Long-distance contribution to the muon-polarization asymmetry in $K^+ \to \pi^+ \mu^+ \mu^-$

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We reexamine the calculation of the long-distance contribution to the muon-polarization asymmetry Δ_{LR}, which arises, in $K^+ \to \pi^+ \mu^+ \mu^-$, from the two-photon intermediate state. The parity-violating amplitude of this process, induced by the local anomalous $K^+ \pi^- \gamma^* \gamma^*$ transition, is analyzed; unfortunately, one cannot expect to predict its contribution to the asymmetry by using chiral perturbation theory alone. Here we evaluate this amplitude and its contribution to Δ_{LR} by employing a phenomenological model called the FMV model (factorization model with vector couplings), in which the use of the vector and axial-vector resonance exchange is important to soften the ultraviolet behavior of the transition. We find that the long-distance contribution is of the same order of magnitude as the standard model short-distance contribution.

I. INTRODUCTION

The measurement of the muon polarization asymmetry in the decay $K^+ \to \pi^+ \mu^+ \mu^-$ is expected to give some valuable information on the structure of the weak interactions and flavor mixing angles [1–6]. The total decay rate for this transition is dominated by the one-photon exchange contribution, which is parity conserving, and the corresponding invariant amplitude can be parametrized in terms of one form factor [1,7]:

$$\mathcal{M}^{\text{PC}} = \frac{s_1 G_F \alpha}{\sqrt{2}} f(s)(p_K + p_\pi)^\mu \bar{u}(p_-, s_-) \gamma_\mu v(p_+, s_+),$$

(1)

where p_K, p_π, and $p_\pm$ are the four-momenta of the kaon, pion, and $\mu^\pm$ respectively, and s_1 is the sine of the Cabibbo angle. The $s_\pm$ are the spin vectors for the $\mu^\pm$, and the quantity $s = (p_+ + p_-)^2$ is the $\mu^+ \mu^-$ pair invariant mass squared.

In the standard model, in addition to the dominant contribution in Eq. (1), the decay amplitude also contains a small parity-violating piece, which generally has the form [2]

$$\mathcal{M}^{\text{PV}} = \frac{s_1 G_F \alpha}{\sqrt{2}} [B(p_K + p_\pi)^\mu + C(p_K - p_\pi)^\mu]$$
$$\times \bar{u}(p_-, s_-) \gamma_\mu \gamma_5 v(p_+, s_+),$$

(2)

where the form factors B and C get contributions from both short- and long-distance physics.

The muon-polarization asymmetry in $K^+ \to \pi^+ \mu^+ \mu^-$ is defined as

$$\Delta_{LR} = \frac{|\Gamma_R - \Gamma_L|}{\Gamma_R + \Gamma_L},$$

(3)

where Γ_R and Γ_L are the rates to produce right- and left-handed μ^+, respectively. This asymmetry arises from the interference of the parity-conserving part of the decay amplitude [Eq. (1)] with the parity-violating part [Eq. (2)], which gives

$$\frac{d(\Gamma_R - \Gamma_L)}{d\cos\theta \, ds} = \frac{-s_1^2 G_F^2 \alpha^2}{2^8 m_K^3 \pi^3} \sqrt{1 - \frac{4m_\mu^2}{s}} \lambda(s, m_K^2, m_\pi^2)$$
$$\times \left\{ \text{Re}[f^*(s)B] \sqrt{1 - \frac{4m_\mu^2}{s}} \right.$$
$$\times \lambda^{1/2}(s, m_K^2, m_\pi^2) \sin^2\theta$$
$$+ 4 \left(\text{Re}[f^*(s)B] \frac{m_K^2 - m_\pi^2}{s} \right.$$
$$\left. + \text{Re}[f^*(s)C] \right) m_\mu^2 \cos\theta \bigg\},$$

(4)

while, as a good approximation, the total decay rate can be obtained from Eq. (1):

$$\frac{d(\Gamma_R + \Gamma_L)}{d\cos\theta \, ds} = \frac{s_1^2 G_F^2 \alpha^2 |f(s)|^2}{2^9 m_K^3 \pi^3} \sqrt{1 - \frac{4m_\mu^2}{s}} \lambda^{3/2}(s, m_K^2, m_\pi^2)$$
$$\times \left[1 - \left(1 - \frac{4m_\mu^2}{s} \right) \cos^2\theta \right],$$

(5)

where $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$, $4m_\mu^2 \leq s \leq (m_K - m_\pi)^2$, and θ is the angle between the three-momentum of the kaon and the three-momentum of the μ^- in the $\mu^+ \mu^-$ pair rest frame. It is easy to see that, when the decay distribution in Eq. (4) is integrated over θ on the full phase space, the contribution to Δ_{LR} from the C part of the amplitude vanishes.

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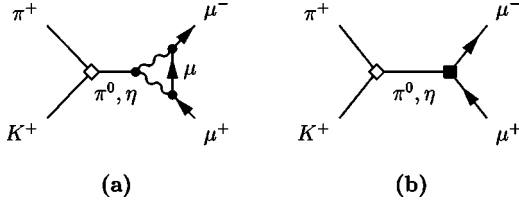

FIG. 1. Feynman diagrams that give the two-photon contribution to the long-distance C part of the parity-violating amplitude of $K^+ \to \pi^+ \mu^+ \mu^-$ in chiral perturbation theory. The wavy line is the photon. The diamond denotes the weak vertex, the full dot denotes the strong or electromagnetic vertex, and the full square in (b) denotes the local $\pi^0 \mu^+ \mu^-$ or $\eta \mu^+ \mu^-$ couplings.

Fortunately, the form factor $f(s)$ in Eq. (1) is now known. In fact chiral perturbation theory dictates the following decomposition [7]:

$$f(s) = a_+ + b_+ \frac{s}{m_K^2} + w_+^{\pi\pi}(s/m_K^2). \tag{6}$$

Here, $w_+^{\pi\pi}$ denotes the pion-loop contribution, which leads to a small imaginary part of $f(s)$ [7,12], and its full expression can be found in Ref. [7]. This structure has been accurately tested and found correct by the E856 Collaboration, which also fixes $a_+ = -0.300 \pm 0.005$, $b_+ = -0.335 \pm 0.022$ [8]. Recently the HyperCP Collaboration also studied this channel in connection with the study of CP-violating width charge asymmetry in $K^\pm \to \pi^\pm \mu^+ \mu^-$ [9]. This channel will be further analyzed by E949 [10] and NA48b [11].

It is known that, within the standard model, the short-distance contributions to $\mathcal{M}^{\text{PV}}$ in Eq. (2) arise predominantly from the W-box and Z-penguin Feynman diagrams, which carry clean information on the weak mixing angles [2]. The authors of Ref. [4] generalized the results in Ref. [2] beyond the leading logarithmic approximation: for the Wolfenstein parameter ρ in the range $-0.25 \leq \rho \leq 0.25$, $|V_{cb}| = 0.040 \pm 0.004$, and $m_t = (170 \pm 20)$ GeV,

$$3.0 \times 10^{-3} \leq \Delta_{LR} \leq 9.6 \times 10^{-3} \tag{7}$$

with the cut $-0.5 \leq \cos\theta \leq 1.0$. Hence, the experimental determination of B and C would be very interesting from the theoretical point of view, provided that the long-distance contributions are under control.

The dominant long-distance contributions to the parity-violating amplitude of $K^+ \to \pi^+ \mu^+ \mu^-$ are from the Feynman diagrams in which the $\mu^+ \mu^-$ pair is produced by two-photon exchange [2]. Since these contributions arise from nonperturbative QCD, they are difficult to calculate in a reliable manner. The contribution to the asymmetry Δ_{LR} from the long-distance C part amplitude, whose Feynman diagrams are shown in Fig. 1, has been estimated in Ref. [2] with the cut $-0.5 \leq \cos\theta \leq 1$, which indicates that it is substantially smaller than the short-distance part contribution in Eq. (7). However, because of the unknown chiral perturbation theory parameters, the long-distance contribution to Δ_{LR} from the B part of the amplitude, which is induced from the direct $K^+ \pi^- \gamma^* \gamma^*$ anomalous transition, has never been

predicted. As mentioned above, when we integrate over θ without any cuts in Eq. (4), only the contribution from the B part of the amplitude will survive. Therefore, it would be interesting to calculate this part of the parity-violating amplitude, and estimate its contribution to the asymmetry Δ_{LR} using phenomenological models.

The paper is organized as follows. In Sec. II, we briefly reconsider the two-photon long-distance contributions to $K^+ \to \pi^+ \mu^+ \mu^-$ within chiral perturbation theory. In order to evaluate the asymmetry Δ_{LR} from the long-distance B part of the amplitude, models are required. So in Sec. III we introduce a phenomenological model involving vector and axial-vector resonances, called the FMV model (factorization model with vector couplings) from Refs. [13,14], for this task. Section IV contains our conclusions.

II. CHIRAL PERTURBATION THEORY

In this section we reexamine the two-photon contributions to the parity-violating amplitude of $K^+ \to \pi^+ \mu^+ \mu^-$ in chiral perturbation theory. There are local terms that can contribute to the amplitude, which can be constructed using standard notation [5]. The pion and kaon fields are identified as the Goldstone bosons of the spontaneously broken $SU(3)_L \times SU(3)_R$ chiral symmetry and are collected into a unitary 3×3 matrix $U = u^2 = \exp(i\sqrt{2}\Phi/f_\pi)$ with $f_\pi \simeq 93$ MeV, and

$$\Phi = \frac{1}{\sqrt{2}} \lambda \cdot \phi(x) = \begin{pmatrix} \dfrac{\pi^0}{\sqrt{2}} + \dfrac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\dfrac{\pi^0}{\sqrt{2}} + \dfrac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\dfrac{2\eta_8}{\sqrt{6}} \end{pmatrix}. \tag{8}$$

Thus, at the leading order, the local terms contributing to the decays $K \to \pi l^+ l^-$ and $K_L \to l^+ l^-$ can be written as [2,15]

$$\mathcal{L}_\chi = \frac{is_1 G_F \alpha}{\sqrt{2}} f_\pi^2 \bar{l} \gamma_\mu \gamma_5 l \{ h_1 \langle \lambda_6 Q^2 (U \partial^\mu U^+ - \partial^\mu U U^+) \rangle$$
$$+ h_2 \langle \lambda_6 Q (UQ \partial^\mu U^+ - \partial^\mu U Q U^+) \rangle$$
$$+ h_3 \langle \lambda_6 (UQ^2 \partial^\mu U^+ - \partial^\mu U Q^2 U^+) \rangle \}, \tag{9}$$

where $\langle A \rangle$ denotes $\text{Tr}(A)$ in the flavor space, and Q is the electromagnetic charge matrix:

$$Q = \begin{pmatrix} \dfrac{2}{3} & 0 & 0 \\ 0 & -\dfrac{1}{3} & 0 \\ 0 & 0 & -\dfrac{1}{3} \end{pmatrix}. \tag{10}$$

Note that each term contains two Q's because the effective Lagrangian in Eq. (9) is from the Feynman diagrams with two photons, and CPS symmetry [16] has been used to obtain this Lagrangian [2,15]. From Eqs. (9) and (2), it is easy to obtain the two-photon contributions to the parity-violating form factors B and C as

$$B = -\frac{2}{9}(h_1 - 2h_2 + 4h_3), \quad C = 0. \quad (11)$$

As pointed out in Ref. [2], CPS symmetry forces the contribution to C from the leading order local terms to vanish. The dominant contribution to C is from the $\pi^0(\eta)$ pole-type diagrams generated by the transitions $K^+ \to \pi^+ \pi^0(\eta)$ and $\pi^0(\eta) \to \mu^+ \mu^-$, via the two-photon intermediate states (see Fig. 1). This contribution to Δ_{LR} has been estimated by Lu, Wise, and Savage [2]: $|\Delta_{LR}| < 1.2 \times 10^{-3}$ for the cut $-0.5 \leqslant \cos\theta \leqslant 1.0$, which is much less than the asymmetry arising from the short-distance physics in Eq. (7).

On the other hand, as shown in Eq. (11), the contribution from the local terms to the parity-violating form factor B is proportional to the constant $h_1 - 2h_2 + 4h_3$. Since h_i's, $i = 1,2,3$, are unknown coupling constants, B cannot be predicted in the framework of chiral perturbation theory. The Lagrangian in Eq. (9) also gives rise to the two-photon contribution to the decay $K_L \to \mu^+ \mu^-$, which has been studied in Ref. [15] within this context. However, it is not possible to use that decay to measure the unknown combination $h_1 - 2h_2 + 4h_3$ because the h_i's in the local terms enter the amplitude for the transition $K_L \to \mu^+ \mu^-$ as a different linear combination $h_1 + h_2 + h_3$. Therefore, in the following section, we have to turn our attention to phenomenological models and try to estimate this part of the contribution to the asymmetry Δ_{LR}.

III. FMV MODEL

In order to evaluate the two-photon contribution to the B part of the amplitude, one has to construct the local $K^+ \pi^- \gamma\gamma$ coupling that contains the total antisymmetric tensor. The lowest order contribution for it starts from $O(p^6)$ in chiral perturbation theory [17]. Therefore, the unknown couplings in the effective Lagrangian will make it impossible to predict this amplitude, which has been shown in the previous section.

Vector meson dominance has proved to be very effective in predicting the coupling constants in the $O(p^4)$ strong Lagrangian [18,19]; the unknown couplings can thus be reduced significantly. However, this is not an easy task in the weak Lagrangian since the weak couplings of spin-1 resonance with pseudoscalars are not yet fixed by experiment. Thus various models implementing weak interactions at the hadronic level have been proposed, yet it is very likely that mechanisms and couplings working for a subset of processes might not work for other processes unless a secure matching procedure is provided. Nevertheless the information provided by the models can be useful to give a general picture of the hadronization process of the dynamics involved.

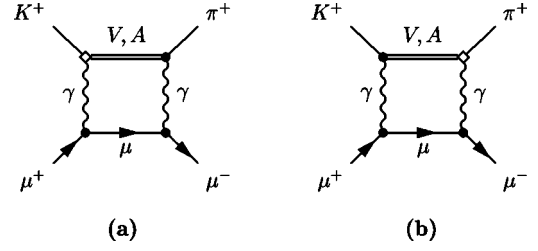The factorization model (FM) has been widely used in the

FIG. 2. One-loop Feynman diagrams that give the two-photon contribution to the long-distance B part of the parity-violating amplitude of $K^+ \to \pi^+ \mu^+ \mu^-$ induced by the vector and axial-vector resonance exchange. The diamond denotes the weak vertex, and the full dot denotes the strong or electromagnetic vertex.

literature [20–23] for this task. An implementation of the FM in vector couplings (FMV) was proposed in Ref. [13]; this seems to be an efficient way of including the $O(p^6)$ vector resonance contributions to the $K \to \pi\gamma\gamma$ and $K_L \to \gamma l^+ l^-$ processes. The basic statement of the FMV model is to use the idea of factorization to construct the weak vertices involving the vector resonances and then integrate out the vectors, i.e. perform the factorization at the scale of the vector mass. An alternative approach is to integrate out the vector degrees of freedom to generate the strong Lagrangian before factorization, i.e. perform the factorization at the scale of the kaon mass. As will be shown below, the vector and axial-vector resonance degrees of freedom play a very important role in our calculation of the one-loop Feynman diagrams of Fig. 2.

Keeping only the relevant terms and assuming nonet symmetry, the strong $O(p^3)$ Lagrangian linear in the vector and axial-vector fields reads [24–26]

$$\mathcal{L}_V = -\frac{f_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + h_V \epsilon_{\mu\nu\alpha\beta}\langle V^\mu \{u^\nu, f_+^{\alpha\beta}\}\rangle, \quad (12)$$

$$\mathcal{L}_A = -\frac{f_A}{2\sqrt{2}}\langle A_{\mu\nu} f_-^{\mu\nu}\rangle + i\alpha_A \langle A_\mu [u_\nu, f_+^{\mu\nu}]\rangle, \quad (13)$$

where

$$u_\mu = iu^+ D_\mu U u^+, \quad (14)$$

$$D_\mu U = \partial_\mu U - ir_\mu U + iUl_\mu, \quad (15)$$

$$f_\pm^{\mu\nu} = u F_L^{\mu\nu} u^+ \pm u^+ F_R^{\mu\nu} u, \quad (16)$$

$F_{R,L}^{\mu\nu}$ being the strength field tensors associated with the external r_μ and l_μ fields. If only the electromagnetic field is considered then $r_\mu = l_\mu = -eQA_\mu$. We set $R_{\mu\nu} = \nabla_\mu R_\nu - \nabla_\nu R_\mu$ ($R = V,A$), and ∇_μ is the covariant derivative defined as

$$\nabla_\mu R = \partial_\mu R + [\Gamma_\mu, R], \quad (17)$$

$$\Gamma_\mu = \frac{1}{2}[u^+(\partial_\mu - ir_\mu)u + u(\partial_\mu - il_\mu)u^+]. \quad (18)$$

The determination of the above couplings in Eqs. (12) and (13) from measurements or theoretical models has been discussed in Refs. [26,25]. In order to generate the anomalous $K^+\pi^-\gamma^*\gamma^*$ vertex from vector and axial-vector exchange, given the strong Lagrangian in Eqs. (12) and (13), we have to construct the nonanomalous weak VPγ and the anomalous weak APγ at $O(p^3)$. By applying the factorization procedure with the FMV model, we obtain (for details, see the Appendix)

$$\mathcal{L}_W(\text{VP}\gamma) = -G_8 f_\pi^2 \frac{f_V}{\sqrt{2}} \eta \langle \Delta\{V_{\mu\nu}, f_-^{\mu\nu}\}\rangle, \qquad (19)$$

$$\mathcal{L}_W(\text{AP}\gamma) = -G_8 f_\pi^2 l_A \eta \epsilon_{\mu\nu\alpha\beta} \langle \{\Delta, A^\mu\}\{u^\nu, f_+^{\alpha\beta}\}\rangle, \qquad (20)$$

where $\Delta = u\lambda_6 u^+$, and

$$l_A = \frac{3}{16\sqrt{2}\pi^2} f_A \frac{m_A^2}{f_\pi^2}. \qquad (21)$$

η is the factorization parameter satisfying $0 < \eta \leq 1.0$ generally, and it cannot be given by the model.

Now, combining the strong $\mathcal{L}_S(\text{VP}\gamma)$ in Eq. (12) with the weak $\mathcal{L}_W(\text{VP}\gamma)$ in Eq. (19) [or the strong $\mathcal{L}_S(\text{AP}\gamma)$ in Eq. (13) with the weak $\mathcal{L}_W(\text{AP}\gamma)$ in Eq. (20)], and attaching the photons to the muons with the usual QED vertices, we can get the spin-1 resonance contribution to the B part of the parity-violating amplitude of $K^+ \to \pi^+ \mu^+ \mu^-$ from the two-photon intermediate state. The corresponding Feynman diagrams have been drawn in Fig. 2. The calculation of the contribution from the vector resonance exchange is straightforward:

$$\mathcal{M}_V^{\text{PV}} = -i32\sqrt{2}e^4 G_8 f_V h_V \eta \frac{1}{d}$$

$$\times \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2-m_\mu^2)(q^2-m_V^2)}$$

$$\times (p_K+p_\pi)^\mu \bar{u}(p_-,s_-)\gamma_\mu\gamma_5 v(p_+,s_+); \quad (22)$$

d is the space-time dimension generated from the integral $\int d^dq\, q_\mu q_\nu = (1/d)\int d^dq\, q^2 g_{\mu\nu}$. Because of the logarithmic divergence in the above equation, we do not set $d=4$ now. Note that we retain the loop momentum q in the Feynman integral of the above equation only because we are concerned about the leading order two-photon contribution to the B part of the amplitude, and the Feynman integrals related to the external momenta are obviously of higher order with respect to Eq. (22). On the other hand, we do not consider the diagrams generated by the weak anomalous $K^+\pi^-V\gamma$ and $K^+\pi^-VV$ couplings, because either they have no contributions to the B part of the amplitude or they are higher order. Likewise, the axial-vector resonance contribution is

$$\mathcal{M}_A^{\text{PV}} = i16\sqrt{2}e^4 G_8 f_A l_A \eta \frac{1}{d}$$

$$\times \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2-m_\mu^2)(q^2-m_A^2)}$$

$$\times (p_K+p_\pi)^\mu \bar{u}(p_-,s_-)\gamma_\mu\gamma_5 v(p_+,s_+), \quad (23)$$

which receives contributions only from the first term in Eq. (13). In fact the contribution from the second term is higher order.

Using the Weinberg sum rules [27] together with the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) sum rule [28], one can obtain [24]

$$f_A = \frac{1}{2}f_V, \qquad m_A^2 = 2m_V^2. \qquad (24)$$

Hence, from Eq. (21), we have

$$f_A l_A = \frac{1}{2}f_V l_V, \qquad (25)$$

with

$$l_V = \frac{3}{16\sqrt{2}\pi^2} f_V \frac{m_V^2}{f_\pi^2}. \qquad (26)$$

Then using the relation

$$l_V = 4h_V, \qquad (27)$$

which, as pointed out in Ref. [13], is exact in the hidden local symmetry model [29] and also well supported phenomenologically, we can get

$$\mathcal{M}_{V+A}^{\text{PV}} = -i32\sqrt{2}e^4 G_8 f_V h_V \eta \frac{1}{d}$$

$$\times \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2-m_\mu^2}\left[\frac{1}{q^2-m_V^2} - \frac{1}{q^2-m_A^2}\right]$$

$$\times (p_K+p_\pi)^\mu \bar{u}(p_-,s_-)\gamma_\mu\gamma_5 v(p_+,s_+). \quad (28)$$

Fortunately, one will find that the logarithmic divergences in Eqs. (22) and (23) cancel each other, provided that the relations (24) and (27) are satisfied. Of course, any violation of these relations will lead to divergent results; a renormalized procedure is thus needed, and further uncertainty will be involved. In this paper, we are concerned only about the leading order two-photon contribution to the B part of the amplitude. Therefore, it is expected that Eqs. (24) and (27) can be regarded as good approximations for this goal. Neglecting m_μ^2 in Eq. (28), whose effect is smaller than 5%, we have

$$\mathcal{M}_{V+A}^{\mathrm{PV}} = 8\sqrt{2}\,\alpha^2 G_8 f_V h_V \eta \ln \frac{m_A^2}{m_V^2}$$

$$\times (p_K + p_\pi)^\mu \bar{u}(p_-, s_-)\gamma_\mu \gamma_5 v(p_+, s_+). \tag{29}$$

Comparing Eq. (29) with Eq. (2), and using

$$G_8 = \frac{s_1 G_F}{\sqrt{2}}\, g_8, \tag{30}$$

one will get the two-photon contribution to the parity-violating form factor B as

$$B^{2\gamma} = 8\sqrt{2}\,\alpha g_8 f_V h_V \eta \ln \frac{m_A^2}{m_V^2}. \tag{31}$$

From the measured $K_S \to \pi^+ \pi^-$ decay rate, we fixed $|g_8| = 5.1$ [12,2]. Note that f_V and h_V can be determined from the phenomenology of vector meson decays, as shown in Ref. [26], $|f_V| = 0.20$, and $|h_V| = 0.037$. Thus, using $m_A^2 = 2m_V^2$, we have

$$|B^{2\gamma}| = 2.16 \times 10^{-3} \eta. \tag{32}$$

Now from Eqs. (3), (4), and (5), we can get the asymmetry Δ_{LR} contributed by $B^{2\gamma}$:

$$\Delta_{LR} = 1.7|B^{2\gamma}| = 3.6 \times 10^{-3} \eta \tag{33}$$

for θ integrated over the full phase space, and

$$\Delta_{LR} = 3.0|B^{2\gamma}| = 6.5 \times 10^{-3} \eta \tag{34}$$

for $-0.5 \le \cos\theta \le 1.0$.

From Eqs. (33) and (34), if the factorization parameter $\eta \simeq 1$, which is implied by naive factorization, we find that the long-distance contributions from $B^{2\gamma}$ can be compared with the short-distance contributions given in Refs. [2,4]. In Ref. [13], $\eta \simeq 0.2$–0.3 is preferred by fitting the phenomenology of $K \to \pi\gamma\gamma$ and $K_L \to \gamma l^+ l^-$. In this case, the contributions from Eqs. (33) and (34) might be small, although not fully negligible. Generally, $0 < \eta \le 1.0$; therefore, this uncertainty may make it difficult to get valuable information on the structures of the weak interaction and the flavor-mixing angles by measurement of Δ_{LR} in $K^+ \to \pi^+ \mu^+ \mu^-$. However effects larger than 1% would be a signal of new physics.

IV. CONCLUSIONS

We have studied the long-distance contribution via the two-photon intermediate state to the parity-violating B part of the amplitude of $K^+ \to \pi^+ \mu^+ \mu^-$. Within chiral perturbation theory, one can calculate it up to an unknown parameter combination, which is shown in Eq. (11). At present, we have no way of estimating this unknown combination, and it is therefore impossible to determine this parity-violating amplitude and to predict its contribution to Δ_{LR} by using chiral perturbation theory alone.

We calculate this amplitude in a phenomenological model called the FMV model. The muon polarization asymmetry Δ_{LR} has been estimated up to the factorization parameter η, which is not given by the model, but may be determined phenomenologically in the future. We have established that the background effect may obscure the standard model prediction but large new physics effects can still be tested.

ACKNOWLEDGMENTS

APPENDIX: NONANANOMALOUS WEAK VPγ AND ANOMALOUS WEAK APγ VERTICES IN THE FMV MODEL

The $\Delta S = 1$ nonleptonic weak interactions are described by an effective Hamiltonian

$$\mathcal{H}_{\mathrm{eff}}^{\Delta S = 1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i \mathcal{Q}_i + \mathrm{H.c.} \tag{A1}$$

in terms of Wilson coefficients C_i and four-quark local operators $\mathcal{Q}_i$. If we neglect the penguin contributions, justified by the $1/N_C$ expansion [22], we can write the octet-dominant piece in $\mathcal{H}_{\mathrm{eff}}^{\Delta S = 1}$ as

$$\mathcal{H}_{\mathrm{eff}}^{\Delta S = 1} = -\frac{G_F}{2\sqrt{2}} V_{ud} V_{us}^* C_- \mathcal{Q}_- + \mathrm{H.c.}, \tag{A2}$$

with

$$\mathcal{Q}_- = 4(\bar{s}_L \gamma^\mu u_L)(\bar{u}_L \gamma_\mu d_L) - 4(\bar{s}_L \gamma^\mu d_L)(\bar{u}_L \gamma_\mu u_L). \tag{A3}$$

The bosonization of the $\mathcal{Q}_-$ can be carried out in the FMV model from the strong action S of a chiral gauge theory. If we split the strong action and the left-handed current into two pieces $S = S_1 + S_2$ and $\mathcal{J}_\mu = \mathcal{J}_\mu^1 + \mathcal{J}_\mu^2$, respectively, the $\mathcal{Q}_-$ operator is represented, in the factorization approach, by

$$\mathcal{Q}_- \leftrightarrow 4[\langle \lambda \{\mathcal{J}_\mu^1, \mathcal{J}_2^\mu\}\rangle - \langle \lambda \mathcal{J}_\mu^1 \rangle \langle \mathcal{J}_2^\mu \rangle - \langle \lambda \mathcal{J}_\mu^2 \rangle \langle \mathcal{J}_1^\mu \rangle], \tag{A4}$$

with $\lambda = (\lambda_6 - i\lambda_7)/2$; for generality, the currents have been supposed to have nonzero trace.

In order to apply this procedure to constructing the factorizable contribution to the $O(p^3)$ nonanomalous weak VPγ Lagrangian, we have to identify in the full strong action the pieces that can contribute at this chiral order. We define, correspondingly,

$$S = S_{V\gamma} + S_2^\chi, \tag{A5}$$

where $S_{V\gamma}$ corresponds to the first term in Eq. (12), and S_2^χ corresponds to the leading order $[O(p^2)]$ effective Lagrangian $\mathcal{L}_2$ in chiral perturbation theory for the strong sector.

Evaluating the left-handed currents and keeping only terms of interest we get

$$\frac{\delta S_{V\gamma}}{\delta l^\mu} = -\frac{f_V}{\sqrt{2}} \nabla^\nu (u^+ V_{\mu\nu} u),$$

$$\frac{\delta S_2^\chi}{\delta l^\mu} = -\frac{f_\pi^2}{2} u^+ u_\mu u. \tag{A6}$$

Then the effective Lagrangian in the factorization approach is

$$\mathcal{L}_W^{fact}(VP\gamma) = 4G_8 \eta \left[\left\langle \lambda \left\{ \frac{\delta S_{V\gamma}}{\delta l^\mu}, \frac{\delta S_2^\chi}{\delta l_\mu} \right\} \right\rangle - \left\langle \lambda \frac{\delta S_{V\gamma}}{\delta l^\mu} \right\rangle \right.$$

$$\left. \times \left\langle \frac{\delta S_2^\chi}{\delta l_\mu} \right\rangle - \left\langle \lambda \frac{\delta S_2^\chi}{\delta l^\mu} \right\rangle \left\langle \frac{\delta S_{V\gamma}}{\delta l_\mu} \right\rangle \right] + \text{H.c.} \tag{A7}$$

The explicit term that will give a contribution in our calculations has been written in Eq. (19).

For the anomalous weak $AP\gamma$ vertex, the corresponding left-handed currents are

$$\frac{\delta S_{A\gamma}}{\delta l^\mu} = -\frac{f_A}{\sqrt{2}} m_A^2 u^+ A_\mu u,$$

$$\frac{\delta S_{WZW}}{\delta l_\mu} = \epsilon^{\mu\nu\alpha\beta} \frac{1}{16\pi^2} \left\{ F_{\nu\alpha}^L + \frac{1}{2} U^+ F_{\nu\alpha}^R U, u^+ u_\beta u \right\}, \tag{A8}$$

where the first term is from the f_A part in Eq. (13), and the second term is from the Wess-Zumino-Witten Lagrangian [30]. An equation similar to Eq. (A7) will be obtained, and the explicit term for our purpose has been shown in Eq. (20).

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