Chiral extrapolation of lattice data for the hyperfine splittings of heavy mesons

X.-H. Guo* and A. W. Thomas†
Department of Physics and Mathematical Physics and Special Research Center for the Subatomic Structure of Matter, Adelaide University, Adelaide, SA 5005, Australia
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Hyperfine splittings between the heavy vector (D^*,B^*) and pseudoscalar (D,B) mesons have been calculated numerically in lattice QCD, where the pion mass (which is related to the light quark mass) is much larger than its physical value. Naive linear chiral extrapolations of the lattice data to the physical mass of the pion lead to hyperfine splittings which are smaller than experimental data. In order to extrapolate these lattice data to the physical mass of the pion more reasonably, we apply the effective chiral perturbation theory for heavy mesons, which is invariant under chiral symmetry when the light quark masses go to zero and heavy quark symmetry when the heavy quark masses go to infinity. This leads to a phenomenological functional form with three parameters to extrapolate the lattice data. It is found that the extrapolated hyperfine splittings are even smaller than those obtained using linear extrapolation. We conclude that the source of the discrepancy between lattice data for hyperfine splittings and experiment must lie in non-chiral physics.

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I. INTRODUCTION

The past few years have seen much progress in lattice gauge theory, which is the only quantitative tool currently available to calculate nonperturbative phenomena in QCD from first principles. These phenomena include the spectrum of light hadrons [1], the spectrum of heavy hadrons [2,3], the decay constants of heavy mesons [4], nucleon structure functions [5], and the Isgur-Wise function for $B \to D(D^*)$ transitions [6].

In Ref. [3] the authors studied extensively the spectra of the D and B mesons using non-relativistic QCD (NRQCD) on the lattice in the quenched approximation. For spin-independent splittings such as the splittings between strange and non-strange mesons, good agreement with experiments was obtained. However, it was found that the hyperfine splittings between $D(B)$ and $D^*(B^*)$ are much smaller than the experimental data. In fact, three lattice data for the mass of each of these mesons were obtained in the region where the mass of the pion is larger than about 680 MeV, which is much larger than the physical mass of the pion. With naive linear extrapolations from the unphysical region to the physical value of the pion mass, the extrapolated hyperfine splitting between D and D^* is about 110 MeV for $\beta=5.7$ compared with the experimental value 140 MeV, whereas for B and B^* the extrapolated hyperfine splitting is about 29 MeV compared with the experimental value 46 MeV. Obviously these large differences between the extrapolated and experimental values merit careful investigation.

It is well known that QCD possesses a chiral $SU(3)_L \times SU(3)_R$ symmetry in the limit where the masses of light quarks u,d, and s go to zero. This symmetry is spontaneously broken into $SU(3)_V$, leading to eight Goldstone bosons. The interactions of these pseudoscalar mesons are described by an effective chiral Lagrangian which is invariant under $SU(3)_L \times SU(3)_R$. Another interesting quark mass limit in QCD is the heavy quark limit where the masses of heavy quarks, c and b, go to infinity. In this limit there are heavy quark flavor symmetry and heavy quark spin symmetry $SU(2)_f \times SU(2)_s$. Based on these symmetries which are not manifest in the full theory of QCD, an effective theory for heavy quark interactions which is called heavy quark effective theory (HQET) was established [7]. With the aid of HQET, the physical processes involving heavy quarks are greatly simplified. The interactions of heavy hadrons containing one heavy quark with the light pseudoscalar mesons π, K, and η should be constrained by both chiral symmetry and heavy quark symmetry. The combination of these two symmetries leads to an effective chiral Lagrangian for heavy hadrons which is invariant under both $SU(3)_L \times SU(3)_R$ and $SU(2)_f \times SU(2)_s$ transformations. There has been considerable work in this direction in recent years [8,9].

In the past few years there has been a series of work dealing with extrapolations of lattice data for hadron properties, such as mass [10], magnetic moments [11], parton distribution functions [12], and charge radii [13], to the physical region. In these extrapolations the inclusion of pion loops yields leading and next-to-leading non-analytic behavior. This leads to rapid variation at small pion masses while lattice data are extrapolated to the physical pion mass. However, when the pion mass is greater than some scale Λ, which characterizes the physical size of the hadrons which emit or absorb the pion, the hadron properties vary slowly and smoothly. It is obvious that extrapolations which ensure the correct non-analytic behavior of QCD in this way should be more reliable than a naive linear extrapolation.

With these considerations in mind, the aim of the present work is to extrapolate the lattice data in Ref. [3] to the physical region, while building in the constraints of chiral perturbation theory for heavy mesons, and to compare the extrapolated hyperfine splittings with experiments.

The remainder of this paper is organized as follows. In Sec. II we give a brief review for chiral perturbation theory for heavy mesons. In Sec. III we apply this theory to calcu-

*Email address: xhguo@physics.adelaide.edu.au
†Email address: athomas@physics.adelaide.edu.au

late self-energy contributions to heavy mesons from pion loops. Based on this, we propose a phenomenological, functional form for extrapolating the lattice data for heavy meson masses to the physical region. Then in Sec. IV we use this form to fit the lattice data and give numerical results. Finally, Sec. V contains a summary and discussion.

II. CHIRAL PERTURBATION THEORY FOR HEAVY MESONS

When the heavy quark mass m_Q ($Q=b$, or c) is much larger than the QCD scale $\Lambda_{\rm QCD}$, the light degrees of freedom in a heavy hadron are blind to the flavor and spin orientation of the heavy quark Q. Therefore, dynamics inside a heavy hadron remains unchanged under $SU(2)_f \times SU(2)_s$ transformations. In the opposite mass limit $m_q \to 0$, the QCD Lagrangian possesses an $SU(3)_L \times SU(3)_R$ chiral symmetry. The light pseudo Goldstone bosons associated with spontaneous breaking of chiral symmetry are incorporated in a 3×3 matrix

$$\Sigma = \exp\left(\frac{2iM}{f_\pi}\right), \tag{1}$$

where f_π is the pion decay constant, $f_\pi = 132$ MeV, and

$$M = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}. \tag{2}$$

Under $SU(3)_L \times SU(3)_R$ transformations

$$\Sigma \to L \Sigma R^+, \tag{3}$$

where $L \in SU(3)_L$ and $R \in SU(3)_R$.

While discussing the interactions of Goldstone bosons with other matter fields it is convenient to introduce

$$\xi = \sqrt{\Sigma}. \tag{4}$$

Under $SU(3)_L \times SU(3)_R$ transformations

$$\xi \to L \xi U^+ = U \xi R^+, \tag{5}$$

where the unitary matrix U is a complicated nonlinear function of L, R, and the Goldstone fields.

In order to discuss the interactions of Goldstone bosons with heavy mesons, which consist of a heavy quark Q and a light antiquark $\bar{q}^a$ ($a=1,2,3$ for u,d,s quarks, respectively), a 4×4 Dirac matrix H_a is introduced [8] as follows:

$$H_a(v) = \frac{1+\not{v}}{2}(P_a^{*\mu}\gamma_\mu - P_a \gamma_5), \tag{6}$$

where $P_a^{*\mu}$ and P_a are field operators which destroy vector and pseudoscalar heavy mesons with fixed four velocity v, respectively. $P_a^{*\mu}$ satisfies the constraint

$$v_\mu P_a^{*\mu} = 0. \tag{7}$$

Since H_a is composed of a heavy quark and a light antiquark, under $SU(3)_L \times SU(3)_R$

$$H_a \to H_b U_{ba}^+, \tag{8}$$

and under heavy quark spin symmetry

$$H_a \to S H_a, \tag{9}$$

where $S \in SU(2)_s$.

Defining

$$\bar{H}_a = \gamma_0 H_a^+ \gamma_0, \tag{10}$$

we have

$$\bar{H}_a(v) = (P_{a\mu}^{*+}\gamma^\mu + P_a^+ \gamma_5)\frac{1+\not{v}}{2}, \tag{11}$$

which transforms as $\bar{H}_a \to U_{ab}\bar{H}_b$ and $\bar{H}_a \to \bar{H}_a S^{-1}$ under chiral symmetry and heavy quark symmetry, respectively.

It is convenient to introduce a vector field V_{ab}^μ,

$$V_{ab}^\mu = \frac{1}{2}(\xi^+ \partial^\mu \xi + \xi \partial^\mu \xi^+)_{ab}, \tag{12}$$

and an axial-vector field A_{ab}^μ,

$$A_{ab}^\mu = \frac{i}{2}(\xi^+ \partial^\mu \xi - \xi \partial^\mu \xi^+)_{ab}. \tag{13}$$

Under $SU(3)_L \times SU(3)_R$, $V^\mu \to U V^\mu U^+ + U \partial^\mu U^+$, and $A^\mu \to U A^\mu U^+$. Defining the covariant derivative

$$(D^\mu H)_a = \partial^\mu H_a - H_b V_{ba}^\mu, \tag{14}$$

we find that $D^\mu H \to (D^\mu H) U^+$ under $SU(3)_L \times SU(3)_R$.

The Lagrangian for the strong interactions of heavy mesons with Goldstone pseudoscalar bosons should be invariant under both chiral symmetry and heavy quark symmetry, since we are working in the limit where light quarks have zero mass and heavy quarks have infinite mass. It should also be invariant under Lorentz and parity transformations as required in general. The most general form for the Lagrangian satisfying these requirements is [8]

$$\mathcal{L} = -\mathrm{Tr}[\bar{H}_a i v_\mu (D^\mu H)_a] + g\,\mathrm{Tr}(\bar{H}_a H_b \gamma_\mu A_{ba}^\mu \gamma_5), \tag{15}$$

where g is the coupling constant describing the interactions between heavy mesons and Goldstone bosons. Obviously, g is universal for D, B, D^*, and B^*. Since g contains information about the interactions at the quark and gluon level, it cannot be fixed from chiral perturbation theory for heavy mesons, but should be determined by experiments.

From Eq. (15), the propagator for the pseudoscalar meson D (or B) is

$$\frac{i}{2v\cdot p}$$

where p is the residual momentum of the meson. For the vector meson D^* (or B^*), the propagator is

$$\frac{-i(g_{\mu\nu}-v_\mu v_\nu)}{2v\cdot p}.$$

In the limit $m_Q\to\infty$, there is no mass difference between pseudoscalar and vector mesons.

Since in this work we will study hyperfine splittings, we need to take $1/m_Q$ corrections into account. At order $1/m_Q$ in HQET, the term which is responsible for hyperfine splittings is the color-magnetic-moment operator, $\overline{h}_v\sigma_{\mu\nu}G^{\mu\nu}h_v$ (where h_v is the heavy quark field operator in HQET and $G^{\mu\nu}$ is the gluon field strength tensor). This leads to the following correction term to $\mathcal{L}$ in Eq. (15):

$$\frac{\lambda_2}{m_Q}\mathrm{Tr}\overline{H}_a\sigma^{\mu\nu}H_a\sigma_{\mu\nu}, \tag{16}$$

where λ_2 is a constant which also contains interaction information at the quark and gluon level, and which is same for D,B,D^*, and B^* at the tree level. (When QCD loop corrections are included, λ_2 depends on m_Q logarithmically.) Finally, we note that the inclusion of the other term at order $1/m_Q$ in HQET, $(1/m_Q)\overline{h}_v(iD)^2h_v$, leads to a slight m_Q dependence of the coupling constant g.

Adding the term (16) to the Lagrangian for HQET, Eq. (15), and using Eqs. (6) and (11), we can easily see that the mass difference between vector and pseudoscalar heavy mesons is

$$\Delta=-\frac{8\lambda_2}{m_Q}, \tag{17}$$

and consequently, the propagators for heavy pseudoscalar and vector mesons become

$$\frac{i}{2\left(v\cdot p+\frac{3}{4}\Delta\right)}$$

and

$$\frac{-i(g_{\mu\nu}-v_\mu v_\nu)}{2\left(v\cdot p-\frac{1}{4}\Delta\right)},$$

respectively.

In order to consider the interactions of heavy mesons with Goldstone bosons, we substitute $\xi=\exp(iM/f_\pi)$ into Eqs. (12), (13) and obtain the following expressions for V^μ and A^μ:

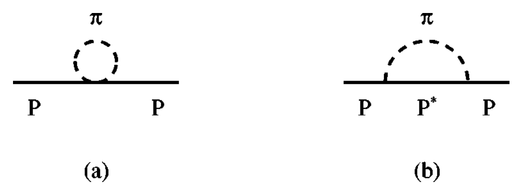

FIG. 1. Pion loop corrections to the propagator of heavy pseudoscalar meson P (P could be D or B).

$$V_\mu=\frac{1}{2f_\pi^2}[M,\partial_\mu M]+O(M^4), \tag{18}$$

$$A_\mu=-\frac{1}{f_\pi}\partial_\mu M+O(M^3). \tag{19}$$

Substituting Eq. (18) and Eq. (19) into Eq. (15) we have the following explicit form for the interactions of heavy mesons with Goldstone bosons:

$$\begin{aligned}
&\mathrm{Tr}[\overline{H}_a i v_\mu V^\mu_{ba}H_b]+g\mathrm{Tr}(\overline{H}_a H_b\gamma_\mu A^\mu_{ba}\gamma_5)\\
&=\frac{i}{f_\pi^2}v^\mu[M,\partial_\mu M]_{ba}(P^{*+}_{a\nu}P^{*\nu}_b-P^+_aP_b)\\
&\quad-\frac{2g}{f_\pi}(P^{*+}_{a\mu}P_b\partial^\mu M_{ba}+P^+_aP^*_{b\mu}\partial^\mu M_{ba}\\
&\quad+i\epsilon^{\mu\nu\rho\sigma}P^{*+}_{a\rho}P^*_{b\sigma}v_\nu\partial_\mu M_{ba}),
\end{aligned} \tag{20}$$

where $O(M^3)$ terms are ignored.

Chiral symmetry can also be broken explicitly by nonzero light quark masses. Under $SU(3)_L\times SU(3)_R$, light quark mass terms transform as $(\overline{3}_L,3_R)+(\overline{3}_R,3_L)$. Then, to leading order in the explicit chiral symmetry breaking from light quark masses, the following terms are added to the Lagrangian in Eq. (15):

$$\begin{aligned}
&\lambda_1\mathrm{Tr}\overline{H}_bH_a(\xi m_q\xi+\xi^+m_q\xi^+)_{ab}\\
&+\lambda_1'\mathrm{Tr}\overline{H}_aH_a(\xi m_q\xi+\xi^+m_q\xi^+)_{bb},
\end{aligned} \tag{21}$$

where λ_1 and λ_1' are parameters which are also independent of the heavy quark mass in the limit $m_Q\to\infty$.

III. FORMULAS FOR THE EXTRAPOLATION OF HEAVY MESON MASSES

In this section we use the Lagrangian for the interactions of heavy mesons with light Goldstone bosons to calculate pion loop corrections to the masses of heavy vector and pseudoscalar mesons. This yields the dependence of heavy meson masses on the pion mass. Then we propose a phenomenological functional form for extrapolating lattice data for heavy meson masses to the physical pion mass.

From Eq. (20) we can see that there are five possible diagrams for pion loop corrections to heavy meson masses. These diagrams are shown in Fig. 1 for D and B mesons, and in Fig. 2 for D^* and B^* mesons. Figures 1(a) and 2(a) arise

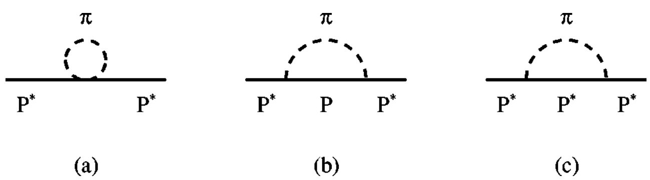

FIG. 2. Pion loop corrections to the propagator of heavy vector meson P^* (P could be D or B).

from the first term in Eq. (20). It can be easily seen that these two diagrams do not contribute to the masses of heavy mesons and we will not consider them from now on.

Figure 1(b) represents the pion loop correction to the heavy pseudoscalar meson propagator (P could be D or B). In momentum space it can be expressed as

$$\frac{i}{2\left(v\cdot p+\frac{3}{4}\Delta\right)}(-2i\Sigma)\frac{i}{2\left(v\cdot p+\frac{3}{4}\Delta\right)}, \tag{22}$$

where p is the residual momentum of the pseudoscalar meson and $-2i\Sigma$ is given by the following integral:

$$-2i\Sigma=-\frac{3g^2}{f_\pi^2}\int\frac{d^4k}{(2\pi)^4}\frac{k^2-(v\cdot k)^2}{\left[v\cdot(p-k)-\frac{1}{4}\Delta\right](k^2-m_\pi^2)}, \tag{23}$$

where k is the momentum of the pion in the loop, and m_π is the pion mass which is not necessarily its physical mass. It can be seen that Σ is a function of $v\cdot p$, m_π^2, and Δ. After the correction from Σ is added, the heavy meson propagator is proportional to

$$\frac{1}{v\cdot p-m_0-\Sigma(v\cdot p)}, \tag{24}$$

where m_0 is the mass term without Σ correction (for the propagator of the pseudoscalar heavy mesons, $m_0=-\frac{3}{4}\Delta$). The physical mass of the heavy meson, m, is defined by

$$[v\cdot p-m_0-\Sigma(v\cdot p)]|_{v\cdot p=m}=0. \tag{25}$$

Therefore, to order $O(g^2)$ we have

$$m=m_0+\Sigma(v\cdot p=m_0). \tag{26}$$

In order to calculate the integral in Eq. (23), we need to deal with the following integral:

$$X^{\mu\nu}\equiv\int\frac{d^4k}{(2\pi)^4}\frac{k^\mu k^\nu}{[v\cdot k-\delta](k^2-m_\pi^2)}, \tag{27}$$

where $\delta=v\cdot p-\frac{1}{4}\Delta$ for Eq. (23). On the grounds of Lorentz invariance, in general we have

$$X^{\mu\nu}=X_1 g^{\mu\nu}+X_2 v^\mu v^\nu, \tag{28}$$

where X_1 and X_2 are Lorentz scalars, which are functions of $v\cdot p$, m_π^2, and Δ. It can be easily seen from Eq. (23) that $\Sigma\sim(g^{\mu\nu}-v^\mu v^\nu)X_{\mu\nu}$. Consequently, only the X_1 term contributes, since $v^2=1$. Multiplying both sides of Eq. (27) with $(g^{\mu\nu}-v^\mu v^\nu)$ we have

$$X_1=\frac{1}{3}\int\frac{d^4k}{(2\pi)^4}\frac{k^2-(v\cdot k)^2}{v^0\left(k_0-\frac{\mathbf{v}\cdot\mathbf{k}+\delta}{v^0}-i\epsilon\right)(k_0^2-W_k^2+i\epsilon)}, \tag{29}$$

where $W_k^2\equiv|\mathbf{k}|^2+m_\pi^2$.

Next we carry out the integration over k_0 by choosing the appropriate contour. From Eq. (26), when we calculate mass corrections from pion loops, we need the value of Σ at $v\cdot p=m_0$. Since $v\cdot p$ is a Lorentz scalar, we are free to choose some special value of v for this purpose. With $v^0=1$ and $\mathbf{v}=\mathbf{0}$, and choosing the contour in the lower half plane, in which there is only one pole for k_0, $W_k-i\epsilon$, we arrive at the following expression:

$$X_1=\frac{i}{6}\int\frac{d^3k}{(2\pi)^3}\frac{|\mathbf{k}|^2}{W_k(W_k-\delta)}. \tag{30}$$

In chiral perturbation theory one usually develops a power expansion systematically. However, in the case of lattice QCD calculations of hadron masses the quark masses for which calculations can be made at present are so large that, even if the series of such an expansion converged (which is unclear) one would have to introduce too many parameters corresponding to the high order of the expansion needed. Therefore, in the present work we choose to follow the approach explained in Refs. [10–13]. It has been argued [10– 13] that when the Compton wavelength of the pion is smaller than the source of the pion, pion loop contributions are suppressed as powers of Λ/m_π where Λ characterizes the finite size of the source of the pion. We follow this argument and introduce a cutoff Λ in the integral (30) to evaluate the pion-baryon loops which yield the leading and next-to-leading non-analytic behavior. Since the leading non-analytic contribution of these loops is associated with the infrared behavior of the integral, it does not depend on the details of the cutoff. This approach has the advantage that when m_π is not far away from the chiral limit, i.e., when m_π is smaller than the cutoff Λ, the expression agrees with chiral perturbation theory for heavy mesons which is constrained by both heavy quark and chiral symmetry, and consequently has the correct chiral limit behavior [see Eq. (34)]. On the other hand, when m_π is bigger the masses are effectively fit using the form which is linear in m_π^2 as indicated by lattice simulations. This will be discussed in more detail later. Although the cutoff Λ is not Lorentz invariant our formalism for evaluating the heavy meson mass is Lorentz invariant and we choose to work in a simple frame $\mathbf{v}=0$ in the present work for the numerical evaluation. In the following, we will treat Λ as a parameter to be fixed by lattice data.

With the cutoff Λ, we obtain the following result for the integral (30):

$$X_1=\frac{i}{72\pi^2}\left\{12(m_\pi^2-\delta^2)^{3/2}\left[\arctan\frac{\Lambda+\sqrt{\Lambda^2+m_\pi^2}-\delta}{\sqrt{m_\pi^2-\delta^2}}-\arctan\frac{m_\pi-\delta}{\sqrt{m_\pi^2-\delta^2}}\right]+3\delta(2\delta^2-3m_\pi^2)\ln\frac{\Lambda+\sqrt{\Lambda^2+m_\pi^2}}{m_\pi}\right.$$
$$\left.+3\delta\Lambda\sqrt{\Lambda^2+m_\pi^2}+6(\delta^2-m_\pi^2)\Lambda+2\Lambda^3\right\}, \tag{31}$$

when $m_\pi^2\geqslant\delta^2$;

$$X_1=\frac{i}{72\pi^2}\left\{6(\delta^2-m_\pi^2)^{3/2}\ln\left(\frac{\Lambda+\sqrt{\Lambda^2+m_\pi^2}-\delta-\sqrt{\delta^2-m_\pi^2}}{\Lambda+\sqrt{\Lambda^2+m_\pi^2}-\delta+\sqrt{\delta^2-m_\pi^2}}\left|\frac{m_\pi-\delta+\sqrt{\delta^2-m_\pi^2}}{m_\pi-\delta-\sqrt{\delta^2-m_\pi^2}}\right|\right)+3\delta(2\delta^2-3m_\pi^2)\ln\frac{\Lambda+\sqrt{\Lambda^2+m_\pi^2}}{m_\pi}\right.$$
$$\left.+3\delta\Lambda\sqrt{\Lambda^2+m_\pi^2}+6(\delta^2-m_\pi^2)\Lambda+2\Lambda^3\right\}, \tag{32}$$

when $m_\pi^2\leqslant\delta^2$. It is noted that when $m_\pi^2\leqslant\delta^2$ (for $\delta>0$) there is a pole in Eq. (30). In this case, we have kept the principal value of the integral which is real. The difference between the integral and its principal value is imaginary and contributes only to the widths of heavy mesons.

In the case where $\delta=0$, we have

$$X_1=\frac{i}{36\pi^2}\left(3m_\pi^3\arctan\frac{\Lambda}{m_\pi}-3m_\pi^2\Lambda+\Lambda^3\right). \tag{33}$$

If we take the chiral limit $m_\pi\to 0$ in Eq. (32), we can see that the leading non-analytic term is

$$X_1|_{m_\pi\to 0}=\frac{i}{32\pi^2}\,\frac{m_\pi^4}{\delta}\ln m_\pi . \tag{34}$$

It can be easily checked that the same chiral limit behavior is obtained if we work with the dimensional regularization method in evaluating $X^{\mu\nu}$. This is because the leading non-analytic contribution of the pion loops is only associated with their infrared behavior.

In Eq. (26), m_0 for a pseudoscalar heavy meson is $-\frac{3}{4}\Delta$, and hence δ in Eq. (27) equals $-\Delta$. Therefore, for pion loop contributions to the mass of a pseudoscalar heavy meson P, we have

$$\sigma_P=-\frac{g^2}{16\pi^2f_\pi^2}\left\{12(m_\pi^2-\Delta^2)^{3/2}\left[\arctan\frac{\Lambda+\sqrt{\Lambda^2+m_\pi^2}+\Delta}{\sqrt{m_\pi^2-\Delta^2}}-\arctan\frac{m_\pi+\Delta}{\sqrt{m_\pi^2-\Delta^2}}\right]-3\Delta(2\Delta^2-3m_\pi^2)\ln\frac{\Lambda+\sqrt{\Lambda^2+m_\pi^2}}{m_\pi}\right.$$
$$\left.-3\Delta\Lambda\sqrt{\Lambda^2+m_\pi^2}+6(\Delta^2-m_\pi^2)\Lambda+2\Lambda^3\right\}, \tag{35}$$

when $m_\pi^2\geqslant\Delta^2$;

$$\sigma_P=-\frac{g^2}{16\pi^2f_\pi^2}\left\{6(\Delta^2-m_\pi^2)^{3/2}\ln\left(\frac{\Lambda+\sqrt{\Lambda^2+m_\pi^2}+\Delta-\sqrt{\Delta^2-m_\pi^2}}{\Lambda+\sqrt{\Lambda^2+m_\pi^2}+\Delta+\sqrt{\Delta^2-m_\pi^2}}\left|\frac{m_\pi+\Delta+\sqrt{\Delta^2-m_\pi^2}}{m_\pi+\Delta-\sqrt{\Delta^2-m_\pi^2}}\right|\right)\right.$$
$$\left.-3\Delta(2\Delta^2-3m_\pi^2)\ln\frac{\Lambda+\sqrt{\Lambda^2+m_\pi^2}}{m_\pi}-3\Delta\Lambda\sqrt{\Lambda^2+m_\pi^2}+6(\Delta^2-m_\pi^2)\Lambda+2\Lambda^3\right\}, \tag{36}$$

when $m_\pi^2\leqslant\Delta^2$.

Now we turn to the pion loop corrections to heavy vector meson masses. First we discuss Fig. 2(b), where P could be D or B. This diagram is caused by the $PP^*\pi$ vertices in Eq. (20). In momentum space, Fig. 2(b) can be expressed as

$$\frac{-i(g_{\mu\rho}-v_\mu v_\rho)}{2\left(v\cdot p-\frac{1}{4}\Delta\right)}(2i\Pi^{\rho\sigma})\frac{-i(g_{\sigma\nu}-v_\sigma v_\nu)}{2\left(v\cdot p-\frac{1}{4}\Delta\right)}, \tag{37}$$

where p is the residual momentum of the heavy vector meson and $2i\Pi^{\rho\sigma}$ is given by the following integral:

$$2i\Pi^{\rho\sigma}=\frac{3g^2}{f_\pi^2}\int\frac{d^4k}{(2\pi)^4}\frac{k^\rho k^\sigma}{\left[v\cdot(p-k)+\frac{3}{4}\Delta\right](k^2-m_\pi^2)}, \tag{38}$$

with k the momentum of the pion in the loop. While we evaluate Eq. (38), δ in Eq. (27) is equal to $v\cdot p+\frac{3}{4}\Delta$. Because of the factor $g_{\mu\rho}-v_\mu v_\rho$ in Eq. (37), only the X_1 term contributes. We define the coefficient of $g^{\rho\sigma}$ in $\Pi^{\rho\sigma}$ to be Π,

$$\Pi^{\rho\sigma}|_{g^{\rho\sigma}}=\Pi, \tag{39}$$

so that after Fig. 2(b) is included, the propagator of a heavy vector meson becomes proportional to Eq. (24), with Σ being replaced by Π and m_0 being equal to $\frac{1}{4}\Delta$. While calculating the pion loop corrections to a heavy vector meson, we have to fix $v\cdot p=\frac{1}{4}\Delta$, as required in Eq. (26). Consequently, δ in Eq. (27) is equal to Δ. Then with the aid of Eqs. (31), (32) we have

$$\begin{aligned}\Pi=&-\frac{g^2}{48\pi^2 f_\pi^2}\Bigg\{12(m_\pi^2-\Delta^2)^{3/2}\left[\arctan\frac{\Lambda+\sqrt{\Lambda^2+m_\pi^2}-\Delta}{\sqrt{m_\pi^2-\Delta^2}}-\arctan\frac{m_\pi-\Delta}{\sqrt{m_\pi^2-\Delta^2}}\right]+3\Delta(2\Delta^2-3m_\pi^2)\ln\frac{\Lambda+\sqrt{\Lambda^2+m_\pi^2}}{m_\pi}\\&+3\Delta\Lambda\sqrt{\Lambda^2+m_\pi^2}+6(\Delta^2-m_\pi^2)\Lambda+2\Lambda^3\Bigg\},\end{aligned} \tag{40}$$

when $m_\pi^2\geqslant\Delta^2$;

$$\begin{aligned}\Pi=&-\frac{g^2}{48\pi^2 f_\pi^2}\Bigg\{6(\Delta^2-m_\pi^2)^{3/2}\ln\left(\left|\frac{\Lambda+\sqrt{\Lambda^2+m_\pi^2}-\Delta-\sqrt{\Delta^2-m_\pi^2}}{\Lambda+\sqrt{\Lambda^2+m_\pi^2}-\Delta+\sqrt{\Delta^2-m_\pi^2}}\right|\left|\frac{m_\pi-\Delta+\sqrt{\Delta^2-m_\pi^2}}{m_\pi-\Delta-\sqrt{\Delta^2-m_\pi^2}}\right|\right)\\&+3\Delta(2\Delta^2-3m_\pi^2)\ln\frac{\Lambda+\sqrt{\Lambda^2+m_\pi^2}}{m_\pi}+3\Delta\Lambda\sqrt{\Lambda^2+m_\pi^2}+6(\Delta^2-m_\pi^2)\Lambda+2\Lambda^3\Bigg\},\end{aligned} \tag{41}$$

when $m_\pi^2\leqslant\Delta^2$.

Now we discuss Fig. 2(c), which arises from the $P^*P^*\pi$ vertices in the Lagrangian (20). In momentum space, Fig. 2(c) can be expressed in the following explicit form:

$$\begin{aligned}&\frac{-i(g_{\mu\rho_1}-v_\mu v_{\rho_1})}{2\left(v\cdot p-\frac{1}{4}\Delta\right)}\Bigg\{\frac{3g^2}{f_\pi^2}\epsilon^{\alpha_1\nu_1\rho_1\sigma_1}\epsilon^{\alpha_2\nu_2\rho_2\sigma_2}(g_{\sigma_1\rho_2}-v_{\sigma_1}v_{\rho_2})v_{\nu_1}v_{\nu_2}\\&\times\int\frac{d^4k}{(2\pi)^4}\frac{k_{\alpha_1}k_{\alpha_2}}{\left[v\cdot(p-k)-\frac{1}{4}\Delta\right](k^2-m_\pi^2)}\Bigg\}\frac{-i(g_{\nu\sigma_2}-v_\nu v_{\delta_2})}{2\left(v\cdot p-\frac{1}{4}\Delta\right)}.\end{aligned} \tag{42}$$

After some algebra, this expression can be written in the form

$$\frac{-i(g_{\mu\nu}-v_\mu v_\nu)}{2\left(v\cdot p-\frac{1}{4}\Delta\right)}\frac{i}{2\left(v\cdot p-\frac{1}{4}\Delta\right)}\frac{6g^2}{f_\pi^2}X_1, \tag{43}$$

where in $X_1,\delta=v\cdot p-\frac{1}{4}\Delta$.

Because of Fig. 2(c), the propagator for a heavy vector meson becomes proportional to

$$\frac{1}{v\cdot p-\frac{1}{4}\Delta-\Pi'}, \tag{44}$$

where

$$\Pi' = i\frac{3g^2}{f_\pi^2}X_1, \tag{45}$$

and again $\delta = v\cdot p - \frac{1}{4}\Delta$ in X_1. When we calculate corrections to a heavy vector meson mass from Fig. 2(c), $v\cdot p$ should be taken to be $\frac{1}{4}\Delta$, as required by Eq. (26). Hence $\delta=0$. Using Eq. (33) we have

$$\Pi' = -\frac{g^2}{12\pi^2 f_\pi^2}\left(3m_\pi^3 \arctan\frac{\Lambda}{m_\pi} - 3m_\pi^2\Lambda + \Lambda^3\right). \tag{46}$$

Adding Π in Eqs. (40) and (41) and Π' in Eq. (46) together, we have the following expression for pion loop contributions to the mass of a heavy vector meson P^*:

$$\sigma_{P^*} = -\frac{g^2}{16\pi^2 f_\pi^2}\Bigg\{4(m_\pi^2-\Delta^2)^{3/2}\left[\arctan\frac{\Lambda+\sqrt{\Lambda^2+m_\pi^2}-\Delta}{\sqrt{m_\pi^2-\Delta^2}} - \arctan\frac{m_\pi-\Delta}{\sqrt{m_\pi^2-\Delta^2}}\right] + 4m_\pi^3\arctan\frac{\Lambda}{m_\pi}$$
$$+\Delta(2\Delta^2-3m_\pi^2)\ln\frac{\Lambda+\sqrt{\Lambda^2+m_\pi^2}}{m_\pi} + \Delta\Lambda\sqrt{\Lambda^2+m_\pi^2} + 2(\Delta^2-3m_\pi^2)\Lambda + 2\Lambda^3\Bigg\}, \tag{47}$$

when $m_\pi^2 \geqslant \Delta^2$;

$$\sigma_{P^*} = -\frac{g^2}{16\pi^2 f_\pi^2}\Bigg\{2(\Delta^2-m_\pi^2)^{3/2}\ln\left(\left|\frac{\Lambda+\sqrt{\Lambda^2+m_\pi^2}-\Delta-\sqrt{\Delta^2-m_\pi^2}}{\Lambda+\sqrt{\Lambda^2+m_\pi^2}-\Delta+\sqrt{\Delta^2-m_\pi^2}}\right|\left|\frac{m_\pi-\Delta+\sqrt{\Delta^2-m_\pi^2}}{m_\pi-\Delta-\sqrt{\Delta^2-m_\pi^2}}\right|\right) + 4m_\pi^3\arctan\frac{\Lambda}{m_\pi}$$
$$+\Delta(2\Delta^2-3m_\pi^2)\ln\frac{\Lambda+\sqrt{\Lambda^2+m_\pi^2}}{m_\pi} + \Delta\Lambda\sqrt{\Lambda^2+m_\pi^2} + 2(\Delta^2-3m_\pi^2)\Lambda + 2\Lambda^3\Bigg\}, \tag{48}$$

when $m_\pi^2 \leqslant \Delta^2$ and where again we keeps the principal value of the integral.

Equations (35), (36), (47), (48) are valid provided m_π is not far away from the chiral limit, i.e., when $m_\pi \leqslant \Lambda$. It can be seen from Eq. (29) that pion loop contributions vanish in the limit $m_\pi \to \infty$. In order to extrapolate lattice data from large pion mass to the physical mass of the pion we need to know the behavior of heavy meson masses at large m_π. As we know, a heavy meson is composed of a heavy quark and a light quark. In HQET, the heavy quark mass, m_Q, is removed. Therefore, as the light quark mass, m_q, becomes large, the heavy meson mass should be proportional to m_q. On the other hand, explicit lattice calculations show that over the range of masses of interest to us, m_π^2 is proportional to m_q [10]. Consequently, as m_π becomes large, the heavy meson mass becomes proportional to m_π^2. Based on these considerations, in order to extrapolate lattice data at large m_π to the physical value of the pion mass, we propose the following phenomenological functional form for the dependence of heavy meson masses on the mass of the pion over the mass range of interest to us:

$$m_P = a_P + b_P m_\pi^2 + \sigma_P, \tag{49}$$

for heavy pseudoscalar mesons, and

$$m_{P^*} = a_{P^*} + b_{P^*} m_\pi^2 + \sigma_{P^*}, \tag{50}$$

for heavy vector mesons. σ_P and σ_{P^*} are given in Eqs. (35), (36) and Eqs. (47), (48), respectively. The advantage of fitting in this way is that we can guarantee that our formalism has the correct chiral limit behavior [as shown in Eq. (34)] and appropriate behavior when m_π is big with only three parameters (a, b, and Λ) to be determined in the fit rather than the excess of parameters in the usual power expansion. In the next section we will use this form of fit to extrapolate lattice data for heavy mesons.

Before turning to the lattice data we comment briefly on the explicit chiral symmetry breaking by the terms in Eq. (21). Substituting Eqs. (1), (4), (6), and (11) into Eq. (21) we have the following explicit expression for Eq. (21):

$$4\lambda_1 \sum_{a=1}^{3} m_{q^a}(P^{*+}_{a\mu}P^{*\mu}_a - P^+_a P_a)$$
$$+4\lambda_1' \sum_{a=1}^{3} m_{q^a} \sum_{a=1}^{3} (P^{*+}_{a\mu}P^{*\mu}_a - P^+_a P_a), \tag{51}$$

where we have made a Taylor expansion for ξ and omitted $O(1/f_\pi^2)$ terms. It can be seen clearly that Eq. (51) contributes equally to the mass of a heavy pseudoscalar meson and that of a heavy vector meson. Therefore, it does not contribute to their mass difference. Corrections to Eq. (51) may modify the propagators of heavy mesons to order

TABLE I. Fitted parameters, masses of D and D^* and their difference at $m_\pi^{\rm phys}$ [numbers without (with) brackets are for D (D^*)].

	$\lambda_2=-0.02$ GeV^2 $g^2=0.3$	$\lambda_2=-0.02$ GeV^2 $g^2=0.5$	$\lambda_2=-0.03$ GeV^2 $g^2=0.3$	$\lambda_2=-0.03$ GeV^2 $g^2=0.5$
Λ (GeV)	0.43 [0.65]	0.38 [0.55]	0.45 [0.63]	0.39 [0.55]
$a_{D[D^*]}$ (GeV)	0.5439 [0.6848]	0.5436 [0.6796]	0.5444 [0.6821]	0.5438 [0.6807]
$b_{D[D^*]}$ (GeV^{-1})	0.2082 [0.1783]	0.2083 [0.1809]	0.2078 [0.1798]	0.2082 [0.1800]
$m_{D[D^*]}^{\rm fit}$ (GeV)	0.5374 [0.6282]	0.5364 [0.6233]	0.5375 [0.6254]	0.5369 [0.6166]
$m_{D^*}^{\rm fit}-m_D^{\rm fit}$ (GeV)	0.0908	0.0869	0.0879	0.0796

$m_q O(1/f_\pi^2)$, with extra suppression from m_q with respect to the pion loop effects in Eqs. (35), (36) and Eqs. (47), (48) and we will ignore them.

IV. EXTRAPOLATION OF LATTICE DATA FOR HEAVY MESON MASSES

The spectra of the D and B mesons were calculated in Ref. [3], where NRQCD was used to treat the heavy quarks. In fact, when the mass of a heavy quark, m_Q, is much larger than $\Lambda_{\rm QCD}$, it becomes an irrelevant scale for the dynamics inside a heavy hadron. Consequently, heavy meson states can be simulated on lattices with the aid of NRQCD (where m_Q is removed from the Hamiltonian) when the lattice spacing is larger than the Compton wavelength of the heavy quark. Two values for the bare gauge coupling β, 5.7 and 6.2, were used. We choose to use the data at $\beta=5.7$, since in this case the inverse lattice spacing a^{-1} is about 1.116 GeV, which is smaller than the masses of either the b or c quarks. The box size is 2.1 fm, corresponding to the volume $12^3\times 24$. In their simulations, three different values for the hopping parameter κ, 0.1380, 0.1390, and 0.1400, were used. The light quark mass is related to κ as $m_q=(1/2a)(1/\kappa-1/\kappa_c)$, with $\kappa_c=0.1434$.

In our model there are three parameters to be fixed, $a_{P(P^*)}$, $b_{P(P^*)}$, and Λ in Eqs. (49) and (50). These parameters are related to g and λ_2 which represent interactions at the quark and gluon level and cannot be determined from chiral perturbation theory for heavy mesons. As discussed in Sec. II, they may depend slightly on m_Q. In our fitting procedure, we treat them as effective parameters and assume that their m_Q dependence has been considered effectively. From the upper limit for the experimental decay width of $D^*\to D\pi$ [14] we have the upper limit $g^2\leqslant 0.5$ [8,15]. In our numerical work we let g^2 vary from 0.3 to 0.5. From Eq. (17) λ_2 is related to the mass splitting between a heavy vector meson and a heavy pseudoscalar meson. From experimental data for the case of B mesons, where $1/m_Q^2$ corrections can be safely neglected, the value of λ_2 should be around -0.03 GeV^2. To see the dependence on λ_2 of our fits, we allow λ_2 to vary between -0.03 GeV^2 and -0.02 GeV^2.

Using the three simulation masses for D, D^*, B, and B^* in Ref. [3], we fix the three parameters, $a_{P(P^*)}$, $b_{P(P^*)}$, and Λ in Eqs. (49) and (50) with the least squares fitting method. In Tables I and II, for different values of λ_2 and g^2, we list the parameters $a_{P(P^*)}$, $b_{P(P^*)}$, and Λ obtained by fitting the lattice data. Furthermore, we give the masses of D, D^*, B, and B^*, and hyperfine splittings at the physical pion mass, $m_\pi^{\rm phys}$, after extrapolation. With these parameters $a_{P(P^*)}$, $b_{P(P^*)}$, and Λ, we obtain the masses of D, D^*, B, and B^* as a function of the pion mass, for different values of λ_2 and g. These results are shown in Figs. 3 and 4. The mass differences between $D(B)$ and D^* (B^*) as a function of the pion mass are shown in Figs. 5 and 6. From these figures we can see that when the pion mass is smaller than about 600 MeV the extrapolations deviate significantly from linear behavior. This is because the pion loop corrections begin to affect the extrapolations around this point. As the pion mass becomes smaller and smaller pion loop corrections become more and more important. Consequently, the dependence of these plots on λ_2 and g^2 is stronger when the pion mass is smaller.

As discussed in Sec. III, the parameter Λ characterizes the size of the source of the pion. Since the sizes of D, D^*, B, and B^* are different, we expect Λ for them are different. This is consistent with what is shown in Tables I and II. Furthermore, we can see that the Λ difference between D and D^* is much bigger than that between B and B^*. This is because the size difference between a heavy pseudoscalar meson and a heavy vector meson is caused by $1/m_Q$ effects.

In the naive linear extrapolations pion loop corrections are

TABLE II. Fitted parameters, masses of B and B^* and their difference at $m_\pi^{\rm phys}$ [numbers without (with) brackets are for B (B^*)].

	$\lambda_2=-0.02$ GeV^2 $g^2=0.3$	$\lambda_2=-0.02$ GeV^2 $g^2=0.5$	$\lambda_2=-0.03$ GeV^2 $g^2=0.3$	$\lambda_2=-0.03$ GeV^2 $g^2=0.5$
Λ (GeV)	0.62 [0.65]	0.53 [0.56]	0.62 [0.65]	0.54 [0.56]
$a_{B[B^*]}$ (GeV)	0.7540 [0.7915]	0.7507 [0.7884]	0.7535 [0.7918]	0.7519 [0.7887]
$b_{B[B^*]}$ (GeV^{-1})	0.1657 [0.1581]	0.1673 [0.1593]	0.1661 [0.1579]	0.1665 [0.1591]
$m_{B[B^*]}^{\rm fit}$ (GeV)	0.7138 [0.7394]	0.7107 [0.7342]	0.7146 [0.7389]	0.7107 [0.7336]
$m_{B^*}^{\rm fit}-m_B^{\rm fit}$ (GeV)	0.0256	0.0236	0.0242	0.0229

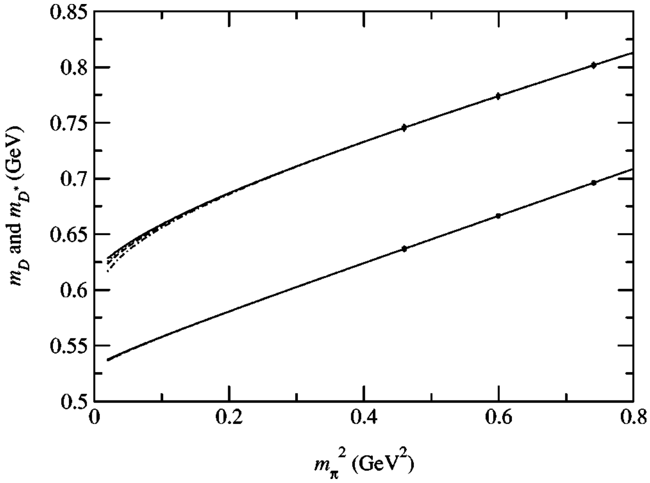

FIG. 3. Phenomenological fits to the lattice data for the masses of D^* (the upper lines) and D (the lower lines) as a function of the pion mass. The solid (dashed) lines correspond to $\lambda_2 = -0.02$ GeV^2 and $g^2=0.3$ ($g^2=0.5$). The dot (dot-dashed) lines correspond to $\lambda_2=-0.03$ GeV^2 and $g^2=0.3$ ($g^2=0.5$).

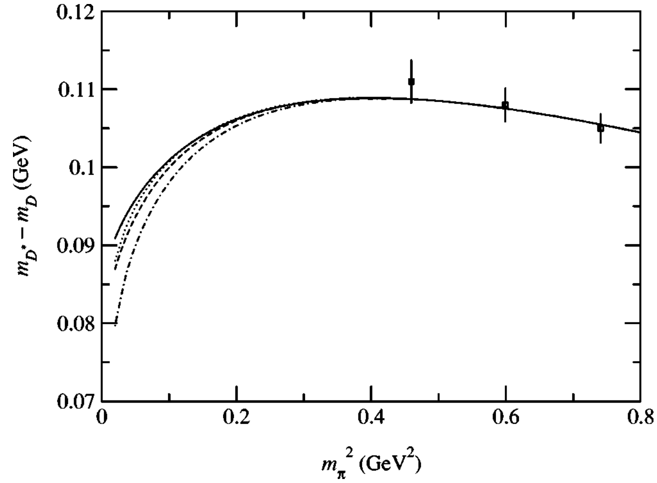

FIG. 5. Phenomenological fits to the lattice data for the hyperfine splitting between D^* and D as a function of the pion mass. The solid (dashed) lines correspond to $\lambda_2=-0.02$ GeV^2 and $g^2=0.3$ ($g^2=0.5$). The dot (dot-dashed) lines correspond to $\lambda_2=-0.03$ GeV^2 and $g^2=0.3$ ($g^2=0.5$).

ignored. Hence the results do not depend on λ_2 and g^2. In Table III we list the results of the linear extrapolations for comparison.

It can be seen clearly that the mass differences between $D(B)$ and $D^*(B^*)$ in our model are even smaller than those obtained in the naive linear extrapolations. Since in the linear extrapolations, the hyperfine splittings at the physical mass of the pion for D and B mesons are already smaller than experimental data, the inclusion of pion loop effects makes the situation even worse. As shown in Table I, the hyperfine splitting between D and D^* is $0.080\sim0.091$ GeV in the range of our parameters, compared with the experimental value 0.14 GeV, while the result from the linear extrapolation is 0.114 GeV in Table III. Similarly, the hyperfine splitting between B and B^* in Table II is $0.023\sim0.026$ GeV in the range of our parameters, compared with the experimental value 0.046 GeV and the result from the linear extrapolation, 0.031 GeV, in Table III.

In our fits, in addition to the uncertainties in the three parameters, $a_{P(P*)}$, $b_{P(P*)}$, and Λ, which are caused by the uncertainties of λ_2 and g, the errors in the lattice data can also lead to some uncertainties in these three parameters. Since all the three lattice data are at large pion masses, and since Λ is mainly related to the data at small pion masses, the error in our determination of Λ is very large. In addition to Λ, $a_{P(P*)}$ and $b_{P(P*)}$ also have some errors. In fact, the numerical results for $a_{P(P*)}$, $b_{P(P*)}$, and Λ in Tables I, II, and III correspond to the central values of the lattice data. As a consequence of heavy quark symmetry, the dynamics in-

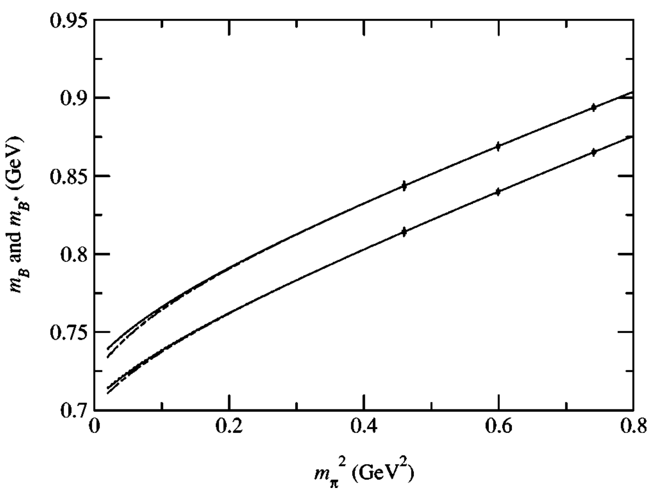

FIG. 4. Phenomenological fits to the lattice data for the masses of B^* (the upper lines) and B (the lower lines) as a function of the pion mass. The solid (dashed) lines correspond to $\lambda_2=-0.02$ GeV^2 and $g^2=0.3$ ($g^2=0.5$). The dot (dot-dashed) lines correspond to $\lambda_2=-0.03$ GeV^2 and $g^2=0.3$ ($g^2=0.5$).

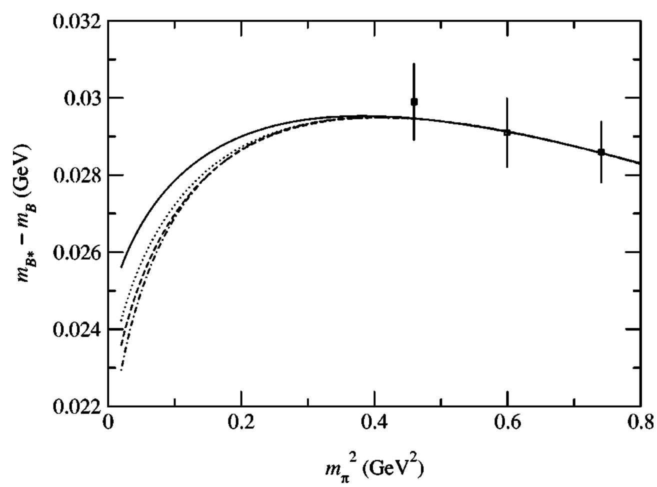

FIG. 6. Phenomenological fits to the lattice data for the hyperfine splitting between B^* and B as a function of the pion mass. The solid (dashed) lines correspond to $\lambda_2=-0.02$ GeV^2 and $g^2=0.3$ ($g^2=0.5$). The dot (dot-dashed) lines correspond to $\lambda_2=-0.03$ GeV^2 and $g^2=0.3$ ($g^2=0.5$).

TABLE III. Fitted parameters, masses of P and P^* and their difference at $m_\pi^{\rm phys}$ for linear extrapolation [numbers without (with) brackets are for P (P^*)].

$a_{D[D*]}$ (GeV)	$b_{D[D*]}$ (GeV^{-1})	$m^{\rm fit}_{D[D*]}$ (GeV)	$m^{\rm fit}_{D*}-m^{\rm fit}_{D}$ (GeV)
0.5397 [0.6540]	0.2112 [0.1995]	0.5438 [0.6579]	0.1141
$a_{B[B*]}$ (GeV)	$b_{B[B*]}$ (GeV^{-1})	$m^{\rm fit}_{B[B*]}$ (GeV)	$m^{\rm fit}_{B*}-m^{\rm fit}_{B}$ (GeV)
0.7311 [0.7621]	0.1812 [0.1780]	0.7346 [0.7656]	0.0310

side a heavy meson does not depend on the mass of the heavy quark, so we expect that the values of Λ for D and B mesons are not very different from those of light mesons [10]. Consequently, we expect that the values of Λ in Tables I and II are quite reasonable. In our fits, we find that the errors of the lattice data cause 1%–2% relative uncertainties for $a_{P(P*)}$ ($P=D$ or B), 6% for b_D, 9% for b_{D*}, 8% for b_B, and 9% for b_{B*}. These lead to about 1.3% uncertainties for m_D, 1.5% for m_D^*, 1.2% for m_B, and 1.2% for m_{B*}. As a result, the relative uncertainties of the hyperfine splittings are about 13% and 54% in the cases of D and B mesons, respectively. In spite of all these errors, our conclusion that the hyperfine splitting obtained after a careful treatment of chiral corrections are smaller than those obtained using naive linear extrapolation, remains unchanged.

V. SUMMARY AND DISCUSSION

QCD possesses chiral symmetry when light quark masses go to zero and heavy quark symmetry when heavy quark masses go to infinity. Combining these two symmetries leads to chiral perturbation theory for heavy mesons which are invariant under both chiral symmetry and heavy quark symmetry. We have evaluated pion loop corrections to heavy meson propagators with the aid of chiral perturbation theory for heavy mesons as the Compton wavelength of the pion becomes larger than the size of the heavy mesons. This leads to the dependence of heavy meson masses on the pion mass. In order to study hyperfine splittings, we took the color-magnetic-moment operator at order $1/m_Q$ in HQET into account. This operator breaks heavy quark spin symmetry and is primarily responsible for the mass difference between a heavy pseudoscalar meson and a heavy vector meson. The small masses of the light quarks break chiral symmetry explicitly. We showed that contributions to the mass difference between a heavy pseudoscalar meson and a heavy vector meson from these terms are suppressed by light quark masses with respect to the pion loop contributions we considered in chiral perturbation theory. When m_π becomes large, lattice data show that heavy meson masses are proportional to m_π^2. Based on these considerations, we proposed a phenomenological functional form with three parameters to extrapolate the lattice data. Because it guarantees the model independent chiral behavior of QCD, our model is more appropriate than a naive linear extrapolation. The parameters in our model are fixed by the least squares fitting method, while fitting the lattice data for the masses of heavy mesons D, D^*, B, and B^* in the large pion mass region ($m_\pi \geqslant 680$ MeV). It is found that the hyperfine splittings extrapolated in this way are even smaller than those obtained in the linear extrapolations, in which the extrapolated hyperfine splittings for both D and B mesons are already smaller than the experimental data.

There are some uncertainties in our model. We have two parameters, λ_2 and g, which are related to the color-magnetic-moment operator at order $1/m_Q$ in HQET and the interactions between heavy mesons and Goldstone bosons in chiral perturbation theory, respectively. In the ranges we choose for these two parameters, we have about 13% and 11% uncertainties for the hyperfine splitting in the D and B cases, respectively. Furthermore, the errors in the lattice data also lead to some uncertainties when we fix the three parameters $a_{P(P*)}$, $b_{P(P*)}$, and Λ in our model. Since all the lattice data are at high pion masses, the error in Λ is very large. However, we believe that the range of Λ we obtained is appropriate because of considerations based on heavy quark symmetry. The errors for $a_{P(P*)}$ and $b_{P(P*)}$ lead to about 13% and 54% uncertainties for hyperfine splittings in the cases of D and B mesons, respectively. Despite all these uncertainties, the hyperfine splittings obtained in our model are smaller than those in the naive linear extrapolations. Our analysis shows that the current lattice data for hyperfine splittings at large pion masses are probably too small to give hyperfine splittings at the physical pion mass which are consistent with experiments.

Some approximations made in current lattice simulations may be the cause of these small hyperfine splittings. The quenched approximation might be one reason. In fact, in lattice simulations for light spectroscopy the hyperfine splittings are also too small [16]. Furthermore, as pointed out in Ref. [3], the lattice results for hyperfine splittings are sensitive to the coefficient of the $\sigma\cdot\mathbf{B}$ term in NRQCD which is at order $1/m_Q$ and which is the leading term to cause hyperfine splittings. The inclusion of radiative corrections beyond tadpole improvement for this coefficient may increase hyperfine splittings. Another reason might be the light quark mass dependence of the clover coefficient in the clover action for light quarks. In addition, finite size effects and higher order terms in NRQCD in lattice simulations may lead to an underestimate of hyperfine splittings at large light quark mass as well. More careful lattice simulations with more data and better accuracy are urgently needed to resolve this important problem.

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