Updating V_{us} from kaon semileptonic decays

G. Calderón and G. López Castro
Departamento de Física, Cinvestav del IPN, Apartado Postal 14-740, 07000 México Distrito Federal, Mexico
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We update the determination of $|V_{us}|$ using semielectronic and semimuonic decays of K mesons. A modest improvement of 15% with respect to its present value is obtained for the error bar of this matrix element when we combine the four available semileptonic decays. The combined effects of long-distance radiative corrections and nonlinear terms in the vector form factors can decrease the value of $|V_{us}|$ by up to 1%. Refined measurements of the decay widths and slope form factors in the semimuonic modes and a more accurate calculation of vector form factors at zero momentum transfer can push the determination of $|V_{us}|$ at a low percent level.

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I. INTRODUCTION

V_{ud} and V_{us} are the most accurate entries of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1] that have been determined up to now. Their values recommended by the Particle Data Group are [2]

$$|V_{ud}|=0.9735\pm 0.0008, \tag{1}$$

$$|V_{us}|=0.2196\pm 0.0023. \tag{2}$$

When they are combined with $|V_{ub}|=0.0036\pm 0.0010$ [2], the most precise test of the unitarity condition of the CKM matrix up to date becomes

$$|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=0.9959\pm 0.0019. \tag{3}$$

Neither the central value nor the error bar quoted for $|V_{ub}|$ play any role in the present test of this unitarity condition. The error bars quoted in Eq. (1) for V_{ud} and V_{us} contribute to 70% and 30% of the total uncertainty in Eq. (3), respectively. A direct inspection of Eq. (3) would indicate that the present experimental values for these entries of the CKM matrix fail to satisfy unitarity by 2.2σ. This problem makes necessary that further efforts are devoted to investigate the sources of uncertainties that play a role in the determination of V_{ud} and V_{us} [3] at the level of 10^{-4} and 10^{-3}, respectively.

The value quoted for V_{ud} in Eq. (1) arises [2] from the average of their values extracted from superallowed Fermi transitions (SFT) in nuclei and from free neutron beta decay. At present, the error bar in V_{ud} from SFT is still dominated by different model-dependent calculations of isospin breaking corrections [4]. Despite the fact that isospin breaking corrections in individual SFT's are at a few times the 10^{-3} level, the resulting uncertainty in the weighted average for V_{ud} is small (5×10^{-4}!) [4] because a large set of nine decays are used in their determination. On the other hand, the determination of V_{ud} from neutron beta decay is reaching the 10^{-3} accuracy due to recent improvements in the measurements of the neutron lifetime [5] and the ratio of its vector and axial vector couplings [6]. We have recently reviewed [7] this determination of V_{ud} by putting careful attention to the sources of uncertainties in the neutron decay rate at the 10^{-4} level. It was concluded [7] that present inconsistencies among the measurements of the axial-vector form factor $g_A(0)$ [6] are behind the main limitations in order to have an alternative accurate determination of V_{ud}.

In the present paper, we focus on the determination of V_{us} from kaon semileptonic decays.[1] The first determination of this mixing element in the three-generation standard model using charged and neutral $K\rightarrow \pi e \nu_e$ (K_{e3}) decays was done in Ref. [9]. The value quoted in Eq. (2) was determined in 1984 by Leutwyler and Roos [10] using those decays and including leading flavor symmetry breaking effects. Several articles (see, for example, [11,12]) and comprehensive review papers have appeared [13,14] that make different updates to the value of $|V_{us}|$ reported in [10]. Some of them (see, for example, Ref. [14]), however, combine old data for the integrated spectra (those of Ref. [10]) with new information on the decay widths of K_{e3}. Because new information on the decay widths and form factors of K_{e3} decays has been accumulated since Leutwyler and Roos' original work (which is based in Ref. [15]), we would like in this paper to explore their impact in the determination of V_{us}. In addition, in the present paper we also include in our analysis the data corresponding to kaon semimuonic decays ($K_{\mu 3}$). We paid particular attention to the effects of long-distance radiative corrections and of non-linearities in the squared momentum-transfer dependence of the form factors in the extraction of $|V_{us}|$. It is found that those combined effects can decrease the central value of $|V_{us}|$ by up to 1%. We are not able to sensibly improve the accuracy in the determination of V_{us} with respect to Eq. (2), but we can identify some elements of the analysis that, if improved, will help to obtain a more refined and consistent value of this CKM matrix element.

[1]The determination of V_{us} from semileptonic hyperon decays is not fully reliable at present because different calculations of SU(3) symmetry breaking effects disagree among themselves [8].

II. DECAY AMPLITUDE AND FORM FACTORS

Let us start by defining the tree-level decay amplitude for the $K(p)\to\pi(p')l^+(p_1)\nu_l(p_2)(K_{l3}$, with $l=e,\mu)$ decays [16]:

$$\mathcal{M}=\frac{G}{\sqrt{2}}V_{us}C_K\langle\pi(p')|\bar{u}\gamma_\mu s|K(p)\rangle \times\bar{v}(p_1)\gamma^\mu(1-\gamma_5)u(p_2), \tag{4}$$

where V_{us} is the CKM matrix element we are interested in, G is the effective weak coupling at the tree-level and $C_{K^0}=\sqrt{2}C_{K^+}=1$ are Clebsch-Gordan coefficients for the hadronic matrix elements. The properties of the hadronic matrix elements have been discussed in numerous papers before (see, for example, [16]). Here we focus on some of their properties under flavor symmetry breaking that are relevant for the determination of V_{us}.

Using Lorentz covariance, we can write the hadronic matrix element as[2]

$$\langle\pi(p')|\bar{u}\gamma_\mu s|K(p)\rangle=f_+(t)\left((p+p')_\mu-\frac{m_K^2-m_\pi^2}{t}q_\mu\right)+f_0(t)\left(\frac{m_K^2-m_\pi^2}{t}\right)q_\mu. \tag{5}$$

The form factors $f_{+,0}$ are Lorentz-invariant functions of the squared momentum transfer $[t=q^2=(p-p')^2]$. They correspond to the $J=1,0$ total angular momentum configuration of the K-π system in the crossed channel, respectively. The kinematical range allowed for the squared momentum transfer in K_{l3} decays is $m_l^2\leqslant t\leqslant(m_K-m_\pi)^2$. The analyticity of the amplitude for low values of t demands that $f_0(0)=f_+(0)$.

In the limit of exact isospin symmetry, we have ($i=+$ or 0)

$$f_i^{K^+\to\pi^0}(t)=f_i^{K^0\to\pi^-}(t). \tag{6}$$

This means that the form factors of charged and neutral K mesons should be equal for all values of t in this limit. If we rewrite the form factors as follows:

$$f_i(t)=f_i(0)\tilde{f}_i(t) \tag{7}$$

we have $\tilde{f}_i(0)=1$. Thus, isospin symmetry would imply

$$f_+^{K^+\to\pi^0}(0)=f_+^{K^0\to\pi^-}(0),$$

and, for all values of t,

$$\tilde{f}_i^{K^+\to\pi^0}(t)=\tilde{f}_i^{K^0\to\pi^-}(t). \tag{8}$$

[2]We will also use superindexes in the form factors to indicate the specific $K\to\pi$ channel when required.

TABLE I. Comparison of observables for K_{l3} decays as reported by the Particle Data Group in 1982 (Ref. [15]) and 2001 (Ref. [2]). The decay widths Γ are given in units of 10^{-15} MeV. The last column displays the long-distance radiative corrections to the decay widths according to Ref. [30].

Experiment	PDG 1982	PDG 2001	Ref. [30]
K_{e3}^+			
Γ	2.5645 ± 0.0271	2.5616 ± 0.0323	
λ_+	0.029 ± 0.004	0.0278 ± 0.0019	
δ_K (%)	–	–	-0.45
K_{e3}^0			
Γ	4.9147 ± 0.0740	4.9385 ± 0.0446	
λ_+	0.0300 ± 0.0016	0.0290 ± 0.0016	
δ_K (%)	–	–	1.5
$K_{\mu3}^+$			
Γ	1.7026 ± 0.0480	1.6847 ± 0.0426	
λ_+	0.026 ± 0.008	0.031 ± 0.008	
λ_0	-0.003 ± 0.007	0.006 ± 0.007	
δ_K (%)	–	–	-0.06
$K_{\mu3}^0$			
Γ	3.4415 ± 0.0573	3.4604 ± 0.0416	
λ_+	0.034 ± 0.006	0.034 ± 0.005	
λ_0	0.020 ± 0.007	0.025 ± 0.006	
δ_K (%)	–	–	2.02

The effects of isospin symmetry breaking will modify these relations.

The form factors $\tilde{f}_{+,0}^{K\to\pi}(t)$ have been measured experimentally [2] for K_{l3} decays. It has been found that a linear parametrization in t,

$$\tilde{f}_{+,0}^{K\to\pi}(t)=1+\frac{\lambda_{+,0}}{m_\pi^2}t, \tag{9}$$

is *sufficient* to describe the data, in most of the kinematical range of K_{l3} decays, to the degree of accuracy attained by experiments. Note that in Eq. (9) the mass scale in the denominator is set by the mass of the pion emitted in the corresponding K decay. Thus, the isospin symmetry relation of Eq. (8) indicates that the dimensionful quantities $\lambda'_{+,0}\equiv\lambda_{+,0}/m_\pi^2$ are the same for charged and neutral kaon decays.

For comparison, the experimental results for the slope constants $\lambda_{+,0}$ as used in the Leutwyler-Roos analysis [10] and in the present paper (using the data of Ref. [2]) are shown in Table I. We observe that isospin violation in present data for λ_+ of K_{e3} and $K_{\mu3}$ decays are at a few percent level, as expected. However, the average value reported for the λ_0 slopes in $K_{\mu3}$ decays, strongly violates isospin symmetry [12]:

$$b_0=\frac{\lambda_0'(K_{\mu3}^0)-\lambda_0'(K_{\mu3}^+)}{\lambda_0'(K_{\mu3}^0)+\lambda_0'(K_{\mu3}^+)}=0.59\pm0.37. \tag{10}$$

This large isospin breaking is usually thought to arise from the small value of λ_0 in $K^+_{\mu 3}$ decay.[3] New high precision measurements of $K_{\mu 3}$ decays as those expected at DaΦne [20] will be very useful to understand the nature of this isospin symmetry breaking effect. In this paper we will use the values of λ_0 reported in Ref. [2] and we will comment on the impact of the new result reported in Ref. [17] on the determination of V_{us}.

Concerning the value of the form factor at $t=0$, we will use the values obtained in Ref. [10] (note, however, that our numerical results in Secs. IV A and IV B are obtained using $f_+^{K^0\to\pi^-}(0)=0.9606$, since the value of the π^- decay constant used to evaluate the chiral corrections is now 0.8% smaller (see p. 395 in [2]) than the one used in [10]):

$$f_+^{K^0\to\pi^-}(0)=0.961\pm0.008, \tag{11}$$

$$f_+^{K^+\to\pi^0}(0)=0.982\pm0.008. \tag{12}$$

These form factors incorporate the second order [21] SU(3) breaking effects arising from the $s-d$, u quark mass difference and the isospin breaking corrections due to the π^0-η mixing in the case of $f_+^{K^+\to\pi^0}(0)$ [10].

The values in Eqs. (11), (12) exhibit a small isospin breaking effect [19,10]:

$$r_{\pi^0-\eta+ChPT}=\frac{f_+^{K^+\to\pi^0}(0)}{f_+^{K^0\to\pi^-}(0)}=1.022. \tag{13}$$

This 2.2% isospin breaking correction is composed of a 1.7% contribution from $\pi^0-\eta$ mixing and 0.5% contribution from isospin violation in the chiral perturbative calculations.

As it was pointed out in Ref. [19], the π^0-η' mixing does not contribute at the leading order corrections in the ratio quoted above. Furthermore [19], at this order, the η' meson contributes only indirectly to Eq. (13) through the mass of the η meson involved in the definition of the π^0-η mixing. However, since weak interactions transform a K^+ meson ($u\bar{s}$ state) into a $u\bar{u}$ state, it is unavoidable to have an η' [actually, a SU(3) singlet η_1] component in the physical π^0 state. Thus, if we write the physical π^0 state as a linear combination of the octet (π_3,η_8) and singlet (η_1) SU(3) eigenstates, we obtain $r_{\pi^0-\eta-\eta'+ChPT}\approx1.026$ [21], to be compared with Eq. (13). The additional 0.4% isospin breaking correction due to the η_1 SU(3) singlet state is a subleading effect, since it requires also that η-η' mixing occurs.

In Secs. IV A and IV B we quote within square brackets our corresponding results obtained using this additional isospin correction. The large error assigned to Eqs. (11), (12) are, at present, the main source of uncertainty in the value reported in Eq. (2). Other calculations of these form factors have been done using either the relativistic constituent quark model [22] or a formalism based on the Schwinger-Dyson equations [23]. Their results are fully consistent with the ones shown in Eqs. (11), (12), but theoretical errors are not provided for them (Ref. [22] provides an error bar $\delta f_+^{K^0\to\pi^-}=^{+0.002}_{-0.006}$ associated with uncertainties in the strange quark mass). Thus, we will restrict ourselves to the results quoted in the previous equations.

Finally, let us focus on the energy dependence of the form factors. The linear parametrizations of the form factors, Eq. (9), are a convenient although arbitrary choice to describe experimental data for all values of t in K_{l3} decays. On the other hand, information about these form factors can also be obtained from $\tau\to K\pi\nu_\tau$ decays in the region $(m_K+m_\pi)^2\leqslant t\leqslant m_\tau^2$. The vector form factors in τ decays display a resonant structure [24] such that when they are extrapolated to low values of t they will naturally induce nonlinear terms. Following the results obtained in $\tau\to\pi\pi\nu_\tau$ decays, which suggest that two or more vector resonances dominate the 2π mass distribution [25], one can model the vector form factor in $\tau\to K\pi\nu_\tau$ decays as follows [26]:

$$\tilde{f}_+(t)=\frac{1}{1+\beta_{K*}}[BW_{K*}(t)+\beta_{K*}BW_{K'*}(t)], \tag{14}$$

where

$$BW_X(t)=\frac{m_X^2}{m_X^2-t-i\sqrt{t}\Gamma_X(t)\,\theta[t-(m_K+m_\pi)^2]}.$$

The subindex $X=K^*(892),K'^*(1410)$ denotes the charged vector resonances in the $l=1$ configuration of the $K\pi$ system, $\Gamma_X(t)$ is the corresponding decay width and β_{K*} is used to denote the relative strength of both contributions.

When we extrapolate Eq. (14) below the $K\pi$ threshold, we obtain

$$\tilde{f}_+(t)=\frac{1}{1+\beta_{K*}}\left[\frac{m_{K*}^2}{m_{K*}^2-t}+\beta_{K*}\frac{m_{K'*}^2}{m_{K'*}^2-t}\right]. \tag{15}$$

Note that if we set $\beta_{K*}=0$ in Eq. (15), and expand the resulting form factor in powers of t, we obtain $\lambda_+=(m_\pi/m_{K*})^2=0.024$. This fact, namely that a single pole $K^*(892)$ underestimates the value of λ_+, was discussed long ago (see, for example, Ref. [27]). Now, if we keep $\beta_{K*}\neq0$ and expand Eq. (15) up to terms of order t, we can find the value of the free parameter β_{K*} from the values of λ_+ of each K_{l3} decay using the expression:

$$\beta_{K*}=-\left(\frac{\lambda_+-r_{K*}}{\lambda_+-r_{K'*}}\right),$$

[3]Note, however, that the value $\lambda_0(K^+_{\mu3})=0.0190\pm0.0064$ measured recently by the KEK-E246 experiment [17] leads to $b_0=0.10\pm0.20$, in better agreement with isospin symmetry. This new measurement of λ_0 seems to validate the predictions based on the Callan-Treiman relation $\lambda_0^{CT}=0.019$ [18], and the results obtained in chiral perturbation models $\lambda_0^{ChPT}=0.017\pm0.004$ [19].

where $r_X=(m_\pi/m_X)^2$. Thus, the vector form factor with two poles given in Eq. (15) is a natural generalization that includes non-linear effects in t and reproduces the correct size of the coefficients in the linear terms. An estimate of the effects of nonlinear terms in the determination of $|V_{us}|$ was given in Ref. [12].

III. DECAY RATES AND RADIATIVE CORRECTIONS

When the radiative corrections are added to the lowest order amplitude, the decay rate of the K_{l3} decays can be written as follows [10]:

$$\Gamma(K_{l3})=\frac{G_F^2 m_K^5}{192\pi^3} S_{EW} C_K^2 |f_+^{K\to\pi}(0) V_{us}|^2 I_K (1+\delta_K), \tag{16}$$

where $G_F=1.16639(1)\times 10^{-5}$ GeV^{-2} [2] is the Fermi coupling constant obtained from μ decay, and the dimensionless integrated spectrum I_K is defined as

$$\begin{aligned} I_K=&\frac{1}{m_K^8}\int_{m_l^2}^{(m_K-m_\pi)^2}\frac{dt}{t^3}(t-m_l^2)^2\lambda^{1/2}(t,m_K^2,m_\pi^2)\\ &\times\{\lambda(t,m_K^2,m_\pi^2)(2t+m_l^2)|\tilde{f}_+(t)|^2\\ &+3m_l^2(m_K^2-m_\pi^2)^2|\tilde{f}_0(t)|^2\}, \end{aligned} \tag{17}$$

where $\lambda(x,y,z)\equiv x^2+y^2+z^2-2xy-2xz-2yz$. The second term in I_K (proportional to m_l^2) shows that $K_{\mu 3}$ decays are more sensitive to the effects of the scalar form factor than K_{e3} decays. In practice, the factor $(m_K^2-m_\pi^2)^2$ in front of the scalar form factor provides a further suppression for this contribution. This fact helps our purposes of using $K_{\mu 3}$ decays in our analysis, because the issue of the isospin breaking in λ_0 discussed in Sec. II is not very critical at the present level of accuracy to determine V_{us}.

As usual, in the above expression for the decay width we have factorized the radiative corrections into a short-distance electroweak piece S_{EW}, and a long-distance model-dependent QED correction δ_K. Since energetic virtual gauge bosons explore the hadronic transitions at the quark level, S_{EW} is, in good approximation, the same for all the K_{l3} decays;[4] by including the resummation of the dominant logarithmic terms one obtains $S_{EW}=1.024$ [28].

The long distance radiative corrections δ_K are different for each process since they depend upon the charges and masses of the particles involved in a given decay. They were computed long ago for different observables associated with K_{l3} decays [30] using the following approximations: (i) point electromagnetic vertices of the pseudoscalar mesons, (ii) fixed values of the hadronic weak form factors, and (iii) the local four-fermion weak interaction. The values of δ_K for each of the four K_{l3} decays [30] are shown in the last column of Table I. It is interesting to observe that $\delta_K(K_{l3}^0)-\delta_K(K_{l3}^+)\approx 2\%$, both for semielectronic and semimuonic kaon decays. The origin of this difference can be traced back to the coulombic interaction between the pion and the charged lepton in K_{l3}^0 decays [30] (see also [11]). We discuss in Sec. IV B the impact of these long-distance radiative corrections in the determination of V_{us}.

A new calculation of the long-distance radiative corrections has been reported very recently [31]. The authors of Ref. [31] focus mainly on the structure of the Dalitz plot observables for kaon semileptonic decays, which can be used to extract the relevant form factors from experiments. In the case of the K_{e3}^+ decay, the corrected integrated rate is provided, which seems to confirm that long-distance radiative corrections reduce the decay rate, although their value ($\delta_{K_{e3}^+}\approx -1.27\%$) [31] is larger than the one displayed in Table I.

TABLE II. Integrated spectrum I_K of K_{l3} decays and values extracted for the product $|f_+(0)V_{us}|$ with (wrc) and without (worc) radiative corrections, using the input data from Ref. [2].

Process	I_K	$\|f_+(0)V_{us}\|$ (worc)	$\|f_+(0)V_{us}\|$ (wrc)
K_{e3}^+	0.1603 ± 0.0011	0.2158 ± 0.0019	0.2160 ± 0.0016
K_{e3}^0	0.1555 ± 0.0008	0.2108 ± 0.0015	0.2093 ± 0.0011
$K_{\mu 3}^+$	0.1054 ± 0.0033	0.2159 ± 0.0041	0.2159 ± 0.0043
$K_{\mu 3}^0$	0.1068 ± 0.0021	0.2130 ± 0.0027	0.2109 ± 0.0024

IV. DETERMINATION OF V_{us}

In this section we extract the quantities $|f_+(0)V_{us}|$ for each decay and provide a determination of $|V_{us}|$. We first ignore the effects of long-distance radiative corrections in order to compare our results with the ones provided in Ref. [10]. Later we evaluate the effects of those radiative corrections and comment on the prospects to improve the accuracy in the determination of V_{us} (see also Refs. [11,12]).

A. $|V_{us}|$ without using long-distance radiative corrections

Using the input data of Table I and the decay rate of Eq. (16), we obtain the values for the integrated spectrum I_K and the product $|f_+(0)V_{us}|$ shown in the second and third columns of Table II, respectively. The error bars quoted for the latter quantities include a 1% uncertainty attributed to long-distance radiative corrections according to the prescription[5] of Ref. [10]. It is interesting to observe that the values of $|f_+^{K^+\to\pi^0}(0)V_{us}|$ obtained from K_{e3}^+ and $K_{\mu 3}^+$ decays are re-

[4]In the case of the strangeness-conserving SFT decays the piece of this correction arising from the axial-induced photonic corrections contributes with one of the important theoretical uncertainties, $\delta|V_{ud}|\approx 0.0004$ [28,29]. Since the accuracy in the determination of V_{us} does not reach this level yet, we ignore here those corrections.

[5]Since long-distance radiative corrections have not been applied to all experimental data used to obtain the average values quoted in Table I for K_{l3} decay observables, a 1% uncertainty is added to the decay widths.

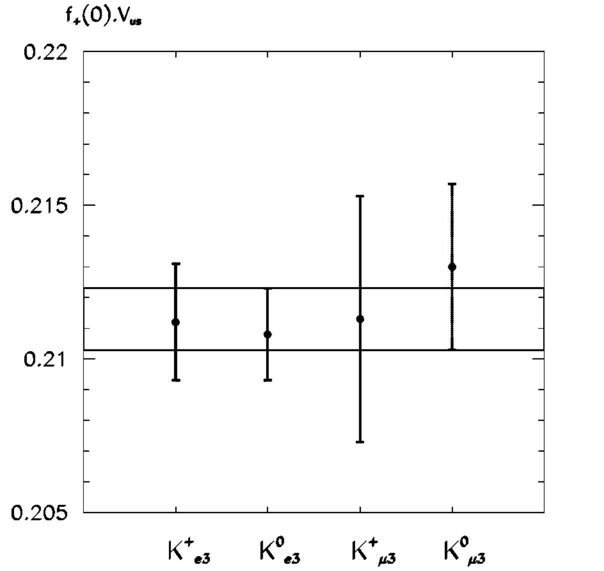

FIG. 1. Values of $|f_+^{K^0\to\pi^-}(0)V_{us}|$ obtained from all K_{l3} decays by ignoring long-distance radiative corrections. The horizontal band is the 1σ weighted average of the four values.

markably consistent among them as demanded by $e-\mu$ universality, despite the fact that isospin symmetry is strongly broken in the slope of the scalar form factor. The same can be stated (within errors) for the corresponding quantities in neutral kaon decays. Contrary to Ref. [10], we have used the experimental values for the slopes of the form factors in K_{e3} decays instead of assuming isospin symmetry [namely, we have not assumed $\lambda_+(K_{l3}^+)=\lambda_+(K_{l3}^0)$ as in [10]]. Observe in Table II that the value of $|f_+(0)V_{us}|$ extracted from K_{e3}^+ is almost at the same level of accuracy as in K_{e3}^0.

Now, if we include the isospin breaking corrections to the form factors at $t=0$ using Eq. (13), we can express the results in Table II in terms of the quantity $|f_+^{K^0\to\pi^-}(0)V_{us}|$ from the four K_{l3} decays. The different values of this quantity can be used as a consistency test of the calculations of the different corrections applied to the semileptonic decays, namely, this quantity must be the *same* for all K_{l3} decays. In Fig. 1 we plot the values of $|f_+^{K^0\to\pi^-}(0)V_{us}|$ obtained from the four semileptonic kaon decays. We observe that their values are consistent among themselves and with their weighted average

$$|f_+^{K^0\to\pi^-}(0)V_{us}|=0.2113\pm0.0010\ \ [0.2110\pm0.0010], \tag{18}$$

which is displayed as a horizontal band in Fig. 1 (for $r=1.022$). In the previous equation and in the results of this and the following subsections, we show within square brackets the figures corresponding to the choice $r=1.026$ of the isospin breaking correction [see discussion after Eq. (13)]. The scale factor (see p. 11 of Ref. [2]) associated with the set of four-independent measurements of $|f_+^{K^0\to\pi^-}(0)V_{us}|$ is $S=0.41$, indicating a good consistency of those results.

From Eqs. (11) and (18) we obtain the following value:[6]

$$|V_{us}|=0.2200\times\left[1\pm\sqrt{(0.0083)^2_{f_+(0)}+(0.0047)^2_{\text{“}exp\text{”}}}\right]$$
$$=0.2200\pm0.0021[0.2197\pm0.0021], \tag{19}$$

where we have used quotation marks on the experimental error to indicate that it contains a 1% uncertainty associated with radiative corrections. The present uncertainty in $|V_{us}|$ is dominated (75%) by the theoretical uncertainty in the calculation of $f_+(0)$ [10]. Thus, any experimental effort aiming to improve the accuracy in measurements of the K_{l3} properties, should be accompanied by an effort to reduce the error bars in the calculation of form factors at $t=0$.

B. $|V_{us}|$ including long-distance radiative corrections

Before we proceed to include the effects of long-distance radiative corrections in the rates of K_{l3} decays, let us first discuss the effects of these corrections in the determination of the slope parameter[7] $\lambda_+(K_{e3}^+)$ from experiments.

If we use the set of measurements of $\lambda_+(K_{e3}^+)$ reported in [2] and include the effects of radiative corrections in all of them, we obtain the weighted average $\lambda_+^{rc}=0.0285\pm0.0019$ (namely, an increase of 2.5% with respect to its value in the third column of Table I). However, if we evaluate the phase space factor with this corrected value of λ_+^{rc} we obtain $I_K(K_{e3}^+)=0.1607$ to be compared with 0.1603 (see Table II). This is an effect of only 0.2%, indicating that attributing a 1% error bar to the decay rates (see footnote 5) due to the effects of long-distance radiative corrections probably overestimates this uncertainty.

Thus, we proceed to include explicitly the effects of δ_K in the decay rate. Using the input data of Table I into Eq. (16), we obtain the values for the product $|f_+(0)V_{us}|$ shown in the fourth column of Table II. Once we include the isospin breaking corrections in the $|f_+(0)|$ values for K^+ decays, we obtain the following weighted average value from the four K_{l3} decays:

$$|f_+^{K^0\to\pi^-}(0)V_{us}|=0.2101\pm0.0008\ \ [0.2099\pm0.0008]. \tag{20}$$

This quantity is plotted as a horizontal band in Fig. 2 (for $r=1.022$), together with the individual values obtained from the four kaon decays after including isospin breaking corrections from Eq. (13). The agreement among these four values is equally good (scale factor $S=0.66$) as in the case where long-distance radiative corrections were excluded (Fig. 1).

[6] If we use only the K_{e3} decays, we would have obtained the weighted average value $|V_{us}|=0.2196\pm0.0022$ [0.2192±0.0022], namely all the new data on K_{l3} decays accidentally combine to give the same value as in Ref. [10].

[7] We restrict ourselves to this particular case because Ref. [2] provides information about the values of the entries for λ_+ obtained with and without radiative correction effects in the Dalitz plot or pion spectrum observables.

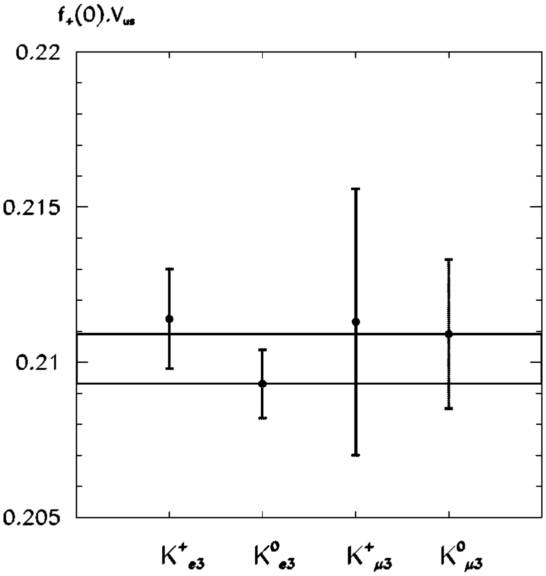

FIG. 2. Same description as in Fig. 1 but when long-distance radiative corrections are included in the decay widths.

Thus, on the basis of the scale factor alone we can conclude that the set of four measurements of the $|f_+^{K^0\to\pi^-}(0)V_{us}|$, obtained with and without radiative corrections, provide an equally consistent set of data.

Using the average value obtained in Eq. (20) we extract the CKM matrix element:

$$|V_{us}|_{rc}=0.2187\times[1\pm\sqrt{(0.0083)^2_{f_+(0)}+(0.0038)^2_{exp}}]$$

$$=0.2187\pm0.0020[0.2185\pm0.0020], \tag{21}$$

which is only 0.6% smaller than the value in Eq. (19). As in the case of Sec. IV A, the error bar is largely dominated by the uncertainty in the calculation of $f_+(0)$. For comparison, the corresponding value obtained from K_{e3} decays alone is $|V_{us}|=0.2186\pm0.0021[0.2183\pm0.0020]$.

In summary, when we include long-distance radiative corrections in the decay rates, the value of $|V_{us}|$ decreases by almost 0.6% and the error bars remain almost the same. If we compare our result for $|V_{us}|$ in Eq. (21) with the one obtained in Ref. [10], Eq. (2), we observe that the overall uncertainty is being reduced by 15%. From Eqs. (19) and (21) we conclude that any experimental effort aiming to improve the precision in measurements of K_{l3} properties would not have a significant impact on the determination of $|V_{us}|$. A reassessment of the SU(3) breaking effects in $f_+(0)$ is compelling to attain a greater accuracy in $|V_{us}|$.

However, an improvement in measurements of the properties of K_{l3} decays would help to assess the requirement of long-distance radiative corrections. In particular, a consistency check of these calculations can be provided by verifying that the quantities $|f_+^{K^0\to\pi^-}(0)V_{us}|$ are the same in all four K_{l3} decays. This quantity plays a similar role as the $\mathcal{F}t$ parameter in SFT, which must be the same for all the $0^+\to0^+$ nuclear transitions after removing (process-dependent) isospin breaking and radiative corrections from the ft value of each decay (see [4]).

TABLE III. Integrated spectrum I_K of K_{l3} decays and values extracted for the product $|f_+(0)V_{us}|$ and without (worc) radiative corrections using the nonlinear form factors of Eq. (15).

Process	β_{K*}	I_K	$\|f_+(0)V_{us}\|$ (worc)
K^+_{e3}	-0.1827	0.1616 ± 0.0012	0.2149 ± 0.0019
K^0_{e3}	-0.3057	0.1568 ± 0.0009	0.2100 ± 0.0016
$K^+_{\mu3}$	-0.3060	0.1065 ± 0.0049	0.2147 ± 0.0054
$K^0_{\mu3}$	-0.4454	0.1079 ± 0.0032	0.2119 ± 0.0036

C. Effects of non-linear form factors

In this section we study the effects on the determination of $|V_{us}|$ due to nonlinear terms that could be present in the vector form factors $f_+(t)$. These nonlinear terms are naturally induced when we extrapolate the vector form factor measured in the resonance region to energies below the threshold for $K\pi$ production (see the discussion in Sec. II).

The strength β_{K*} of the relative contributions of the two resonances in the model of Eq. (15) can be fixed either (i) from the slope of the $\tilde{f}_+(t)$ form factor at low momentum transfer or (ii) from the decay rate of $\tau\to K\pi\nu$ decays. Using the first method, we can find the values of β_{K*} using the expression given after Eq. (15). The values obtained in this way are shown in the second column of Table III. These values of β_{K*} are small and negative as expected from SU(3) symmetry[8] considerations, since the corresponding parameter β_ρ measured in $\tau\to\pi\pi\nu$ decays is also small and negative [25].

The corresponding integrated spectrum factor I_K computed by using Eqs. (15) and (17) is shown in the third column of Table III. A comparison of these results and the values of I_K found for the linear case (second column in Table II) indicates that in the nonlinear case, the values are shifted upwards by around 1%. Consequently, these nonlinearities in t would decrease the individual values of $|f_+(0)V_{us}|$ (and of $|V_{us}|$) by an amount of 0.5% (see Table IV). Thus, instead of quoting a value of $|V_{us}|$ in this case, we would like to stress that nonlinear effects in the vector form factors of K_{l3} decays would be very important in the precise determination of this CKM matrix element. In this case, more refined measurements of the this form factor both from the Dalitz plot or π spectrum of K_{l3} decays would be suitable.

V. CONCLUSIONS

In this paper we have used the updated information on semileptonic decay properties of kaons to determine the

[8] In a vector dominance model we would expect $\beta_{K*}\sim g_{K'^*K\pi}f_{K*}/(g_{K*K\pi}f_{K'^*})$ and a similar expression for β_ρ with $K^*(K'^*)$ and K replaced by $\rho(\rho')$ and π, respectively. Using SU(3) symmetry one can relate both β_V constants and expect an equality of their magnitudes within roughly 40%.

TABLE IV. Values of $|V_{us}|$ extracted for two values of the isospin-breaking parameter r (see Sec. II) from different combinations of K_{l3} decays and including (fourth column) or not (third column) the long-distance radiative corrections. Also shown are the values obtained using non-linear form factors but excluding radiative corrections (fifth column).

r	Source	$\lvert V_{us}\rvert$ (without r.c.) linear form factors	$\lvert V_{us}\rvert$ (with r.c.) linear form factors	$\lvert V_{us}\rvert$ (without r.c.) nonlinear form factors
	K_{e3}	0.2196 ± 0.0022	0.2186 ± 0.0021	0.2187 ± 0.0022
1.022	$K_{\mu3}$	0.2212 ± 0.0030	0.2196 ± 0.0029	0.2200 ± 0.0036
	All decays	0.2200 ± 0.0021	0.2187 ± 0.0020	0.2189 ± 0.0021
	K_{e3}	0.2192 ± 0.0022	0.2183 ± 0.0020	0.2184 ± 0.0022
1.026	$K_{\mu3}$	0.2209 ± 0.0030	0.2194 ± 0.0029	0.2198 ± 0.0036
	All decays	0.2197 ± 0.0021	0.2185 ± 0.0020	0.2186 ± 0.0021

$|V_{us}|$ entry of the CKM mixing matrix. We have employed both, the semielectronic (K_{e3}) and semimuonic ($K_{\mu3}$) decays of charged and neutral kaons. In addition to the original work of Ref. [10], we have explicitly included the effects of long-distance radiative corrections in our analysis and have studied the impact of non-linear vector form factors.

Our results are summarized in Table IV. We observe that the determination of $|V_{us}|$ from the semielectronic, the semimuonic and from the combined modes are consistent among them. The values of $|V_{us}|$ obtained from the muonic modes are larger than the ones obtained using the semielectronic modes, although they are less accurate. This difference becomes smaller when long-distance radiative corrections are included in the decay widths. On the other hand, long-distance radiative corrections tend to reduce the values of $|V_{us}|$ by a $0.3\%(0.7\%)$ in the electronic (muonic) channel. The error bars in the case of the semimuonic channels are still dominated by the experimental uncertainties in the decay widths and form factor slopes, while the corresponding error bars from the semielectronic modes are largely dominated by the theoretical uncertainty in the calculation of the form factors at zero momentum transfer. When we combine all the four K_{l3} decay channels, we obtain a determination of $|V_{us}|$ which modestly improves the accuracy obtained by Ref. [10], Eq. (2).

Concerning the effects of nonlinear form factors at low momentum transfer, we have considered a vector dominance model with two resonances, which turns out to be adequate in the resonance region. We fix the relative contributions of the two resonances by matching the form factor with experimental values at low energies. The overall effect of the nonlinear terms is to reduce the value of $|V_{us}|$ by a 0.5%. New measurements of the vector form factors of K_{l3} decays, particularly their energy dependence for soft pions (large values of the momentum transfer), will be very useful to improve the determination of $|V_{us}|$.

Finally, we would like to stress that the set of four kaon semileptonic decays turns out to be very useful to make a consistency test of the measurements and the different corrections applied to the decay rates. In particular, we mean that when isospin breaking corrections are removed from the vector form factors at zero momentum transfer, we can extract the product $|f_+^{K^0\to\pi^-}(0)V_{us}|$ which must be the *same* for all the four K_{l3} decays. In other words, this parameter plays the same role as the process-independent $\mathcal{F}t$ values used in superallowed Fermi nuclear transitions to determine $|V_{ud}|$.

Summarizing, the combined effect of long-distance radiative corrections and nonlinear form factors contributes to a decrease of the value of $|V_{us}|$ by up to 1%. This conclusion suggests that the present test of the unitarity of the CKM matrix [see Eq. (3)] points to a further investigation of the $|V_{ud}|$ entry as a potential solution of the problem pointed out in the Introduction of this paper.

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