Effective Lagrangian description of the lepton flavor violating decays $Z \to l_i^{\mp} l_j^{\pm}$

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A comprehensive analysis of the lepton flavor violating (LFV) decays $Z \to l_i^{\mp} l_j^{\pm}$ is presented within the effective Lagrangian approach. Both the decoupling and nondecoupling scenarios are explored. The experimental constraints from $l_i \to l_j l_k \bar{l}_k$ and $l_i \to l_j \gamma$ as well as some relationships arising from the gauge invariance of the effective Lagrangian are used to put constraints on $Z \to l_i^{\mp} l_j^{\pm}$. It is found that while current experimental data impose very strong constraints on $Z \to \mu^{\mp} e^{\pm}$, the channel $Z \to \tau^{\mp} l^{\pm}$ may still be at the reach of the planned DESY TESLA collider.

I. INTRODUCTION

Recent neutrino experimental data, such as those coming from Super-Kamiokande [1], have shown evidence of atmospheric and solar neutrino oscillations. This class of effects points to physics beyond the standard model (SM) and has immediate consequences on some sectors of the theory. For instance, the conservation of lepton number and lepton flavor cannot be taken for granted anymore, as in the SM with its massless neutrinos. Clearly, some lepton flavor violating (LFV) processes such as $Z \to l_i^{\mp} l_j^{\pm}$ ($l_i = e, \mu, \tau$) may occur and be observable at the future particle colliders. The neutrino experimental data have thus renewed interest in LFV transitions. Moreover, the prospect of the DESY e^+e^- TeV Energy Superconducting Linear Accelerator (TESLA) with its giga-Z option [2] opens up the possibility of studying in the near future some LFV Z boson decays, which might be at the reach of that collider [3]. Currently, the best direct experimental bounds on the $Z \to l_i^{\mp} l_j^{\pm}$ rates, obtained by the search at the CERN e^+e^- collider LEP-I, are [4]

$$\text{BR}(Z \to e^{\mp} \mu^{\pm}) < 1.7 \times 10^{-6}, \tag{1a}$$

$$\text{BR}(Z \to e^{\mp} \tau^{\pm}) < 9.8 \times 10^{-6}, \tag{1b}$$

$$\text{BR}(Z \to \mu^{\mp} \tau^{\pm}) < 1.2 \times 10^{-5}, \tag{1c}$$

whereas the expectations at TESLA are [2]

$$\text{BR}(Z \to e^{\mp} \mu^{\pm}) < 2.0 \times 10^{-9}, \tag{2a}$$

$$\text{BR}(Z \to e^{\mp} \tau^{\pm}) < (1.3 - 6.25) \times 10^{-8}, \tag{2b}$$

$$\text{BR}(Z \to \mu^{\mp} \tau^{\pm}) < (0.44 - 2.2) \times 10^{-8}. \tag{2c}$$

In order to disentangle the origin of any possible LFV effect, TESLA expectations must be confronted with the predictions of the diverse available models. Considerable work has been done along these lines, but here we will only refer to the most recent studies. For instance, the authors of Ref. [3] reviewed diverse scenarios that enlarge the SM by just adding massive neutrinos. It turns out that, after considering the most recent experimental data for neutrino masses, one can have at most BR$(Z \to l_i^{\mp} l_j^{\pm}) \sim 10^{-54}$ for light neutrinos. On the other hand, $Z \to l_i^{\mp} l_j^{\pm}$ might be at the reach of TESLA in some models with heavy neutrinos whose mass is of the order of 200–1000 GeV. This decay has also been studied within the general two Higgs doublet model [5]. It was found that the channel $Z \to \mu^{\mp} e^{\pm}$ is the only one that may be at the reach of TESLA. Studies within the Zee model [6] and theories with a heavy Z' boson with family nonuniversal couplings [7] gave results that are far from the experimental limits. Further works have been realized within other models, such as supersymmetry, leptoquark theories, left-right symmetric models, etc. [8].

All of the aforementioned studies have focused on specific models, which share the common feature of being of a weakly coupled nature, i.e., when the masses of the heavy particles become large they decouple from low-energy physics. Therefore, it is convenient to take a more general approach that allows us to make a model-independent analysis. We will consider thus the effective Lagrangian approach (ELA), which is suitable for this purpose. In particular, the ELA has been extensively used to study some quantities that are forbidden or highly suppressed within the SM. In this approach, there are two well-motivated schemes to parametrize virtual effects of particles lying beyond the Fermi scale via effective operators involving only the SM fields, namely the linear and nonlinear realizations of the electroweak group.

In the linear realization or decoupling scenario [9], it is assumed that the spontaneous symmetry breaking (SSB) of the electroweak group takes place in the usual way, thereby implying the existence of at least one physical Higgs boson. In addition, the light particles (the SM ones) fill out multiplets of $\text{SU}_L(2) \times \text{U}_Y(1)$. Although one can only expect marginal contributions from the heavy fields to low-energy physics, there are indeed some processes in which the new physics effects may compete with the SM ones, such as those involving flavor-changing neutral current (FCNC) and LFV transitions [10]. The latter are forbidden in the SM at any order of perturbation theory. The decoupling scenario is suit-

able to parametrize any virtual effect arising from a fundamental gauge theory that is assumed to be renormalizable and of a weakly coupled nature. This hypothesis is fundamental to establish a hierarchy among those operators of a particular dimension: gauge invariance allows us to infer the order at which the effective operators may be generated in perturbation theory [11]. In particular, a loop-generated operator is suppressed by a factor of $(4\pi)^{-2}$ with respect to a tree-level induced one. Throughout this work we will make systematic use of this fact when studying LFV processes mediated by the Z boson.

As to the nonlinear realization or nondecoupling scenario [12], in this case it is assumed that the low-energy processes are affected by unknown residual strong-dynamics effects. In this effective (chiral) theory, the SSB of the electroweak group is accomplished by introducing a unitary matrix field U that replaces the SM doublet. It is also assumed that the physical Higgs boson is either very heavy or does not exist at all. The scalar sector is comprised only of Goldstone bosons that define the U field, which in turn transforms nonlinearly under the $SU_L(2) \times U_Y(1)$ group. In the unitary gauge, where the Goldstone bosons are absent, we have that $U = 1$ and the chiral Lagrangian reproduces the SM without the Higgs field. Due to the fact that a strongly interacting regimen implies that loop effects can be as important as the tree-level ones, one cannot establish *a priori* what operators are the most relevant. We will bear this in mind when we discuss the general structure of the Zl_il_j couplings within the nondecoupling scenario.

Our main aim is thus to present a model-independent study of the LFV decay $Z \to l_i^{\mp} l_j^{\pm}$ within the ELA. We will make general predictions for the respective rates in both the decoupling and nondecoupling scenarios. Further, the impact on this decay of the experimental constraints on $l_i \to l_j l_k \bar{l}_k$ and $l_i \to l_j \gamma$ will be analyzed, and the expectations at the future TESLA collider [13], running at the Z peak (giga-Z), will be discussed.

The rest of our presentation is organized as follows. In Sec. II, we consider the decoupling scenario and discuss the most general structure of the Zl_il_j vertex. It is argued that the contribution from the monopole structure γ_μ dominates over that from the dipole structure $\sigma_{\mu\nu}k^\nu$: the latter can only arise at the one-loop level in any renormalizable theory. Therefore, the most stringent bounds on $Z \to l_i^{\mp} l_j^{\pm}$ can be obtained from the three-body decay $l_i \to l_j l_k \bar{l}_k$, which receives contributions from the Zl_il_j coupling via a virtual Z. We will also consider the contribution from the Zl_il_j and $Wl_i\nu_{l_j}$ couplings to the one-loop decay $l_i \to l_j \gamma$. In Sec. III, a similar analysis is performed within the nondecoupling scenario. We would like to stress that, in contrast to what is observed in the decoupling case, in the nondecoupling scenario the contributions from the monopole and dipole structures may be equally important due to the presence of strong-dynamics effects arising from the underlying theory, thereby allowing two possible scenarios. In the first case, it is assumed that the monopole structure gives the dominant contribution, which means that the most stringent bounds on $Z \to l_i^{\mp} l_j^{\pm}$ can be obtained from the three-body decay $l_i \to l_j l_k \bar{l}_k$. In the second

scenario, it is assumed that the dipole contribution is the dominant one, which implies that, due to the $SU_L(2) \times U_Y(1)$ symmetry, it is possible to obtain bounds on $Z \to l_i^{\mp} l_j^{\pm}$ by using the tree-level decays $l_i \to l_j \gamma$. It turns out that the bounds obtained in this way are the most stringent. Finally, we present our conclusions in Sec. IV.

II. LFV IN THE DECOUPLING SCENARIO

In this section, we assume that the underlying theory is of a decoupled nature. The effective operators inducing LFV couplings were presented in a previous work [10]. These operators can be classified according to whether they induce the $\gamma l_i l_j$ coupling or not.

A. Effective operators that only induce the Zl_il_j vertex

We can classify these operators in two classes. In the first place, we have those operators that can be generated at tree-level in a fundamental theory. They are given by

$$\mathcal{O}_{\phi l}^{ij} = i(\phi^\dagger D_\mu \phi)(\bar{l}_{Ri} \gamma^\mu l_{Rj}), \tag{3a}$$

$$\mathcal{O}_{\phi L}^{(1)ij} = i(\phi^\dagger D_\mu \phi)(\bar{L}_i \gamma^\mu L_j), \tag{3b}$$

$$\mathcal{O}_{\phi L}^{(3)ij} = i(\phi^\dagger \tau^a D_\mu \phi)(\bar{L}_i \tau^a \gamma^\mu L_j), \tag{3c}$$

where L_i and l_{Ri} stand for the left-handed doublet and the right-handed singlet of $SU_L(2) \times U_Y(1)$, respectively, τ^a are the Pauli matrices, and lepton flavors are denoted by i and j. The first two operators induce the Zl_il_j and Hl_il_j couplings, whereas the third one also induces the $Wl_i\nu_{l_j}$ vertex. Both the Zl_il_j and $Wl_i\nu_{l_j}$ couplings contribute to the one-loop induced decay $l_i \to l_j \gamma$.

There is also another set of operators that can be generated at the one-loop level or at a higher order:

$$\mathcal{O}_{Dl}^{ij} = (\bar{L}_i D_\mu l_{Rj}) D^\mu \phi, \tag{4a}$$

$$\mathcal{O}_{DL}^{ij} = (\overline{D_\mu L_i} l_{Rj}) D^\mu \phi. \tag{4b}$$

Both of these sets of operators contribute to the three-body decay $l_i \to l_j l_k \bar{l}_k$ via a virtual Z.

B. Effective operators that induce both the Zl_il_j and $\gamma l_i l_j$ vertices

Owing to gauge invariance, operators of this kind can only arise at the one-loop level in any fundamental theory. According to the Lorentz structure of these operators, we can classify them in two categories:

$$\mathcal{O}_{LW}^{ij} = ig(\bar{L}_i \mathbf{W}^{\mu\nu} \gamma_\mu D_\nu L_j), \tag{5a}$$

$$\mathcal{O}_{LB}^{ij} = ig'(\bar{L}_i B^{\mu\nu} \gamma_\mu D_\nu L_j), \tag{5b}$$

$$\mathcal{O}_{lB}^{ij} = ig'(\bar{l}_{Ri} B^{\mu\nu} \gamma_\mu D_\nu l_{Rj}), \tag{5c}$$

and

$$\mathcal{O}_{lW\phi}^{ij} = g(\bar{L}_i \sigma_{\mu\nu} \mathbf{W}^{\mu\nu} l_{Rj}) \phi, \tag{6a}$$

$$\mathcal{O}_{lB\phi}^{ij} = g'(\bar{L}_i \sigma_{\mu\nu} B^{\mu\nu} l_{Rj}) \phi, \tag{6b}$$

where $\mathbf{W}^{\mu\nu} = \tau^\alpha W^{\alpha\mu\nu}$. It is understood that the Hermitian conjugate of each operator is to be added in the respective Lagrangian, i.e., $\mathcal{L}_{\text{eff}} = (\alpha^{ij}/\Lambda^2) \mathcal{O}_{ij} + \text{H.c.}$ We have assumed that all the effective matrices α cannot be simultaneously diagonalized by the unitary matrices V_L^l and V_R^l that define the mass eigenstates. Note that these groups of operators give rise to both $Z l_i l_j$ and $\gamma l_i l_j$ couplings as a direct consequence of the $\text{SU}_L(2) \times \text{U}_Y(1)$ gauge invariance of the effective theory. Therefore, the experimental constraints on $l_i \to l_j \gamma$ can be easily translated into bounds on $Z \to l_i^\mp l_j^\pm$. However, we will see below that these operators play a marginal role in this decay, though the situation may be different in the nondecoupling scenario.

C. The most general $Zl_i l_j$ vertex and the decay $Z \to l_i^\mp l_j^\pm$

The effective operators shown in Eqs. (3)–(6) induce the most general $Z l_i l_j$ vertex. In the case of on-shell leptons, it is possible to make use of the Dirac equation along with the Gordon identity to transform the Lorentz structure induced by the operators of Eq. (5) into a dipole structure. It turns out that the contribution from these operators has terms that are proportional to m_i/m_Z or m_j/m_Z, with $m_{i,j}$ the lepton masses. This means that these operators give a very suppressed contribution, as compared to that from the operators of Eq. (6). Therefore, from now on we will not consider the operators of Eq. (5). We thus can write the most general structure of the $Z l_i l_j$ vertex in the following way:

$$\mathcal{M}_\mu^{Z l^i l^j} = \frac{ig}{2c_W} \bar{u}(p_i) \left[\gamma_\mu (F_{1L}^{ij} P_L + F_{1R}^{ij} P_R) \right.$$
$$+ \frac{1}{m_Z} (F_{2L}^{ij} P_L + F_{2R}^{ij} P_R) k_\mu$$
$$\left. + \frac{i}{m_Z} F_{3R}^{ij} P_R \sigma_{\mu\nu} k^\nu \right] v(p_j), \tag{7}$$

where k_μ is the Z four-momentum. We have defined the following matrices in the flavor space:

$$F_{1L} = -\left(\frac{v}{\Lambda}\right)^2 V_L^l (\alpha_{D\phi}^{(1)} + \alpha_{D\phi}^{(3)}) V_L^{l\dagger}, \tag{8a}$$

$$F_{1R} = -\left(\frac{v}{\Lambda}\right)^2 V_R^l \alpha_{D\phi} V_R^{l\dagger}, \tag{8b}$$

$$F_{2L} = \frac{g^2}{2\sqrt{2}c_W^2} \left(\frac{v}{\Lambda}\right)^2 V_L^l \alpha_{DL} V_R^{l\dagger}, \tag{9a}$$

$$F_{2R} = -\frac{g^2}{2\sqrt{2}c_W^2} \left(\frac{v}{\Lambda}\right)^2 V_L^l \alpha_{Dl} V_R^{l\dagger}, \tag{9b}$$

$$F_{3R} = \frac{\sqrt{2}g}{c_W} \left(\frac{v}{\Lambda}\right)^2 V_L^l (c_W^2 \alpha_{lW\phi} + s_W^2 \alpha_{lB\phi}) V_R^{l\dagger}. \tag{10}$$

It is evident that the terms proportional to k_μ in Eq. (7) do not contribute when the Z boson is on-shell. We thus can conclude that the contributions to $Z \to l_i^\mp l_j^\pm$ can only arise from the operators given in Eqs. (3) and (6), i.e., only through the monopole and dipole structures. Since the monopole structure can be generated at tree level by the underlying theory, its contribution will dominate that from the dipole structure because the latter can only arise at the one-loop level and has a suppression factor of $(4\pi)^{-2}$. It is thus a good approximation to consider only the contributions arising from the operators of Eq. (3). In contrast, the $\gamma l_i l_j$ coupling is only induced by the operators of Eqs. (5) and (6) since the monopole contribution is forbidden because of electromagnetic gauge invariance, i.e., the $\gamma l_i l_j$ coupling can only arise at the one-loop level in any renormalizable theory. In order to obtain bounds on $Z \to l_i^\mp l_j^\pm$, we will use the experimental bounds on the three-body decays $l_i \to l_j l_k \bar{l}_k$, which may receive contributions from the $Z l_i l_j$ coupling through a virtual Z, mainly via the monopole structure. We will also calculate the contributions from the $Z l_i l_j$ and $W l_i \nu_{l_j}$ couplings to the one-loop induced decay $l_i \to l_j \gamma$ in order to analyze if this mode could be useful to obtain bounds on $Z \to l_i^\mp l_j^\pm$. All these results can be translated readily into the nondecoupling scenario, where the dipole contribution to $Z \to l_i^\mp l_j^\pm$ will be studied also. It turns out that, in that scenario, the dipole contribution may be as important as that from the monopole structure.

Taking into account just the contribution from the tree-level-generated operators, the branching fraction for the decay $Z \to l_i^\mp l_j^\pm$ can be written as

$$\text{BR}(Z \to l_i^\mp l_j^\pm) = \frac{\alpha}{3s_{2W}^2} \left(\frac{m_Z}{\Gamma_Z}\right) (|F_{1L}^{ij}|^2 + |F_{1R}^{ij}|^2), \tag{11}$$

where we have neglected the lepton masses. We have also introduced the definition $s_{2W} = 2c_W s_W$.

D. Bounds from the three-body decay $l_i \to l_j l_k \bar{l}_k$

The contribution from the $Z l_i l_j$ coupling to the decay $l_i \to l_j l_k \bar{l}_k$ (viz., Fig. 1) can be written as

$$\text{BR}(l_i \to l_j l_k \bar{l}_k) = \frac{a\alpha^2}{96\pi s_{2W}^4} \frac{m_i}{\Gamma_{l_i}} \left(\frac{m_i}{m_Z}\right)^4$$
$$\times (|F_{1L}^{ij}|^2 + |F_{1R}^{ij}|^2), \tag{12}$$

with $a = 1 - 4s_W^2 + 8s_W^4$ and Γ_{l_i} being the full l_i width. Again we have neglected the final lepton masses, i.e., $m_j = m_k = 0$. From Eqs. (11) and (12), we can obtain the following expression:
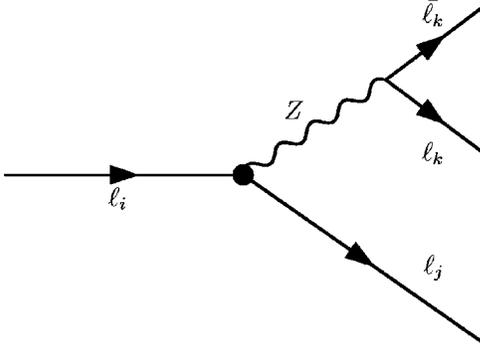
FIG. 1. Feynman diagram for the three-body decay $l_i \to l_j l_k \bar{l}_k$ in the effective Lagrangian approach. The dot denotes an effective LFV coupling.

$$\mathrm{BR}(Z \to l_i^\mp l_j^\pm) \lesssim \frac{48\pi s_{2W}^2}{a\,\alpha}\left(\frac{\Gamma_{l_i}}{\Gamma_Z}\right)\left(\frac{m_Z}{m_i}\right)^5$$

$$\times \mathrm{BR}_{\mathrm{expt.}}(l_i \to l_j l_k \bar{l}_k) \qquad (13)$$

where $\mathrm{BR}_{\mathrm{expt.}}(l_i \to l_j l_k \bar{l}_k)$ stands for the experimental constraints [4]:

$$\mathrm{BR}_{\mathrm{expt.}}(\mu^- \to e^- e^- e^+) < 10^{-12}, \qquad (14a)$$

$$\mathrm{BR}_{\mathrm{expt.}}(\tau^- \to l_j l_k \bar{l}_k) < \kappa_{jk} 10^{-6}, \qquad (14b)$$

and κ_{jk} is a factor of order $O(1)$ corresponding to each different channel [4]. These equations allow us to obtain the following bounds:

$$\mathrm{BR}(Z \to \mu^\mp e^\pm) \lesssim 1.04 \times 10^{-12}, \qquad (15a)$$

$$\mathrm{BR}(Z \to \tau^\mp e^\pm) \lesssim 1.7 \times 10^{-5}, \qquad (15b)$$

$$\mathrm{BR}(Z \to \tau^\mp \mu^\pm) \lesssim 1.0 \times 10^{-5}. \qquad (15c)$$

These results are in agreement with those obtained from unitarity-inspired arguments in Ref. [14].

E. Bounds from the two-body decay $l_i \to l_j \gamma$

We now study the contributions from the Zl_il_j and $Wl_i\nu_{l_j}$ couplings to the one-loop decay $l_i \to l_j \gamma$ (viz., Fig. 2). While the three-body decay $l_i \to l_j l_k \bar{l}_k$ gets naturally suppressed by

the three-body phase space and the exchange of a virtual Z boson, the one-loop decay $l_i \to l_j \gamma$ gets a suppression factor of $(4\pi)^{-2}$ plus an extra power of α. Since the current experimental constraints on both decay modes are of the same order of magnitude, the only way in which the radiative decay can compete with the three-body decay is if the former arises from a nondecoupling effect. However, we will see below that the $l_i \to l_j \gamma$ amplitude is dominated by the virtual Z and vanishes when $m_i/m_Z \to 0$.

The respective Feynman diagrams for the decay $l_i \to l_j \gamma$ are shown in Fig. 2. We have used the unitary gauge in our calculation. The expression for the Zl_il_j coupling was given in Eq. (7), though we will only consider the monopole contribution here. As for the $Wl_i\nu_{l_j}$ coupling, which can be induced by the operators of Eq. (3c), it is expressed as

$$\mathcal{M}^{W\nu_i l_j} = \frac{ig\,\epsilon_L^{ij}}{\sqrt{2}}\bar{u}(p_i)P_L\gamma_\mu v(p_j)W^\mu, \qquad (16)$$

with

$$\epsilon_L = \left(\frac{v}{\Lambda}\right)^2 V_L^l \alpha_{D\phi}^{(3)} V_L^{l\dagger}. \qquad (17)$$

After some calculation, the decay amplitude can be expressed as

$$\mathcal{M}^\mu(l_i \to l_j \gamma) = \bar{u}(p_j)(f_V - f_A\gamma^5)\sigma^{\mu\nu}q^\nu v(p_i), \qquad (18)$$

where q is the photon four-momentum and the coefficients $f_{V,A}$ are given as follows:

$$f_{V,A} = \frac{\alpha}{4\pi s_{2W}^2}(A_R^{Vij} \pm A_L^{Vij}), \qquad (19)$$

where the superscript Vij denotes the contribution from the virtual boson (Z or W). As to the coefficients $A_{L,R}^{Vij}$, they are given, in terms of scalar integrals, by

$$A_{L,R}^{Zij} = \frac{1}{4m_i^3 m_Z^2}\{4m_Z^4 g_{L,R}^l(1 + B_0^1 + B_0^2 - B_0^3 - B_0^4)$$
$$- 2m_i^2 m_Z^2[\pm 3(B_0^2 - B_0^4) + 2g_{L,R}^l(B_0^1 - B_0^3)]$$
$$- m_i^4(\pm 1 - 4g_{L,R}^l m_Z^2 C_0^1)\}F_{1L,1R}^{ij}, \qquad (20)$$

FIG. 2. Feynman diagrams for the radiative decay $l_i \to l_j \gamma$ in the effective Lagrangian approach. The dot denotes an effective LFV coupling. There is another set of diagrams where the flavor-changing effective vertex is inserted in the opposite end of the Z boson or the neutrino.

with $g_L^l = -1 + 2s_W^2$ and $g_R^l = 2s_W^2$. For simplicity we neglected the final lepton mass. The sign $+(-)$ holds for the $L(R)$ term. As far as the A_L^{Wij} coefficient is concerned, we have

$$A_L^{Wij} = \frac{2\,\epsilon_L^{ij}c_W^2}{m_i^3}[2m_W^2 - 3m_i^2$$
$$- 2(m_W^2 - m_i^2)(B_0^6 - B_0^5 + m_i^2\,C_0^2)], \qquad (21)$$

whereas $A_R^{Wij} = 0$. The scalar integrals B_0^i and C_0^i are given, in the notation of Ref. [15], as follows:

$$B_0^1 = B_0(0, m_Z^2, m_Z^2),$$

$$B_0^2 = B_0(0, m_i^2, m_Z^2),$$

$$B_0^3 = B_0(m_i^2, 0, m_Z^2),$$

$$B_0^4 = B_0(m_i^2, m_i^2, m_Z^2),$$

$$B_0^5 = B_0(0, m_W^2, m_W^2),$$

$$B_0^6 = B_0(m_i^2, 0, m_W^2),$$

$$C_0^1 = C_0(m_i^2, 0, 0, m_i^2, m_Z^2, m_i^2),$$

and

$$C_0^2 = C_0(m_i^2, 0, 0, m_W^2, 0, m_W^2).$$

It is interesting to note that, although an effective vertex was inserted into a one-loop diagram, from the above expressions it is evident that the calculation renders a finite result. It can be explained from the fact that the Zl_il_j coupling has a renormalizable structure. Our result is very general in the sense that it can also be applied to theories with an extra Z' boson with LFV couplings of the monopole-structure form.

The branching ratio for the radiative decay $l_i \rightarrow l_j\gamma$ is given by

$$\mathrm{BR}(l_i \rightarrow l_j\gamma) = \frac{m_i^3}{8\pi\Gamma_{l_i}}(|f_V|^2 + |f_A|^2)$$

$$= \frac{m_i^3\alpha^2}{64\pi^3\Gamma_{l_i}s_{2W}^4}(|A_L^{Vij}|^2 + |A_R^{Vij}|^2). \qquad (22)$$

The scalar functions involved in the coefficients $A_{L,R}^{Vij}$ can be numerically evaluated [15] or expanded in powers of m_i. We will end up with an expression of the form $\mathrm{BR}(l_i \rightarrow l_j\gamma) = \beta_1|F_{1L}^{ij}|^2 + \beta_1|F_{1R}^{ij}|^2 + \beta_3|\epsilon_L^{ij}|^2$, where the β_k are some numerical coefficients.

From the experimental side, we have

$$\mathrm{BR}(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11}, \qquad (23a)$$

$$\mathrm{BR}(\tau \rightarrow e\gamma) < 2.9 \times 10^{-6}, \qquad (23b)$$
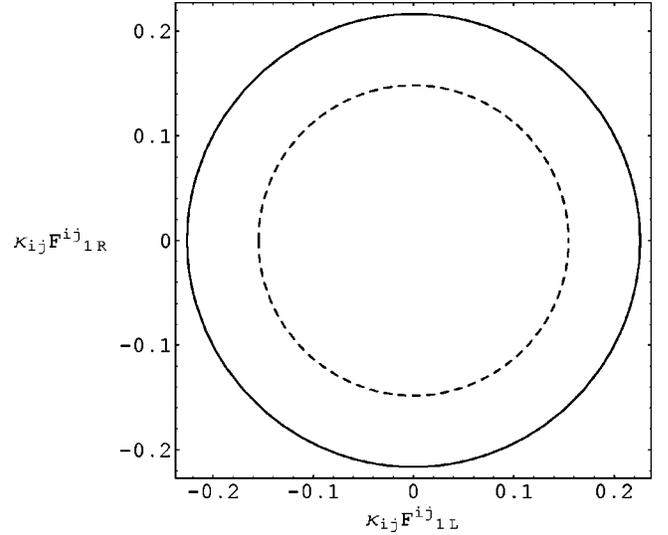
FIG. 3. Bounds on the coefficients $F_{1L,1R}^{ij}$ from $\mu \rightarrow e\gamma$ (solid line) and $\tau \rightarrow \mu\gamma$ (dashed line). The allowed region lies inside the curves. $\kappa_{\mu e} = 10^{-8}$ and $\kappa_{\tau\mu} = 10^{-2}$.

$$\mathrm{BR}(\tau \rightarrow \mu\gamma) < 1.1 \times 10^{-6}. \qquad (23c)$$

Therefore, from Eqs. (22) and (23) we can obtain an upper bound on the coefficients $F_{1L,1R}^{ij}$ and ϵ_L^{ij}, which in turn can be used to put constraints on the decay $Z \rightarrow l_i^{\mp}l_j^{\pm}$. In Fig. 3, we show the allowed region for the coefficients $F_{1L,1R}^{ij}$, as obtained from the decays $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$. As we are interested in obtaining upper bounds on $F_{1L,1R}^{ij}$, we set $\epsilon_L^{ij} = 0$. In the plot of Fig. 3, the allowed regions, which interestingly are almost circular in shape, lie inside the curves. From these results and Eq. (11) we can obtain the following bounds:

$$\mathrm{BR}(Z \rightarrow \mu^{\pm}e^{\pm}) \lesssim 6.12 \times 10^{-11}, \qquad (24a)$$

$$\mathrm{BR}(Z \rightarrow \tau^{\pm}l^{\pm}) \lesssim 2.8 \times 10^{-5}, \qquad (24b)$$

where $l = e$ or μ. Although these bounds are weaker than the ones obtained from the three-body decay $l_i \rightarrow l_jl_k\bar{l}_k$, they show that the one-loop decay $l_i \rightarrow l_j\gamma$ may also be useful to obtain bounds on $Z \rightarrow l_i^{\mp}l_j^{\pm}$.

III. LVF IN THE NONDECOUPLING SCENARIO

In the scenario where the underlying new physics effects arise from a strongly interacting sector, the relevant LFV operators are similar to those given in the decoupling scenario, but now with the Higgs doublet replaced by the following unitary matrix:

$$U = \exp\left(\frac{2i\tau^a\varphi^a}{v}\right), \qquad (25)$$

where φ^a stands for the Goldstone bosons. In this realization of the $\mathrm{SU}_L(2) \times \mathrm{U}_Y(1)$ group, the covariant derivative is de-

fined as $\mathbf{D}_\mu U = \partial_\mu U + ig\mathbf{W}_\mu U - ig'U\mathbf{B}_\mu$, with the Abelian field given by $\mathbf{B}_\mu = (\tau^3/2)B_\mu$. From the discussion presented before, it is clear that the relevant operators are analogous to those shown in Eqs. (3) and (6), although the operator $\mathcal{O}_{\phi L}^{(3)ij}$ has no nonlinear counterpart. These operators can be written as

$$\mathcal{L}_{UR} = i\lambda_{UR}^{ij}\mathrm{Tr}[\tau^3 U^\dagger \mathbf{D}_\mu U]\bar{R}_i\gamma^\mu R_j + \mathrm{H.c.}, \qquad (26a)$$

$$\mathcal{L}_{UL} = i\lambda_{UL}^{ij}\mathrm{Tr}[\tau^3 U^\dagger \mathbf{D}_\mu U]\bar{L}_i\gamma^\mu L_j + \mathrm{H.c.}, \qquad (26b)$$

$$\mathcal{L}_{lWU} = g\frac{\lambda_{lWU}^{ij}}{\Lambda}(\bar{L}_i\sigma_{\mu\nu}\mathbf{W}^{\mu\nu}UR_j) + \mathrm{H.c.}, \qquad (27a)$$

$$\mathcal{L}_{lBU} = g'\frac{\lambda_{lBU}^{ij}}{\Lambda}(\bar{L}_i\sigma_{\mu\nu}UR_j)B^{\mu\nu} + \mathrm{H.c.}, \qquad (27b)$$

where $R_i = (0, l_{Ri})$. In this scenario, there is an upper bound on the new physics scale, i.e., $\Lambda \sim 4\pi v$, which will be adopted below. Notice that the first group of operators have dimension 4 in mass units, whereas the last ones have dimension 5.

The formulas given in Eqs. (7)–(11) also hold, but the F_i matrices are to be replaced by

$$A_{1L} = -2V_L^l\lambda_{UL}V_L^{l\dagger}, \qquad (28a)$$

$$A_{1R} = -2V_R^l\lambda_{UR}V_R^{l\dagger}, \qquad (28b)$$

$$A = \frac{g^2}{2\pi c_W^2}V_L^l(c_W^2\lambda_{lWU} - s_W^2\lambda_{lBU})V_R^{l\dagger}. \qquad (28c)$$

Since we are assuming a strong interaction is responsible for the LFV effects, two scenarios are of interest. In the first case we will assume that the monopole contribution dominates over the dipole contribution, whereas in the second case we will take the dipole moment contribution as being dominant.

A. Monopole dominance

In this scenario, it is assumed that the structure induced by the operators (26a) and (26b) gives the dominant contribution. From the above discussion, it is clear that the most stringent bounds can be obtained from the three-body decay $l_i \to l_j l_k \bar{l}_k$. It is also clear that the relation given in Eq. (13) still holds. Consequently, the respective bounds are the same as those given in Eqs. (15). Finally, the bounds arising from the decay $l_i \to l_j\gamma$ also hold.

B. Dipole moment dominance

We now neglect the monopole term and focus on the contribution arising from the operators (27a) and (27b). Due to $SU_L(2) \times U_Y(1)$ gauge invariance, these operators induce both the Zl_il_j and γl_il_j vertices, which means that the decay $l_i \to l_j\gamma$ can give more stringent bounds than the ones arising from the three-body decay $l_i \to l_j l_k \bar{l}_k$. This is because the electromagnetic decay has a phase-space factor less restricted than the three-body decay and does not involve the factor $(m_i/m_Z)^4$ coming from the inclusion of the virtual Z but only the kinematic one $(m_i/m_Z)^2$.

The branching fractions for the decays $Z \to l_i^\mp l_j^\pm$ and $l_i \to l_j\gamma$ can now be written as

$$\mathrm{BR}(Z \to l_i^\pm l_j^\pm) = \frac{\alpha}{6s_{2W}^2}\left(\frac{m_Z}{\Gamma_Z}\right)|A^{ij}|^2, \qquad (29)$$

$$\mathrm{BR}(l_i \to l_j\gamma) = \frac{3\alpha}{4s_{2w}^2}\left(\frac{m_i}{\Gamma_{l_i}}\right)\left(\frac{m_i}{m_Z}\right)^2|B^{ij}|^2, \qquad (30)$$

where

$$B = \frac{e}{2\pi c_W^2}V_L^l(\lambda_{lBU} + \lambda_{lWU})V_R^{l\dagger}. \qquad (31)$$

After introducing the experimental constraints on the electromagnetic decays, we have

$$\mathrm{BR}(Z \to l_i^\mp l_j^\pm) \leqslant \frac{2}{9}\left(\frac{\Gamma_{l_i}}{\Gamma_Z}\right)\left(\frac{m_Z}{m_i}\right)^3$$

$$\times \mathrm{BR}_{\mathrm{expt.}}(l_i \to l_j\gamma) \begin{cases} \left(\dfrac{c_W}{s_W}\right)^2 & \text{for } \mathcal{L}_{lWU}^{ij} \\ \left(\dfrac{s_W}{c_W}\right)^2 & \text{for } \mathcal{L}_{lBU}^{ij}. \end{cases}$$

$$(32)$$

By using the respective experimental constraints [4], we get

$$\mathrm{BR}(Z \to \mu^\mp e^\pm) \leqslant (8.64 \times 10^{-22}, \ 7.81 \times 10^{-23}), \qquad (33a)$$

$$\mathrm{BR}(Z \to \tau^\mp e^\pm) \leqslant (3.09 \times 10^{-13}, \ 2.79 \times 10^{-14}), \qquad (33b)$$

$$\mathrm{BR}(Z \to \tau^\mp \mu^\pm) \leqslant (1.25 \times 10^{-12}, \ 1.13 \times 10^{-13}), \qquad (33c)$$

where the first (second) figure in the parentheses corresponds to the operator $\mathcal{L}_{lWU}$ ($\mathcal{L}_{lBU}$). It should be noticed that the same bounds apply in the decoupling case, in the unlikely scenario in which the dipole contribution dominates over that from the monopole. The above bounds have severe consequences. They imply that the existing experimental constraints on the decays $l_i \to l_j\gamma$, together with $SU_L(2) \times U_Y(1)$ gauge invariance, are enough to rule out any pos-

TABLE I. Constraints on the LFV decays $Z \to l_i^{\mp} l_j^{\pm}$ as obtained from the experimental bounds on $l_i \to l_j l_k \bar{l}_k$ and $l_i \to l_j \gamma$. The third column corresponds to the monopole term of the $Z l_i l_j$ coupling, which can induce the decay $l_i \to l_j \gamma$ at the one-loop level. The last column is obtained in the scenario in which the new physics LFV effects only contribute to the dipole term of $Z l_i l_j$. The operators that induce this term can also give rise to $l_i \to l_j \gamma$ at tree level.

	$l_i \to l_j l_k \bar{l}_k$	$l_i \to l_j \gamma$ (one-loop level)	$l_i \to l_j \gamma$ (tree level)
BR$(Z \to \mu^{\mp} e^{\pm})$	$\leqslant 1.04 \times 10^{-12}$	$\leqslant 6.12 \times 10^{-11}$	$\leqslant (10^{-22} - 10^{-23})$
BR$(Z \to \tau^{\mp} e^{\pm})$	$\leqslant 1.7 \times 10^{-5}$	$\leqslant \sim 10^{-5}$	$\leqslant (10^{-13} - 10^{-14})$
BR$(Z \to \tau^{\mp} \mu^{\pm})$	$\leqslant 1.0 \times 10^{-5}$	$\leqslant \sim 10^{-5}$	$\leqslant (10^{-12} - 10^{-13})$

sible detection of a LFV transition of the Z boson if it arises via a $Z l_i l_j$ coupling of the form of a dipole moment.

IV. FINAL DISCUSSION

Until now, the LFV decay $Z \to l_i^{\mp} l_j^{\pm}$ has been studied within a large variety of models [3,5–8]. These studies show that, at least for some values of the model parameters, the respective decay rates might be at the reach of the planned TESLA collider. However, all of these analyses rely on several assumptions about the parameters of the model under study. We have shown in this work that an ELA analysis is well suited to studying this LFV decay. We have considered both the linear and nonlinear realizations of the ELA. This approach has allowed us to make some general predictions about the $Z \to l_i^{\mp} l_j^{\pm}$ rates starting from the current experimental bounds on the low-energy processes $l_i \to l_j l_k \bar{l}_k$ and $l_i \to l_j \gamma$. We summarize our results in Table I. We also analyzed the impact of the LFV couplings $Z l_i l_j$ and $W l_i \nu_{l_j}$ on the muon anomalous magnetic moment $(g-2)_{\mu}$, but our calculation showed that the bounds obtained this way are rather weak. So, we refrain from showing the respective results here. In the same context, there are other processes that could be useful to obtain constraints on the LFV Z boson couplings, such as μ-e conversion and muonium-antimuonium conversion. We preferred the decays $l_i \to l_j l_k \bar{l}_k$ and $l_i \to l_j \gamma$ since they do not imply any extra assumption.

In this work, we have examined some potential scenarios for the contributions arising from the two Lorentz structures associated with the on-shell $Z l_i l_j$ vertex, namely the mono-

pole and dipole terms. It was shown that, in the decoupling scenario, the decay $Z \to l_i^{\mp} l_j^{\pm}$ arises mainly from the monopole term. In this case, the strongest constraints on these processes are obtained from the current bounds on the decays $l_i \to l_j l_k \bar{l}_k$, though the constraints on $l_i \to l_j \gamma$ are also useful for the same purpose. On the other hand, in the nondecoupling scenario, in which the LFV effects have a strongly interacting origin, gauge invariance as the main ingredient of the effective theory induces simultaneously both $Z l_i l_j$ and $\gamma l_i l_j$ vertices. In this scenario, it might be that both the monopole and dipole contributions have the same strength. If the main contribution came from the dipole term, the current limits on the decay $l_i \to l_j \gamma$ would place severe constraints on $Z \to l_i^{\mp} l_j^{\pm}$ [see Eqs. (33)], which clearly are far from the reach of the planned TESLA collider. These results suggest indeed that the dipole contribution is unlikely to be observed.

In summary, if the new physics LFV effects are of a decoupled nature, the most stringent bound on $Z \to \mu^{\mp} e^{\pm}$ is of the order of 10^{-12}, which suggests that this mode would be out of the reach of TESLA. Since the current experimental limits on $\tau \to l_j l_k \bar{l}_k$ are less stringent than those on $\mu^- \to e^- e^- e^+$, the resulting bounds on $Z \to \tau^{\mp} l^{\pm}$ are also weaker than those on $Z \to \mu^{\mp} e^{\pm}$. As a consequence, the decay $Z \to \tau^{\mp} l^{\pm}$ may still be at the reach of the TESLA collider. In this respect, it has been conjectured that LFV effects might be more evident in transitions involving the τ lepton.

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