Explicit CP violation in the general two-doublet model, with real CKM matrix

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Taking the most general two-doublet model with explicit CP violation in the Higgs sector but no phase in the Cabibbo-Kobayashi-Maskawa matrix, we analyze the K^0 CP observables with pure new physics, the CP violation coming from the Higgs sector only. It is found that there is a direct correlation between the strength of FCNC couplings and the Higgs boson masses, and the lightest Higgs boson is nearly pure CP-even, but its small CP-odd composition gives a large enough contribution to ϵ_K.

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I. INTRODUCTION

The phenomenon of CP violation is one of the key problems in particle physics from both experimental and theoretical points of views [1]. The observed CP violation in the neutral kaon system [2] as well as the electric dipole moment of the neutron [3] severely constrain the sources and the strength of CP violation in the underlying model. It is well known that in the standard electroweak theory (SM), where CP is explicitly broken at the Lagrangian level through the complex Yukawa couplings, the single phase in the Cabibbo-Koayashi-Maskwa (CKM) matrix is the unique source of CP violation which can appear only in the charged current interactions [4]. However, the available experimental information, namely, ϵ_K and ϵ'/ϵ_K, is not enough to establish this phase as the only source of CP violation.

Actually, the very rich phenomena of CP violation provide some motivations to go beyond the SM. Electroweak baryogenesis, for instance, is one of the reasons for searching for new sources of CP violation. Indeed, it has been long understood that in order to generate the observed baryon asymmetry of the universe, the conservation of the baryon number, CP violation and the existence of nonequilibrium processes [5] are the three requirements to be satisfied [6]. In spite of satisfying the desired properties, the amount of CP violation within the SM may not be sufficient to induce the observed baryon asimmetry of the universe [7]. In the case of spontaneous CP violation electroweak baryogenesis constraints imply a light Higgs pseudoscalar [8].

In the literature, various extensions of the SM have been introduced. The simplest such extension is the two-Higgs doublet model (2HDM) where the scalar sector of the SM is extended by adding a second doublet without modifying the gauge structure [9]. On the other hand, the Higgs sector of the minimal version of the supersymmetric model [10] mimics that of 2HDM in many respects except for the fact that various parameters are fixed by supersymmetry in terms of the gauge couplings [9].

The strong CP problem, which exists in the SM, still persists in the supersymmetric (SUSY) extensions of the standard model. Moreover, in the SUSY extensions of the standard electroweak theory there appear novel sources of CP violation coming from the soft symmetry breaking mass terms. Though the phases of the soft terms have been shown to relax to CP-conserving points in the minimal model (MSSM) [11], this is not necessarily true in the nonminimal model (NMSSM) [12] containing a singlet. These soft terms contribute to known CP-violating observables [13] (EDM's and neutral meson mixings); however, they also induce CP violation in the Higgs sector as already noted in Refs. [14,15] and Refs. [16,17]. Furthermore, the novel sources of CP violation contributing to the CP violation observables offer quite useful tools in search for new physics in near future B factories such as CERN Large Hadron Collider (LHC-B) [18]. Clearly, for a given model comprising the SM, one has to saturate all the CP-violating as well as the CP-conserving observables.

From the earlier literature it is known that in the various nonsupersymmetric multi-Higgs doublet models, for instance, containing two [19] or three [20] Higgs doublets, there are flavor changing currents, and a vacuum leads to spontaneous CP violation with a real CKM matrix [21]. In the MSSM, since the tree level vacua are CP conserving, spontaneous CP violation can only occur if the radiative corrections are taken into account leading to a very light Higgs boson which has been discarded by the experiment [22]. In the next-to-minimal supersymmetric standart model (NSSM), however, spontaneous CP violation can occur even at the tree level [23].

On the other hand, the possibility of the observed CP violation in the kaon sector arising solely from supersymmetry has been investigated in Refs. [24,25] with a real CKM matrix, discussing whether or not the required experimental values of ϵ_K and ϵ'/ϵ_K can be accomodated by the SUSY sources of CP violation. An interesting scenario was considered in Ref. [26], where charged weak interactions are CP conserving, with all the CP -violating phenomena stemming from physics beyond SM. The authors of Ref. [26] have particularly addressed the issue whether or not the constraints from K^0 and B^0 systems offered by the current experimental data allow for a real CKM matrix, assuming that CP is spontaneously broken. In this work, we also assume that the CKM matrix is real. However, we differ from [26] in the sense that we have explicit CP violation in the Higgs potential. Thus, in our model all CP violation effects originate from the explicit CP violation in the Higgs potential. At this point, we would like to explain the differences which arise from this distinction and in which sense our analysis and results differ from those obtained by the authors of [26].

Although, in both of the works V_{CKM} is assumed to be real, in the case of the spontaneous CP violation which guarantees the CP invariance of the Lagrangian before the electroweak breaking, it is possible to keep the CKM matrix real naturally by introducing appropriate discrete symmetries. Indeed, the authors of [26] have chosen to break the CP symmetry spontaneously by introducing additional discrete symmetries, in order to attain the reality of the CKM matrix in a natural way. It is clear that with explicit CP violation in the Higgs sector alone, one cannot generate a real CKM matrix naturally. Thus in the model presented here, taking the reality of the CKM matrix for granted, we attempt to answer the following question: "Under such circumtances, what is the correlation between the Higgs sector and K^0 CP observables?" In other words, we would like to understand how the assumption of the real CKM matrix constrain the CP-violating sources stemming from the new physics in the framework of explicit CP violation in the Higgs potential only. As strong as it may sound, this line of reasoning is widely used in supersymmetric models. However general it might be, any two Higgs doublet model should follow from SUSY above the weak scale. Therefore, it would not be inappropriate to mimic this line of reasoning in this work as well.

Another difference between the two works resides in the fact that the underlying models are not the same. In this work, we consider the most general two-doublet model with explicit CP violation. The main difference between the model used in our work and that of [26] is that the Higgs potential in our model can be regarded as a remnant of the supersymmetric models broken at the weak scale, and the quartic couplings are identified with the tree-level supersymmetric expressions, to reduce the unknowns in a viable manner.

Moreover, although the structure of the Yukawa matrices are the same in both works, the Yukawa structures of [26] are as a result of the requirement of a real CKM matrix in the case of spontaneous symmetry breaking in the Higgs sector. That is, they proceed from the additional discrete symmetries imposed, as mentioned above. Clearly, with explicit CP violation in the Higgs potential, such Yukawa structures cannot be justified as in [26], as these additional discrete symmetries cannot be introduced. Therefore we will use the Yukawa structures of [26] as ansatzes, for the limited purpose of studying the correlation effects between the Higgs sector and K^0 CP observables, instead of the other rather general parametrizations

The authors of [26] discuss the possibility of a real CKM by taking into account all constraints from the K^0 and B^0 systems and they investigate how the experimental constraints on ΔM_K, ϵ_K, and ΔM_{B_d} can be satisfied for this case. They have shown that the present experimental constraints allow for a real CKM matrix, provided that the physics beyond the SM contributes to both K^0 and B_d^0 systems. (In particular, the new physics should contribute to ΔM_{B_d} by at least 20% of the SM contribution, besides generating CP violation in the kaon system). The main aim of this work is to study the correlation effects between the Higgs sector and K^0 CP observables only. Therefore, relying on the detailed analysis of [26] for the B^0 system constraints, we mainly concentrate on the K^0 system with the aim of saturating ΔM_K, ϵ_K, and observing the resulting constraints on the Higgs sector of the model.

The organization of this work is as follows: In Sec. II, starting from the Higgs potential of the general two-doublet model with explicit CP violation, we calculate the mass-squared matrix of the Higgs scalars in the (H,h,A) basis. In Sec. III we focus on the K^0 system for saturating ΔM_K and ϵ_K, and we analyze the implications of these quantities on the Higgs sector of the theory. Then, using the experimental values of ΔM_K and ϵ_K, we carry out the numerical analysis to determine the possible correlating effects between Higgs and K^0 systems in Sec. IV. Finally, in Sec. V we summarize our results.

II. HIGGS SECTOR

We adopt a general two-doublet model [27] with most general CP-violating soft and hard operators:

$$\begin{aligned} V(\Phi_1,\Phi_2) = {} & \mu_1^2\Phi_1^\dagger\Phi_1 + \mu_2^2\Phi_2^\dagger\Phi_2 + [\mu_{12}^2\Phi_1^\dagger\Phi_2 + \text{H.c.}] \\ & + \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 \\ & + \lambda_3(\Phi_1^\dagger\Phi_1\Phi_2^\dagger\Phi_2) \\ & + \lambda_4(\Phi_1^\dagger\Phi_2\Phi_2^\dagger\Phi_1) + \tfrac{1}{2}[\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \text{H.c.}] \\ & + [(\lambda_6\Phi_1^\dagger\Phi_1 + \lambda_7\Phi_2^\dagger\Phi_2)\Phi_1^\dagger\Phi_2 + \text{H.c.}], \end{aligned} \tag{1}$$

where the dimensionless couplings $\lambda_1,\ldots,4$ are all real whereas $\lambda_{5,6,7}$ as well as the soft mass parameter μ_{12}^2 can have nontrivial phases. One can regard Eq. (1) as a direct extension of the SM Higgs sector to two Higgs doublets [27], or as a remnant of the supersymmetric models broken above the weak scale [16,17,28,29]. Although our framework is completely nonsupersymmetric, to reduce the unknowns in a viable manner we will take $\lambda_1,\ldots,4$ from the tree level minimal supersymmetric model [17]

$$\begin{aligned} & \lambda_1 = \lambda_2 = -\tfrac{1}{8}(g_1^2+g_2^2), \quad \lambda_3 = -\tfrac{1}{4}(g_1^2-g_2^2), \\ & \lambda_4 = -\tfrac{1}{2}g_1^2, \end{aligned} \tag{2}$$

but vary $\lambda_{5,6,7}$ freely.

As usual we expand the neutral components of the Higgs doublets around the electroweak vacuum as

$$\Phi_i = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_i + \phi_i + i a_i \end{pmatrix} \quad (i=1,2), \tag{3}$$

where $v_1 = v\cos\beta$, $v_2 = v\sin\beta$ with $v = 246.2$ GeV for the correct electroweak breaking.

In the weak basis (G_0, a, ϕ_1, ϕ_2), the mass-squared matrix of the neutral Higgs bosons, whose entries can be expressed in terms of the parameters of the potential (1), assumes the form

$$M_0^2 = \begin{pmatrix} M_a^2 & v^2\Im\{\lambda_{56}\} & v^2\Im\{\lambda_{75}\} \\ v^2\Im\{\lambda_{56}\} & M_1^2 - 2s_\beta v^2\Re\{\lambda_{56}\} & M_{34}^2 - v^2\Re\{\lambda_{76}\} \\ v^2\Im\{\lambda_{75}\} & M_{34}^2 - v^2\Re\{\lambda_{76}\} & M_2^2 - 2c_\beta v^2\Re\{\lambda_{75}\} \end{pmatrix}, \tag{4}$$

where we use the usual short-hand notations $s_\beta \equiv \sin\beta$, $c_\beta \equiv \cos\beta$, and

$$M_1^2 = M_a^2 s_\beta^2 - 2v^2\lambda_1 c_\beta^2,$$
$$M_2^2 = M_a^2 c_\beta^2 - 2v^2\lambda_2 s_\beta^2, \tag{5}$$
$$M_{34}^2 = -s_\beta c_\beta[M_a^2 + v^2(\lambda_3 + \lambda_4)].$$

The entries of Eq. (4) depend on three new combinations of $\lambda_{5,6,7}$:

$$\lambda_{56} = s_\beta\lambda_5 + c_\beta\lambda_6,$$
$$\lambda_{75} = s_\beta\lambda_7 + c_\beta\lambda_5, \tag{6}$$
$$\lambda_{76} = s_\beta^2\lambda_7 + c_\beta^2\lambda_6.$$

It is understood that, in Eq. (4), the soft mass parameters μ_1^2, μ_2^2, and μ_{12}^2 are expressed in terms of the other parameters of the potential using the minimization condition. It can be easily checked that the mass-squared matrix given in Eq. (4) is in agreement with the one presented in Ref. [28] in the *CP*-conserving limit.

To obtain the mass-squared matrix of the Higgs scalars in the (H,h,A) basis, we introduce the *CP*-even scalars h, H, the *CP*-odd scalar A, and the Goldstone boson G^0 (eaten by the Z boson in acquiring the mass) via the unitary rotation

$$\begin{pmatrix} a_1 \\ a_2 \\ \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta & 0 & 0 \\ \sin\beta & -\cos\beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} G_0 \\ A \\ h \\ H \end{pmatrix}, \tag{7}$$

Then, in (H,h,A) basis the mass-squared matrix of the Higgs scalars becomes

$$M^2 = \begin{pmatrix} M_{11}^2 & M_{12}^2 & M_{13}^2 \\ M_{12}^2 & M_{22}^2 & M_{23}^2 \\ M_{13}^2 & M_{23}^2 & M_a^2 \end{pmatrix}. \tag{8}$$

The entries of M^2 can be expressed, in terms of λ_i and λ_{ij} defined in Eqs. (1) and (6), respectively, as

$$M_{11}^2 = M_a^2 - (1/2)v^2 s_{2\beta}^2[\lambda_1 + \lambda_2 - (\lambda_3 + \lambda_4)] - 2v^2[s_\beta^3\Re\{\lambda_{56}\} + c_\beta^3\Re\{\lambda_{75}\} - (1/2)s_{2\beta}\Re\{\lambda_{76}\}], \tag{9}$$

$$M_{12}^2 = v^2 s_{2\beta}[-\lambda_1 c_\beta^2 + \lambda_2 s_\beta^2 + (1/2)c_{2\beta}(\lambda_3 + \lambda_4)] + v^2 s_{2\beta}[c_\beta\Re\{\lambda_{75}\} - s_\beta\Re\{\lambda_{56}\} + t_{2\beta}^{-1}\Re\{\lambda_{76}\}], \tag{10}$$

$$M_{13}^2 = v^2[c_\beta\Im\{\lambda_{75}\} - s_\beta\Im\{\lambda_{56}\}], \tag{11}$$

$$M_{22}^2 = -2v^2[\lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + (1/4)s_{2\beta}^2(\lambda_3 + \lambda_4)] - v^2 s_{2\beta}[c_\beta\Re\{\lambda_{56}\} + s_\beta\Re\{\lambda_{75}\} + \Re\{\lambda_{76}\}], \tag{12}$$

$$M_{23}^2 = -v^2[c_\beta\Im\{\lambda_{56}\} + s_\beta\Im\{\lambda_{75}\}], \tag{13}$$

where $t_\beta^{-1} \equiv \cot\beta$. The Higgs mass-squared matrix (8) will be diagonalized numerically via

$$O^T \cdot M^2 \cdot O = \mathrm{diag}(M_{H_1}^2, M_{H_2}^2, M_{H_3}^2), \tag{14}$$

where O is the orthonormal Higgs mixing matrix. Perhaps the most spectacular property of the Higgs mass-squared matrix (8) is that the *CP*-even (H,h) and *CP*-odd (A) components are mixed [15–17,30]. The entries responsible for this mixing are M_{13}^2 and M_{23}^2, both of which depend on the imaginary parts of λ_{56} and λ_{75}. Hence, the explicit *CP* violation in the Higgs potential (1) shuffles the opposite *CP* components so that the mass eigenstate Higgs scalars have no longer definite *CP* quantum numbers. In what follows, this Higgs sector *CP* violation will be the sole source of *CP* violation in saturating the (K^0, B^0) *CP* observables through the exchange of neutral Higgs scalars inducing flavor-changing neutral currents (FCNC) at tree level.

III. CORRELATING EFFECTS OF *CP* VIOLATION BETWEEN THE HIGGS AND KAON SECTORS

The existence of correlating effects that may exist between a two-doublet model with tree level FCNC's and the K^0-$\bar{K}^0$ system is a generic feature of theories with Higgs bosons that mediate FCNC interactions at the tree level [31].

In general, FCNC's induced by the neutral Higgs exchanges depend on the model employed for the Yukawa Lagrangian [26,32].

On the other hand, in the case of spontaneous CP violation in the Higgs sector with nonvanishing phases of the Higgs doublets [20,26,33], the requirement of a real CKM matrix leads at once to the following mass matrices for quarks:

$$\Gamma^u = \tan\beta \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \tag{15}$$

$$\Gamma^d = \begin{pmatrix} m_d(\tan\beta - Sx^2) & -m_d S\beta_K & -m_d S\beta_d \\ -m_s S\beta_K & m_s(\tan\beta - Sy^2) & -m_s S\beta_s \\ -m_b S\beta_d & -m_b S\beta_s & m_b(\tan\beta - Sz^2) \end{pmatrix}, \tag{16}$$

where $S=\tan\beta+\cot\beta$, $\beta_K = xy$, $\beta_d = xz$, and $\beta_s = yz$ with $x^2+y^2+z^2=1$ determine the strengths for FCNC transitions in K^0, B_d^0, and B_s^0 systems, respectively. This flavor structure forbids FCNC's among up-type quarks, that is, there is no D^0-$\overline{D^0}$ mixing. This particular flavor structure follows from the requirement of a real CKM matrix with spontaneous CP violation in the Higgs sector [33]. Obviously, one can deviate the picture presented here in several aspects. One possibility would be to consider nondiagonal Γ^u in which case one would automatically obtain D^0-$\overline{D^0}$ mixing. But, as we concentrate on the K^0 system, such a structure is out of the scope of this work.

One notes that the flavor-changing couplings of the three neutral Higgs fields are free parameters which are constrained to obey the condition $|\beta_{K,s,d}| < 1/2$ [26]. This is an important feature, because the strength of the neutral Higgs contributions to ΔM_K and ΔM_{B_d} are proportional to $|\beta_K|$ and $|\beta_d|$, respectively.

In a two-doublet model with explicit CP violation as given in Eq. (1), such Yukawa structures such as Eqs. (15) and (16) cannot be justified as in Ref. [26]. Thus, we will use them as *ansatzes*, as they will prove useful in analyzing $(K^0,B^0)CP$ systems instead of the other rather general parametrizations [32].

The detailed analysis in Ref. [26] shows that for saturating the present experimental bounds, at least 20% of ΔM_{B_d} must come from the new physics contribution. This requirement eventually boils down to $(\beta_d/\beta_K)^2 \sim 1$, which we will assume. Therefore, relying on the analysis of [26], which has discussed B-meson system constraints in detail, we concentrate only on the K^0 system with the aim of saturating ΔM_K, ϵ_K, and observing the resulting constraints on the Higgs sector of the theory.

As mentioned in the Introduction, the main aim of this work is to study the correlation effects between the Higgs and K^0 systems only. However, for completeness one should also comment upon the consequences of such a model for the B meson system. Indeed, in the B-meson system, experimentally known quantities are ΔM_{B_d} (which needs no CP violation) and $\sin 2\beta$ (which must be saturated by new physics CP violation). As the form of the down quark mixing matrix Γ^d (16) shows, there are three fundamental parameters: β_K (for the kaon system), β_d (for the B_d meson system), and β_s (for the B_s meson system). Among these parameters, the analysis of the kaon system is sensitive to β_K only. On the other hand, the B_d meson system is sensitive to β_d, and the ratio of β_d/β_K is in general arbitrary. As the detailed analysis of [26] shows, by varying β_d/β_K, both ΔM_{B_d} and $\sin 2\beta$ can be saturated. For instance, the CP asymmetry in $B_d \to J/\psi K_s$ decay can be as large as unity ($\sin 2\beta \sim 1$), which covers the present experimental interval fully [34]. Moreover, ΔM_{B_d} is also saturated for the same range of the parameter space. In conclusion, having $(\beta_d/\beta_K)^2 \sim O(1)$ is sufficient to saturate CP-violating as well as CP-conserving experimental data in the B_d meson system.

The effective $\Delta S = 2$ Hamiltonian for the K^0-$\overline{K^0}$ system receives contributions from the exchange of charged bosons $W^\pm W^\pm$ (the SM contribution), $W^\pm H^\pm$, $H^\pm H^\pm$ (the two-doublet model contribution), as well as the neutral Higgs scalars h,H,A (the two-doublet model with tree level FCNC). Since the CKM matrix is real, only neutral Higgs bosons contribute to ϵ_K

$$\epsilon_K \equiv \frac{1}{\sqrt{2}} \frac{\Im\langle \overline{K^0}|M_{12}|K^0\rangle}{\Delta M_K}, \tag{17}$$

whereas the neutral kaon mass difference depends on the exchange of all bosons above,

$$\Delta M_K \approx 2|\Re\langle \overline{K^0}|M_{12}|K^0\rangle|. \tag{18}$$

The $W^\pm$ and $H^\pm$ contributions to $\langle \overline{K^0}|M_{12}|K^0\rangle$ can be found in Refs. [32] and [35], and the exchange of the neutral scalars contribute by [26,32]

$$\langle \overline{K^0}|M_{12}|K^0\rangle = \frac{G_F^2}{12\pi^2} f_K^2 M_K \frac{m_d}{m_s}\left(1+\frac{m_d}{m_s}\right)^{-1} \times \sum_k \left(\frac{2\sqrt{3}\pi v M_K}{M_{H_k}}\right)^2 (U_{k,12})^2, \quad (19)$$

where $U_{k,ij}$ is defined by

$$U_{k,ij} = -\frac{1}{2}(S_{k,ij} - S^*_{k,ji}) \quad (20)$$

$$\text{with}\quad S_{k,ij} = -\frac{\Gamma^d_{ij}}{\sqrt{m_i m_j}}(O_{1k} + iO_{3k}),$$

using the convention that $k=1,2,3$ counts the mass eigenstate neutral Higgs scalars, whereas $i,j=1,2,3$ are the generation indices. As Eq. (19) suggests, since $U_{k,ij} \propto \beta_K^2$, an increase in β_K^2 must be compensated by the corresponding increase in $M^2_{H_k}$. Therefore, (ΔM_K) is maintained within the experimental bounds. Although cancellations among different Higgs contributions are important, in general one expects the lightest Higgs boson mass (whose contribution is the dominant one) to be more sensitive to β_K^2 as compared to heavier Higgs bosons.

IV. NUMERICAL ANALYSIS

Equipped with the necessary formulas for the Higgs and K^0 sectors, we now turn to a numerical tracing of the parameter space. We restrict the experimental constraints to the following bands:

$$0.98 \leqslant \left|\frac{\epsilon_K}{(\epsilon_K)^{exp}}\right| \leqslant 1.02, \quad (21)$$

$$0.98 \leqslant \left|\frac{\Delta M_K}{(\Delta M_K)^{exp}}\right| \leqslant 1.02,$$

together with the positivity of the Higgs boson masses. By doing this, we wander in the parameter space varying $|\lambda_5|$, $|\lambda_6|$, $|\lambda_7|$, Arg$[\lambda_5]$, Arg$[\lambda_6]$, Arg$[\lambda_7]$, β_K, $\tan\beta$, and M_a freely.

We would like to note at this point that the kaon data have large hadronic uncertainities, in that the vacuum insertion approximation used in (17)–(19) receives large nonperturbative corrections due to the neglect of the nonfactorizable contributions. These corrections are not presently predictable, and are outside the scope of this work. We are interested in testing the model; we have adopted in close vicinity (at the 2% level) of the experimental result, which is the rationale behind the expression (21). As one moves away from the experimental values, it is clear that there will be more points in the parameter space which satisfy our constraints. For the purpose of constraining the model, it is sufficient to look for the allowed points in close vicinity of the experimental mean value.

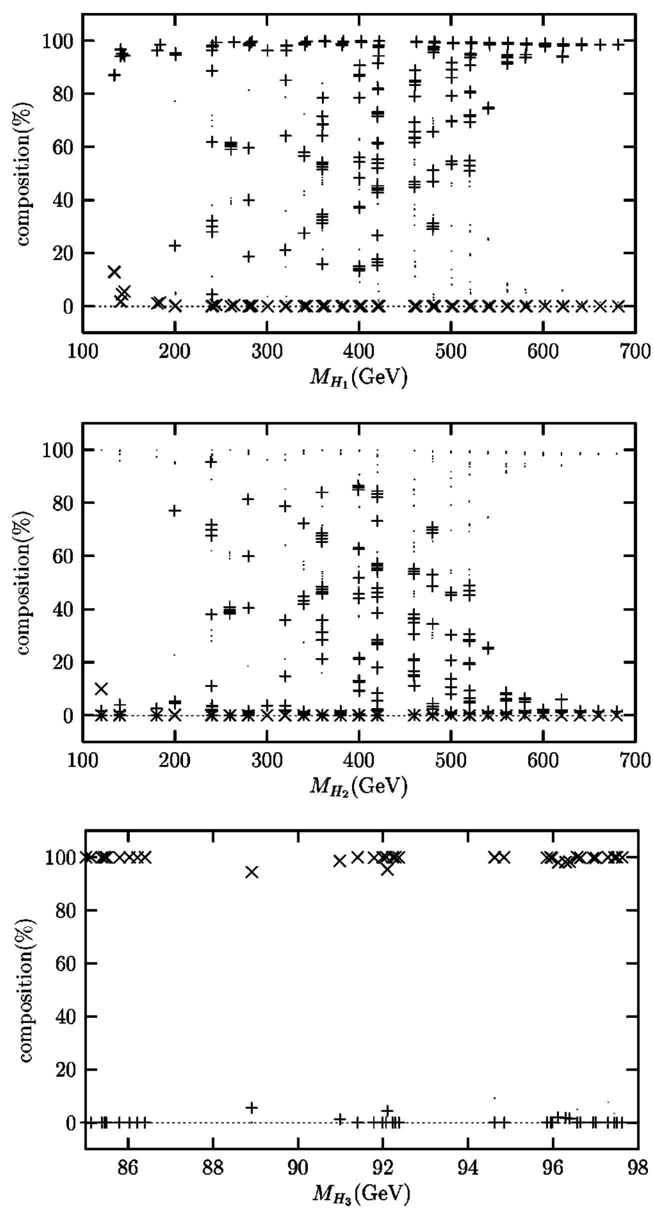

FIG. 1. Variation of $H\,(+)$, $h\,(\times)$, and $A\,(\cdot)$ compositions (%) of H_1, H_2, and H_3 with their masses. H_3 is the lightest Higgs boson and its CP-odd component is negligibly small.

We first analyze the Higgs sector within the experimental band of Eq. (21). Depicted in Fig. 1 is the scatter plot of such an analysis for H_1 (top), H_2 (middle), and H_3 (bottom). Each window here shows the variation of the CP composition (in %) of a mass-eigenstate Higgs boson with its mass. It is clear that H_1 and H_2 are heavy Higgs bosons and they have approximately alternate CP properties: H_1 is composed mostly of CP-even elements H (''+'') and h (''×'') though it has also considerable CP-odd composition (''·'') for $M_{H_1} \lesssim 550$ GeV. On the other hand, the other heavy Higgs H_2 has complementary properties compared to H_1 with a similar range of masses. The third Higgs boson,

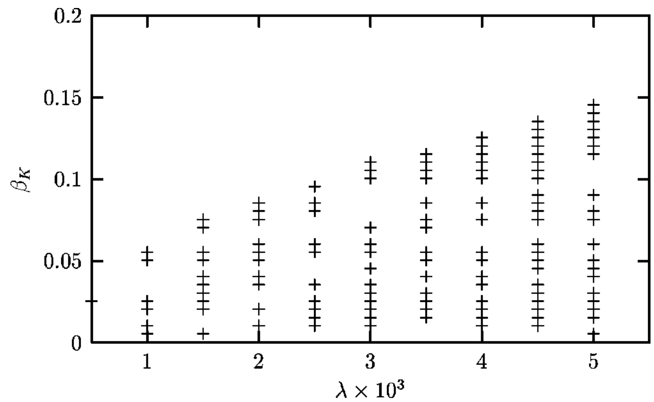

FIG. 2. Variation of β_K with $\lambda \equiv |\lambda_5| = |\lambda_6| = |\lambda_7|$.

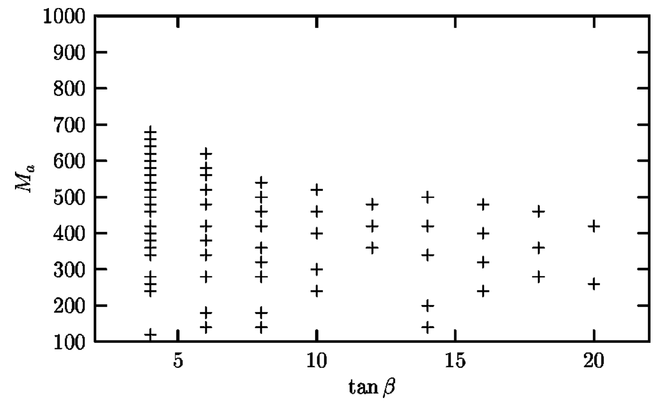

FIG. 4. Variation of M_a with $\tan\beta$.

whose mass lies in the range between 85 GeV$\lesssim M_{H_3} \lesssim 100$ GeV, is the lightest of all three and it has exclusively even CP, that is, its odd CP composition is below 0.1% in the entire parameter space. As is clear from Eq. (19), the contribution of H_3 to $\langle \overline{K^0}|M_{12}|K^0\rangle$ is large due to its relatively small mass; however, its CP-odd composition is also small. As mentioned above, the lightest Higgs boson predicted in our model is predominantly CP-even and lighter than about 100 GeV. One notes that in view of the present CERN e^+e^- Collider LEP2 data, a SM-type CP-even Higgs boson must be heavier than 110 GeV. However, it is worthy of emphasizing that this bound does not constrain the model at hand since both the parameter space and the particle spectrum differ from the one in the SM.

In Fig. 2 we show the variation of β_K (which is already constrained to be less than 1/2 [26]) with the absolute magnitudes of $\lambda_{5,6,7}$ for the special case of $|\lambda| = |\lambda_5| = |\lambda_6| = |\lambda_7|$. The general tendency of the solution points is that the required value of β_K increases roughly linearly with λ. It can be seen from Eq. (19) that $\langle \overline{K^0}|M_{12}|K^0\rangle$ is proportional to $\beta_K^2/M_{H_k}^2$. Therefore, in order (ΔM_K) to stay within the experimental bounds, any small increase in the value of β_K should be compensated for by a corresponding increase in the Higgs boson masses. Equation (19) makes it clear that the dominant contribution comes from the lightest Higgs boson, although, it is possible to realize partial cancellations among several terms. Therefore, the sensitivity of β_K to the lightest Higgs is stronger than those of two heavier bosons. Moreover, it is also known that lightest Higgs boson mass is much more sensitive to the quartic couplings than the heavier Higgs bosons. Consequently, β_K has a sensitivity to λ via mainly the contribution of the lightest Higgs boson.

In Fig. 3 the variation of β_K with $\tan\beta$ is indicated. As is seen from the figure, with the increasing values of β_K $\tan\beta$ decreases. Depicted in Fig. 4 is the dependence of M_a on $\tan\beta$. As the figure suggests, higher the value of $\tan\beta$ lower the value of M_a. This, in particular, implies that K^0 constraints do not allow the Higgs sector be in the decoupling regime in which the heavy Higgs bosons become degenerate and the lightest Higgs mass becomes maximal ($\sim$100 GeV), and more importantly, the lightest Higgs boson approaches the pure CP-even composition [16,17,36]. It is with this figure that one arrives at the necessity of a small but sufficient CP-odd composition of the lightest Higgs boson for saturating $(\epsilon_K)^{exp}$. Namely, the CP-odd composition of the lightest Higgs boson, though small, is important for $(\overline{K^0})CP$ constraints due to its low mass, enhancing $\langle \overline{K^0}|M_{12}|K^0\rangle$.

Finally, in Fig. 5 we show the dependence of $\epsilon_K/(\epsilon_K)^{exp}$ on $\Delta M_K/(\Delta M_K)^{exp}$, when all the free parameters of the model vary. As the figure suggests, there are solutions in any close proximity of the experimental results.

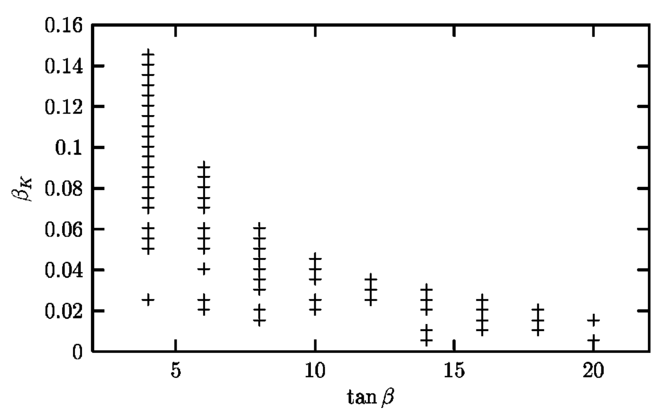

FIG. 3. Variation of β_K with $\tan\beta$.

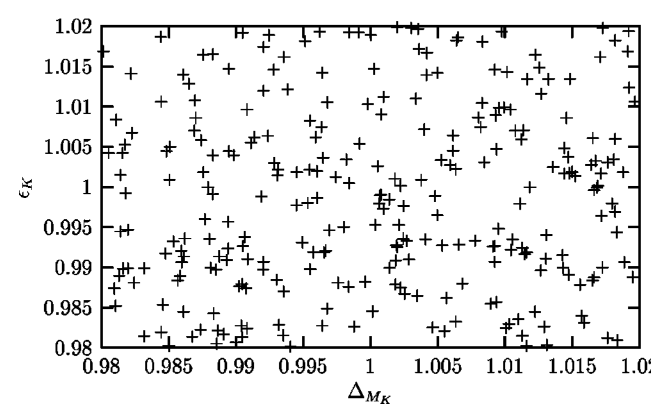

FIG. 5. Variation of ϵ_K with ΔM_K.

V. CONCLUSION

In concluding the work, we now come back to the question mentioned in the Introduction: "The correlation between the Higgs and K^0 systems." That is, (1) the neutral Higgs exchange is sufficient to saturate $(K^0)CP$ with the contribution of that Higgs boson which is the *lightest* of all three, and which is *nearly* pure CP-even, (2) the larger the Γ^d_{12} the higher the masses of the Higgs bosons to meet $(\Delta M_K)^{exp}$, (3) the Higgs sector should not slide to the decoupling limit, that is, the heavy and the light Higgs bosons should not decouple as otherwise the CP-odd composition of the lightest Higgs boson is washed out, and (4) the results depend only on β_K and β_d and, as long as $\beta_K \sim \beta_d$ (for saturating ΔM_{B_d} and $\sin 2\beta$) and FCNC's in the up-quark sector are neglected, one can take the parametrization of Γ^d in Eq. (16) as an ansatze for the flavor structure with free parameters β_K and β_d, with the constraint $|\beta_{K,s,d}| < 1/2$.

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