any'' geodesic will have finite past proper time as long as its ''locally measured Hubble constant'' always exceeds some positive minimum value [9], implying that inflating spaces are generically past geodesically incomplete.

This is an odd result as it applies to—and hence appears to forbid—the seemingly physically reasonable classical SS cosmology (which can itself be considered a form of eternal inflation). In this paper we attempt to resolve this incongruity by carefully examining the implications of the singularity theorems and providing a construction that allows for a physically reasonable geodesically complete eternally inflating spacetime in which physical observers can have indefinitely long past proper time. In Sec. II we examine the classical SS cosmology in light of the singularity theorems, and show how to construct a self-consistent and physically reasonable model with its essential features. Eternal inflation is based on the same (de Sitter) spacetime as the SS cosmology, and in Sec. III we show how our construction might be used to formulate a viable truly eternal model of inflation with big-bang-like regions embedded in an eternally inflating background. We conclude in Sec. IV.

II. THE STEADY-STATE UNIVERSE

Let us now review the classical SS model. The backbone of this theory is the PCP, which holds that an observer at a randomly chosen position in space and time measures physical properties of the universe that are isotropic and that are statistically indistinguishable from any other such observer. This principle places strong conditions on a cosmology that satisfies it. The spatial part of the PCP implies that spacetime can be described by the Friedmann-Robertson-Walker (FRW) metric with scale factor a. The measurable Hubble parameter $H \equiv (1/a)da/dt \equiv \dot{a}/a$ must be a constant in cosmic time t, implying exponential expansion. The universe must be spatially flat, lest there be a time-varying ratio of the curvature scale to the Hubble radius. The physical matter

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density must also be constant in time. Because of the Hubble expansion, this implies that particles must be continually created so as to keep this density constant. This requires that the number of particles created in a given four-volume be proportional to the four-volume itself. (Mechanisms for doing this are generally considered somewhat artificial, detracting from the aesthetic appeal of the SS; but this is not important for the present argument.)

A subtle question that should be asked at this point is: why have we stated that the universe is expanding (rather than contracting) and that particles are being created (not destroyed)? This is necessary if the arrow of time (AOT) is to point in the direction of entropy creation: to maintain a SS, both microscopic and coarse-grained entropy must, on average, be created as the universe expands at a constant rate per unit physical volume. In other words, if either microscopic or coarse-grained entropy were created as the universe contracted, the entropy *density* would change in time, violating the PCP. The ''thermodynamic AOT'' defined by entropy creation is in turn linked to the electromagnetic AOT. The latter specifies, for example, that while moving charges can emit radiation (creating an asymptotically spherical outgoing wavefront that is either eventually absorbed or propagates to infinity), an incoming spherical wavefront cannot ''miraculously'' assemble into a local electromagnetic field that can move charges. Conditions sufficient to ensure this behavior are that the ''retarded'' rather than ''advanced'' potentials are appropriate (which is tied to the thermodynamic AOT) and that there be no radiation incoming from infinity (the ''Sommerfield radiation condition'') [10,11]. This last condition is discussed below; for now note only that in a SS model the thermodynamic and (hence) electromagnetic AOTs are explicitly linked to the direction of the expansion.

The metric for an exponentially expanding FRW universe can be written

$$ds^2 = -dt^2 + e^{2Ht}(dx^2 + dy^2 + dz^2), \tag{1}$$

with H constant chosen positive so that $a = e^{Ht}$ increases as t increases as required by the preceding argument. This is a well studied metric: that of the flat slicing of de Sitter space [12]. 4-dimensional de Sitter spacetime can be thought of as a hyberboloid in 5-dimensional Minkowski space; see Fig. 1. One may coordinatize this hyperboloid in a variety of ways to yield slices of constant time that are open, flat or closed. In our case the PCP has singled out as physically appropriate the flat expanding system shown in Fig. 1: slices of constant time are the nearly diagonal parabolas, with the $t \to -\infty$ surface $\mathcal{J}^-$ a null surface. Comoving observers emerge from past timelike infinity i_{I}^- in the lower-right corner, with future light cones that open in the direction of expansion, which is toward future timelike infinity at the top of the diagram.

However, as we can see from the figure, this slicing does not cover the entire hyperboloid. So the universe described by Eq. (1) is geodesically incomplete; geodesics like that labeled ''X'' in Fig. 1 can go ''through $t = -\infty$'' into the uncharted region. This singularity at $\mathcal{J}^-$ is exactly what the singularity theorems [8,9] point to, so let us look at the equations here. Consider a massive particle (the analogous argument can be made for light). Its geodesic equations in the metric (1) read

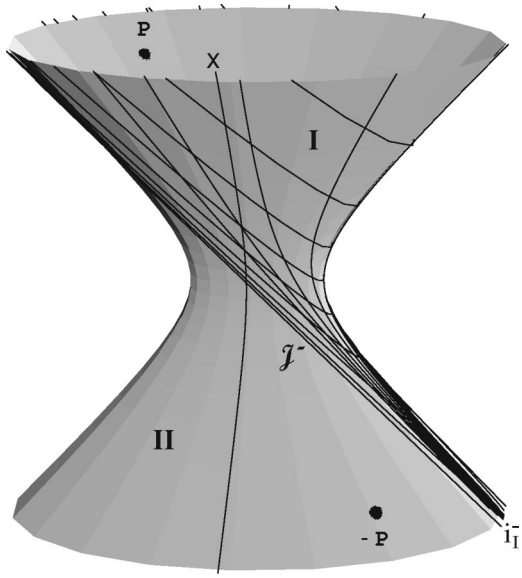

FIG. 1. A diagram of de Sitter space, with lines of equal time in the flat slicing (the nearly diagonal parabolas) and comoving geodesics (emerging from i_{I}^- in the lower-right) shown. The $t \to -\infty$ lightlike surface is labeled $\mathcal{J}^-$. The nearly vertical line labeled ''X'' represents a comoving geodesic in the closed slicing, and ''P'' and ''$-$P'' are, respectively, points in ''region I'' (the portion of the spacetime covered by the shown flat slicing coordinates) and ''region II'' (the uncovered portion).

$$t'' + He^{2Ht}\mathbf{x}'^2 = 0, \qquad (e^{2Ht}\mathbf{x}')' = 0, \tag{2}$$

where prime denotes a derivative with respect to the proper time τ of the particle. Integrating the latter gives $e^{2Ht}\mathbf{x}' = \mathbf{v}$, with $\mathbf{v}$ constant. Substituting into the former leads to

$$t'^2 - e^{-2Ht}\mathbf{v}^2 = 1, \tag{3}$$

remembering the normalization of the proper time. Now consider the particular class of geodesics with $\mathbf{v} = \mathbf{0}$. Then Eq. (3) simply reads $t'^2 = 1$ so that $\Delta t = \Delta\tau$ and for any value of τ the particle stays in the region of spacetime covered by the coordinates of the flat slicing. So these (comoving) geodesics are in fact complete in the region of spacetime described by Eq. (1). Now consider a geodesic with $\mathbf{v} \neq \mathbf{0}$. Equation (3) leads to

$$\int_{t_i}^{t_f} \frac{dt}{1 + \mathbf{v}^2 e^{-2Ht}} = \tau_f - \tau_i . \tag{4}$$

The LHS is unbounded as $t_f \to +\infty$, showing that the region of spacetime described by Eq. (1) is geodesically complete to the future. But it tends to a finite constant as $t_i \to -\infty$, implying that along this geodesic there is only a finite proper time since $t = -\infty$. Hence the region described by Eq. (1) is geodesically incomplete to the past. This sort of argument is the basis for the recent singularity theorem of Ref. [9].

What does this mean? Is the steady-state model badly defined? We will address this question from several perspectives. First, consider what a particle X on such a trajectory that came ''from the outside'' into the region described by the metric (1) would look like to the geodesically complete comoving observer it passes at time t. It would appear to be a particle with energy $m_0\sqrt{1+\mathbf{v}^2e^{-2Ht}}$, where m_0 is its rest mass. This is time dependent and as $t\to-\infty$ particle X has an arbitrarily large energy. That particles like X are forbidden can be seen in several complementary ways. First, consider X to be propagating ''backwards in time'' toward $\mathcal{J}^-$. If X has *any* nonzero interaction cross section with any particle in the universe that has nonzero physical number density, then particle X will interact with an infinite number of them with arbitrarily high energy. It would create, then, a ''spray'' of particles in a light-cone opening toward $\mathcal{J}^-$, violating the CP to an abritrarily great degree as $\mathcal{J}^-$ is approached.[1] Now consider X to be propagating forward in time, starting ''from'' $\mathcal{J}^-$. Then for the same reason, X would interact with an abitrarily large number of particles, yielding a spray of high-energy particles filling a light-cone opening *away* from $\mathcal{J}^-$, rendering any time-slice inhomogeneous. Even supposing the particle to somehow avoid all interactions, it would—simply by virtue of its asymptotically infinite energy—still pick out a preferred position, and violate the PCP.[2] In short, enforcing the PCP as $t\to-\infty$ acts a boundary condition on $\mathcal{J}^-$ that forbids any physical particles from entering the SS universe from ''elsewhere.'' The only allowed physical things in the SS are particles or photons or observers that are created within the spacetime or particles or observers that have world lines approaching the inextendible comoving geodesics in their infinite past.

Another way of looking at the situation is by asking what is in the region on the other side of $\mathcal{J}^-$, labeled as ''region II'' in Fig. 1. Note first that no signal or particle created in region I can escape into region II, because to do so it would have to travel along a spacelike path, or somehow backward in time so as to exist before it was created. Thus region II will see $\mathcal{J}^-$ as a boundary from which no particle or information emerges. This is exhibited in the conformal diagram for the flat slicing of de Sitter spacetime (Fig. 2): the future light-cone of any point in region I fails to intersect $\mathcal{J}^-$. Conceive now some physical beings residing in region II. In what sort of universe do they live? Consider first the electromagnetic AOT. A point in region II could *not* experience an incoming spherical wave from infinity traveling along a light cone opening toward $\mathcal{J}^-$ (because no particles can emerge from there). It could, however, send such a wavefront *away* from $\mathcal{J}^-$. This provides the Sommerfield radiation condition in region II *as long as* the electromagnetic AOT points away from $\mathcal{J}^-$, i.e. toward the *bottom* of Figs. 1 and 2, with comoving observers emerging from the point labeled $i_{\rm II}^-$. Now,

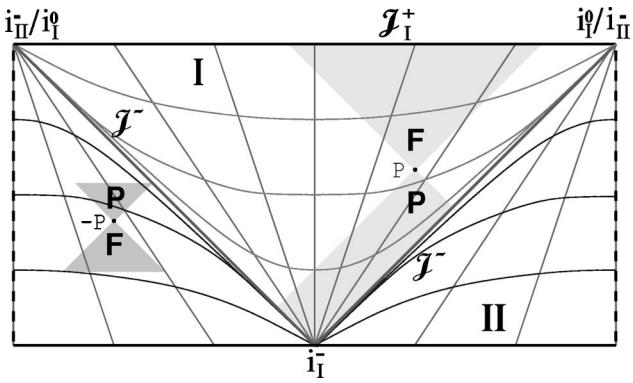

FIG. 2. Conformal diagram for de Sitter space. Equal-time flat slices are curved and spacelike; comoving geodesics are straight and timelike. The null surface $\mathcal{J}^-$ represents $t\to-\infty$ in both region I (above $\mathcal{J}^-$) and region II (below $\mathcal{J}^-$). Shaded regions represent future (''**F**'') and past (''**P**'') light cones of point ''P'' and its antipode ''−P,'' and $\mathcal{J}_{\rm I}^+$ is future timelike infinity for region I, while $i_{\rm I}^-$ and $i_{\rm I}^0$ are its past-timelike and spacelike infinities. The left and right (dotted) edges are identified.

noting that the boundary condition on $\mathcal{J}^-$ picks out the flat slicing as preferred in region II, just as the PCP did in region I, we are strongly motivated to apply the PCP in region II as in region I (which would also link the thermodynamic and cosmological AOTs as before).[3] Turning the page upside-down, we see that region II now closely resembles region I, and that the ''no incoming particles'' boundary condition on $\mathcal{J}^-$ (which, recall, followed simply from causality in region I) is exactly the necessary condition to prevent the sort PCP-violating particles previously discussed in the context of the geodesic completeness of region I. And further, the inability of particles created in region II to travel along spacelike paths or into their own past prevents any particles from traveling from region II into region I, completing the circle.

In essence, this construction partitions the full de Sitter spacetime into a self-consistent set of two non-communicating SS universes. An observer in region I does not see anything in its past light cone from an observer in region II because that other observer cannot signal into its past, and vice versa. Seen in this way the boundary condition forbidding physical particles from following geodesics across $\mathcal{J}^-$ into one universe is in no way strange or unreasonable, as it follows directly from the forbidding of causality violations in the other universe. (One could similarly partition de Sitter spacetime by any non-timelike boundary $\mathcal{B}$ away from which time flows. But $\mathcal{J}^-$ is special: any spacelike $\mathcal{B}$ would allow no eternal observers, and any other null $\mathcal{B}$ would be less symmetric; moreover, $\mathcal{J}^-$ is the only $\mathcal{B}$ that

[1]Note also that when viewed forward in time, this requires collective, anti-entropic behavior by an increasingly large number of particles as $t\to-\infty$.

[2]A homogeneous family of such incoming particles can satisfy the CP but only on one of the flat spatial sections.

[3]It does not appear strictly necessary to enforce the PCP in region II, though this makes the construction simpler; all that is really necessary is that—as suggested but not required by the Sommerfield condition—there is a globally well-defined AOT that prevents particles created in region II from passing through $\mathcal{J}^-$. The PCP is a sufficient but probably not necessary condition.

can be ''irrelevant'' by allowing interesting physics to occur throughout the full de Sitter spacetime even while no information flows from $\mathcal{B}$.)

The two universes resulting from the partition may not be identical, despite sharing the null boundary, because i_{II}^{-} (which is the beginning point of all region II comoving geodesics) is not necessarily the same as i_{I}^{-}. The universes can be *made* identical, however, providing a more economical and perhaps more elegant construction, through the identification of antipodal points on the hyperboloid (demonstrated by equating the two points P and $-P$ in Figs. 1 and 2). This identification maps region I onto region II (and vice versa), and $\mathcal{J}^{-}$ onto itself. The resulting spacetime was called the ''elliptic interpretation'' of de Sitter spacetime by Schrödinger [13] (who advocated a fully ''timeless'' model distinct from our own), and was studied mathematically in Ref. [14]. It has some appealing features: for example, unlike in the usual de Sitter spacetime, *any* event is connectable via a non-spacelike geodesic to the world line of an immortal observer following a timelike geodesic. For our SS construction, this implies that a physical observer that has existed forever can affect (i.e., can send signals to) all of the spacetime (though there exist events that cannot signal *to* the immortal observer, because the required signal would be spacelike, or travel ''back in time'' through $\mathcal{J}^{-}$).

Neither the identified nor non-identified spacetimes contain closed timelike curves. Without the identification, the spacetime manifold is time-orientable in the mathematical sense that it is possible to continuously divide non-spacelike vectors into two classes which can be labeled ''future'' and ''past.'' In our construction these labels will only correspond to the physical AOT in one of the two regions. With the identification the spacetime is still a manifold but is not mathematically time-orientable. The physical AOT is, however, still well-defined and no physical observer will see it reverse.

While our construction is self-consistent for any physical observer with an origin in the spacetime, one might nevertheless ask what a meta-physical invisible observer (with its own arrow of time) might see as it follows for example one of the comoving geodesics of the closed slicing which covers the full de Sitter spacetime (shown in Fig. 1 and labeled as ''X''). Moving in region I toward $\mathcal{J}^{-}$, it would observe that the clocks on the comoving observers it passes [at huge velocities as per Eq. (3)] would read earlier and earlier times, but that the clock of any one such observer would be turning at an ever-slower rate. Passing through $\mathcal{J}^{-}$ it would see the *times* diverge to minus infinity but the *rates* freeze. Emerging into region II (or in the identified case into another part of region I), it would see the times increasing from minus infinity and the rates increasing again. It would perceive nothing singular happening, interpreting the time reaching minus infinity and back as due to the infinite length contraction between the comoving observers as its speed relative to them momentarily becomes the speed of light.

III. ETERNAL INFLATION

The construction we have outlined gives either one or two past- and future-eternally inflating regions of spacetime, but is not a viable cosmological model for the same (observational) reasons the classical SS is not. It could, however, provide the background for events that create big-bang-like regions, one of which could describe our environment.

One possibility, based on ''old inflation'' [15,16], was discussed by Vilenkin in Ref. [17] but considered not to be viable because the background spacetime (the flat de Sitter slicing) is geodesically incomplete—the very problem to which we have outlined a solution. To construct such a steady-state inflation model, one may simply take the SS universe described above and replace the particles by bubbles[4] in which the scalar field will eventually roll down to the true vacuum [18]; the interior of each bubble looks like an open FRW cosmology to observers inside it [19]. For a suitably flat inflaton potential (as in ''open inflation'' [18,20]), the FRW regions can be nearly flat, homogeneous, and have scale-invariant density perturbations just as in standard inflationary cosmology. Like the particles in the SS model, the number of bubble nucleation events in a given four-volume is proportional to the four-volume itself [19,21,22]. For non-overlapping bubbles, this would yield a model obeying the PCP, as the physical number density of the bubbles of a given size on each space-like surface would be independent of time. However, the bubbles do tend to overlap: given a volume V at a time t, bubbles formed after some earlier time t_0 at a rate λ per unit 4-volume fill all of V except a fraction

$$f_{\mathrm{inf}} = \exp[-\lambda Q] = \exp\left[\frac{-4\pi\lambda(t-t_0)}{3H^3}\right] \tag{5}$$

for $(t-t_0) \gg H^{-1}$ and $V \to \infty$, where Q is the 4-volume between t_0 and t in the past light cone of a point in V [16,17]. Then $f_{\mathrm{inf}} \to 0$ as $t_0 \to -\infty$ and inflation seemingly halts. However, as argued in Ref. [17] this is not necessarily the case. One can show that *if* a large but finite sphere of comoving volume $\tilde{V}$ contains a fraction $f_{\mathrm{inf}} > 0$ of inflating volume, then the inflating phase's physical volume increases with time in that comoving volume, and its distribution at any time is a self-similar fractal of dimension $D = 3 - 4\pi\lambda/3H^4$ up to a scale $L \propto \log(t-t_0)$ [that is, $V(r) f_{\mathrm{inf}}(r) \propto r^D$ for $r < L$]. As $t_0 \to -\infty$ the distribution becomes fractal on arbitrarily large scales and, because $D < 3$, the inflating fraction of an arbitrarily large region tends to zero even though parts inflate indefinitely. The global structure of the spacetime is still apparently de Sitter, however, as all inflating regions are connected in spacetime and the fractal ''skeleton'' of inflating phase cannot be in any way affected by the regions of true vacuum. Note also that—although bubble intersections are common—the interior of any given bubble formed at time t

[4]Not all of the arguments carry over directly. For example, unlike the X particle, a physical observer (somehow) beginning in an inflating region could, without encountering anything else, follow a timelike geodesic (necessarily always passing through locally de Sitter space) toward $\mathcal{J}^{-}$ and (noticing nothing) pass through it. The observer would then quickly encounter a bubble and realize that it was traveling into the future of its surrounding region.

will be essentially homogeneous if λ/H^4 is small enough: it can be shown [16] that throughout all time, on average only $(80\pi/9)(\lambda/H^4)$ bubbles formed prior to t will intersect the chosen bubble, and that the fractional volume of the chosen bubble filled with bubbles formed after t is also $(80\pi/9)(\lambda/H^4)$.

If the disquieting feature of this model that events of zero probability occur in the universe (infinitely often) is accepted,[5] there is still cause for concern because an infinite fractal does *not* satisfy the CP. Note, however, that the distribution $f_{\rm inf}(r)$ about each *inflating* point is the same, and is independent of time. Thus the inflating part of the universe does satisfy a ''perfect'' (stationary in time) version of the ''conditional cosmographic principle'' (proposed by Mandelbrot [23] as a generalization of the CP) that the statistical distribution of inflating volume around any point is isotropic and does not depend upon that point *on the condition* that the point is itself inflating. This principle, which holds for both the inflating and thermalized regions, could therefore serve as a replacement for the PCP in constructing an SS universe, though it is a radical departure from the CP that implictly or explicitly lies at the heart of almost all known cosmological models.

The fractally inflating model just described is, however, not the only conceivable possibility, and we can sketch out a number of possible variations that could satisfy the usual CP and might (or might not) improve upon the model.

First, relaxing the PCP to the CP, the nucleation rate λ could tend to zero at ''early times'' so that the number of nucleation events in the past light cone of any point is finite. This would prevent $f_{\rm inf}$ from vanishing at any finite time (and the fractal distribution would, as in future-eternal inflation, have an outer scale above which it becomes homogeneous). This would, however, come at the great expense of introducing a preferred point in time.

Second, eternal inflation could occur in some $4+1$ or higher-dimensional manifold and somehow nucleate $(3+1)$-dimensional bubbles incapable of filling the space. This would also prevent $f_{\rm inf}$ from vanishing. (A related possibility is that big-bang cosmologies could be formed within the intersection of bubble walls in a higher-dimensional spacetime. A model of this sort has been proposed in Ref. [24]. Our construction might be applied to make the inflating bulk eternal.)

Third, thermalized or slowly-rolling regions could reenter inflation. In the ''recycling'' scenario of Ref. [25] this occurs due to quantum fluctuations of the inflaton. There might be some way in which the fraction of inflating space could be a finite fraction of ''all'' space but is extremely difficult to see how to compute this volume fraction in an unambiguous way.

[5]Constructing a steady-state model is different from constructing a model evolved from an initial condition in that one must choose the state of the system at some particular time, and then show that the state maintains itself and does not ''self-destruct'' by evolving into anything else. Thus *assuming* that some region is inflating is allowed (despite the odds) as long as this assumption self-consistently leads to the same configuration at a later time.

Fourth, in the recent ''cyclic'' model of Ref. [26] the universe consists of two repeatedly colliding $(3+1)$-dimensional flat branes embedded in a $(4+1)$-dimensional bulk. An (indestructible) observer on one of the branes sees a flat space which is almost always exponentially expanding, and in which particles are periodically created when an ''ekpyrotic'' collision occurs between the branes. Averaged over sufficient time, the expansion is exponential and the model comes to resemble a quasisteady-state to an observer on the brane. The argument of Ref. [9] applies to geodesics on the brane, all of which cannot then be fully extended (because they encounter $\mathcal{J}^-$ a finite proper time in their past.) Here as in the classical SS all the matter that is created in this model (in the brane collisions) originates at rest in the comoving frame (defined here by those collisions), so no physical particles follow the geodesics intersecting $\mathcal{J}^-$. Nevertheless, as in the SS the construction described herein may aid in constructing a global, geodesically complete spacetime for this scenario.

IV. CONCLUSION

We have argued that the geodesically incomplete flat slicing of de Sitter spacetime can be completed in a self-consistent and physically sensible way by considering it to be one of two similar or identical regions of a full de Sitter spacetime that is partitioned by the flat slicing $t\rightarrow-\infty$ null surface $\mathcal{J}^-$. Our construction follows naturally from causality constraints which forbid each region from sending particles or information into the other region. It also suggests intimate links between the arrows of time, the cosmic expansion, and the (perfect) cosmological principle.

Although our arguments have been largely classical, they may have interesting implications for the formulation of quantum field theories (QFTs) in de Sitter space. Both the ''two universe'' and identified models are geodesically complete and seem therefore to provide a more satisfactory background for QFTs than would an eternally inflating spacetime with a boundary. How (and whether) this quantum mechanical formulation can be achieved constitutes an interesting subject for future research [27].

Our construction may be applied to extend standard theories of future-eternal inflation into the eternal past, though we do not claim that such models are problem-free. In particular, on any equal-time slice the inflating regions form a fractal distribution of infinite volume and yet a vanishing volume *fraction*, and the cosmological principle must be replaced by some sort of ''conditional'' cosmological principle that can hold for infinite fractals. For those deeming these features undesirable, we have listed a number of possible ways in which they might be avoided by other models.

Speaking more generally, what makes constructing eternal models of the universe both appealing and difficult is that almost all, at bottom, have the same essential features. To avoid a preferred time (as seems highly desirable), the model must enforce some sort of (quasi)steady state. For the 2nd law of thermodynamics to hold universally, the universe must then expand lest it be always in equilibrium, and to be (quasi)steady this expansion must be (quasi)exponential.

Though not rigorous, these arguments lead somewhat unavoidably to the flat slicing of de Sitter spacetime or some variant of it.[6] Thus we suspect that a construction like that proposed here may be necessary in any reasonable model for an eternal universe that avoids a beginning of time.

[6]Chaotic ''eternal'' inflation [28] may conceivably be an exception to this, as the inflaton potential in the inflating region tends to approach the Planck energy and it is unclear how one is to talk about a global structure of the universe at all.

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