Updated analysis of some two-body charmless B decays

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New data from the BaBar, Belle, and CLEO Collaborations on B decays to two-body charmless final states are analyzed, with the following consequences. (1) The penguin amplitude which dominates the decay $B^+\to\pi^+K^{*0}$ has a magnitude similar to that dominating $B^+\to\pi^+K^0$. (2) The decay $B^+\to\pi^+\eta$, a good candidate for observing direct CP violation, should be detectable at present levels of sensitivity. (3) The decays $B^+\to\eta'K^+$ and $B^+\to\eta K^{*+}$ are sufficiently similar in rate to the corresponding decays $B^0\to\eta'K^0$ and $B^0\to\eta K^{*0}$, respectively, that one cannot yet infer the need for "tree" amplitudes t' contributing to the B^+ but not the B^0 decays. Statistical requirements for observing this and other examples of tree-penguin interference are given. (4) Whereas the $B^+\to\eta'K^+$ and $B^0\to\eta'K^0$ rates cannot be accounted for by the penguin amplitude p' alone but require an additional flavor-singlet penguin contribution s', no such flavor-singlet penguin contribution is yet called for in the decays $B^+\to\eta K^{*+}$ or $B^0\to\eta K^{*0}$. Predictions for the rates for $B^+\to\eta'K^{*+}$ and $B^0\to\eta'K^{*0}$ are given which would allow one to gauge the importance of these flavor-singlet penguin amplitudes.

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I. INTRODUCTION

The decays of B mesons are rich sources of information on fundamental aspects of weak couplings as described by the Cabibbo-Kobayashi-Maskawa (CKM) matrix, and on potential effects of physics beyond the standard model. Especially useful information can be obtained from B decays to pairs of light charmless mesons, both pseudoscalar (P) and vector (V). A number of questions can now be addressed more incisively in the light of recent data from the CLEO, BaBar, and Belle detectors. In the present paper we shall discuss several of these, showing that progress is being made and setting goals of data samples for more definitive answers. We limit our discussion to a few topics.

(1) Recent measurements of the branching ratio for $B^+\to\pi^+K^{*0}$ indicate that the penguin amplitude dominating this decay has a magnitude not too much smaller than that of the penguin amplitude dominating $B^+\to\pi^+K^0$. We use this information, as well as new information on the decays $B\to K(\rho,\omega,\phi)$, to discuss several open questions associated with penguin contributions to $B\to PV$ decays. These include a conjectured relation between two types of penguin amplitudes called p'_P and p'_V in Ref. [1] in which the spectator quark is incorporated into a pseudoscalar or a vector meson, respectively. Arguments first proposed by Lipkin [2] suggest that such amplitudes would be equal and opposite. The contribution of electroweak penguin diagrams in suppressing the decays $B\to K(\omega,\phi)$ is also noted.

(2) In Refs. [3] and [4], the decays $B^+\to\pi^+(\eta,\eta')$ were proposed as good candidates for detecting direct CP violation. Present data samples are approaching the sensitivity for observing these modes, whose branching ratios are expected to be a few parts in 10^6. We update estimates for the branching ratio for these decays and indicate the possible range of likely direct CP asymmetries.

(3) It has been suggested by several sets of authors (see, e.g., Refs. [1,5–7]) that the decays $B^+\to\eta'K^+$ and $B^+\to\eta K^{*+}$ might be enhanced with respect to the corresponding decays $B^0\to\eta'K^0$ and $B^0\to\eta K^{*0}$, respectively, as a consequence of constructive interference between tree and penguin amplitudes. We review this suggestion in light of the latest data and find that this conclusion is not yet warranted. We indicate the statistical precision that is likely to be needed in order to establish tree-penguin interference in this and other processes reliably. For $B^{+,0}$ decays to charmless nonstrange final states such interference involves the product $\cos\alpha\cos\delta$, while for decays to charmless strange final states it involves $\cos\gamma\cos\delta$, where α and γ are weak phases of the unitarity triangle, while δ is a relative strong phase between tree and penguin amplitudes.

(4) Lipkin [2] has argued for the enhancement of the decays $B\to\eta'K$ and $B\to\eta K^*$ as a result of constructive interference between nonstrange and strange quark components of the η' or η, and for the suppression of the decays $B\to\eta K$ and $B\to\eta'K^*$ because of correspondingly destructive interference. However, an additional amplitude associated with the flavor-singlet part of the η and η' is both allowed [4] and required for the proper description of the $B\to\eta'K$ decay rates [8]. The status of this amplitude, called s', is reviewed. It is pointed out that it does not need to be as large as the penguin amplitude p' in order to explain the data if it interferes constructively with p'. At present, while no such singlet contribution is needed to explain the data on B

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$\to \eta K^*$, the flavor-singlet component of η is small. A much more incisive test would be available once the decays $B \to \eta' K^*$ (both charged and neutral) are available, since the penguin contributions of nonstrange and strange quarks in the η' partially cancel one another, while the flavor-singlet component of the η' is dominant. Predictions for these rates are given.

We discuss our notation in Sec. II. Experimental data, their averages, and the corresponding inputs to our determination of amplitudes are treated in Sec. III. We then discuss the above four questions in turn: penguin contributions in $B \to PV$ decays (Sec. IV), direct CP violation in $B \to \pi^+(\eta, \eta')$ (Sec. V), tree-penguin interference (Sec. VI), and the role of the flavor-singlet amplitude (Sec. VII). We summarize in Sec. VIII. An Appendix contains details of decay constant calculations.

II. NOTATION

We use the following quark content and phase conventions:

Bottom mesons: $B^0 = d\bar{b}$, $\bar{B}^0 = b\bar{d}$, $B^+ = u\bar{b}$, $B^- = -b\bar{u}$, $B_s = s\bar{b}$, $\bar{B}_s = b\bar{s}$.

Charmed mesons: $D^0 = -c\bar{u}$, $\bar{D}^0 = u\bar{c}$, $D^+ = c\bar{d}$, $D^- = d\bar{c}$, $D_s^+ = c\bar{s}$, $D_s^- = s\bar{c}$.

Pseudoscalar mesons: $\pi^+ = u\bar{d}$, $\pi^0 = (d\bar{d} - u\bar{u})/\sqrt{2}$, $\pi^- = -d\bar{u}$, $K^+ = u\bar{s}$, $K^0 = d\bar{s}$, $\bar{K}^0 = s\bar{d}$, $K^- = -s\bar{u}$, $\eta = (s\bar{s} - u\bar{u} - d\bar{d})/\sqrt{3}$, $\eta' = (u\bar{u} + d\bar{d} + 2s\bar{s})/\sqrt{6}$.

Vector mesons: $\rho^+ = u\bar{d}$, $\rho^0 = (d\bar{d} - u\bar{u})/\sqrt{2}$, $\rho^- = -d\bar{u}$, $\omega = (u\bar{u} + d\bar{d})/\sqrt{2}$, $K^{*+} = u\bar{s}$, $K^{*0} = d\bar{s}$, $\bar{K}^{*0} = s\bar{d}$, $K^{*-} = -s\bar{u}$, $\phi = s\bar{s}$.

In the present approximation there are seven types of independent amplitudes: a "tree" contribution t; a "color-suppressed" contribution c; a "penguin" contribution p; a "singlet penguin" contribution s, in which a color-singlet $q\bar{q}$ pair produced by two or more gluons or by a Z or γ forms an SU(3) singlet state; an "exchange" contribution e, an "annihilation" contribution a, and a "penguin annihilation" contribution pa. These amplitudes contain both the leading-order and electroweak penguin contributions:

$$t \equiv T + P_{EW}^C, \quad c \equiv C + P_{EW},$$

$$p \equiv P - \tfrac{1}{3} P_{EW}^C, \quad s \equiv S - \tfrac{1}{3} P_{EW}, \tag{1}$$

$$a \equiv A, \quad e + pa \equiv E + PA,$$

where the capital letters denote the leading-order contributions [4,9,10] while P_{EW} and P_{EW}^c are, respectively, color-favored and color-suppressed electroweak penguin amplitudes [10]. We shall neglect smaller terms [11,12] P_{EW}^E and P_{EW}^A [(γ,Z)-exchange and (γ,Z)-direct-channel electroweak penguin amplitudes]. We shall denote $\Delta S = 0$ transitions by unprimed quantities and $|\Delta S| = 1$ transitions by primed quantities. For PV decay modes, the subscript P or V denotes the final-state meson (pseudoscalar or vector) incorporating the spectator quark. Although one $B \to VV$ decay ($B^0 \to \phi K^{*0}$) has been seen, we shall not discuss such processes further here.

TABLE I. Hierarchies among magnitudes of flavor-SU(3) amplitudes in powers of a parameter $\lambda \equiv |V_{us}| \simeq 0.22$.

	$O(1)$	$O(\lambda)$	$O(\lambda^2)$	$O(\lambda^3)$	$O(\lambda^4)$
$\Delta S = 0$	T	C, P	E, A, P_{EW}	PA, P_{EW}^C	PA_{EW}
	t	c, p	e, a, s	pa	
$\|\Delta S\| = 1$	P'	T', P'_{EW}	C', PA', P'^C_{EW}	E', A', PA'_{EW}	
	p'	t', c', s'	pa'	e', a'	

For the $\bar{b} \to \bar{d}$ and $\bar{b} \to \bar{u} u \bar{d}$ transitions, an educated guess of the hierarchies among the amplitudes [10] is given in Table I. One notices that for $|\Delta S| = 1$ transitions, c' contains an electroweak penguin amplitude at the next order. Therefore, we put c' together with t' at the same order. Similarly, since part of the singlet amplitude is the electroweak penguin, s' is at least of order P'_{EW}.

III. AMPLITUDE DECOMPOSITIONS AND EXPERIMENTAL RATES

We list theoretical predictions and averaged experimental data for interesting charmless B decays involving $\Delta S = 0$ transitions in Table II and those involving $|\Delta S| = 1$ transitions in Table III. Amplitudes of order λ^2 and smaller in Table I are omitted unless dominant. Detailed experimental values are listed in Tables IV and V. We will assume [1] $p_V = -p_P$ and $p'_V = -p'_P$. The averaged rates are obtained by combining the data recently reported from CLEO, BaBar, and Belle groups [13–32]. In this section we shall comment on some of the methods used to determine the invariant amplitudes, deferring discussions of others to subsequent sections.

In Table II the values of $|t| \simeq |T| = 2.7 \pm 0.6$ and $|p| \simeq |P| = 0.72 \pm 0.14$ for the $\pi^+\pi^-$ decay mode are based on the detailed analysis in Ref. [33]. Here amplitudes are defined such that their squares give B^0 branching ratios in units of 10^{-6}. In estimating $\mathcal{B}(B^+ \to \pi^+\pi^0)$ from T, we take into account the lifetime difference between B^+ and B^0, $\tau_{B^+}/\tau_{B^0} = 1.068 \pm 0.016$ [34], and assume a constructively interfering amplitude $c \simeq 0.1t$. The branching ratio thus computed is $\simeq 4.7 \times 10^{-6}$, consistent with the averaged data. The penguin contribution to $\mathcal{B}(B^+ \to K^+\bar{K}^0)$ is then about 0.55×10^{-6}.

The magnitude of $|p'|^2$ can be directly obtained from the π^+K^0 decay mode to have a central value ~ 17.2. This result is used to compute $|p|^2$ using the relation $|p/p'|^2 = |V_{td}/V_{ts}|^2 \simeq 0.032$, giving the number quoted above from Ref. [33]. Here the bounds $0.66 \leqslant |V_{td}/\lambda V_{ts}| = |1 - \rho - i\eta| \leqslant 0.96$ on parameters of the CKM matrix are taken from the analysis of Ref. [35].

The contributions of $|t'|^2$ are estimated using the relation $|t'/t|^2 = |V_{us}/V_{ud}|^2 |f_K/f_\pi|^2 \simeq 0.076$. We use [36] $f_\pi = 130.7$ MeV, $f_K = 159.8$ MeV, $V_{us} = 0.2205$, and $V_{ud} \simeq 1$

TABLE II. Summary of predicted contributions to selected $\Delta S=0$ decays of B mesons. Branching ratios ($\mathcal{B}$) are quoted in units of 10^{-6}. Numbers in italics are assumed inputs. Experimental values are averaged over results in Refs. [13–32].

Mode	Amplitudes	$\|t(+c)\|^2$	$\|p\|^2$	$\|s\|^{2\,a}$	$\|s\|^{2\,b}$	Expt.
$B^+\to\pi^+\pi^0$	$-\frac{1}{\sqrt{2}}(t+c)$	4.7	0	0	0	5.7 ± 1.5
$K^+\overline{K^0}$	p	0	0.55	0	0	<2.4
$\pi^+\eta$	$-\frac{1}{\sqrt{3}}(t+c+2p+s)$	3.1	0.73	0.04	0.18	<5.7
$\pi^+\eta'$	$\frac{1}{\sqrt{6}}(t+c+2p+4s)$	1.6	0.37	0.35	1.4	<7
$\pi^+\rho^0$	$-\frac{1}{\sqrt{2}}(t_V+c_P+p_V-p_P)$	7.9	0.78	0	0	12.8 ± 3.6
$\pi^+\omega$	$\frac{1}{\sqrt{2}}(t_V+c_P+p_P+p_V+2s_P)$	*7.9* [c]	$\simeq0$	~0.01 [d]	–	7.9 ± 1.8
$\pi^+\phi$	s_P	0	0	0.02	–	<1.4
$B^0\to\pi^+\pi^-$	$-(t+p)$	*7.3*	0.51	0	0	4.4 ± 0.9
$\pi^0\pi^0$	$-\frac{1}{\sqrt{2}}(c-p)$	0.04	0.26	0	0	<5.7
K^+K^-	$-(e+pa)$	0	0	0	0	<1.9
$\pi^\pm\rho^\mp$	$-(t_{(V,P)}+p_{(V,P)})$	14.7 [e]	0.36 [f]	0	0	25.8 ± 4.5 [g]
$\pi^0\omega$	$\frac{1}{2}(c_P-c_V+p_P+p_V+2s_P)$	–	$\simeq0$	<0.01 [d]	–	<3

[a]Assuming constructive interference between s' and p' in $B\to\eta'K$ (Table III).
[b]Assuming no interference between s' and p' in $B\to\eta'K$ (Table III).
[c]Neglecting other contributions to decay rate.
[d]$(c_P+2s_P)/\sqrt{2}$ contributes a term $\frac{1}{3}P_{EWP}/\sqrt{2}$ to amplitude.
[e]$|t_V|^2=14.7\pm3.3$ contribution to $\mathcal{B}(B^0\to\pi^+\rho^-)$ estimated from $B^+\to\pi^+\omega$, neglecting c_P and s_P, leaving $|t_P|^2=11.1\pm5.6$ contributing to $\mathcal{B}(B^0\to\pi^-\rho^+)$.
[f]$|p_P|^2$ contribution to $\mathcal{B}(B^0\to\pi^-\rho^+)$ and $|p_V|^2$ contribution to $\mathcal{B}(B^0\to\pi^+\rho^-)$.
[g]Combined branching ratio for $\pi^+\rho^-$ and $\pi^-\rho^+$.

$-V_{us}^2/2$. It should be noted that the lifetime difference has to be taken into account when going from B^0 to B^+ decays. For $|\Delta S|=1$ decays, the presence of a substantial electroweak penguin contribution in c' means that one cannot simply take $c'/t'=0.1$ as in the $\Delta S=0$ decays, but must consider the relative magnitude and weak phase of the electroweak penguin and tree terms, as in Refs. [12,37]. Predictions of the branching ratios for πK modes other than π^+K^0 depend on both CKM phases and on final-state phases, which are not yet measured but are likely to be small [38]. Extraction of CKM phases from the πK modes is a rich area which we do not address in the present paper.

Two new measurements of the $\pi^+\rho^0$ and $\pi^\pm\rho^\mp$ decay modes are reported in Ref. [19]. The measurement in the latter mode does not distinguish between the two final states, while the former contains a possible penguin contribution. If we assume $p_V=-p_P$, then $A(B^+\to\pi^+\rho^0)\simeq-(1/\sqrt{2})(t_V+c_P-2p_P)$, while $A(B^+\to\pi^+\omega)\simeq(1/\sqrt{2})(t_V+c_P+2s_P)$. Thus, neglecting the s_P and c_P contributions as in Ref. [1], we may use $\mathcal{B}(B^+\to\pi^+\omega)$ to estimate the $|t_V|^2$ contribution, obtaining $(7.9\pm1.8)\times10^{-6}$. [If we had neglected the penguin contribution in $B^+\to\pi^+\rho^0$ and averaged its branching ratio with that of $B^+\to\pi^+\omega$ we would have obtained instead $(8.8\pm1.6)\times10^{-6}$, not very different.] We shall return to the possibility of a measurable difference between the $\pi^+\rho^0$ and $\pi^+\omega$ modes in Sec. VI.

The inferred $|t_V|^2$ contribution to $\mathcal{B}(B^0\to\pi^+\rho^-)$ (neglecting c_P) is $(14.7\pm3.3)\times10^{-6}$, or approximately half of $\mathcal{B}(B^0\to\pi^\pm\rho^\mp)=(25.8\pm4.5)\times10^{-6}$. This leaves a contribution of $\mathcal{B}(B^0\to\pi^-\rho^+)=(11.1\pm5.6)\times10^{-6}$ to be supplied by $|t_P|^2$, if we neglect penguin contributions. A value of $|t_P|^2$ comparable to $|t_V|^2$, but with large errors, thus is allowed by present data. A better measurement of $\mathcal{B}(B^0\to\pi^\pm\rho^\mp)$ is needed to reduce the uncertainty. The magnitude of t_P is of particular interest because of the possibility that the smaller $|\Delta S|=1$ amplitude t'_P, related to t_P by flavor SU(3), could contribute to a rate difference between $B^+\to\eta K^{*+}$ and $B^0\to\eta K^{*0}$ (Sec. VI).

We take into account SU(3) breaking in estimating $t'_{V,P}$ by noting the meson to which the current gives rise: pseudoscalar in t'_V and vector in t'_P. Thus, we have $|t'_V/t_V|^2=|V_{us}/V_{ud}|^2|f_K/f_\pi|^2$ and $|t'_P/t_P|^2=|V_{us}/V_{ud}|^2|f_{K*}/f_\rho|^2$.

TABLE III. Same as Table II for $|\Delta S|=1$ decays of B mesons.

Mode	Amplitudes	$\|t'\|^2$	$\|p'\|^2$	$\|s'\|^2$ [a]	$\|s'\|^2$ [b]	Expt.
$B^+\to\pi^+K^0$	p'	0	*17.2*	0	0	17.2 ± 2.6
π^0K^+	$-\frac{1}{\sqrt{2}}(p'+t'+c')$	0.30	8.6	0	0	12.0 ± 1.6
ηK^+	$-\frac{1}{\sqrt{3}}(t'+c'+s')$	0.20	0	1.4	5.6	<6.9
$\eta' K^+$	$\frac{1}{\sqrt{6}}(3p'+t'+c'+4s')$	0.10	25.9	10.9	44.4	75 ± 7
π^+K^{*0}	p'_P	0	*12.2*	0	0	12.2 ± 2.4
ηK^{*+}	$-\frac{1}{\sqrt{3}}(p'_P-p'_V+t'_P+c'_V+s'_V)$	0.22	16.2	–	–	24.5 ± 7.1
$\eta' K^{*+}$	$\frac{1}{\sqrt{6}}(p'_P+2p'_V+t'_P+c'_V+4s'_V)$	0.11	2.0	–	–	<35
$K^+\omega$	$\frac{1}{\sqrt{2}}(p'_V+t'_V+c'_P+2s'_P)$	0.60	6.1	0.24 [c]	–	<4
$K^+\phi$	$p'_P+s'_P$	0	12.2	0.48	–	7.7 ± 1.2
$B^0\to\pi^-K^+$	$-(p'+t')$	0.56	16.1	0	0	17.3 ± 1.5
π^0K^0	$\frac{1}{\sqrt{2}}(p'-c')$	0	8.1	0	0	10.4 ± 2.6
ηK^0	$-\frac{1}{\sqrt{3}}(c'+s')$	0	0	1.3	5.2	<9.3
$\eta' K^0$	$\frac{1}{\sqrt{6}}(3p'+c'+4s')$	0	24.2	10.2	41.6	56 ± 9
π^-K^{*+}	$-(p'_P+t'_P)$	0.62	11.4	0	0	23.8 ± 6.1
ηK^{*0}	$-\frac{1}{\sqrt{3}}(p'_P-p'_V+c'_V+s'_V)$	0	15.2	–	–	18.0 ± 3.2
$\eta' K^{*0}$	$\frac{1}{\sqrt{6}}(p'_P+2p'_V+c'_V+4s'_V)$	0	1.9	–	–	<24
$K^+\rho^-$	$-(p'_V+t'_V)$	1.13	11.4	0	0	15.9 ± 4.4
$K^0\omega$	$\frac{1}{\sqrt{2}}(p'_V+c'_P+2s'_P)$	0	5.7	0.23 [c]	–	<13
$K^0\phi$	$p'_P+s'_P$	0	11.4	0.45	–	7.5 ± 1.8

[a] Maximal interference between p' and s' amplitudes assumed: constructive for ηK and $\eta' K$; destructive for $K\phi$.
[b] No interference between p' and s' amplitudes assumed.
[c] $(c'_P+2s'_P)/\sqrt{2}$ contributes a term $\frac{1}{3}P'_{EWP}/\sqrt{2}\simeq-0.20p'_V/\sqrt{2}$ to amplitude.

We estimate $f_{K^*}/f_\rho=1.04\pm0.02$ using standard kinematic factors (see the Appendix) and branching ratios for $\tau\to\rho\nu_\tau$ and $\tau\to K^*\nu_\tau$ quoted in Ref. [36].

IV. PENGUIN AND ELECTROWEAK PENGUIN AMPLITUDES

A. $B\to\eta' K$ decays

The decays $B^+\to\eta' K^+$ and $B^0\to\eta' K^0$ have quite large branching ratios. A large fraction of the amplitudes are contributed by penguin (p') terms, but these are not sufficient. One must include also singlet penguin contributions, as introduced in Refs. [4] and [8].

Neglecting t' contributions (to be discussed below), the branching ratios of $\eta' K^+$ and $\eta' K^0$ modes should have a ratio roughly equal to the lifetime ratio. Averaging these two sets of data, we obtain $\mathcal{B}(B^0\to\eta' K^0)\simeq(65.8\pm5.2)\times10^{-6}$, whose central value implies $(8/3)|s'|^2\simeq10.2$ for constructive interference and 41.6 for no interference between p' and s'. The corresponding average numbers for $B^+\to\eta' K^+$ can

TABLE IV. Experimental branching ratios of selected $\Delta S=0$ decays of B mesons. Branching ratios are quoted in units of 10^{-6}. Numbers in parentheses are upper bounds at 90% C.L. References are given in square brackets.

Mode	CLEO	BaBar	Belle
$B^+\to\pi^+\pi^0$	$5.6^{+2.6}_{-2.3}\pm1.7$ (<12.7) [14]	$5.1^{+2.0}_{-1.8}\pm0.8$ (<9.6) [22]	$7.8^{+3.8+0.8}_{-3.2-1.2}$ (<13.4) [27]
$K^+\overline{K}^0$	<5.1 [14]	$-1.3^{+1.4}_{-1.0}\pm0.7$ (<2.4) [22]	<5.0 [27]
$\pi^+\eta$	$1.2^{+2.8}_{-1.2}$ (<5.7) [15]	–	–
$\pi^+\eta'$	$1.0^{+5.8}_{-1.0}$ (<12) [15]	$5.4^{+3.5}_{-2.6}\pm0.8$ (<12) [24]	<7 [29]
$\pi^+\rho^0$	$10.4^{+3.3}_{-3.4}\pm2.1$ [16]	$24\pm8\pm3$ [20]	<14.5 [31]
$\pi^+\omega$	$11.3^{+3.3}_{-2.9}\pm1.4$ [16]	$6.6^{+2.1}_{-1.8}\pm0.7$ [24]	<9.4 [28]
$\pi^+\phi$	–	$0.21^{+0.49}_{-0.21}\pm0.05$ (<1.4) [21]	–
$B^0\to\pi^+\pi^-$	$4.3^{+1.6}_{-1.4}\pm0.5$ [14]	$4.1\pm1.0\pm0.7$ [22]	$5.6^{+2.3+0.4}_{-2.0-0.5}$ [27]
$\pi^0\pi^0$	$2.2^{+1.7+0.7}_{-1.3-0.7}$ (<5.7) [18]	–	–
K^+K^-	<1.9 [14]	$0.85^{+0.81}_{-0.66}\pm0.37$ (<2.5) [22]	<2.7 [27]
$\pi^\pm\rho^\mp$	$27.6^{+8.4}_{-7.4}\pm4.2$ [16]	$28.9\pm5.4\pm4.3$ [26]	$20.2^{+8.3}_{-6.6}\pm3.3$ (<35.7) [28]
$\pi^0\omega$	$0.8^{+1.9+1.0}_{-0.8-0.8}$ (<5.5) [16]	$-0.3\pm1.1\pm0.3$ (<3) [24]	–

thus be obtained by the lifetime ratio: e.g., $\mathcal{B}(B^+\to\eta' K^+)\simeq(70.3\pm5.5)\times10^{-6}$. When s' and p' interfere constructively, one needs a relatively small value of $s'\simeq0.49p'$ to obtain the observed branching ratios.

B. $B\to K\phi$ decays

The branching ratios $\mathcal{B}(B^+\to K^+\phi)$ and $\mathcal{B}(B^0\to K^0\phi)$, when compared with the p'_P contributions, suggest a destructively interfering s'_P. We associate its contribution with the electroweak penguin component rather than the S'_P amplitude, which would involve a violation of the Okubo-Iizuka-Zweig rule unusual for ω and ϕ mesons.

The average of the charged and neutral $B\to K\phi$ modes $\mathcal{B}(B^+\to K^+\phi)=(7.8\pm1.0)\times10^{-6}$ and $\mathcal{B}(B^0\to K^0\phi)=(7.3\pm0.9)\times10^{-6}$ are used to extract s'_P. The result is $s'_P/p'_P=-0.20\pm0.11$, consistent with the result found in Ref. [1] (see Table III there) and with the predictions of Ref. [39]. However, better measurements of these decay modes and of the mode $B^+\to\pi^+K^{*0}$ providing $|p'_P|$ would be worthwhile to confirm the result.

C. $B\to K\omega$ decays

Electroweak penguin terms arise in $B\to K\omega$ from c'_P and s'_P amplitudes, leading to an overall contribution

TABLE V. Same as Table IV for $|\Delta S|=1$ decays of B mesons.

Mode	CLEO	BaBar	Belle
$B^+\to\pi^+K^0$	$18.2^{+4.6}_{-4.0}\pm1.6$ [14]	$18.2^{+3.3}_{-3.0}\pm2.0$ [22]	$13.7^{+5.7+1.9}_{-4.8-1.8}$ [27]
π^0K^+	$11.6^{+3.0+1.4}_{-2.7-1.3}$ [14]	$10.8^{+2.1}_{-1.9}\pm1.0$ [22]	$16.3^{+3.5+1.6}_{-3.3-1.8}$ [27]
ηK^+	$2.2^{+2.8}_{-2.2}$ (<6.9) [15]	–	–
$\eta' K^+$	$80^{+10}_{-9}\pm7$ [15]	$70\pm8\pm5$ [24]	$79^{+12}_{-11}\pm9$ [29]
π^+K^{*0}	$7.6^{+3.5}_{-3.0}\pm1.6$ (<16) [16]	$15.5\pm3.4\pm1.8$ [25]	$16.7^{+3.7+2.1+3.0}_{-3.4-2.1-5.9}$ [31]
ηK^{*+}	$26.4^{+9.6}_{-8.2}\pm3.3$ [15]	$22.1^{+11.1}_{-9.2}\pm3.3$ (<33.9) [23]	<49.9 [32]
$\eta' K^{*+}$	$11.1^{+12.7}_{-8.0}$ (<35) [15]	–	–
$K^+\omega$	$3.2^{+2.4}_{-1.9}\pm0.8$ (<7.9) [16]	$1.4^{+1.3}_{-1.0}\pm0.3$ (<4) [24]	<10.5 [28]
$K^+\phi$	$5.5^{+2.1}_{-1.8}\pm0.6$ [17]	$7.7^{+1.6}_{-1.4}\pm0.8$ [21]	$11.2^{+2.2}_{-2.0}\pm1.4$ [30]
$B^0\to\pi^-K^+$	$17.2^{+2.5}_{-2.4}\pm1.2$ [14]	$16.7\pm1.6\pm1.3$ [22]	$19.3^{+3.4+1.5}_{-3.2-0.6}$ [27]
π^0K^0	$14.6^{+5.9+2.4}_{-5.1-3.3}$ [14]	$8.2^{+3.1}_{-2.7}\pm1.2$ [22]	$16.0^{+7.2+2.5}_{-5.9-2.7}$ [27]
ηK^0	$0.0^{+3.2}_{-0.0}$ (<9.3) [15]	–	–
$\eta' K^0$	$89^{+18}_{-16}\pm9$ [15]	$42^{+13}_{-11}\pm4$ [24]	$55^{+19}_{-16}\pm8$ [29]
π^-K^{*+}	22^{+8+4}_{-6-5} [13]	–	$26.0\pm8.3\pm3.5$ [31]
ηK^{*0}	$13.8^{+5.5}_{-4.6}\pm1.6$ [15]	$19.8^{+6.5}_{-5.6}\pm1.7$ [23]	$21.2^{+5.4}_{-4.7}\pm2.0$ [30,32]
$\eta' K^{*0}$	$7.8^{+7.7}_{-5.7}$ (<24) [15]	–	–
$K^+\rho^-$	$16.0^{+7.6}_{-6.4}\pm2.8$ (<32) [16]	–	$15.8^{+5.1+1.7}_{-4.6-3.0}$ [31]
$K^0\omega$	$10.0^{+5.4}_{-4.2}\pm1.4$ (<21) [16]	$6.4^{+3.6}_{-2.8}\pm0.8$ (<13) [24]	–
$K^0\phi$	$5.4^{+3.7}_{-2.7}\pm0.7$ (<12.3) [17]	$8.1^{+3.1}_{-2.5}\pm0.8$ [21]	$8.9^{+3.4}_{-2.7}\pm1.0$ [30]

$+\frac{1}{3}P'_{EWP}/\sqrt{2}\simeq -0.20p'_V/\sqrt{2}$ to each amplitude. Thus, as in $B\to K\phi$ decays, the electroweak penguin amplitude reduces the contribution of the dominant penguin amplitude to the rate by about 30%, and one has the predictions

$$\mathcal{B}(B^+\to K^+\omega)\simeq\frac{1}{2}\mathcal{B}(B^+\to K^+\phi)=(3.9\pm0.5)\times10^{-6}, \tag{2}$$

$$\mathcal{B}(B^0\to K^0\omega)=\frac{1}{2}\mathcal{B}(B^0\to K^0\phi)=(3.7\pm0.5)\times10^{-6}. \tag{3}$$

The former result could be significantly modified by tree-penguin interference, as noted in Ref. [1] and as we shall see in Sec. VI.

V. RATES AND *CP* ASYMMETRIES IN $B^+\to\pi^+(\eta,\eta')$

The decays $B^+\to\pi^+\eta$ and $B^+\to\pi^+\eta'$ could be detectable at present levels of sensitivity. Measurements of the branching ratios and CP asymmetries of these modes can provide information on strong and weak phases and on the relative importance of singlet amplitude contributions, which are estimated using s' in the $\eta' K^+$ mode as discussed above.

We shall give an illustrative example of the possibilities for large rates and CP asymmetries in $B^+\to\pi^+\eta$ and $B^+\to\pi^+\eta'$ decays. We shall assume that the singlet amplitude s interferes constructively with p. Their electroweak phases are likely to be the same, and a quite modest s' interfering constructively with p' in the decays $B\to\eta' K$ can account for the observed rate. We thus take $s/p=s'/p'=0.49$, leading to the entries on column (a) of Table II.

Using flavor SU(3) to estimate p from the dominant amplitude p' in $B^+\to\pi^+K^0$ and $t+c$ as mentioned earlier, we then reconstruct the $B^+\to\pi^+(\eta,\eta')$ amplitudes as follows:

$$\begin{aligned}
A(B^+\to\pi^+\eta)&=-(1.77e^{i\gamma}+1.06e^{-i\beta}e^{i\delta}),\\
A(B^-\to\pi^-\eta)&=-(1.77e^{-i\gamma}+1.06e^{i\beta}e^{i\delta}),\\
A(B^+\to\pi^+\eta')&=1.25e^{i\gamma}+1.19e^{-i\beta}e^{i\delta},\\
A(B^-\to\pi^-\eta')&=1.25e^{-i\gamma}+1.19e^{i\beta}e^{i\delta},
\end{aligned} \tag{4}$$

where β and γ are CKM phases, δ is a relative strong phase between the penguin and tree amplitudes, and amplitudes are defined such that their squares give branching ratios in units of 10^{-6}.

The CP rate asymmetries

$$A(f)\equiv\frac{\mathcal{B}(B^-\to\overline{f})-\mathcal{B}(B^+\to f)}{\mathcal{B}(B^-\to\overline{f})+\mathcal{B}(B^+\to f)} \tag{5}$$

and the CP-averaged branching ratios

$$\overline{\mathcal{B}}(f)\equiv\frac{\mathcal{B}(B^-\to\overline{f})+\mathcal{B}(B^+\to f)}{2} \tag{6}$$

then are found to be

$$\begin{aligned}
A(\pi^+\eta)&=\frac{-0.88\sin\delta\sin\alpha}{1-0.88\sin\delta\sin\alpha},\\
A(\pi^+\eta')&=\frac{-\sin\delta\sin\alpha}{1-\sin\delta\sin\alpha},
\end{aligned} \tag{7}$$

$$\begin{aligned}
\overline{\mathcal{B}}(\pi^+\eta)&=(4.3\times10^{-6})(1-0.88\cos\delta\cos\alpha),\\
\overline{\mathcal{B}}(\pi^+\eta')&=(3.0\times10^{-6})(1-\cos\delta\cos\alpha).
\end{aligned} \tag{8}$$

Measurement of both CP asymmetries and branching ratios would allow one to obtain values of δ and $\alpha=\pi-\beta-\gamma$, given our assumption about s/p.

VI. TREE-PENGUIN INTERFERENCE

A. $B\to\eta' K$ decays

The central values of the measured rates for $B^+\to\eta' K^+$ and $B^0\to\eta' K^0$ are roughly 1.5σ away from each other. One can attribute part of this difference to a contribution the tree amplitude in the former mode, if the tree and penguin amplitudes happen to interfere constructively. We estimate the $|t'|^2$ term to contribute an amount 0.10×10^{-6} to the branching ratio (see Table III), which by itself would be insignificant. However, with fully constructive interference with the p' and $s'\simeq0.49p'$ terms, we would have

$$\mathcal{B}(B^+\to\eta' K^+)=[70.2+0.10+2\sqrt{(70.2)(0.10)}]\times10^{-6}=75.7\times10^{-6}. \tag{9}$$

Thus, in order to demonstrate such interference, one has to conclusively establish the $(B^+\to\eta' K^+)$ branching ratio with an error of less than a couple of parts in 10^6. At present the errors on the branching ratios are still too large to give a conclusive answer to whether t' plays an important role here.

B. $B\to\eta K^*$ decays

The results for $\mathcal{B}(B^+\to\pi^+K^{*0})$ give $|p'_P|^2\simeq12.2\times10^{-6}$, implying $\mathcal{B}(B^+\to\eta K^{*+})=16.2\times10^{-6}$ and $\mathcal{B}(B^0\to\eta K^{*0})=15.2\times10^{-6}$. Both experimental values are a bit more than 1σ above these predictions. The question was raised in Ref. [1] whether tree-penguin interference could be responsible for the slightly higher ηK^{*+} branching ratio. The t'_P contribution here is related to t_P inferred from $B^0\to\pi^-\rho^+$ by the ratio $|V_{us}/V_{ud}|^2|f_{K*}/f_\rho|^2\tau_{B^+}/\tau_{B^0}\simeq0.059$. With maximal constructive interference we could have a modest enhancement:

$$\mathcal{B}(B^+\to\eta K^{*+})=[16.2+0.22+2\sqrt{(16.2)(0.22)}]\times10^{-6}=20.2\times10^{-6}. \tag{10}$$

To see such an effect, as for $B\to\eta' K$ decays, it would be necessary to achieve an error on branching ratios of a couple of parts in 10^6.

Ignoring the contribution from t'_P, charged and neutral modes are predicted to have the same rates. Taking the average of the current data, we obtain $\mathcal{B}(B^+\to\eta K^{*+})\simeq(20.3\pm3.1)\times10^{-6}$ and $\mathcal{B}(B^0\to\eta K^{*0})\simeq(19.0\pm2.9)\times10^{-6}$. Therefore, at the present level of sensitivity there is no indication of significant effects due to the interference of the t'_P amplitude with the dominant penguin contribution. These data would favor a slightly larger penguin contribution than extracted from the π^+K^{*0} mode.

C. $B\to\omega K$ decays

We mentioned above the possibility of tree-penguin interference in $B^+\to\omega K^+$. To give one example of such effects, let us recall the assumption $p'_V=-p'_P$ but assume the signs of t'_P and t'_V are the same. Then if one has constructive interference in $B^+\to\eta K^{*+}$ as suggested above, one would have *destructive* interference in $B^+\to\omega K^+$. The t'_V contribution here is related to t_V in $B^+\to\omega\pi^+$ by $|t'_V/t_V|^2=|V_{us}/V_{ud}|^2|f_K/f_\pi|^2\simeq0.076$. In the case of maximal destructive interference one would have

$$\mathcal{B}(B^+\to\omega K^+)=[3.9+0.6-2\sqrt{(3.9)(0.6)}]\times10^{-6}=1.4\times10^{-6}, \tag{11}$$

a significant effect.

D. $B^0\to\pi^-K^{*+}$ and $B^0\to K^+\rho^-$ decays

The signs of tree-penguin interference terms in the decays $B^0\to\pi^-K^{*+}$ and $B^0\to K^+\rho^-$ are correlated with those in $B^+\to K^+\omega$. If the interference is destructive in $B^+\to K^+\omega$, it will also be destructive in $B^0\to K^+\rho^-$, since both processes involve the combination $p'_V+t'_V$. If t'_P and t'_V have the same sign (as is likely), but if p'_P and p'_V are equal and opposite (as has been proposed), one then expects constructive tree-penguin interference in $B^0\to\pi^-K^{*+}$. This pattern was noted in Refs. [1] and [40].

In the cases of maximal interference in the directions suggested, one would then have

$$\mathcal{B}(B^0\to\pi^-K^{*+})=[11.4+0.6+2\sqrt{(11.4)(0.6)}]\times10^{-6}=17.3\times10^{-6}, \tag{12}$$

consistent with the experimental branching ratio of $(23.8\pm6.1)\times10^{-6}$, but also

$$\mathcal{B}(B^0\to K^+\rho^-)=[11.4+1.1-2\sqrt{(11.4)(1.1)}]\times10^{-6}=5.4\times10^{-6}, \tag{13}$$

which is well below the experimental branching ratio of $(15.9\pm4.4)\times10^{-6}$. In each case the deviation from pure penguin dominance amounts to 6×10^{-6}, so measurement of each of these branching ratios with an error of no more than 2×10^{-6} should be enough to see whether the interference terms form a consistent pattern, or indeed are present at all.

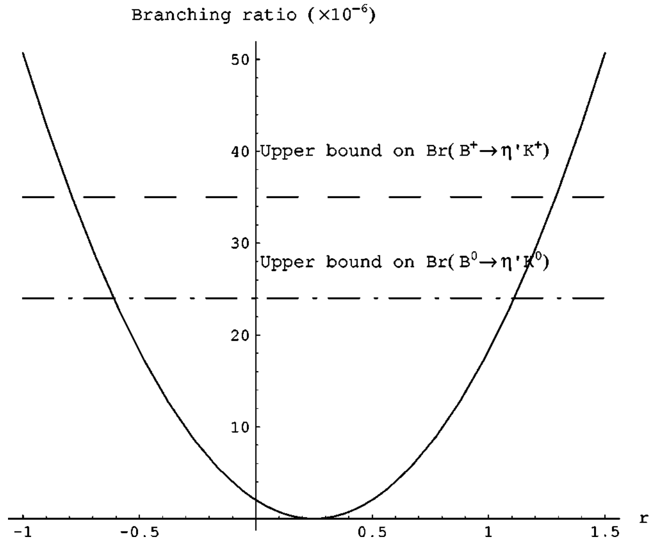

FIG. 1. The branching ratios of $B\to\eta' K^*$ for varying s'_V related to p'_P by the parameter $-1\leqslant r\leqslant1.5$.

E. $B^+\to\pi^+\rho^0$ and $B^+\to\pi^+\omega$ decays

More precise measurements for the $B^+\to\pi^+\rho^0$ and $B^+\to\pi^+\omega$ modes could help to determine whether there is a difference between their branching ratios, which would be ascribed to contributions of the p_P and/or s_P amplitudes. The chance of a detectable s_P contribution to $B^+\to\pi^+\phi$, for which BaBar has presented an upper bound [21], is remote, as one sees from the predicted branching ratio of about 2×10^{-8} in Table II. Consequently, one would most likely ascribe a difference to constructive tree-penguin interference, which would be consistent with the pattern mentioned earlier [1,40], leading to a prediction

$$\mathcal{B}(B^+\to\pi^+\rho^0)=[7.9+0.8+2\sqrt{(7.9)(0.8)}]\times10^{-6}=13.6\times10^{-6}. \tag{14}$$

As in previous cases, the effects of maximal interference amount to a change in the predicted branching ratio of a few parts in 10^6.

VII. FURTHER SINGLET AMPLITUDE CONTRIBUTIONS

We have already noted in Sec. IV the importance of the singlet contribution s' in the decays $B\to\eta' K$. However, no such contribution is yet called for in $B\to PV$ decays. Here we show how to demonstrate its presence.

A contribution from the singlet amplitude s'_V has to come from the comparison between the ηK^* and $\eta' K^*$ modes. If we neglect t'_P, as suggested from the above analysis, and c'_V, as suggested by the hierarchy in the amplitudes, we can assume $s'_V=rp'_P$ and get

$$\mathcal{B}(B^+\to\eta' K^{*+})\simeq\mathcal{B}(B^0\to\eta' K^{*0})\frac{\tau_{B^0}}{\tau_{B^+}}$$
$$=\frac{1}{6}(1-4r)^2 p_P'^2, \qquad (15)$$

where p_P' is the penguin amplitude for the charged modes. Figure 1 shows the branching ratio of $B\to\eta' K^*$ as a parabolic function of r with a minimum at $r=1/4$. To avoid confusion, we only plot the one for $B\to\eta' K^{*+}$ as the difference is tiny in the range of the plot. The dashed and dash-dotted lines give the current upper bounds on the branching ratios of the $\eta' K^{*+}$ and $\eta' K^{*0}$ modes, respectively. Observation of these modes with branching ratios significantly different from $\sim 2\times 10^{-6}$ would provide conclusive evidence for the singlet contribution s_V'. We note that $\mathcal{B}(B^+\to\eta' K^{*+})$ by itself is unable to distinguish between r and $r'\equiv\frac{1}{2}-r$, so if this branching ratio is consistent with $\sim 2\times 10^{-6}$, that does not yet rule out the possibility of a singlet term with $s_V'/p_P'\simeq 1/2$. This is just the value of s'/p' which would accommodate the decays $B\to\eta' K$.

VIII. SUMMARY

New data on B decays to pairs of light mesons are shedding light on a number of interesting questions. We have shown that the penguin contribution in the decay $B^+\to\pi^+ K^{*0}$ is only a bit smaller than that contributing to $B\to\pi K$ decays. Although a similar penguin contribution occurs in $B\to K\phi$ decays, it is partially cancelled by an electroweak penguin contribution, leading to a 30% reduction in rate in accord with predictions [39]. A similar cancellation is expected in the decays $B\to K\omega$.

The prospects for observing $B^+\to\pi^+\eta$ and $B^+\to\pi^+\eta'$, suggested as promising modes for direct CP rate asymmetries [3,4], are excellent. Branching ratios of a few parts in 10^6 are expected. By studying both rates and CP asymmetries, one can determine both the relative strong phases of penguin and tree amplitudes and the weak phase α.

Tree-penguin interference can be studied by comparing B^+ and B^0 branching ratios for processes such as $B\to\eta' K$, $B\to\eta K^*$, and $B\to K\omega$. Anticipated differences in branching ratios in these three cases could be as large as several parts in 10^6, but are unlikely to be more. Other processes which can be examined for this interference include the decays $B^0\to\pi^- K^{*+}, B^0\to K^+\rho^-$, and a comparison of $B^+\to\pi^+\omega$ and $B^+\to\pi^+\rho^0$. Present data are not yet at the required level of accuracy, but will be so soon, providing valuable information on the products $\cos\gamma\cos\delta$ ($|\Delta S|=1$ decays) and $\cos\alpha\cos\delta$ ($\Delta S=0$ decays).

Although a flavor-singlet penguin contribution is needed in describing $B\to\eta' K$, no such amplitude is called for yet in $B\to\eta' K^*$. We have shown that significant deviations of the branching ratio for this process (for both charged and neutral B's) from 2×10^{-6} would provide evidence for such a term. However, a branching ratio equal to this value does not yet rule out a singlet term.

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APPENDIX: DECAY CONSTANT CALCULATIONS

We define the decay constant of a vector meson V $(=u\bar{q})$ through the matrix element between one particle and vacuum of the vector current V_μ: $\langle 0|V_\mu|V(p)\rangle = m_V f_V \epsilon_\mu(p)$. The partial width of the τ lepton into $V\nu_\tau$ is then

$$\Gamma(\tau\to V\nu_\tau)=\frac{(G_F f_V p^*|V_{uq}|)^2}{4\pi}m_\tau\left(1+\frac{2m_V^2}{m_\tau^2}\right), \quad \text{(A1)}$$

where $p^*=(m_\tau^2-m_V^2)/(2m_\tau)$ is the magnitude of the c.m. three-momentum of either final particle, and $|V_{uq}|=|V_{ud}|$ for $\rho\nu_\tau$ or $|V_{us}|$ for $K^*\nu_\tau$. Using [36] $\tau_\tau=(290.6\pm 1.1)$ fs, $\mathcal{B}(\tau\to\rho\nu_\tau)=(25.1\pm 0.3)\%$, and $\mathcal{B}(\tau\to K^*\nu_\tau)=(1.29\pm 0.05)\%$, we find $f_\rho=208$ MeV, $f_{K^*}=217$ MeV, and $f_{K^*}/f_\rho=1.04\pm 0.02$.

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