Phenomenological analysis of properties of the right-handed Majorana neutrino in the seesaw mechanism

Haijun Pan*
Lab of Quantum Communication and Quantum Computation, and Center of Nonlinear Science, University of Science and Technology of China, Hefei, Anhui 230026, People's Republic of China

G. Cheng†
CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, People's Republic of China
and Department of Astronomy and Applied Physics, University of Science and Technology of China, Hefei, Anhui 230026, People's Republic of China

(Received 22 August 2001; published 14 March 2002)

As an extension of our previous work in the seesaw mechanism, we analyze the influence of nonzero U_{e3} on the properties (masses and mixing) of the right-handed Majorana neutrinos in three flavors. The quasidegenerate light neutrinos case is also considered. Assuming the hierarchical Dirac neutrino masses, we find the heavy Majorana neutrino mass spectrum is either hierarchical or partially degenerate if θ_{23}^{ν} is large. We show that degenerate right-handed (RH) Majorana masses correspond to a maximal RH mixing angle while hierarchical ones correspond to the RH mixing angles which scale linearly with the mass ratios of the Dirac neutrino masses. An interesting analogue to the behavior of the matter-enhanced neutrino conversion and their difference is presented.

DOI: 10.1103/PhysRevD.65.073009 PACS number(s): 14.60.Pq, 12.15.Ff

I. INTRODUCTION

By adding the heavy right-handed (RH) neutrinos ν_R, the seesaw mechanism [1] provides a very natural and attractive explanation of the smallness of the neutrino masses compared to the masses of the charged fermions. In the seesaw mechanism, the mass matrix of the left-handed (LH) neutrino has the following form [2]:

$$m_{\nu}=m_D M^{-1} m_D^T, \tag{1}$$

where m_D is the Dirac mass matrix and M is the Majorana mass matrix for the right-handed neutrino components. The scale of the RH Majorana neutrino masses is not precisely known. In various theoretical models, ν_R can be the unification scale ($\sim 10^{16}$ GeV) or the intermediate scale ($\sim 10^{9}-10^{13}$ GeV) [3]. To understand the possible unification of particles and interactions, it is crucial to know if this scale is associated with some new physics and at what energy it eventually happens [3].

On the basis that the Dirac mass matrix of the charged leptons is diagonal, m_ν can be written as $m_\nu = U\mathrm{diag}(m_1,m_2,m_3)U^T$. Here U is the Maki-Nakagawa-Sakata (MNS) mixing matrix. Similarly, M can be expressed as $M=V_M\mathrm{diag}(M_1,M_2,M_3)V_M^T$, where V is a 3×3 unitary matrix. According to some kind of quark-lepton analogy suggested in grand unified theories (GUTs), the structure of the Dirac mass matrix m_D is similar to that in the quark sector [2], which can be diagonalized by a biunitary transformation: $D_L^{\dagger} m_D D_R = \mathrm{diag}(m_{1D},m_{2D},m_{3D})$.

The seesaw mechanism can give us hints about the neutrino properties in two aspects [1,4]. The first is fixing M by some ansatz and predicting masses and mixing of the light neutrinos. On the contrary, especially with the increase of the data from the low-energy neutrino experiments, it becomes pressing and practical to determine the structure of the RH Majorana neutrinos from the light neutrino masses and mixing implied by experiments [3–5].

The knowledge of neutrino masses and mixing comes mainly from three kinds of neutrino oscillation experiments: the solar [6] and atmospheric [7] neutrino deficits, Liquid Scintillation Neutrino Detector (LSND) reactor experiments [8]. The Super-Kamiokande (SK) atmospheric neutrino data are best interpreted in terms of $\nu_\mu \leftrightarrow \nu_\tau$ and $\overline{\nu}_\mu \rightarrow \overline{\nu}_\tau$ with almost maximal mixing $\sin^2 2\theta_{23}^{\nu}=1$ and the following mass-squared difference (best-fit value) [7]:

$$\Delta m_{32}^2 = m_3^2 - m_2^2 = 5.9\times 10^{-3}\ \mathrm{eV}^2. \tag{2}$$

In contrast, there exist two different oscillation mechanisms yielding four possible solutions [11] to the solar neutrino problem: The "just-so" mechanism (i.e., the long-wavelength vacuum oscillations) with

$$(\Delta m_{21}^2, \sin^2 2\theta_{12}^{\nu}) = (6.5\times 10^{-11}\ \mathrm{eV}^2, 0.75) \quad (\mathrm{VO}) \tag{3}$$

and the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism [12] (the matter-enhanced oscillation) with one of the three following values of the neutrino oscillation parameters, respectively:

$$\begin{aligned}(\Delta m_{21}^2, \sin^2 2\theta_{12}^{\nu}) &= (1.8\times 10^{-5}\ \mathrm{eV}^2, 0.76)\ \ (\mathrm{LMA}),\\ &= (7.9\times 10^{-8}\ \mathrm{eV}^2, 0.96)\ \ (\mathrm{LOW}),\\ &= (5.4\times 10^{-6}\ \mathrm{eV}^2, 6.0\times 10^{-3}\ \ (\mathrm{SMA}).\end{aligned} \tag{4}$$

*Email address: phj@mail.ustc.edu.cn
†Email address: gcheng@ustc.edu.cn

Here $\Delta m_{21}^2 = m_2^2 - m_1^2$, LMA (SMA) refers to the large (small) mixing angle, and LOW stands for low mass or possibility. All of the above are best-fit values. For later discussion, we present here the regions of the mass-squared differences Δm_{21}^2 from Bahcall [11] and Δm_{32}^2 obtained from SK [7]:

$$\begin{aligned}
\Delta m_{21}^2: &\quad (4\times 10^{-12})-(6\times 10^{-9})\ \ \mathrm{eV}^2 \quad (\mathrm{VO}),\\
&\quad (6\times 10^{-6})-(3\times 10^{-4})\ \ \mathrm{eV}^2 \quad (\mathrm{LMA}),\\
&\quad (3\times 10^{-8})-(2\times 10^{-7})\ \ \mathrm{eV}^2 \quad (\mathrm{LOW}),\\
&\quad (4\times 10^{-6})-(1\times 10^{-5})\ \ \mathrm{eV}^2 \quad (\mathrm{SMA}),\\
\Delta m_{32}^2: &\quad (1\times 10^{-3})-(1\times 10^{-1})\ \ \mathrm{eV}^2 \quad (\mathrm{ATM.}).
\end{aligned} \tag{5}$$

Another neutrino oscillation experiment, LSND, indicates the following mass-squared difference:

$$\Delta m_{\mathrm{LSND}}^2 \sim 1 \ \ \mathrm{eV}^2 \tag{6}$$

with the mixing angle $\sin^2 2\theta_{\mathrm{LSND}}^{\nu} \sim 10^{-3}-10^{-2}$. It is obvious that four neutrinos are needed to accommodate all three mass-squared differences. However, the LSND results were not confirmed by the recent KARMEN experiment [13] and we will just set it aside. In addition, the CHOOZ Collaboration [9] has obtained the following rather stringent upper bound on the value of the third mixing angle θ_{13}^{ν} [10]:

$$|U_{e3}| = \sin\theta_{13}^{\nu} \leqslant 0.13-0.23. \tag{7}$$

It is noteworthy that, although θ_{13}^{ν} is small, it may be comparable with and/or even far larger than θ_{12}^{ν} in the SMA region and so may have important influence on the RH Majorana neutrino properties (masses and mixing). In Sec. II, we present the main formulas obtained in Ref. [5] and the motivation for this paper. The quasidegenerate mass case is briefly discussed in Sec. III. In Sec. IV, we study the effect of θ_{13}^{ν} in various regions for the hierarchical light neutrino masses in two possibilities according to whether θ_{12}^{ν} is large or small. Finally, we summarize our main results in Sec. V.

II. FORMULAS

By inverting the seesaw formula (1), we can write it in terms of diagonalized matrices and the rotations in the form

$$(m_D^{\mathrm{diag}})^{-1} S m_\nu^{\mathrm{diag}} S^T (m_D^{\mathrm{diag}})^{-1} = V^* (M^{\mathrm{diag}})^{-1} V^{\dagger}. \tag{8}$$

Here $V = D_R^T V_M$ and the seesaw matrix [2] $S = D_L^{\dagger} U$. In this work, we ignore the CP-violating effect (i.e., all the mixing matrices entered in the seesaw mechanism are real orthogonal). One of the standard parametrizations of the MNS matrix reads

$$\begin{aligned}
U &= e^{i\theta_{23}^{\nu}\lambda_7} e^{i\theta_{13}^{\nu}\lambda_5} e^{i\theta_{12}^{\nu}\lambda_2}\\
&= \begin{pmatrix}
\cos\theta_{13}^{\nu}\cos\theta_{12}^{\nu} & \cos\theta_{13}^{\nu}\sin\theta_{12}^{\nu} & \sin\theta_{13}^{\nu}\\
-\sin\theta_{23}^{\nu}\sin\theta_{13}^{\nu}\cos\theta_{12}^{\nu} - \cos\theta_{23}^{\nu}\sin\theta_{12}^{\nu} & -\sin\theta_{23}^{\nu}\sin\theta_{13}^{\nu}\sin\theta_{12}^{\nu} + \cos\theta_{23}^{\nu}\cos\theta_{12}^{\nu} & \sin\theta_{23}^{\nu}\cos\theta_{13}^{\nu}\\
-\cos\theta_{23}^{\nu}\sin\theta_{13}^{\nu}\cos\theta_{12}^{\nu} + \sin\theta_{23}^{\nu}\sin\theta_{12}^{\nu} & -\cos\theta_{23}^{\nu}\sin\theta_{13}^{\nu}\sin\theta_{12}^{\nu} - \sin\theta_{23}^{\nu}\cos\theta_{12}^{\nu} & \cos\theta_{23}^{\nu}\cos\theta_{13}^{\nu}.
\end{pmatrix}.
\end{aligned} \tag{9}$$

The RH mixing matrix V has a similar parametrization,

$$V = e^{i\beta_{23}\lambda_7} e^{i\beta_{13}\lambda_5} e^{i\beta_{12}\lambda_2}. \tag{10}$$

In Ref. [5], Eq. (8) has been reduced to the following three equations:

$$\bar{M}_1 \bar{M}_2 \bar{M}_3 = 1, \tag{11a}$$

$$\bar{M}_1^{-1} + \bar{M}_2^{-1} + \bar{M}_3^{-1} = X_{11} + X_{22} + X_{33}, \tag{11b}$$

$$\bar{M}_1 + \bar{M}_2 + \bar{M}_3 = Y_{11} + Y_{22} + Y_{33}, \tag{11c}$$

and relations between the diagonal and nondiagonal elements of V have been obtained,

$$V_{ij} = \frac{Y_{ij} + \bar{M}_j X_{ij}}{(Y_{jj} + \bar{M}_j X_{jj}) + \bar{M}_j^{-2} - \mathrm{Tr}Y} V_{jj}$$

$$(i,j = 1,2,3 \ \ \text{and} \ \ i \neq j), \tag{12}$$

where X_{ij} and Y_{ij} have the form

$$X_{ij} = \frac{1}{\bar{m}_{iD}\bar{m}_{jD}} \sum_{k=1}^{3} \bar{m}_k S_{ik} S_{jk}, \tag{13a}$$

$$Y_{ij} = \bar{m}_{iD}\bar{m}_{jD} \sum_{k=1}^{3} \frac{1}{\bar{m}_k} S_{ik} S_{jk} \tag{13b}$$

with $\bar{m}_{iD} = m_{iD}(m_{1D}m_{2D}m_{3D})^{-1/3}$ and $\bar{m}_k = m_i(m_1 m_2 m_3)^{-1/3}$. Then, the three RH Majorana masses can be expressed as

$$M_1 = F\overline{M}_1, \quad M_2 = F\overline{M}_2, \quad M_3 = F\overline{M}_3 \tag{14}$$

with $F=(m_{1D}m_{2D}m_{3D})^{2/3}/(m_1m_2m_3)^{1/3}$. The three RH rotation angles can be obtained from Eq. (12):

$$\tan\beta_{23}=V_{23}/V_{33}, \quad \cos\beta_{13}\sin\beta_{12}=V_{12}, \quad \sin\beta_{13}=V_{13}. \tag{15}$$

In this paper, we consider how a nonzero U_{e3} affects the RH neutrino masses and mixing as an extension of our previous analysis [5]. There are three main reasons for us to devote attention to this topic. One is that θ_{13}^{ν} can be comparable with and/or even far larger than θ_{12}^{ν} in the allowed region of the small mixing MSW effect. The second is that S contains contributions from both U and D_L^T. Assuming m_D has the same structure (not only the mass scale but also the mixing matrix) as that in the quark sector, one has [3]

$$D_L=\begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & \lambda^4 \\ -\lambda & 1-\frac{1}{2}\lambda^2 & \lambda^2 \\ \lambda^3-\lambda^4 & -\lambda^2 & 1-\frac{1}{2}\lambda^2 \end{pmatrix} \tag{16}$$

with $\lambda \approx 0.22$. Then S_{12} and S_{13} have the following values:

$$S_{12}=U_{e2}-\lambda U_{\mu 2}-\frac{1}{2}\lambda^2 U_{e2}+(\lambda^3-\lambda^4)U_{\tau 2}, \tag{17a}$$

$$S_{13}=U_{e3}-\lambda U_{\mu 3}-\frac{1}{2}\lambda^2 U_{e3}+(\lambda^3-\lambda^4)U_{\tau 3}. \tag{17b}$$

Since $\lambda \gg U_{e2}\sim 10^{-2}$ in the SMA region, one can see that $S_{12}\sim S_{13}$ even when $\theta_{13}^{\nu}=0$. The third reason is that the scale and the structure of the RH Majorana neutrino may be sensitive, particularly in the SMA region (i.e., when θ_{12}^{ν} is small), to the value of U_{e3} and $U_{\tau 1}$, that is, to the values of θ_{12}^{ν} and θ_{13}^{ν}. This will be clearer in later discussion.

III. QUASIDEGENERATE MASS PATTERN

When $m_0=m_1\approx m_2\approx m_3$, we define δ_{21} and δ_{31} as

$$m_2=m_0(1+\delta_{21}), \quad m_3=m_0(1+\delta_{31}). \tag{18}$$

From the condition $\Delta m_{21}^2 \ll \Delta m_{31}^2$ implied from the solar and atmospheric neutrino experiments, one has $\delta_{21}\ll\delta_{31}\ll 1$. Exploiting this relation, the RH Majorana masses can be obtained from the formulas given in Sec. II:

$$M_i \approx \frac{m_{iD}^2}{m_0}, \quad i=1,2,3. \tag{19}$$

The RH mixing angles are easy to deduce:

$$\beta_{ij}\approx -\frac{m_{iD}}{m_{jD}}(\delta_{21}S_{i2}S_{j2}+\delta_{31}S_{i3}S_{j3}) \quad (1\leqslant i<j\leqslant 3). \tag{20}$$

We can see that the RH mixing angles are small. However, their scales are sensitive to the inputs of S_{12}, S_{13}, δ_{21}, and δ_{31}. For example, when $S_{13}=0$ one has

$$\beta_{12}\approx -\frac{m_{1D}}{m_{2D}}\delta_{21}S_{12}S_{22}, \quad \beta_{13}\approx \frac{m_{1D}}{m_{3D}}\delta_{21}S_{12}S_{32},$$

$$\beta_{23}\approx \frac{m_{2D}}{m_{3D}}\delta_{31}S_{23}S_{33}, \tag{21}$$

and when $S_{13}\gtrsim S_{12}$,

$$\beta_{12}\approx -\frac{m_{1D}}{m_{2D}}\delta_{31}S_{13}S_{23}, \quad \beta_{13}\approx \frac{m_{1D}}{m_{3D}}\delta_{31}S_{13}S_{33},$$

$$\beta_{23}\approx \frac{m_{2D}}{m_{3D}}\delta_{31}S_{23}S_{33} \tag{22}$$

in the SMA region.

Assuming $m_0\sim 1$ eV, one has $M_1=1.6\times 10^3$ GeV, $M_2=1.8\times 10^8$ GeV, and $M_3=1.2\times 10^{13}$ GeV. Note that $M_{2,3}$ are in the intermediate scale while M_1 is in the electric-weak scale. Here, we have taken $m_D=m^{\rm up}(10^9 \text{ GeV})$.

IV. HIERARCHICAL MASS PATTERN

Assuming $m_1\ll m_2\ll m_3$, the two heavier neutrino masses can be approximated as $m_2\approx\sqrt{\Delta m_{21}^2}$ and $m_3\approx\sqrt{\Delta m_{32}^2}$. In the following, we first consider the small mixing solution to the solar neutrino problem and then VO, LMA, and LOW are embodied in a unitized framework since they all have a large θ_{12}^{ν}. To discuss the influence of different (1,3) element of the seesaw matrix, it is convenient to set $D_L=I$. This approximation would not affect the qualitative results obtained in the following analysis.

A. Small θ_{12}^{ν} (SMA)

We will discuss the case in which $\sin^2 2\theta_{12}^{\nu}\gtrsim 10^{-3}$ so that it remains in the SMA region. When θ_{12}^{ν} is small, there are three possibilities by comparing θ_{13}^{ν} with θ_{12}^{ν}: $\theta_{13}^{\nu}\ll\theta_{12}^{\nu}$, $\theta_{13}^{\nu}\gg\theta_{12}^{\nu}$, and $\theta_{13}^{\nu}\sim\theta_{12}^{\nu}$.

1. $\theta_{13}^{\nu}\ll\theta_{12}^{\nu}$

In the limit $\theta_{13}^{\nu}\to 0$, we have obtained, in Ref. [5],

$$M_1\approx f_1\frac{m_{1D}^2}{m_2}, \quad M_2\approx\frac{2m_{2D}^2}{m_3}, \quad M_3\approx f_1^{-1}\frac{m_{3D}^2}{2m_1} \tag{23}$$

and

$$\beta_{12}\approx -f_1\frac{m_{1D}\sin\theta_{12}^{\nu}}{\sqrt{2}m_{2D}}, \quad \beta_{13}\approx f_1\frac{\sqrt{2}m_{1D}\sin\theta_{12}^{\nu}}{m_{3D}},$$

$$\beta_{23}\approx -\frac{m_{2D}}{m_{3D}}, \tag{24}$$

where $f_1=r_{21}/(r_{21}\sin^2\theta_{12}^\nu+1)$ and $r_{21}=m_2/m_1\gg 1$. The dependence of the RH Majorana neutrino masses on the light neutrino masses is different from what one would expect when no mixing occurs, $M_i\propto m_i^{-1}$. As noted in Refs. [4,5], the RH mixing angles scale linearly with the ratios of the Dirac neutrino masses $\beta_{ij}\sim m_{iD}/m_{jD}$ $(1\leqslant i<j\leqslant 3)$, which is different from the LH quark mixing angles, where one has $\tan\theta^{\rm quark}\approx\sqrt{m_d/m_s}$ in the two-generation case [14].

2. $\theta_{13}^\nu\gg\theta_{12}^\nu$

In this case, from Eq. (13a) we obtain $X_{ii}\sim\overline{m}_3/(\overline{m}_{iD}^2S_{i3}^2)$ $(i=1,2,3)$. Then, from Eqs. (11a) and (14), we have

$$M_1\approx\frac{m_{1D}^2}{m_3\sin^2\theta_{13}^\nu},\quad M_2\approx f_1\frac{2m_{2D}^2\sin^2\theta_{13}^\nu}{m_2},\quad M_3\approx f_1^{-1}\frac{m_{3D}^2}{2m_1}, \tag{25}$$

and from Eqs. (12) and (15),

$$\beta_{12}\approx-\frac{m_{1D}}{\sqrt{2}m_{2D}\sin\theta_{13}^\nu},\quad \beta_{13}\approx f_1\frac{\sqrt{2}m_{1D}\sin\theta_{13}^\nu}{m_{3D}},$$
$$\beta_{23}\approx\frac{m_{2D}}{m_{3D}}. \tag{26}$$

Comparing Eqs. (23) and (25), the dependence of M_i on m_i has changed, and from Eqs. (24) and (26) one can see that the three RH mixing angles β_{ij} have the same dependence on the Dirac neutrino mass ratios, respectively.

3. $\theta_{13}^\nu\sim\theta_{12}^\nu$

Taking $\theta_{23}^\nu=\pi/4$, we have $U_{\tau 1}=(\sin\theta_{12}^\nu-\sin\theta_{13}^\nu\cos\theta_{12}^\nu)/\sqrt{2}\sim 0$ when $\theta_{13}^\nu\sim\theta_{12}^\nu$. In this limit, the three RH Majorana masses are given by

$$M_1\approx\frac{m_{1D}^2}{m_3\sin^2\theta_{12}^\nu}, \tag{27a}$$

$$M_2\approx\begin{cases}\dfrac{2m_{2D}^2\sin^2\theta_{12}^\nu}{m_1} & \text{if}\quad r_{21}<r_{21}^{\rm res}\\[2ex] \dfrac{m_{3D}^2}{2m_2} & \text{if}\quad r_{21}>r_{21}^{\rm res},\end{cases} \tag{27b}$$

$$M_3\approx\begin{cases}\dfrac{m_{3D}^2}{2m_2} & \text{if}\quad r_{21}<r_{21}^{\rm res}\\[2ex] \dfrac{2m_{2D}^2\sin^2\theta_{12}^\nu}{m_1} & \text{if}\quad r_{21}>r_{21}^{\rm res},\end{cases} \tag{27c}$$

where $r_{21}^{\rm res}=\frac{1}{4}(m_{3D}^2/m_{2D}^2)\csc^2\theta_{12}^\nu$. The two heavier RH Majorana neutrino masses are degenerate, $M_2=M_3$, when $r_{21}=r_{21}^{\rm res}$. We should point out that M_2 and M_3 are only nearly degenerate when $r_{21}=r_{21}^{\rm res}$ in strict numerical results.

The second RH mixing angle β_{23} can be expressed as

$$\tan 2\beta_{23}\approx-\frac{2m_{2D}/m_{3D}}{1-r_{21}/r_{21}^{\rm res}} \tag{28}$$

or

$$\sin^2 2\beta_{23}\approx\frac{4m_{2D}^2/m_{3D}^2}{(1-r_{21}/r_{21}^{\rm res})^2+4m_{2D}^2/m_{3D}^2}. \tag{29}$$

The other two RH mixing angles are both small and can be expressed in β_{23} as follows:

$$\beta_{12}\approx\frac{1}{\sqrt{2}\sin\theta_{12}^\nu}\left(-\frac{m_{1D}}{m_{2D}}\cos\beta_{23}+\frac{m_{1D}}{m_{3D}}\sin\beta_{23}\right), \tag{30a}$$

$$\beta_{13}\approx\frac{1}{\sqrt{2}\sin\theta_{12}^\nu}\left(-\frac{m_{1D}}{m_{2D}}\sin\beta_{23}-\frac{m_{1D}}{m_{3D}}\cos\beta_{23}\right). \tag{30b}$$

The behaviors of $M_{2,3}$, β_{ij}, and $\sin^2 2\beta_{23}$ near $r_{21}^{\rm res}$ as functions of r_{21} are shown in Fig. 1. The behavior of $\sin^2 2\beta_{23}$ as a function of r_{21} is clearly that of a resonance peaked at $r_{21}=r_{21}^{\rm res}$. We can define the resonance width $\delta_{r_{21}}$ as that of r_{21} around $r_{21}^{\rm res}$ for which $\sin^2 2\beta_{23}$ becomes $\frac{1}{2}$ instead of the maximum value, unity. It is given by

$$\delta_{r_{21}}\approx 4\frac{m_{2D}}{m_{3D}}r_{21}^{\rm res}. \tag{31}$$

Substituting the SMA data in Eq. (4) into Eq. (27), we have $\sin^2 2\beta_{23}=1$, $M_1\approx 1.4\times 10^7$ GeV, and $M_2\approx M_3\approx 5\times 10^{15}$ GeV when $r_{21}=r_{21}^{\rm res}$ (that is, when $m_1\approx 1.2\times 10^{-10}$ eV since we assume the hierarchical mass pattern). The situation is very like that in the matter-enhanced $\nu_e\leftrightarrow\nu_\mu$ oscillation in the sun while here r_{21} plays the part of the effective potential A. In the matter, a level crossing from one mass eigenstate to another takes place in the resonance region. In our case, the dependence of $M_{2,3}$ on m_i and m_{iD} changes through the resonance point. We should point out that the resonancelike behavior of the RH Majorana neutrino parameters, unlike the MSW effect, is not a kinetic process since r_{21} should be a fixed value. Why such resonancelike behavior happens is just due to our ignorance of the absolute neutrino mass m_1. However, such behavior may shed light on the region of m_1. For example, if all the RH rotation angles are required to be small (that is, $\beta_{ij}\leqslant\pi/4$), one has $m_1\gtrsim 1.2\times 10^{-10}$ eV in the SMA region when $\theta_{13}^\nu\sim\theta_{12}^\nu$, and if one of the three angles is large (here β_{23}), one has $m_1\lesssim 1.2\times 10^{-10}$ eV. However, if β_{23} happens to be maximal, we have two near-degenerate heavier RH Majorana neutrino masses and the corresponding electric neutrino mass $m_1\approx 1.2\times 10^{-10}$ eV.

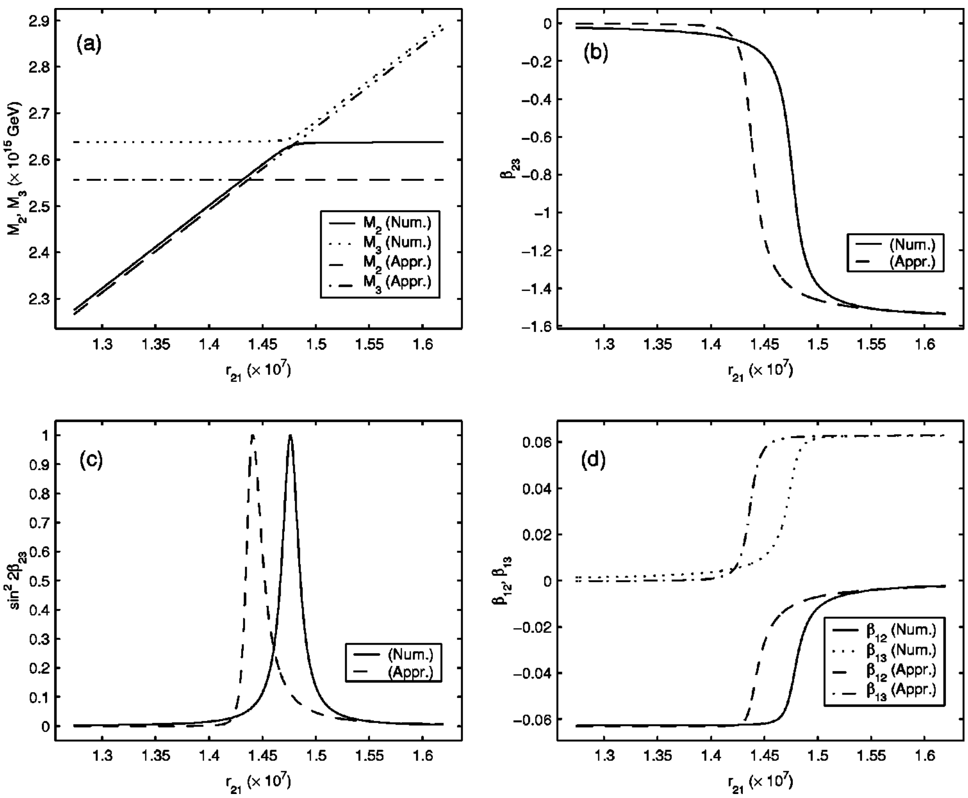

FIG. 1. The behavior of the RH Majorana masses (a), the RH mixing angles (b,d), and $\sin^2 2\beta_{23}$ (c) as functions of r_{21} $(=m_2/m_1)$ for the SMA solution to the solar neutrino anomaly when $\theta_{13}^{\nu} \sim \theta_{12}^{\nu}$. The values of the neutrino parameters are taken to be $m_2^2 = 5.4\times 10^{-6}$ eV^2, $m_3^2 = 5.9\times 10^{-3}$ eV^2, $\sin^2 2\theta_{12}^{\nu} = 6.0\times 10^{-3}$, and $\sin^2 2\theta_{12}^{\nu} = 1.0$.

B. Large θ_{12}^{ν}

In this case, there is no need to consider the relative magnitude of θ_{12}^{ν} and θ_{13}^{ν} since one always has $U_{\tau 1} \approx \sin\theta_{12}^{\nu}/\sqrt{2}$. We shall therefore consider this problem in the following two possibilities according to the magnitude of θ_{13}^{ν}.

1. θ_{13}^{ν} is tiny

From Sec. II, we find, when $\sin^2\theta_{13}^{\nu} \ll \frac{1}{2} m_{1D}^2/m_{2D}^2$, that the RH Majorana masses are given by

$$M_1 \approx \begin{cases} \dfrac{m_{1D}^2}{m_2 \sin^2\theta_{12}^{\nu}}, & r_{32} < r_{32}^{\rm res} \\ \dfrac{2m_{2D}^2}{m_3}, & r_{32} > r_{32}^{\rm res}, \end{cases} \tag{32a}$$

$$M_2 \approx \begin{cases} \dfrac{2m_{2D}^2}{m_3} & r_{32} < r_{32}^{\rm res} \\ \dfrac{m_{1D}^2}{m_2 \sin^2\theta_{12}^{\nu}}, & r_{32} > r_{32}^{\rm res}, \end{cases} \tag{32b}$$

$$M_3 \approx \frac{m_{3D}^2 \sin^2\theta_{12}^{\nu}}{2m_1} \tag{32c}$$

and the RH mixing angles by

$$\sin^2 2\beta_{12} \approx \frac{4m_{1D}^2/m_{2D}^2}{(1 - r_{32}/r_{32}^{\rm res})^2 + 4m_{1D}^2/m_{2D}^2}, \tag{33a}$$

$$\beta_{13} \approx \frac{\sqrt{2} m_{1D} \cot\theta_{12}^{\nu}}{m_{3D}}, \quad \beta_{23} \approx -\frac{m_{2D}}{m_{3D}}. \tag{33b}$$

Here $r_{32} = m_3/m_2$ and $r_{32}^{\rm res} \approx 2(m_{2D}^2/m_{1D}^2)\sin^2\theta_{12}^{\nu}$. We also plot the behaviors of $M_{1,2}$, β_{ij}, and $\sin^2 2\beta_{12}$ near $r_{32}^{\rm res}$ as functions of r_{32} in Fig. 2. From this one can see that β_{12} reaches to maximal and M_1 and M_2 are nearly degenerate when $r_{32} \approx r_{32}^{\rm res}$. For the best values of Δm_{21}^2 and Δm_{32}^2, we always have $r_{32} \ll r_{32}^{\rm res}$ for the VO, LMA, and LOW solutions to the solar neutrino problem and so

$$M_1 \approx \frac{m_{1D}^2}{m_2} \frac{1}{\sin^2\theta_{12}^{\nu}}, \quad M_2 \approx 2\frac{m_{2D}^2}{m_3}, \quad M_3 \approx \frac{1}{2}\frac{m_{3D}^2}{m_1}\sin^2\theta_{12}^{\nu}; \tag{34}$$

$$\beta_{12} \approx -\frac{1}{\sqrt{2}}\frac{m_{1D}}{m_{2D}}\cot\theta_{12}^{\nu}, \quad \beta_{13} \approx \sqrt{2}\frac{m_{1D}}{m_{3D}}\cot\theta_{12}^{\nu},$$

$$\beta_{23} \approx -\frac{m_{2D}}{m_{3D}}. \tag{35}$$

However, taking the possible regions of Δm_{21}^2 and Δm_{32}^2 listed in Eq. (5) into account, r_{32} in the VO region may pass the resonance point and the corresponding RH masses have the values $M_1 \approx M_2 \approx 1.2\times 10^9$ GeV and $M_3 \approx 1.8 \times 10^{17} r_{21}$ GeV when $r_{32} = r_{32}^{\rm res}$. With a similar discussion af-

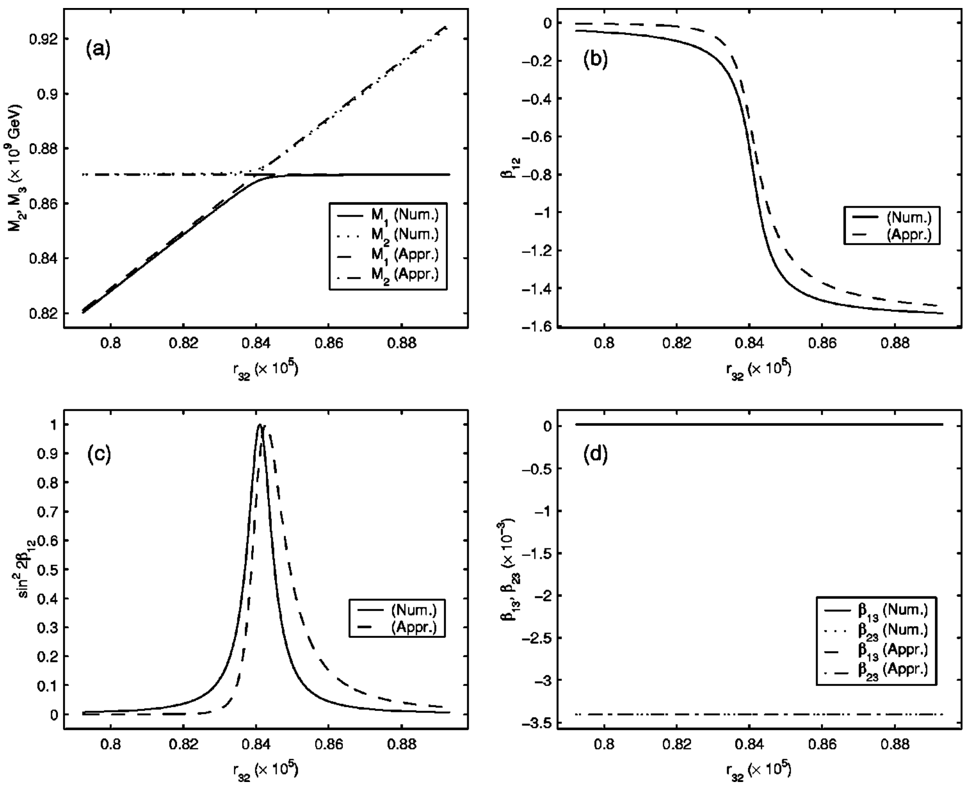

FIG. 2. The behavior of the RH Majorana masses (a), the RH mixing angles (b,d), and $\sin^2 2\beta_{12}$ (c) as functions of r_{32} $(=m_3/m_2)$ for the vacuum oscillation solution to the solar neutrino anomaly when $U_{e3}=0$. In (d), the approximate and the numerical results match so well that their differences cannot be shown in the figure. The values of the neutrino parameters are taken to be $m_2/m_1=1000$, $m_3^2=0.1\ \mathrm{eV}^2$, and $\sin^2 2\theta_{12}^\nu=\sin^2 2\theta_{23}^\nu=1.0$.

ter Eq. (30), the resonancelike behavior together with some supposition of the RH mixing angles could restrict us to the region of m_3/m_2, and thus the region of Δm_{21} and Δm_{32}, in the VO solution.

2. θ_{13}^ν is not so small

We find, when θ_{13}^ν is small but satisfying $\sin^2\theta_{13}^\nu \gtrsim m_2 \sin^2\theta_{12}^\nu/m_3$, that the RH Majorana neutrino masses are hierarchical and have the form

$$M_1 \approx f_2^{-1}\frac{m_{1D}^2}{m_2}, \quad M_2 \approx f_2\frac{m_{2D}^2}{m_3 U_{\tau 1}^2}, \quad M_3 \approx \frac{m_{3D}^2 U_{\tau 1}^2}{m_1}, \tag{36}$$

where $f_2 = U_{e2}^2 + r_{32}U_{e3}^2 + r_{32}(m_{1D}^2/m_{2D}^2)U_{\mu 3}^2$ and all three RH mixing angles are small scale linearly with the ratios of the Dirac neutrino masses,

$$\beta_{12} \approx -\frac{m_{1D}}{m_{2D}}\,\frac{U_{e2}U_{\mu 2}+r_{32}U_{e3}U_{\mu 3}}{U_{e2}^2+r_{32}U_{\tau 3}^2},$$

$$\beta_{13} \approx \frac{m_{1D}U_{e1}}{m_{3D}U_{\tau 1}}, \quad \beta_{23} \approx \frac{m_{2D}U_{\mu 1}}{m_{3D}U_{\tau 1}}. \tag{37}$$

V. SUMMARY AND DISCUSSION

Separating the solution regions of the solar neutrino problem in two cases according to the value of θ_{12}^ν, we have derived simple relations between parameters of the RH and LH Majorana neutrino masses and mixing in the context of the seesaw mechanism and quark-lepton symmetry within the framework of three families. Especially, as an extension of our previous work, we have embodied the quasidegenerate light neutrino mass case and the influence of nonzero U_{e3} on the properties (masses and mixing) of the RH Majorana neutrinos. The *CP*-violating effect has not been included. We find the following.

(i) The quasidegenerate neutrino spectrum leads to hierarchical RH Majorana masses and small RH mixing angles which scale linearly with the ratios of the Dirac masses $\beta_{ij} \sim m_{iD}/m_{jD}, 1 \leqslant i < j \leqslant 3$.

(ii) For SMA, resonancelike behavior of $\sin^2 2\beta_{23}$ is found when $\theta_{13}^\nu \sim \theta_{12}^\nu$. We find when $m_1 \approx 1.2\times 10^{-10}$ eV one has $\sin^2 2\beta_{23}=1$, $M_1 \approx 1.4\times 10^7$ GeV, and $M_2 \approx M_3 \approx 5\times 10^{15}$ GeV. For a wide range of r_{21}, quantitatively, for r_{21} below r_{21}^{res}, all three RH mixing angles are small and $M_3 \approx 5\times 10^{15}$ GeV is near the scale of GUT. Assuming that all three RH mixing angles are small, the lower bound of m_1 could be obtained $(\sim 1.2\times 10^{-10}\ \mathrm{eV})$.

(iii) The behavior $\sin^2 2\beta_{12}$ as functions of m_3/m_2 is a resonance peaked at $r_{32}=r_{32}^{\mathrm{res}}$ for large θ_{12}^ν when θ_{13}^ν is tiny. One has two lighter degenerate RH Majorana masses at this point which lie in the VO region. In Ref. [5] we have not discussed this case.

(iv) The resonancelike behaviors of the RH Majorana neutrino parameters and the MSW effect, although an interesting analogy, are different physically. The former is just a relation for different values of the parameters while the latter is a kinetic behavior that physical parameters display when neutrinos propagate in the matter.

(v) In addition, we have presented a numerical method [embodied in Eq. (11a)] that is generally applicable for calculating the physical parameters in the seesaw mechanism, which usually involves extremely large and extremely small quantities simultaneously.

The results are dependent on the precise determination of U_{e3}. Such a goal is expected to be reached in the neutrino long baseline experiment, registration of the neutrino bursts from the Galactic supernova by existing detectors SK and SNO, and the neutrino factories [15]. In this paper, we have not discussed the case in which M_R and m_D are complex. Also, the renormalization effect has not been included. We hope to return to these topics in future.

ACKNOWLEDGMENTS

We would like to express our sincere thanks to Professor T.K. Kuo for his warm help in the progress of this research and for his kindness in giving us his papers before publication. We are grateful to Dr. Du Taijiao and Dr. Tu Tao for useful discussions. The authors are also grateful for the help provided by the Chinese High Performance Computing Center at Hefei. G.C. is supported in part by the National Science Foundation in China under Grant No. 19875047 and No. 10175058.

[1] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Freedman (North Holland, Amsterdam, 1979); T. Yanagida, in *Proceedings of the Workshop on Unified Theory and Baryon Number in the Universe*, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, 1979); R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).

[2] A.Yu. Smirnov, Phys. Rev. D **48**, 3264 (1993); Nucl. Phys. **B466**, 25 (1996).

[3] D. Falcone, Phys. Rev. D **61**, 097302 (2000).

[4] T.K. Kuo, Guo-Hong Wu, and Sadek W. Mansour, Phys. Rev. D **61**, 111301(R) (2000).

[5] Haijun Pan and G. Cheng, Phys. Rev. D **65**, 013002 (2002).

[6] R. Davis, Jr., D.S. Harmer, and K.C. Hoffman, Phys. Rev. Lett. **20**, 1205 (1968); GALLEX Collaboration, P. Anselmann *et al.*, Phys. Lett. B **342**, 440 (1995); GALLEX Collaboration, W. Hampel *et al.*, *ibid.* **388**, 364 (1996); KAMIOKANDE Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **77**, 1683 (1996); Super-Kamiokande Collaboration, Y. Fukuda *et al.*, *ibid.* **86**, 5651 (2001); **86**, 5656 (2001).

[7] Super-Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1562 (1998); **82**, 2644 (1999).

[8] LSND Collaboration, C. Athanassopoulos *et al.*, Phys. Rev. Lett. **75**, 2650 (1995).

[9] Chooz Collaboration, M. Apollonio *et al.*, Phys. Lett. B **420**, 397 (1998).

[10] F. Boehm *et al.*, Phys. Rev. D **62**, 072002 (2000); E.Kh. Akhmedov, G.C. Branco, and M.N. Rebelo, Phys. Rev. Lett. **84**, 3535 (2000).

[11] J. Bahcall, P. Krastev, and A.Yu. Smirnov, Phys. Rev. D **58**, 096016 (1998); **60**, 093001 (1999).

[12] L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); S.P. Mikheyev and A.Yu. Smirnov, Nuovo Cimento Soc. Ital. Fis., C **9**, 17 (1986).

[13] KARMEN Collaboration, K. Eitel, Nucl. Phys. B (Proc. Suppl.) **91**, 191 (2000).

[14] H. Fusaoka and Y. Koide, Phys. Rev. D **57**, 3986 (1998).

[15] A.Yu. Smirnov, Nucl. Phys. B (Proc. Suppl.) **91**, 306 (2000).