Frame dependence of spin-one angular conditions in light front dynamics

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We elaborate the frame dependence of the angular conditions for spin-1 form factors. An extra angular condition is found in addition to the usual angular condition relating the four helicity amplitudes. Investigating the frame dependence of angular conditions, we find that the extra angular condition is in general as complicated as the usual one, although it becomes very simple in the $q^+=0$ frame involving only two helicity amplitudes. It is confirmed that the angular conditions are identical in frames that are connected by kinematical transformations. The high-Q^2 behavior of the physical form factors and the limiting behavior in special reference frames are also discussed.

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I. INTRODUCTION

Bosons with spin 1 are ubiquitous in modern particle physics. In the standard model the fundamental interactions are described by gauge bosons, such as the photon, $W^{\pm}$ and Z, and the gluon. These particles are considered to be truly elementary; i.e., they occur as quanta of local fields.

In hadron physics many vector mesons composed of a quark and an antiquark are found and understanding their structure is a challenging problem in quantum chromodynamics (QCD), related to the mechanism of confinement and the detailed nature of the interactions between the constituents.

Moreover, the deuteron is an interesting laboratory for the application of QCD to nuclear physics. At large distances the deuteron is evidently well described as a spin-1 composite of two nucleon clusters with binding energy $\sim$2.2 MeV, together with small admixtures of $\Delta\Delta$ and virtual meson components. However, at short distances, in the region where all six quarks overlap within a distance $R\sim 1/Q$, one can show rigorously that the deuteron state in QCD necessarily has ''fractional parentage'' 1/9 (np), 4/45 ($\Delta\Delta$), and 4/5 ''hidden color'' (nonnuclear) components [1]. At any momentum scale, the deuteron cannot be described solely in terms of conventional nuclear physics degrees of freedom, but in principle any dynamical property of the deuteron is modified by the presence of non-Abelian ''hidden color'' components [2]. Alternatively one may describe the deuteron structure in terms of uncolored degrees of freedom only, but then a tower of excited nucleons and Δ's is involved [3,4].

Although these spin-1 systems (e.g., $W^{\pm}$, the ρ meson, and the deuteron) do not seem to share a common internal structure, the universality of spin-1 systems [5] severely constrains them. According to this universality, the fundamental constraints on the magnetic and quadrupole moments of hadronic and nuclear states imposed by the Compton-scattering sum rules [6] and the behavior of the electromagnetic form factors of composite spin-1 systems [7] at large momentum transfer are the same as those of a corresponding elementary particle of the same spin and charge. (For a review and references to the early calculations of the deuteron form factors in the 1970s within light-front dynamics, see Frankfurt and Strikman [8].) At $Q^2=0$, the charge $[G_C(Q^2)]$, magnetic $[G_M(Q^2)]$, and quadrupole $[G_Q(Q^2)]$ form factors define the charge e, the magnetic moment μ_1, and the quadrupole moment Q_1, respectively. In the limit of zero radius of the bound states (or large binding energies), whether confined or nonconfined, the values of μ_1 and Q_1 approach the canonical values [5] of a spin-1 object with mass m and charge e:

$$\mu_1=\frac{e}{m}, \quad Q_1=-\frac{e}{m^2}. \tag{1.1}$$

Universality requires that the values obtained in Eq. (1.1) must be the same as those of the fundamental gauge bosons $W^{\pm}$ in the tree approximation to the standard model. Also, at large Q^2 (in the limit $Q\gg\sqrt{2m\Lambda_{\rm QCD}}$), these form factors are required to approach the universal ratios given by [5]

$$G_C(Q^2):G_M(Q^2):G_Q(Q^2)\to\left(1-\frac{Q^2}{6m^2}\right):2:-1, \tag{1.2}$$

which were obtained in a light-cone frame with $q^+=0$. Equation (1.2) should hold at large momentum transfers in the case of composite systems such as the ρ meson and deuteron, with corrections of order $\Lambda_{\rm QCD}/Q$ and $\Lambda_{\rm QCD}/m$ according to QCD. The ratios are the same as those predicted for the electromagnetic couplings of the $W^{\pm}$ for all Q^2 in the standard model at the tree level.

Furthermore, there are constraints on the current matrix elements, since there are only three form factors for the spin-1 systems. A constraint well known from the literature [9] is the angular condition obtained by demanding rotational covariance for the current matrix elements given by

$$G^{\mu}_{h'h}=\langle p'h'|J^{\mu}|ph\rangle, \tag{1.3}$$

where $|ph\rangle$ is an eigenstate with momentum p and helicity h. For example, in the Drell-Yan-West (DYW) frame and the frames that are connected to the DYW frame by only kinematic transformations, the angular condition is given as [5,9,10]

$$(1+2\eta)G^+_{++}+G^+_{+-}-\sqrt{8\eta}G^+_{+0}-G^+_{00}=0, \qquad (1.4)$$

where $\eta=Q^2/4m^2$. Kondratyuk and Strikman [11] have shown that the additive model for the current operator of interacting constituents is consistent with the angular condition only for the first two terms of the expansion of J^+ in powers of the momentum transfer Q. If the angular condition is not satisfied, an identical extraction of form factors (G_C, G_M, G_Q) from the light-front current matrix elements $G^+_{h'h}$ is not attained. As a consequence, there are indeed different extraction schemes for the spin-1 form factors in the literature [5,12–14]. As an example, G_C, G_M, and G_Q can be given in terms of G^+_{+0}, G^+_{00}, and G^+_{+-} in the DYW frame $q^+=0$, $q_x=Q$, and $q_y=0$ as follows [5]

$$G_C=\frac{1}{2p^+(2\eta+1)}\left[\frac{16}{3}\eta\frac{G^+_{+0}}{\sqrt{2\eta}}-\frac{2\eta-3}{3}G^+_{00}+\frac{2}{3}(2\eta-1)G^+_{+-}\right],$$

$$G_M=\frac{2}{2p^+(2\eta+1)}\left[(2\eta-1)\frac{G^+_{+0}}{\sqrt{2\eta}}+G^+_{00}-G^+_{+-}\right],$$

$$G_Q=\frac{1}{2p^+(2\eta+1)}\left[2\frac{G^+_{+0}}{\sqrt{2\eta}}-G^+_{00}+\frac{\eta+1}{\eta}G^+_{+-}\right]. \qquad (1.5)$$

However, other choices of the current matrix elements can be made to express the right-hand side of Eq. (1.5) and the expression also depends on the reference frame. A few examples of other expressions on the right-hand side of Eq. (1.5) can be found in Ref. [15]. The angular conditions are also useful in testing the validity of model calculations. Especially, as stressed in the recent literature [16–20], the zero-mode contribution is necessary to get the correct result for the form factors unless the good component of the current is used. Even if the good component of the current is used, it was noted that the zero-mode contribution is necessary for the calculation of spin-1 form factors [21]. Such an observation of zero-mode necessity has been made by checking the angular conditions and the degree of necessity can be quantified by examining the angular conditions.

As discussed above, the constraints from universality and the angular conditions are in principle very useful for model building and a self-consistency check of theoretical or phenomenological models for spin-1 objects. However, these constraints do depend on the reference frame. For example, in the Breit frame where $q^+\neq 0$, a less informative prediction of asymptotic form factors is made [22] instead of Eq. (1.2):

$$G_C(Q^2):G_Q(Q^2)\to\frac{Q^2}{6m^2}:1 \qquad (1.6)$$

in the limit $Q\gg 2m$. Thus, it is important to examine the frame dependence of the constraints that are useful for model building and phenomenology.

In this work, we analyze the frame dependence of the angular conditions for spin-1 systems. Interestingly, in addition to the angular condition given by Eq. (1.4), we find another one. Elaborating the frame dependence of these angular conditions in the generalized Breit and target rest frames as well as the DYW frame, we confirm the advantage of using the DYW frame in the calculation of exclusive processes. The complexity of each angular condition in general depends on the reference frame. In the DYW frame, the extra angular condition is particularly simple so that most theoretical models are expected to satisfy it without any difficulty. We also substantiate that the angular conditions are identical in reference frames that are connected by kinematical transformations. Such an investigation is also important in analyzing the high-Q^2 behavior of spin-1 form factors. We confirm that the angular conditions are consistent with the high-Q^2 behavior predicted by perturbative QCD (PQCD) for the three physical form factors [5,9,10].

In the next section (Sec. II), the front-form (LFD) polarization vectors are presented in arbitrary frames. In Sec. III, we derive the relation between the current operator and the form factors and starting from general grounds obtain the most general angular conditions. We show that there are indeed two angular conditions and discuss the reason why they should be regarded as consistency conditions. In Sec. IV, we elaborate the details of the angular conditions in the DYW, generalized Breit, and target-rest frames. In Sec. V, we discuss the large momentum transfer behavior of the form factors in each reference frame. We also consider the limiting behavior of the form factors in approaching special Breit and target-rest frames. Conclusions follow in Sec. VI. In Appendix A, the front-form boost and helicity operators generating the polarization vectors used in this work are summarized. In Appendix B, the kinematical Lorentz transformations that connect the different frames are detailed in specific cases.

II. POLARIZATION VECTORS IN LIGHT-FRONT DYNAMICS

For the polarization vectors in three dimensions we use the standard spherical tensors for spin 1 [23]:

$$\vec{e}(0)=(0,0,1), \quad \vec{e}(\pm)=\mp\frac{1}{\sqrt{2}}(1,\pm i,0). \qquad (2.1)$$

We define the polarization vectors in a specific frame by boosting the four-vectors $(0,\vec{e}(M))$ to that specific frame. The vectors we obtain will depend on the Lorentz transformation. In the front form we need the kinematical front-form boosts. They are given in Appendix A.

We note that the LF components we use satisfy the following relations:

$$\vec{p}_\perp^{\,2}=-2p^rp^l, \quad p\cdot q=p^+q^-+p^-q^++p^rq^l+p^lq^r, \qquad (2.2)$$

where we use the spherical components of the three momentum vectors to simplify the notation. They are defined as follows:

$$p^r=-\frac{p_x+ip_y}{\sqrt{2}},\quad p^l=\frac{p_x-ip_y}{\sqrt{2}}. \tag{2.3}$$

Occasionally, we use the notation p^h with $h=+1,0,-1$ for p^r, p_z, and p^l, respectively. Then the usual relation for spherical tensors applies:

$$(p^h)^*=(-1)^h p^{-h}. \tag{2.4}$$

The polarization vectors in the rest system, where the four-momentum has the LF components $(p^+,p^1,p^2,p^-)=(m/\sqrt{2},0,0,m/\sqrt{2})$, are

$$\mathring{\varepsilon}_{\rm ff}(\pm)=(0,\mp 1/\sqrt{2},-i/\sqrt{2},0),\quad \mathring{\varepsilon}_{\rm ff}(0)=(1/\sqrt{2},0,0-1/\sqrt{2}). \tag{2.5}$$

Upon application of the front-form boost, Eq. (A5), we find the polarization vectors

$$\left.\begin{matrix}\varepsilon_{\rm ff}(p^+,p^l,p^2;+)\\ \varepsilon_{\rm ff}(p^+,p^1,p^2;0)\\ \varepsilon_{\rm ff}(p^+,p^1,p^2;-)\end{matrix}\right\}=\begin{cases}\left(0,\dfrac{-1}{\sqrt{2}},\dfrac{-i}{\sqrt{2}},\dfrac{p^r}{p^+}\right),\\ \left(\dfrac{p^+}{m},\dfrac{p^1}{m},\dfrac{p^2}{m},\dfrac{\vec{p}^{\perp 2}-m^2}{2mp^+}\right),\\ \left(0,\dfrac{1}{\sqrt{2}},\dfrac{-i}{\sqrt{2}},\dfrac{p^l}{p^+}\right).\end{cases} \tag{2.6}$$

It is easy to check that these are mutually orthogonal, transverse, and satisfy the closure property if one uses the front form for the metric.

III. CURRENTS

For a spin-1 particle the current operator has the form

$$J^\mu_{\alpha\beta}(p',p)=-g_{\alpha\beta}(p'+p)^\mu F_1(q^2)+(g^\mu_\beta q_\alpha-g^\mu_\alpha q_\beta)F_2(q^2)+\frac{q_\alpha q_\beta(p'+p)^\mu}{2m^2}F_3(q^2), \tag{3.1}$$

where the momenta p and p' are the momenta of the particle before and after absorption of a photon with momentum $q=p'-p$. The coefficient functions $F_i(Q^2)$ in Eq. (3.1) are given by the physical form factors: i.e.,

$$F_1=G_C-\frac{2}{3}\eta G_Q,$$

$$F_2=-G_M,$$

$$F_3=\frac{1}{1+\eta}\left[-G_C+G_M+\left(1+\frac{2}{3}\eta\right)G_Q\right]. \tag{3.2}$$

A spin tensor G is obtained by taking matrix elements with the polarization vectors, viz.,

$$G^\mu_{h'h}=\varepsilon^{*\alpha}(p';h')J^\mu_{\alpha\beta}\varepsilon^\beta(p;h). \tag{3.3}$$

This form can be derived on very general grounds. First, we write down all tensors of third rank that can be constructed using $g_{\alpha\beta}$, p'^μ, and p^μ alone. There are 14 possible structures. As the matrix elements are obtained by contracting with the polarization vectors $\varepsilon^{*\alpha}(p';h')$ and $\varepsilon^\beta(p;h)$, the structures containing a factor p'_α or p_β do not contribute to the matrix element. Therefore, only six remain and we write

$$J^\mu_{\alpha\beta}(p',q)=f_1 g_{\alpha\beta}p'^\mu+f_2 g_{\alpha\beta}p^\mu+f_3 g^\mu_\alpha p'_\beta+f_4 g^\mu_\beta p_\alpha+f_5 p'^\mu p_\alpha p'_\beta+f_6 p^\mu p_\alpha p'_\beta. \tag{3.4}$$

Second, we require current conservation, which means $q_\mu G^\mu_{h'h}(p',p)=0$ for all μ, h', and h. Contracting with q gives

$$0=(f_1-f_2)g_{\alpha\beta}(m^2-p'\cdot p)+f_3 q_\alpha p'_\beta+f_4 q_\beta p_\alpha+(f_5-f_6)p_\alpha p'_\beta(m^2-p'\cdot p). \tag{3.5}$$

We can immediately conclude that $f_1=f_2$ and $f_5=f_6$. In order to reduce the number of terms further, we again contract with the polarization vectors and see that

$$\varepsilon^*(p';h')\cdot q=-\varepsilon^*(p';h')\cdot p,\quad \varepsilon(p;h)\cdot q=\varepsilon(p;h)\cdot p'. \tag{3.6}$$

So we are left with the term $(f_4-f_3)p_\alpha p'_\beta$. This structure is independent of the one containing (f_5-f_6), because the latter originates from a term that contains the factor $p'^\mu+p^\mu$ while the former does not. So we conclude that $f_3=f_4$, which means that only three independent form factors remain.

Next we impose Hermiticity: i.e.,

$$\langle p'h'|J^\mu|ph\rangle=\langle ph|J^\mu|p'h'\rangle^*, \tag{3.7}$$

which gives, after some rearrangement,

$$\varepsilon^{*\alpha}(p';h')J^\mu_{\alpha\beta}(p',p)\varepsilon^\beta(p;h)=\varepsilon^{*\alpha}(p';h')J^{\mu *}_{\beta\alpha}(p,p')\varepsilon^\beta(p;h). \tag{3.8}$$

This is an identity for all p, p', h, and h', so we find

$$J^\mu_{\alpha\beta}(p',p)=J^{\mu *}_{\beta\alpha}(p,p'). \tag{3.9}$$

If we apply this identity to the structures we found, we see that the coefficients of the tensors must be real, which means that F_1, F_2, and F_3 in Eq. (3.1) are real.[1]

The symmetry of $J^\mu_{\alpha\beta}(p',p)$ entails relations between the matrix elements too. If we, in addition, apply $\vec{e}(h)^*=(-1)^h\vec{e}(-h)$, which we owe to the fact that the polarization vectors are standard spherical tensors, we can deduce

[1]Note that the kinematic region for this discussion is space like, i.e., $q^2<0$.

$$G^{\mu *}_{h'h}(p',p)=(-1)^{h'+h}G^{\mu}_{-h'-h}(p',p). \qquad (3.10)$$

The explicit expressions we are writing down in the next sections of course satisfy these identities.

Equation (3.1) can be split in an obvious way into the pieces $J(1)F_1$, $J(2)F_2$, and $J(3)F_3$. Then we find for the polarization tensor $G=G(1)F_1+G(2)F_2+G(3)F_3$ with the partial tensors

$$G^{\mu}_{h'h}(1)=-(p'+p)^{\mu}\varepsilon^*(p';h')\cdot\varepsilon(p;h),$$

$$G^{\mu}_{h'h}(2)=-p'\cdot\varepsilon(p;h)\varepsilon^{*\mu}(p';h') -p\cdot\varepsilon^*(p';h')\varepsilon^{\mu}(p;h),$$

$$G^{\mu}_{h'h}(3)=-\frac{(p'+p)^{\mu}}{2m^2}p'\cdot\varepsilon(p;h)p\cdot\varepsilon^*(p';h'). \qquad (3.11)$$

Clearly, we need three simple scalar products which we shall write in the front form only:

$$\varepsilon^*(p';0)\cdot\varepsilon(p;0)=\frac{p'^{+2}(\vec{p}_\perp^{\,2}-m^2)+p^{+2}(\vec{p}_\perp'^{\,2}-m^2)-2p'^{+}p^{+}\vec{p}'_\perp\cdot\vec{p}_\perp}{2m^2p'^{+}p^{+}},$$

$$\varepsilon^*(p';0)\cdot\varepsilon(p;h)=\frac{p'^{+}p^{h}-p^{+}p'^{h}}{mp^{+}},$$

$$\varepsilon^*(p';h')\cdot\varepsilon(p;h)=-\frac{1+h'h}{2},$$

$$\varepsilon^*(p';0)\cdot p=\frac{p'^{+2}\vec{p}_\perp^{\,2}+p^{+2}\vec{p}_\perp'^{\,2}+m^2(p'^{+2}-p^{+2})-2p'^{+}p^{+}\vec{p}'_\perp\cdot\vec{p}_\perp}{2mp'^{+}p^{+}},$$

$$\varepsilon^*(p';h)\cdot p=\frac{p'^{+}p^{-h}-p^{+}p'^{-h}}{p'^{+}}, \qquad (3.12)$$

where we made the obvious identification $p^{h=+1}\leftrightarrow p^r$, $p^{h=-1}\leftrightarrow p^l$.

Using these expressions, the matrix elements of the polarization tensors can be easily found. Hermiticity follows from the simultaneous replacements $p\leftrightarrow p'$ and $p^l\leftrightarrow -p^r$.

A. Symmetries of the polarization tensor

The formulas above tell us that the polarization tensor has the following forms

$$G(i)=\begin{pmatrix} a_i & c_i & e_i^* \\ b_i & d_i & -b_i^* \\ e_i & -c_i^* & a_i \end{pmatrix}, \qquad (3.13)$$

which is valid for all three contributions $G(i)$, $i=1,2,3$. Using an obvious notation, we find for the complete polarization tensor the form

$$G=\begin{pmatrix} a_1F_1+a_3F_3 & c_1F_1+c_2F_2+c_3F_3 & e_3^*F_3 \\ b_1F_1+b_2F_2+b_3F_3 & d_1F_1+d_2F_2+d_3F_3 & -(b_1F_1+b_2F_2+b_3F_3)^* \\ e_3F_3 & -(c_1F_1+c_2F_2+c_3F_3)^* & a_1F_1+a_3F_3 \end{pmatrix}$$

$$=\begin{pmatrix} G^+_{++} & G^+_{+0} & G^+_{+-} \\ G^+_{0+} & G^+_{00} & G^+_{0-} \\ G^+_{-+} & G^+_{-0} & G^+_{--} \end{pmatrix}. \qquad (3.14)$$

Apparently, the tensor components we obtain here satisfy an additional identity

$$G^+_{++}=G^+_{--}=G^{+*}_{++}. \qquad (3.15)$$

This result is specific for the choice of μ: it is true for the good current J^+, but does not apply to the terrible current J^-. The matrix elements G^-_{++} and G^-_{--} are not real, but they are complex conjugates.

Using the explicit expressions, we see that the nine matrix elements of G have four relations that involve a phase factor only, viz.

$$G^+_{++}=G^+_{--}, \quad G^+_{+-}=G^{+*}_{-+}, \quad G^+_{0-}=-G^{+*}_{0+},$$
$$G^+_{+0}=-G^{+*}_{-0}. \tag{3.16}$$

We need two more equations that express the fact that there are only three independent form factors. These consistency conditions are the two angular conditions proper. Since we are working only with the + component of the current, we shall use the following shorthand notation:

$$G_a=G^+_{++}=G^{+*}_{--}, \quad G_b=G^+_{0+}=-G^{+*}_{0-},$$
$$G_c=G^+_{+0}=-G^{+*}_{-0}, \quad G_d=G^+_{00},$$
$$G_e=G^+_{-+}=G^{+*}_{+-}. \tag{3.17}$$

We can now solve for F_i in an obvious way. First, we obtain F_3 from G_e, then F_1 from G_a and F_3. Then, we have a choice whether we want to obtain F_2 from G_b, G_c, or G_d; these solutions we denote by F_2^b, F_2^c, and F_2^d, respectively. As these results must coincide, the identity of these three results form the angular conditions: $F_2^b=F_2^c=F_2^d$. We find

$$F_1=\frac{1}{a_1}G_a-\frac{a_3}{a_1e_3}G_e,$$
$$F_3=\frac{1}{e_3}G_e,$$
$$F_2^b=\frac{1}{b_2}\left[-\frac{b_1}{a_1}G_a+G_b+\frac{a_3b_1-a_1b_3}{a_1e_3}G_e\right],$$
$$F_2^c=\frac{1}{c_2}\left[-\frac{c_1}{a_1}G_a+G_c+\frac{a_3c_1-a_1c_3}{a_1e_3}G_e\right],$$
$$F_2^d=\frac{1}{d_2}\left[-\frac{d_1}{a_1}G_a+G_d+\frac{a_3d_1-a_1d_3}{a_1e_3}G_e\right]. \tag{3.18}$$

The relations, Eq. (3.10), reduce the nine complex elements of the polarization tensor to nine real numbers. As there are only three real independent form factors, we need six linear relations to realize the reduction from nine to three. The equations above serve this purpose. By equating the real and imaginary parts of the two sides of the first three of Eqs. (3.18), we find six relations that must hold for the components of $G^{\mu}_{h'h}$. Having thus achieved the reduction to the minimum number of independent functions, the other equations must be considered to be *consistency conditions*. As the three equations expressing F_2 in terms of the tensor components are not independent, but form a system of rank 2 only one complex equation or two real ones remain.

In the literature usually only one is given, said to be the *angular condition*. From our considerations it must be clear that there are indeed two conditions.

B. Angular conditions

The angular conditions (AC) can now be formulated succinctly:

$$F_2^b=F_2^c, \quad F_2^b=F_2^d, \quad F_2^c=F_2^d. \tag{3.19}$$

We shall write these conditions explicitly for unspecified kinematics.

The first one, denoted henceforth by AC 1, is

$$F_2^b-F_2^c=0$$
$$=\frac{p'^+-p^+}{p'^+p^+}\left[G_a+\frac{m^2}{2}\frac{(p'^++p^+)^2}{(p'^+p^r-p^+p'^r)^2}G_e\right]$$
$$-m\frac{p^+}{p'^+}\frac{1}{p'^+p^r-p^+p'^r}G_b$$
$$+m\frac{p'^+}{p^+}\frac{1}{p'^+p^l-p^+p'^l}G_c. \tag{3.20}$$

The second one, AC 2, is

$$F_2^b-F_2^d=0$$
$$=\left[-\frac{1}{p'^+}+\frac{m^2(p'^{+2}+p^{+2})+2(p'^+p^r-p^+p'^r)(p'^+p^l-p^+p'^l)}{(p'^++p^+)[m^2(p'^+-p^+)^2+2(p'^+p^r-p^+p'^r)(p'^+p^l-p^+p'^l)]}\right]G_a-m\frac{p^+}{p'^+}\frac{1}{p'^+p^r-p^+p'^r}G_b$$
$$-\frac{2m^2p'^+p^+}{p'^++p^+}\frac{1}{m^2(p'^+-p^+)^2+2(p'^+p^r-p^+p'^r)(p'^+p^l-p^+p'^l)}G_d+\left[\frac{m^2}{2p'^+}\frac{p'^{+2}-p^{+2}}{(p'^+p^r-p^+p'^r)^2}\right.$$
$$\left.-\frac{m^2}{2(p'^++p^+)(p'^+p^r-p^+p'^r)^2}\frac{m^2(p'^{+2}-p^{+2})^2+2(p'^{+2}+p^{+2})(p'^+p^r-p^+p'^r)(p'^+p^l-p^+p'^l)}{[m^2(p'^+-p^+)^2+2(p'^+p^r-p^+p'^r)(p'^+p^l-p^+p'^l)]}\right]G_e. \tag{3.21}$$

The last one is

$$
\begin{aligned}
F_2^c - F_2^d = 0 \\
&= \left[-\frac{1}{p^+} + \frac{m^2(p'^{+2}+p^{+2}) + 2(p'^+p^l - p^+p'^l)(p'^+p^l - p^+p'^l)}{(p'^+ + p^+)[m^2(p'^+ - p^+)^2 + 2(p'^+p^r - p^+p'^r)(p'^+p^l - p^+p'^l)]} \right] G_a - m\frac{p'^+}{p^+}\frac{1}{p'^+p^r - p^+p'^r} G_c \\
&\quad - \frac{2m^2p'^+p^+}{p'^+ + p^+}\frac{1}{m^2(p'^+ - p^+)^2 + 2(p'^+p^r - p^+p'^r)(p'^+p^l - p^+p'^l)} G_d \\
&\quad + \left[-\frac{m^2}{2p^+}\frac{p'^{+2} - p^{+2}}{(p'^+p^r - p^+p'^r)^2} - \frac{m^2}{2(p'^+p^+)(p'^+p^r - p^+p'^r)^2} \right. \\
&\quad \left. \times \frac{m^2(p'^{+2} - p^{+2})^2 + 2(p'^{+2} + p^{+2})(p'^+p^r - p^+p'^r)(p'^+p^l - p^+p'^l)}{[m^2(p'^+ - p^+)^2 + 2(p'^+p^r - p^+p'^r)(p'^+p^l - p^+p'^l)]} \right] G_e . \qquad (3.22)
\end{aligned}
$$

If we substitute Eq. (3.20) into Eq. (3.21), we see that it is equivalent to Eq. (3.22), as it must be, because these equations are not independent as there are only two independent angular conditions.

Clearly, these conditions are quite complicated. We can simplify them by factoring out some common factors, at the same time avoiding denominators that may vanish. Instead of Eqs. (3.20),(3.21) we get the conditions AC 1,

$$
\begin{aligned}
&2(p'^+ - p^+)(p'^+p^r - p^+p'^r)^2(p'^+p^l - p^+p'^l)G_a - 2mp^{+2}(p'^+p^r - p^+p'^r)(p'^+p^l - p^+p'^l)G_b \\
&+ 2mp'^{+2}(p'^+p^{r'} - p^+p'^r)^2 G_c + m^2(p'^+ - p^+)(p'^+ + p^+)^2(p'^+p^l - p^+p'^l)G_e = 0 \qquad (3.23)
\end{aligned}
$$

and AC 2,

$$
\begin{aligned}
&2(p'^+p^r - p^+p'^r)^2[m^2(p^{+2} - 2p'^+p^+ - p'^{+2}) + 2(p'^+p^r - p^+p'^r)(p'^+p^l - p^+p'^l)]G_a + 2m(p'^+ + p^+)(p'^+p^r - p^+p'^r) \\
&\times [m^2(p'^+ - p^+)^2 + 2(p'^+p^r - p^+p'^r)(p'^+p^l - p^+p'^l)]G_b + 4m^2p'^{+2}(p'^+p^r - p^+p'^r)^2 G_d + m^2[m^2(p'^{+2} - p^{+2})^2 \\
&+ 2(p^{+2} + 2p'^+p^+ - p'^{+2})(p'^+p^r - p^+p'^r)(p'^+p^l - p^+p'^l)]G_e = 0. \qquad (3.24)
\end{aligned}
$$

Clearly, these conditions are minimal, as we cannot eliminate any of the five tensor components to obtain a simpler one.

It is useful to realize the phase relations that occur. In addition to the relations expressed in Eqs. (3.13), (3.16) we can use the fact that $(p^l)^* = -p^r$ and the fact that G_a and G_d are real to infer that both angular conditions have the form

$$
C_a G_a + C_b e^{-i\phi} G_b + C_c e^{i\phi} G_c + C_d G_d + C_e e^{-2i\phi} G_e = 0, \qquad (3.25)
$$

where the coefficients $C_a, \ldots, C_e$ are real and ϕ is the argument of the complex number $p'^+p^r - p^+p'^r$, given by

$$
\tan\phi = \frac{p^+p'_y - p'^+p_y}{p^+p'_x - p'^+p_x}. \qquad (3.26)
$$

This angle can be set to zero by a rotation of the reference frame about the z axis. This rotation being kinematical in LFD, we may expect the phase relations to be satisfied always.

It may turn out for some kinematics that these relations simplify. This happens to be the case in, e.g., the DYW frame, where $p'^+ = p^+$ and $\vec{p}_\perp = 0$. Moreover, when $\vec{q}$ is purely longitudinal, i.e., $\vec{q}_\perp = 0$, we can rotate the reference frame such that $\vec{p}_\perp = \vec{p}'_\perp = 0$. Then, both angular conditions are identically satisfied, as all coefficients vanish.

IV. SPECIFIC FRAMES

We consider three specific frames: the Drell-Yan-West frame, Breit frame, and target-rest frame (TRF). For simplicity, only the kinematics and the angular conditions in the form $F_2^b - F_2^c = 0$ (AC 1) and $F_2^b - F_2^d = 0$ (AC 2) are presented in this section.

A. Drell-Yan-West frame

1. Kinematics

For the DYW frame,

$$
\begin{aligned}
p &= (p^+, 0, 0, m^2/(2p^+)), \\
q &= (0, q_x, q_y, \vec{q}_\perp^{\,2}/(2p^+)), \\
p' &= p + q \\
&= (p^+, q_x, q_y, (\vec{q}_\perp^{\,2} + m^2)/(2p^+)), \qquad (4.1)
\end{aligned}
$$

with the identification $q_x = Q\cos\phi$, $q_y = Q\sin\phi$, one finds the explicit formulas

$$p=(p^+,0,0,m^2/(2p^+)),$$
$$q=(0,Q\cos\phi,Q\sin\phi,Q^2/(2p^+)),$$
$$p'=(p^+,Q\cos\phi,Q\sin\phi,(Q^2+m^2)/(2p^+)), \tag{4.2}$$

and

$$q^r=-\frac{Q}{\sqrt{2}}e^{i\phi},\quad q^l=\frac{Q}{\sqrt{2}}e^{-i\phi}. \tag{4.3}$$

2. Angular conditions

We write the angular conditions mentioned in the previous section.

AC 1:

$$e^{-i\phi}G_b+e^{i\phi}G_c=0. \tag{4.4}$$

AC 2:

$$(2m^2+Q^2)G_a+2\sqrt{2}mQe^{-i\phi}G_b-2m^2G_d+2m^2e^{-2i\phi}G_e=0. \tag{4.5}$$

B. Breit frame

1. Kinematics

We define the quantity β as

$$\beta=\sqrt{1+\left(\frac{Q}{2m}\right)^2}. \tag{4.6}$$

Then,

$$p=\left(\frac{2m\beta-Q\cos\theta}{2\sqrt{2}},-\frac{Q\sin\theta\cos\phi}{2},-\frac{Q\sin\theta\sin\phi}{2},\frac{2m\beta+Q\cos\theta}{2\sqrt{2}}\right),$$
$$p'=\left(\frac{2m\beta+Q\cos\theta}{2\sqrt{2}},\frac{Q\sin\theta\cos\phi}{2},\frac{Q\sin\theta\sin\phi}{2},\frac{2m\beta-Q\cos\theta}{2\sqrt{2}}\right),$$
$$q=\left(\frac{Q\cos\theta}{\sqrt{2}},Q\sin\theta\cos\phi,Q\sin\theta\sin\phi,-\frac{Q\cos\theta}{\sqrt{2}}\right). \tag{4.7}$$

2. Angular conditions

By now, we give only the two linearly independent conditions. We simplify the expressions as much as possible by dividing out common factors to find the two conditions.

AC 1:

$$-2\sqrt{2}\beta Q^2\cos\theta\sin^2\theta G_a+(2\beta m-Q\cos\theta)^2\sin\theta e^{-i\phi}G_b+(2\beta m+Q\cos\theta)^2\sin\theta e^{i\phi}G_c-8\sqrt{2}\beta m^2\cos\theta e^{-2i\phi}G_e=0. \tag{4.8}$$

AC 2:

$$-[4\beta mQ\cos\theta-Q^2\cos^2\theta+2\beta^2(2m^2+Q^2\sin^2\theta)]\sin^2\theta G_a-4\sqrt{2}mQ(\beta^2\sin^2\theta-\cos^2\theta)\sin\theta e^{-i\phi}G_b+(2\beta m+Q\cos\theta)^2\sin^2\theta G_d+[(8m^2+Q^2\sin^2\theta)\cos^2\theta+4\beta mQ\cos\theta\sin^2\theta-4\beta^2m^2\sin^2\theta]e^{-2i\phi}G_e=0. \tag{4.9}$$

We note that Eqs. (4.8) and (4.9) are reduced to results in the DYW frame, Eqs. (4.4) and (4.5), respectively, if $\theta=\pi/2$ as they should, because the two frames are related by a kinematical transformation in that case and the angular conditions do not change under any kinematical transformation.

C. Target-rest frame

1. Kinematics

Using again β and κ, defined as

$$\kappa=\frac{Q^2}{2m}, \tag{4.10}$$

we find

$$p=\left(\frac{m}{\sqrt{2}},0,0,\frac{m}{\sqrt{2}}\right).$$

$$q=\left(\frac{\kappa+\beta Q\cos\theta}{\sqrt{2}},\beta Q\sin\theta\cos\phi,\beta Q\sin\theta\sin\phi,\frac{\kappa-\beta Q\cos\theta}{\sqrt{2}}\right),$$

$$p'=p+q=\left(\frac{m+\kappa+\beta Q\cos\theta}{\sqrt{2}},\beta Q\sin\theta\cos\phi,\beta Q\sin\theta\sin\phi,\frac{m+\kappa-\beta Q\cos\theta}{\sqrt{2}}\right). \tag{4.11}$$

2. Angular conditions

We give again only the two conditions after simplification by dividing out as many common factors as possible.

AC 1:

$$-\beta^2Q^2(\kappa+\beta Q\cos\theta)\sin^2\theta G_a+\sqrt{2}\beta m^2Q\sin\theta e^{-i\phi}G_b+\sqrt{2}\beta Q(m+\kappa+\beta Q\cos\theta)^2\sin\theta e^{i\phi}G_c$$
$$-(\kappa+\beta Q\cos\theta)(2m+\kappa+\beta Q\cos\theta)^2e^{-2i\phi}G_e=0. \tag{4.12}$$

AC 2:

$$-\beta^2Q^2[\kappa^2+4\kappa m+2m^2+\beta^2Q^2+2\beta(2m+\kappa)Q\cos\theta]\sin^2\theta G_a+\sqrt{2}\beta Q(2m+\kappa+\beta Q\cos\theta)$$
$$\times[\kappa^2+2\beta\kappa Q\cos\theta+\beta^2Q^2\cos 2\theta]\sin\theta e^{-i\phi}G_b+2\beta^2Q^2(m+\kappa+\beta Q\cos\theta)^2\sin^2\theta G_d$$
$$+[(\kappa+\beta Q\cos\theta)^2(2m+\kappa+\beta Q\cos\theta)^2+\beta^2Q^2(\kappa^2-2m^2+2\beta\kappa Q\cos\theta+\beta^2Q^2\cos^2\theta)]\sin^2\theta e^{-2i\phi}G_e=0. \tag{4.13}$$

We note that Eqs. (4.12) and (4.13) are identical to Eqs. (4.4) and (4.5) if $\beta\sin\theta=1$.

V. LIMITING CASES

In order to be able to interprete the angular conditions, we studied the dependence on Q in the limits $Q\to 0$ and $Q\to\infty$. We shall use the notation

$$\text{AC } 1\Leftrightarrow R_a^1G_a+R_b^1G_b+R_c^1G_c+R_e^1G_e=0,$$

$$\text{AC } 2\Leftrightarrow R_a^2G_a+R_b^2G_b+R_d^2G_d+R_e^2G_e=0. \tag{5.1}$$

A. $Q\to 0$ limit

Using the definition of the physical form factors at $Q^2=0$, i.e.,

$$eG_C(0)=e,\quad eG_M(0)=2m\mu_1,\quad eG_Q(0)=m^2Q_1, \tag{5.2}$$

we find, from Eq. (3.2),

$$F_1(0)=1,\quad F_2(0)=-\frac{2m\mu_1}{e},$$

$$F_3(0)=-1+\frac{2m\mu_1}{e}+\frac{m^2Q_1}{e}. \tag{5.3}$$

According to the universality condition given by Eq. (1.1), in the limit of bound-state radius $R\to 0$ the form factors $F_i(0)$ for $i=2,3$ are reduced to

$$F_2(0)=-2,\quad F_3(0)=0. \tag{5.4}$$

Since the target is intact in the $Q\to 0$ limit, $p^\mu=p'^\mu$ and thus we find $G_a=G_d$ or $G_{++}^+=G_{00}^+$. All other spin-flip amplitudes vanish in this limit regardless of reference frames. This can be understood because the spin would not flip if the target is intact and also the direction of spin would not matter in this limit. Moreover, all the coefficients (R_a^i, etc.) in Eq. (5.1) vanish in $Q\to 0$ limit, and thus both angular conditions AC 1 and AC 2 are trivially satisfied.

B. Behavior for $Q\to\infty$

Imposing a naturalness condition—namely, all three terms in Eq. (3.1) should be of the same order in Q—one can find that the form factors $F_i(Q^2)$ behave as $F_1(Q^2)\sim F_2(Q^2)\sim(Q^2/m^2)F_3(Q^2)$ in the large-Q^2 limit. Using this, we can derive the high-Q^2 behaviors of the helicity amplitudes $G_{h'h}^+$ and the coefficients (R_a^i, etc.) of the angular conditions. In Table I, we summarize the results.

As we can see from Table I, the high-Q behavior of each helicity amplitude in general depends on the reference frame. This is so because the helicities and components of the current do mix in general, although the physical form factors are of course identical for any Q regardless of the reference frame. Only in frames connected by a kinematic transforma-

TABLE I. Leading behavior for $Q\to\infty$ of the tensor components $G_a,\ldots,G_e$, and the coefficients $R_a^1,\ldots,R_e^2$ in the different reference frames considered. The Breit frame (BF) and TRF are kinematically connected to the DYW frame only in particular angles $\theta_{\rm BF}=\pi/2$ and $\theta_{\rm TRF}=\theta_0=\sin^{-1}(1/\beta)$, respectively.

		$Q\to\infty$					
	DYW	Breit			TRF		
		$\theta\neq 0,\pi/2$	$\theta=\pi/2$	$\theta=0$	$\theta\neq 0,\theta_0$	$\theta=0$	$\theta=\theta_0$
G_a	1	Q	Q	Q	Q^2	Q^2	1
G_b	Q	Q^2	Q^2	0	Q^4	0	Q
G_c	Q	Q^2	Q^2	0	Q^2	0	Q
G_d	Q^2	Q^3	Q^3	Q^3	Q^4	Q^4	Q^2
G_e	1	Q	Q	0	Q^2	0	1
R_a^1	0	Q^3	0	0	Q^6	0	0
R_b^1	1	Q^2	Q^2	0	Q^2	0	Q
R_c^1	1	Q^2	Q^2	0	Q^6	0	Q
R_e^1	0	Q	0	Q	Q^6	Q^6	0
R_a^2	Q^2	Q^4	Q^4	0	Q^8	0	Q^4
R_b^2	Q	Q^3	Q^3	0	Q^8	0	Q^3
R_d^2	1	Q^2	Q^2	0	Q^8	0	Q^2
R_e^2	1	Q^2	Q^2	1	Q^8	0	Q^2

tion that keeps the light-front time $\tau=t+z/c$ $(=0)$ invariant are the helicity amplitudes $G^+_{h'h}$ the same [24]. Indeed, our results summarized in Table I are essentially identical in kinematically connected frames such as DYW, Breit ($\theta=\pi/2$), and TRF ($\theta=\theta_0$).[2] Note that $\theta_0\to\pi$ in the limit $Q\to\infty$. It is interesting to find that in all cases the behavior of the helicity amplitudes in these frames is consistent with the perturbative QCD predictions obtained in the $q^+=0$ frame. Indeed, PQCD predicts [25] that the helicity-0 to helicity-0 matrix element G^+_{00} (or G_d) is the dominant helicity amplitude at large Q^2 [5]. For example, in the deuteron form factor [9] calculation using the factorization theorem of PQCD one can show that the five intermediate gluons connecting the six quarks can be arranged in such a way that the gluon polarizations and quark helicities alternate to allow a maximum amplitude when the initial helicity-0 state transits to the final helicity-0 state. Further, in the $q^+=0$ frame, PQCD predicts that the helicity-flip amplitudes $G^+_{+0}(G_c)$ and $G^+_{+-}(G_e)$ are suppressed by factors of Q^1 and Q^2, respectively:

$$G_c=a\,\frac{\Lambda_{\rm QCD}}{Q}G_d \quad G_e=b\left(\frac{\Lambda_{\rm QCD}}{Q}\right)^2 G_d, \tag{5.5}$$

where a and b are constants and there are also corrections of order $\Lambda_{\rm QCD}/m$ [5,22]. Our results, based on the naturalness condition, coincide with these PQCD predictions. From the table, we also find that $G^+_{++}(G_a)$ should be suppressed by two powers of Q compare to the dominant G^+_{00} in the high-Q limit. However, neither our analysis nor PQCD can fix the constants a and b. Both angular conditions, AC 1 and AC 2, are satisfied independent of a and b. Thus both angular conditions are consistent with the PQCD predictions.

On the other hand, in the frames that are not connected to DYW by a kinematical transformation the results are not consistent with the PQCD predictions as one can see from Table I. Since there are contributions from embedded states [26] in $q^+\neq 0$ frames, there are no reasons why they should be consistent with the leading-order PQCD predictions. Nevertheless, it is interesting to note that G_d dominates regardless of the reference frame. We now discuss some details of AC 1 and AC 2 in each reference frame.

1. Drell-Yan-West frame

The first angular condition, AC 1, is simple. It reads

$$e^{-i\phi}G_b+e^{i\phi}G_c=0. \tag{5.6}$$

The leading-Q behavior of the left-hand side (LHS) of AC 1 is

$$\frac{m}{Q}(R_b^1G_b+R_c^1G_c)\overset{Q\to\infty}{\sim}-\frac{p^+}{2\sqrt{2}}\left(4F_1+2F_2+\frac{m^2}{Q^2}F_3\right)+\frac{p^+}{2\sqrt{2}}\left(4F_1+2F_2+\frac{m^2}{Q^2}F_3\right). \tag{5.7}$$

So if we assume $F_3\overset{Q\to\infty}{\sim}(Q^2/m^2)H_3$ and F_1, F_2, and H_3 are of the same order in Q^2 for $Q\to\infty$, then both terms are equal in magnitude.

AC 2 is more involved, but still easy. Its LHS behaves for $Q\to\infty$ to leading order as follows:

[2]The reason for an extra power Q for the Breit ($\theta=\pi/2$) and TRF ($\theta=\theta_0$) frames in comparison to DYW can be understood by the kinematic factors in the relation between (G_C,G_M,G_Q) and $G^+_{h'h}$.

$$\frac{m^2}{Q^2}(R_a^2G_a+R_b^2G_b+R_d^2G_d+R_e^2G_e)\overset{Q\to\infty}{\sim} m^2p^+\left[\frac{1}{2}(4F_1+H_3)-(4F_1+2F_2+H_3)+\frac{1}{2}(4F_1+4F_2+H_3)\right]. \tag{5.8}$$

The term involving G_e does not contribute in leading order.

2. Breit frame

First AC 1. We multiply by $(m/Q)^4$:

$$\begin{aligned}\frac{m^4}{Q^4}(R_a^1G_a+R_b^1G_b+R_c^1G_c+R_e^1G_e)\overset{Q\to\infty}{\sim} m^3\Bigg[&-\frac{1}{4}(4F_1+H_3)\sin^2\theta\cos\theta\\&-\frac{(4F_1+4F_2+H_3)\cos\theta+\{4F_1+F_2(3+\cos 2\theta)+H_3\}}{8(1+\cos\theta)^2}\sin^4\theta\\&-\frac{(4F_1+4F_2+H_3)\cos\theta-\{4F_1+F_2(3+\cos\theta)+H_3\}}{8(1-\cos\theta)^2}\sin^4\theta\Bigg].\end{aligned} \tag{5.9}$$

Actually, $R_e^1G_e$ is two orders Q/m down compared to the other three terms. The contributions of the three terms that remain in leading order will depend on the angle θ. For example, for $\theta=0$ all vanish identically and we find that then the leading order is lower than $(Q/m)^4$. For $\theta=\pi/2$ only the terms $R_b^1G_b$ and $R_c^1G_c$ survive and cancel each other.

The leading order of AC 2 is $(Q/m)^5$. We find

$$\begin{aligned}\frac{m^5}{Q^5}(R_a^2G_a+R_b^2G_b+R_d^2G_d+R_e^2G_e)\overset{Q\to\infty}{\sim} m^3\Bigg[&-\frac{4F_1+H_3}{8\sqrt{2}}\sin^4\theta+\frac{4F_1+2F_2(1+\cos\theta)+H_3}{4\sqrt{2}(1-\cos\theta)^2(1+\cos\theta)}\sin^6\theta\\&-\frac{(4F_1+4F_2+H_3)(3-4\cos\theta+\cos 2\theta)}{16\sqrt{2}(1-\cos\theta)^4}\sin^6\theta\Bigg],\end{aligned} \tag{5.10}$$

and again the term with G_e is not of leading order. For $\theta\to 0$, the first term is of order θ^4, while the two others are of order θ^2 and cancel each other exactly at this order. So for small θ the contributions of G_b and G_d dominate. For $\theta=\pi/2$, all three terms are of the same order. This situation corresponds exactly with AC 2 in the DYW frame.

3. Target rest frame

Since the leading term in AC 1 is of order $(Q/m)^8$, we multiply it with $(m/Q)^8$ and find

$$\begin{aligned}\frac{m^8}{Q^8}(R_a^1G_a+R_b^1G_b+R_c^1G_c+R_e^1G_e)\overset{Q\to\infty}{\sim}&\frac{m^4\sin^2\theta}{512\sqrt{2}}[(-48-64\cos\theta-16\cos 2\theta)F_1+(-2+2\cos 2\theta-2\cos\theta\\&+2\cos\theta\cos 2\theta)H_3+(48+64\cos\theta+16\cos 2\theta)F_1\\&+(-12\cos\theta-4\cos\theta\cos 2\theta-16\cos^2\theta)H_3\\&+(10+15\cos\theta+6\cos 2\theta+\cos 3\theta)H_3].\end{aligned} \tag{5.11}$$

The contribution from G_e is not of leading order. The other three terms are comparable in size, but the details depend on the angle θ.

AC 2 is different, as only G_b and G_d contribute in leading order, which is $(Q/m)^{12}$. We find

$$\frac{m^{12}}{Q^{12}}(R_b^2 G_b + R_d^2 G_d) \overset{Q\to\infty}{\sim} \frac{m^5 \sin^2\theta}{256\sqrt{2}}[-(4F_1+4F_2+H_3)+(4F_1+4F_2+H_3)]\cos\theta(1+\cos\theta)(3+4\cos\theta+\cos 2\theta). \tag{5.12}$$

VI. CONCLUSIONS

In this work we elaborated the frame dependence of the angular conditions for spin-1 systems. We found that there is an additional angular condition in addition to the well-known one given by Eq. (1.4). In the $q^+=0$ frame including DYW, Breit ($\theta=\pi/2$), and TRF ($\theta=\theta_0$), we find that the additional condition is very simple involving only two helicity amplitudes as shown in Eq. (4.4) and most quark models rather easily satisfy it. Thus it does not seem to provide as strong a constraint as the usual condition, Eq. (4.5). However, in $q^+\neq 0$ frames, the additional condition is generally as complicated as the usual one. Since the $q^+=0$ frame (e.g., DYW) is in principle restricted to the spacelike region of the form factors, it may be useful to impose the additional condition in processes involving the timelike region. As a result of the recent development [27] of the effective treatment in timelike exclusive processes, we can see that the range of applicability for the angular conditions in $q^+\neq 0$ frames is quite broad. Nevertheless, it seems rather clear from our spin-1 form factor discussion that the analysis of exclusive processes is greatly simplified in the DYW frame and in general $q^+=0$ frames. We note that the angular conditions given by Eqs. (4.4) and (4.5) are identical in any frame connected to the DYW frame by kinematical transformations.

We also find that both angular conditions in the $q^+=0$ frame are consistent with the PQCD predictions. Our predictions for the Q dependence of the helicity amplitudes based on the naturalness condition as well as the angular condition are also consistent with the PQCD predictions given by Eq. (5.5). However, the proportionality constants a and b can be fixed neither by our analysis nor by PQCD. Some other inputs such as experimental data are needed to find these values. For example, in the deuteron analysis a value near 5 was obtained for a [28]. Nevertheless, it is interesting to note that for some particular values of a and b the relations among F_1, F_2, and F_3 are greatly simplified. For $a=b=0$, we find that $F_2/F_1=-2$ and $F_3/F_1=0$, which are identical to Eq. (5.4) for a point particle. Since the form factors for a point particle do not depend on Q^2 at the tree level, one can understand this universality result rather easily. Also, for $a=\sqrt{2}m/\Lambda_{\rm QCD}$ and $b=2m^2/\Lambda_{\rm QCD}^2$ we find that $F_2/F_1=-1$ and $F_3/F_1=-1$. Even though the results are simple for these particular values of a and b, it is not yet clear what their importance is. In order to analyze the values of a and b, one may need to have some sort of bound-state information for the spin-1 system. Work along this line, using a simple but exactly solvable model, is in progress.

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APPENDIX A: CONVENTIONS OF POLARIZATION VECTORS AND LF BOOST AND HELICITY OPERATORS

We note that sometimes in the literature calculations are performed using LF dynamics, but at the same time employing instant-form polarization vectors. In this work, we have used the LF polarization vectors [see Eq. (2.6)] generated by the LF boost and helicity operators [29,30] briefly summarized below.

In order to define the conventions in this work, we define the front-form components $p_{\rm ff}^\mu=(p^+,p^1,p^2,p^-)$ with the definition

$$p^\pm=\frac{p^0\pm p^3}{\sqrt{2}}. \tag{A1}$$

The metric tensor $g_{\rm ff}$ is then

$$g_{\rm ff}=\eta g\,\eta=\begin{pmatrix}0&0&0&1\\0&-1&0&0\\0&0&-1&0\\1&0&0&0\end{pmatrix}. \tag{A2}$$

The kinematical front-form boost is given by

$$L_{\rm ff}(\vec{v}_\perp;\chi)=\exp(-i\sqrt{2}\vec{v}^\perp\cdot\vec{E}^\perp)\exp(-i\chi K^3), \tag{A3}$$

where $K^3=M^{-+}$ is the third component of the boost generator and the generators E^1 and E^2 are given by

$$E^1=M^{+1}=\frac{1}{\sqrt{2}}(K^1+J^2),\quad E^2=M^{+2}=\frac{1}{\sqrt{2}}(K^2-J^1). \tag{A4}$$

In the same convention, the LF helicity operator is given by

$$h_{\rm ff}=\exp(-i\sqrt{2}\vec{v}^\perp\cdot\vec{E}^\perp)J^3\exp(i\sqrt{2}\vec{v}^\perp\cdot\vec{E}^\perp). \tag{A5}$$

One can write it in operator form as

$$h_{\rm ff}=\frac{W^+}{P^+}=J^3-\frac{P^1E^2-P^2E^1}{P^+}. \tag{A6}$$

This operator is clearly a kinematic one, as J^3, P^1, P^2, E^1, and E^2 all belong to the stability group of $x^+=0$. The helicity has the eigenvectors $\varepsilon_{\rm ff}(h)$ with $h=0,\pm 1$, Eq. (2.6), and a fourth eigenvector (0,0,0,1). The latter does not correspond to a genuine polarization vector. It has only a minus component, which means that it is orthogonal to all four vectors with $p^-=0$, i.e., $p^+\to\infty$.

APPENDIX B: SYMMETRIES OF FRAMES AND RELATIONS BETWEEN DIFFERENT FRAMES

In this section we give the kinematical Lorentz transformations that connect the different frames in specific cases. We stress that the frames can be transformed into each other by general Lorentz transformation, but only in special cases can this be done using elements from the kinematical subgroup alone.

The kinematical group is generated by J^3, K^3, and E^1 and E^2. As all frames are invariant under rotations about the z axis, we shall not discuss J^3. We can use this kinematical rotation to remove the ϕ dependence of the angular conditions. The interesting transformations are $L_{\rm ff}(0;\chi)$ and $L_{\rm ff}(\vec{v}_\perp;0)$.

1. Symmetries of frames

a. Boosts along the z axis

$L_{\rm ff}(0;\chi)$ is a symmetry of the Drell-Yan-West frame, but not of the Breit frame or target rest frame.

b. Transverse boosts

We write $L_{\rm ff}(\vec{v}_\perp;0)$ explicitly

$$L_{\rm ff}(\vec{v}_\perp;0)=\begin{pmatrix} 1 & 0 & 0 \\ \sqrt{2}\vec{v}_\perp & 1 & 0 \\ \vec{v}_\perp^{\,2} & \sqrt{2}\vec{v}_\perp & 1 \end{pmatrix}. \tag{B1}$$

The transverse boosts are not symmetries of the target rest frame.

If we apply it to the DYW momentum transfer, we find

$$L_{\rm ff}(\vec{v}_\perp;0)q_{\rm DYW}=\left(0,Q\hat{n},\frac{Q^2}{2p^+}+\sqrt{2}Q\vec{n}\cdot\vec{v}_\perp\right). \tag{B2}$$

If one generalizes the definition of the DYW frame to $q^+=0$, then this transformation is a symmetry of this frame, if one allows for a perpendicular momentum in the initial state,

$$\vec{p}_\perp=\sqrt{2}p^+\vec{v}_\perp, \tag{B3}$$

otherwise, insisting on $\vec{p}_\perp=0$ in the DYW frame, it is not.

In the Breit frame we find for the transformed momentum transfer

$$L_{\rm ff}(\vec{v}_\perp;0)q_{\rm Breit}=(Q\cos\theta/\sqrt{2},Q(\sin\theta\hat{n}+\cos\theta\vec{v}_\perp),Q(-\cos\theta+2\sin\theta\hat{n}\cdot\vec{v}_\perp+\cos\theta\vec{v}_\perp^{\,2})/\sqrt{2}). \tag{B4}$$

If we require this vector to have the form

$$q'_{\rm Breit}=(Q\cos\theta/\sqrt{2},Q\sin\theta\hat{n}',-Q\cos\theta/\sqrt{2}), \tag{B5}$$

then we must find a vector $\vec{v}_\perp$ that satisfies

$$(\sin\theta\hat{n}+\cos\theta\vec{v}_\perp)^2=\sin^2\theta. \tag{B6}$$

There are two classes of solutions: either $\cos\theta=0$ and $\hat{n}\cdot\vec{v}_\perp=0$ or $\cos\theta\vec{v}_\perp^{\,2}+2\sin\theta\hat{n}\cdot\vec{v}_\perp=0$. In the latter case the length of the velocity vector is correlated with its direction through the relation

$$v=-2\tan\theta\hat{n}\cdot\vec{v}_\perp. \tag{B7}$$

If we denote the azimuthal angles of $\hat{n}$ and $\vec{v}_\perp$ by ϕ and ψ, respectively, then the vector $\hat{n}'$ in Eq. (B5) is given by

$$\hat{n}'=(-\cos(2\psi-\phi),-\sin(2\psi-\phi)). \tag{B8}$$

We conclude that there is a class of transverse boosts that leaves the Breit frame invariant.

2. Relations between different frames

If we want two reference frames to be connected by a Lorentz transformation, we need to verify that both the initial momenta (p) and the momentum transfers (q) are related by the same transformation.

In the case of the TRF and DYW frames the two are identical if $p^+=m/\sqrt{2}$ and in addition $\beta\sin\theta=1$. The corresponding angle we denote by θ_0. The latter condition ensures that the momentum transfer in the TRF frame has vanishing plus component. Clearly, for every value of Q there is an angle θ_0 for which the TRF and DYW frames are kinematically connected.

If we try the same for the TRF and Breit frames, we find that they are kinematically related for all Q at $\theta=0$.

The DYW and Breit frames can only be related for $\theta=\pi/2$. Then the momentum transfer in the Breit frame has the form

$$q_{\rm Breit}=(0,Q\hat{n},0). \tag{B9}$$

We now try to find the transformation that transforms the momentum transfer in the DYW frame into this special vector. If we write $\vec{v}_\perp=v\hat{v}_\perp$, then we find the parameters

$$\hat{v}_\perp = -\hat{n}, \quad v = \frac{Q}{2m\beta}, \quad e^\chi = \frac{m\beta}{\sqrt{2}p^+}. \tag{B10}$$

We see that for any value of Q we can connect the DYW frame to the Breit frame with $\theta = \pi/2$.

The main conclusion from this exercise is that the three frames considered here are only in special cases related by kinematical Lorentz transformations. In these cases the angular conditions are the same. In all other cases we find nonequivalent angular conditions.

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