Supersymmetric model of the muon anomalous magnetic moment and neutrino masses

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We propose the novel lepton-number relationship $L_\tau = L_e + L_\mu$, which is uniquely realized by the interaction $(\hat{\nu}_e \hat{\mu} - \hat{e} \hat{\nu}_\mu)\hat{\tau}^c$ in supersymmetry and may contribute to the muon anomalous magnetic moment. Neutrino masses (with bimaximal mixing) may be generated from the spontaneous and soft breaking of this lepton symmetry.

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In the minimal standard model of particle interactions, the 3 lepton numbers L_e, L_μ, L_τ are separately conserved automatically. If it is extended to include supersymmetry, the assignment of L_e, L_μ, and L_τ becomes more complicated. However, it has been shown some time ago [1] that there are actually 17 well-defined models: 1 with 3 lepton numbers, i.e. the minimal supersymmetric standard model (MSSM), 6 with 2 lepton numbers, 9 with 1 lepton number, and 1 with no lepton number, i.e. the general *R*-parity violating (but baryon-number conserving) supersymmetric model. Three such models are particularly interesting because they require only one additional term in the superpotential beyond that of the MSSM, i.e.,

$$\hat{W} = h(\hat{\nu}_e \hat{\mu} - \hat{e} \hat{\nu}_\mu)\hat{\tau}^c, \tag{1}$$

and its two obvious permutations. These terms are unique because they are the only ones allowed by the conservation of two lepton numbers [1] with the pattern $e \sim (1,0)$, $\mu \sim (0,1)$, and $\tau \sim (1,1)$ for the example given above.

In this paper we will show that this extra term allows a significant contribution to the anomalous magnetic moment of the muon [2], independent of other possible MSSM contributions [3]. We then break this symmetry softly and spontaneously, and show that neutrino masses (with bimaximal mixing) are easily obtained for an explanation of the atmospheric [4] and solar [5] neutrino observations.

The interaction terms of the Lagrangian resulting from Eq. (1) are given by

$$\mathcal{L}_{int} = h(\nu_e \mu - e \nu_\mu)\tilde{\tau}^c + h(\nu_e \tau^c \tilde{\mu} - e \tau^c \tilde{\nu}_\mu) + h(\mu \tau^c \tilde{\nu}_e - \nu_\mu \tau^c \tilde{e}) + \text{H.c.} \tag{2}$$

Hence there are 2 contributions to the muon anomalous magnetic moment as shown in Fig. 1. They are easily evaluated [6] and we obtain

$$\Delta a_\mu = \frac{h^2 m_\mu^2}{96\pi^2}\left(\frac{2}{m_{\tilde{\nu}_e}^2} - \frac{1}{m_{\tilde{\tau}^c}^2}\right). \tag{3}$$

Similarly,

$$\Delta a_e = \frac{h^2 m_e^2}{96\pi^2}\left(\frac{2}{m_{\tilde{\nu}_\mu}^2} - \frac{1}{m_{\tilde{\tau}^c}^2}\right), \tag{4}$$

$$\Delta a_\tau = \frac{h^2 m_\tau^2}{96\pi^2}\left(\frac{2}{m_{\tilde{\nu}_e}^2} + \frac{2}{m_{\tilde{\nu}_\mu}^2} - \frac{1}{m_{\tilde{e}}^2} - \frac{1}{m_{\tilde{\mu}}^2}\right). \tag{5}$$

Of all the possible effective four-fermion interactions which can be derived from Eq. (2), only two are easily accessible experimentally: $\mu \to e \nu_\mu \bar{\nu}_e$ through $\tilde{\tau}^c$ exchange [7] and $e^+ e^- \to \tau^+ \tau^-$ through $\tilde{\nu}_\mu$ exchange. For simplicity, both $\tilde{\tau}^c$ and $\tilde{\nu}_\mu$ may be assumed to be heavy, say a few TeV, so the coupling h is allowed to be of order unity in Eq. (2). To obtain $\Delta a_\mu \sim 10^{-9}$ to account for the possible discrepancy of the experimental value [2] with the standard-model expectation [8], we need $\tilde{\nu}_e$ to be relatively light, say around 200 GeV. We have no understanding why $\tilde{\tau}^c$ and $\tilde{\nu}_\mu$ should be so much heavier than $\tilde{\nu}_e$. However, given that we have chosen Eq. (1) as our model, significant differences among the lepton families are to be expected.

Our model as it stands forbids neutrino masses because it conserves L_e and L_μ (with $L_\tau = L_e + L_\mu$). Consider now the *soft* breaking of these lepton numbers by the terms

$$\mu_\alpha(\hat{l}_\alpha \hat{h}_2^+ - \hat{\nu}_\alpha \hat{h}_2^0) \tag{6}$$

in the superpotential, i.e. the so-called bilinear *R*-parity violation [9]. In that case, the 4×4 neutralino mass matrix of the MSSM must be expanded to include the 3 neutrinos as well as to form a 7×7 mass matrix. It is well-known that

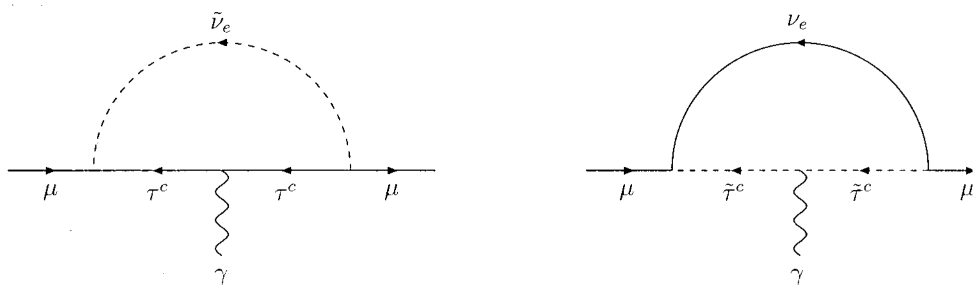

FIG. 1. Contributions to the muon anomalous magnetic moment.

one tree-level mass, corresponding to a linear combination of ν_e, ν_μ, and ν_τ is now obtained. In this scenario, the scalar neutrinos also acquire nonzero vacuum expectation values [10] and one-loop radiative neutrino masses are possible [11]. To fit the present data on atmospheric [4] and solar [5] neutrino oscillations, restrictions on the parameters of the MSSM are implied.

In our model there is another, unrestricted source of radiative neutrino mass, as shown in Fig. 2. This gives a contribution only to the off-diagonal $\nu_e \nu_\mu$ term. Hence our effective 3×3 neutrino mass matrix in the basis $(\nu_e, \nu_\mu, \nu_\tau)$ is of the form

$$\mathcal{M}_\nu = \begin{bmatrix} a_1^2 & a_1 a_2 + b & a_1 a_3 \\ a_1 a_2 + b & a_2^2 & a_2 a_3 \\ a_1 a_3 & a_2 a_3 & a_3^2 \end{bmatrix}, \tag{7}$$

where we have assumed that the usual one-loop contributions from bilinear *R*-parity violation [11] are actually negligible. This matrix has 4 parameters and yields 3 eigenvalues and 3 mixing angles. Consider for example $a_3 = a_2$ and define $x \equiv 1 + (b/a_1 a_2)$. We then have

$$\mathcal{M}_\nu = \begin{bmatrix} a_1^2 & x a_1 a_2 & a_1 a_2 \\ x a_1 a_2 & a_2^2 & a_2^2 \\ a_1 a_2 & a_2^2 & a_2^2 \end{bmatrix}. \tag{8}$$

Assuming that a_1 and $x a_1$ are much smaller than a_2, the eigenvalues are easily determined to be

$$m_1 = -\frac{(1-x) a_1 a_2}{\sqrt{2}} + \frac{(1-x)(3+x) a_1^2}{8}, \tag{9}$$

$$m_2 = \frac{(1-x) a_1 a_2}{\sqrt{2}} + \frac{(1-x)(3+x) a_1^2}{8}, \tag{10}$$

$$m_3 = 2 a_2^2 + \frac{(1+x)^2 a_1^2}{4}, \tag{11}$$

corresponding to the eigenstates

$$\nu_1 = \frac{1}{\sqrt{2}}\left[1 - \frac{(3+x) a_1}{8\sqrt{2} a_2}\right] \nu_e + \frac{1}{2}\left[1 - \frac{(1+3x) a_1}{8\sqrt{2} a_2}\right] \nu_\mu - \frac{1}{2}\left[1 + \frac{(7+5x) a_1}{8\sqrt{2} a_2}\right] \nu_\tau, \tag{12}$$

$$\nu_2 = \frac{1}{\sqrt{2}}\left[1 + \frac{(3+x) a_1}{8\sqrt{2} a_2}\right] \nu_e - \frac{1}{2}\left[1 + \frac{(1+3x) a_1}{8\sqrt{2} a_2}\right] \nu_\mu + \frac{1}{2}\left[1 - \frac{(7+5x) a_1}{8\sqrt{2} a_2}\right] \nu_\tau, \tag{13}$$

$$\nu_3 = \frac{(1+x) a_1}{2\sqrt{2} a_2} \nu_e + \frac{1}{\sqrt{2}} \nu_\mu + \frac{1}{\sqrt{2}} \nu_\tau, \tag{14}$$

which is of course very near the case of bimaximal mixing. Atmospheric neutrino oscillations are thus explained by $\nu_\mu \to \nu_\tau$ with $\sin^2 2\theta \simeq 1$ and

$$\Delta m_{23}^2 \simeq \Delta m_{13}^2 \simeq 4 a_2^4 + \frac{1}{2}(1 + 6x + x^2) a_1^2 a_2^2, \tag{15}$$

and solar neutrino oscillations by $\nu_e \to (\nu_\mu - \nu_\tau)/\sqrt{2}$ with $\sin^2 2\theta \simeq 1$ and

$$\Delta m_{12}^2 \simeq \frac{(1-x)^2 (3+x)}{2\sqrt{2}} a_1^3 a_2. \tag{16}$$

Using $a_2 = 0.16$ eV$^{1/2}$, $a_1 = 0.05$ eV$^{1/2}$, and $x = -1$, we find $\Delta m_{atm}^2 \simeq 2.5 \times 10^{-3}$ eV^2, and $\Delta m_{sol}^2 \simeq 5.7 \times 10^{-5}$ eV^2, in very good agreement with data.

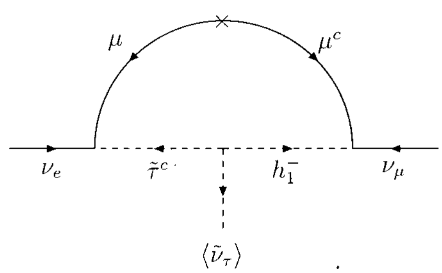

FIG. 2. Radiative contribution to the $\nu_e \nu_\mu$ mass.

Referring back to Fig. 2, we calculate the parameter b to be given by

$$b=\frac{G_F m_\mu^2}{4\pi^2\sqrt{2}}\frac{hAm_\tau\langle\tilde{\nu}_\tau\rangle}{m_{eff}^2\cos^2\beta}, \tag{17}$$

where m_{eff} is a function of $m_{\tilde{\tau}^c}$ and $m_{h^\pm}$. Using $h=1$ and $m_{eff}^2/A=1$ TeV, we find that in order to obtain $b=-2a_1a_2\simeq 0.016$ eV, we need $\langle\tilde{\nu}_\tau\rangle\simeq 3.86\cos^2\beta$ GeV. This relatively small value is negligible compared to $v=(2\sqrt{2}G_F)^{-1/2}=174$ GeV (especially for large values of $\tan\beta$), and consistent with all present low-energy phenomenology.

The salient feature of our model is that $\tilde{\nu}_e$ must be relatively light, say around 200 GeV, to explain a large Δa_μ. In that case, $\tilde{e}$ must also be light, because of the well-known MSSM relationship

$$m_{\tilde{e}}^2=m_{\tilde{\nu}_e}^2-M_W^2\cos 2\beta. \tag{18}$$

Now both $\tilde{\nu}_e$ and $\tilde{e}$ can be produced by electroweak interactions, such as $Z\to\tilde{\nu}_e^*\tilde{\nu}_e$ and $W^-\to\tilde{\nu}_e^*\tilde{e}$. They must then decay according to Eq. (2), i.e.,

$$\tilde{\nu}_e\to\mu^+\tau^-, \qquad \tilde{e}\to\overline{\nu_\mu}\tau^-. \tag{19}$$

These are very distinctive signatures and if observed, the two masses may be reconstructed and the value of β determined by Eq. (18).

If the MSSM neutralinos $\tilde{\chi}_i^0$ and charginos $\tilde{\chi}_i^+$ are produced, as decay products of squarks for example, then the decays

$$\tilde{\chi}_i^0\to\tilde{\nu}_e\overline{\nu_e}(\tilde{\nu}_e^*\nu_e),\ \ \tilde{e}e^+(\tilde{e}^*e^-), \qquad \tilde{\chi}_i^+\to\tilde{\nu}_e e^+,\ \ \tilde{e}^*\nu_e \tag{20}$$

are possible. The subsequent decays of Eq. (19) would again be indicative of our model. In a future muon collider, the process

$$\mu^+\mu^-\to\tilde{\nu}_e^*\tilde{\nu}_e \tag{21}$$

(through τ exchange) is predicted, by which the $\tilde{\nu}_e$ decay of Eq. (19) could be studied with precision.

Single production of $\tilde{\nu}_e$ and $\tilde{e}$ is also possible in an e^+e^- collider. There are 4 different final states: $\tau^+\mu^-\tilde{\nu}_e$, $\tau^+\nu_\mu\tilde{e}$, and their conjugates. With the subsequent decays given by Eq. (19), the experimental signatures are 4 charged leptons ($\tau^+\tau^-\mu^+\mu^-$) and 2 charged taus+missing energy ($\tau^+\tau^-\overline{\nu_\mu}\nu_\mu$). The absence of such events at LEP up to 207 GeV constrains h and $m_{\tilde{\nu}_e}$. Although a quantitative analysis is not available at present, we estimate the likely mass bound (on the basis that it would be similar to that of single scalar leptoquark production) to be around 180 GeV for $h=1$. To get $\Delta a_\mu\sim 10^{-9}$, we have thus chosen $h=1$ and $m_{\tilde{\nu}_e}=200$ GeV (which puts $\tilde{\nu}_e$ beyond the production capability of LEP) as representative values.

Lepton-flavor violating processes are very much suppressed in our model, because they have to be proportional to the small parameters μ_α in Eq. (6) or the small vacuum expectation values $\langle\tilde{\nu}_\alpha\rangle$. For example, the rare decay $\tau\to e\gamma$ proceeds in one-loop order through $\tilde{\nu}_e$ exchange and the mixing of μ_L with $\tilde{w}^-$, i.e.,

$$\left(\frac{\mu_\mu}{\mu_0}-\frac{\langle\tilde{\nu}_\mu\rangle}{v\cos\beta}\right)\frac{M_W\cos\beta\sqrt{2}}{m_{\tilde{w}}}, \tag{22}$$

and through $\tilde{e}$ exchange and the mixing of ν_μ with $\tilde{B}$ and $\tilde{w}^0$, i.e.,

$$\begin{gathered}-\left(\frac{\mu_\mu}{\mu_0}-\frac{\langle\tilde{\nu}_\mu\rangle}{v\cos\beta}\right)\frac{M_Z\sin\theta_W\cos\beta}{m_{\tilde{B}}},\\ \left(\frac{\mu_\mu}{\mu_0}-\frac{\langle\tilde{\nu}_\mu\rangle}{v\cos\beta}\right)\frac{M_Z\cos\theta_W\cos\beta}{m_{\tilde{w}}},\end{gathered} \tag{23}$$

where μ_0 is the coefficient of the $(\hat{h}_1^-\hat{h}_2^+-\hat{h}_1^0\hat{h}_2^0)$ term in the superpotential of the MSSM. Its amplitude is approximately given by

$$\begin{aligned}\mathcal{A}=&\frac{ehg(5-\tan^2\theta_W)}{96\pi^2\sqrt{2}}\left(\frac{\mu_\mu}{\mu_0}-\frac{\langle\tilde{\nu}_\mu\rangle}{v\cos\beta}\right)\\ &\times\frac{M_W\cos\beta}{m_{eff}^2}\epsilon^\alpha q^\beta\overline{e}\sigma_{\alpha\beta}\left(\frac{1+\gamma_5}{2}\right)\tau,\end{aligned} \tag{24}$$

where m_{eff} is a function of all the heavy masses in the loop and normalized so that if all of them are equal, then they are all equal to m_{eff}. (Contributions due to the mixing of scalar leptons and charged Higgs scalars are suppressed because they involve the Yukawa couplings of the latter to the leptons.)

Since the neutrino mass parameter a_2 used earlier is given by

$$\begin{aligned}a_2^2=&\left(\frac{\mu_\mu}{\mu_0}-\frac{\langle\tilde{\nu}_\mu\rangle}{v\cos\beta}\right)^2\\ &\times\frac{M_Z^2\cos^2\beta}{m_{\tilde{B}}m_{\tilde{w}}}(m_{\tilde{B}}\cos^2\theta_W+m_{\tilde{w}}\sin^2\theta_W),\end{aligned} \tag{25}$$

the $\tau\to e\gamma$ branching fraction is related to it by

$$\begin{aligned}B(\tau\to e\gamma)=&\frac{\sin^2\theta_W\cos^2\theta_W}{48\pi^2}\\ &\times\frac{(5-\tan^2\theta_W)^2h^2a_2^2m_{\tilde{B}}m_{\tilde{w}}M_W^4}{(m_{\tilde{B}}\cos^2\theta_W+m_{\tilde{w}}\sin^2\theta_W)m_\tau^2m_{eff}^4}\\ &\times B(\tau\to e\nu\overline{\nu}).\end{aligned} \tag{26}$$

Using $h=2$, $a_2=0.16$ eV$^{1/2}$, and assuming that $m_{\tilde{B}}=m_{\tilde{w}}=m_{eff}=200$ GeV, we find $B(\tau\to e\gamma)\simeq 2.5\times 10^{-13}$, which is many orders of magnitude below the experimental upper bound of 2.7×10^{-6}.

The $\mu\to e\gamma$ rate is even more suppressed because it has to violate both L_μ and L_e, whereas $\tau\to e\gamma$ only needs to violate L_μ. We note that if we had chosen the extra term in Eq. (1) to be $h(\hat{\nu}_e\hat{\tau}-\hat{e}\hat{\nu}_\tau)\hat{\mu}^c$ or $h(\hat{\nu}_\mu\hat{\tau}-\hat{\mu}\hat{\nu}_\tau)\hat{e}^c$, then $\mu\to e\gamma$ would not be doubly suppressed and would have a branching fraction of about 4×10^{-10}, in contradiction with the present experimental bound [12] of 1.2×10^{-11}. We note also that $m_{ee}=a_1^2$ of Eq. (7) is the effective neutrino mass measured in neutrinoless double beta decay. It is of order 10^{-3} eV in our model, which is well below the present experimental bound [13] of 0.2 eV.

In conclusion, we have shown how a novel minimal extension of the MSSM with $L_\tau=L_e+L_\mu$ allows it to have a significant contribution to the muon anomalous magnetic moment without otherwise constraining the usual MSSM parameter space. With the soft and spontaneous breaking of this lepton symmetry, realistic neutrino masses (with bimaximal mixing) are generated for an explanation of atmospheric and solar neutrino oscillations. The scalar electron doublet $(\tilde{\nu}_e,\tilde{e})$ is predicted to be light (perhaps around 200 GeV) and has distinctive experimental signatures.

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