Kaluza-Klein theories and the anomalous magnetic moment of the muon

Xavier Calmet* and Andrey Neronov†

Ludwig-Maximilians-University Munich, Sektion Physik, Theresienstraße 37, D-80333 Munich, Germany
(Received 5 July 2001; published 15 February 2002)

We discuss nonminimal couplings of fermions to the electromagnetic field, which generically appears in models with extra dimensions. We consider models where the electromagnetic field is generated by the Kaluza-Klein mechanism. The nonminimal couplings contribute at the tree level to the anomalous magnetic and electric dipole moments of fermions. We use recent measurements of these quantities to put limits on the parameters of models with extra dimensions.

The measurements of the electric and anomalous magnetic dipole moments of fermions provide a very stringent test of the standard model of particle physics. The latest measurement of $g-2$ of the muon [1] seems to indicate a deviation from the standard model. The difference between the experimental value and the theoretical value calculated in the framework of the standard model is

$$\Delta a_\mu = a_\mu(\exp) - a_\mu(\mathrm{SM}) = 43(16) \times 10^{-10}. \qquad (1)$$

For a theoretical review see [2]. This effect does not necessarily imply a conflict with the standard model as there are theoretical uncertainties in the calculations involving hadronic quantum corrections [3]. Interpretations of this 2.6σ effect were immediately proposed in different frameworks such as compositeness and technicolor [4], supersymmetry [5], extra dimensions [6], massive neutrinos [7], leptoquarks [8], additional gauge bosons [9] and others [10]. In this Brief Report we want to discuss the contribution to the electric and to the anomalous magnetic dipole moments of fermions which arises due to the nonminimal coupling

$$\mathcal{S}_{int} = \int d^4x\, F_{\mu\nu} \bar{\psi}(A + B\gamma^5)\sigma^{\mu\nu}\psi \qquad (2)$$

of fermions to the electromagnetic field $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ ($\sigma_{\mu\nu} = \frac{1}{4}[\gamma_\mu, \gamma_\nu]$). This nonminimal coupling generically appears in Kaluza-Klein type theories [11], e.g., Kaluza-Klein supergravity [12]. As is shown in [16,17], the Kaluza-Klein interpretation of gauge fields can be revived in the framework of models with brane universe [13,14]. Whether the nonminimal coupling (2) appears depends on the particular assumptions made about the mechanism responsible for the localization of gauge fields and fermions on the brane.

In models with extra dimensions [12–14] the space-time is taken to be a product of the four-dimensional Minkowski space M^4 and a Riemannian manifold K^n. In a coordinate chart (x^μ, y^α) on $M^4 \times K^n$, a general Lorentz-invariant metric can be written in the form

$$ds^2 = f(y)\eta_{\mu\nu}dx^\mu dx^\nu + g_{\alpha\beta}(y)dy^\alpha dy^\beta, \qquad (3)$$

where $\eta_{\mu\nu}$ is the four-dimensional Minkowski metric and $g_{\alpha\beta}$ is a metric on K^n. Depending on the type of model under consideration, the observable matter fields of the standard model of particle physics are either allowed to propagate in the bulk or confined to live on a four-dimensional surface $\mathcal{M}^4 \subset M^4 \times K^n$. In the former case the typical size L of compact manifold K^n is restricted to be small enough, so that the energy scale of Kaluza-Klein excitations of standard model particles $M \sim L^{-1}$ is higher than the experimentally reachable one, $M \gtrsim 1$ TeV. In the latter case the size of K^n can be large and even infinite.

If the manifold K^n possesses a nontrivial isometry group $\mathcal{G}$, perturbations of the background metric (3) which have the form

$$g_{AB} = \begin{pmatrix} f(y)\eta_{\mu\nu} + g_{\alpha\beta}A_\mu^\alpha A_\nu^\beta & g_{\alpha\beta}A_\mu^\beta \\ g_{\alpha\beta}A_\mu^\beta & g_{\alpha\beta} \end{pmatrix} \qquad (4)$$

where the vector fields A_μ^α,

$$A_\mu^\alpha = \mathcal{A}_\mu^{(p)}\xi_{(p)}^\alpha, \qquad (5)$$

are proportional to Killing vector fields $\xi_{(p)}^\alpha$, $p = 1,\ldots,\dim\mathcal{G}$. One can, in principle, interpret these fields as observable gauge fields of the standard model if the group $\mathcal{G}$ contains $SU(3) \times SU(2) \times U(1)$ as a subgroup. In this case the minimal dimension of the compact manifold K^n is $n = 7$ [15].

In general, in models with large or infinite extra dimensions, when all the fields of the standard model are supposed to be localized on a four-dimensional brane, the fields A_μ^α cannot be related to the standard model gauge fields since they are generally not confined to the brane $\mathcal{M}^4$. But, it turns out that a certain subset of these fields, which is associated to the symmetry of rotations around the brane can be naturally localized on the brane in models with warped extra dimensions [that is, when $f(y) \neq 1$] [16]. It is possible to obtain the $SU(2) \times U(1)$ group as a group of rotations around the brane [17]. Thus, with some modifications, Kaluza-Klein's idea of relating gauge fields to isometries of higher-dimensional space-time can also be implemented in this type of models.

If the observable electromagnetic field originates from isometries of higher-dimensional space-time, it differs from the electromagnetic field of Maxwell theory in a crucial

---

*Email address: calmet@theorie.physik.uni-muenchen.de
†Email address: neronov@theorie.physik.uni-muenchen.de

point. Although this field is minimally coupled to scalars, the coupling to spin-1/2 fields contains nonminimal terms (2) [11]. This coupling results in an additional contribution to the anomalous magnetic moment (if $A \neq 0$) and to the electric (if $B \neq 0$) dipole moment of fermions. The term proportional to γ_5 is T and P violating.

In the simplest (unrealistic) five-dimensional Kaluza-Klein theory the parameters A and B are determined by the five-dimensional gravitational constant G_5 and the size L of the only extra dimension. G_5 and L are, in turn, expressed through the four-dimensional gravitational constant G_4 $\sim 10^{-38}$ GeV^{-2} and four-dimensional fine structure constant $\alpha_{QED} \sim 10^{-2}$. An order-of-magnitude estimate shows that the contribution of the nonminimal coupling (2) to the anomalous magnetic moment and to the electric dipole moment is, in this case, too small to be experimentally detected.

In higher dimensional theories, as we will show below, the strength of the anomalous coupling (2) depends not only on the geometry of K^n and the $(4+n)$-dimensional gravitational constant G_{4+n} but also on the higher-dimensional profiles of fermions. In this case, the available experimental data on anomalous magnetic and electric dipole moments can provide certain restrictions on parameters of the higher-dimensional theory.

Couplings of Kaluza-Klein gauge fields to the fermion fields can be found from the higher-dimensional Dirac action

$$\mathcal{S}_D = i \int d^4x \, d^n y (\det E_N^{\hat{M}}) \bar{\Psi} \Gamma^A D_A \Psi \qquad (6)$$

where $E_N^{\hat{M}}$ is the vielbein field (hat denotes the local Lorentz indexes and the bulk metric is expressed through the vielbein as $g_{AB} = E_A^{\hat{C}} E_B^{\hat{D}} \eta_{\hat{C}\hat{D}}$ where $\eta_{\hat{C}\hat{D}}$ is $(4+n)$-dimensional Minkowski metric). The curved space gamma matrices Γ^A are related to the flat space ones as $\Gamma^A = E_{\hat{B}}^A \Gamma^{\hat{B}}$ where $\{\Gamma^{\hat{B}}, \Gamma^{\hat{C}}\} = 2\eta^{\hat{B}\hat{C}}$. The covariant derivative of Dirac spinor is defined as

$$D_A \Psi = \partial_A \Psi + \frac{1}{2} \omega_A^{\hat{B}\hat{C}} \sigma_{\hat{B}\hat{C}} \Psi, \qquad (7)$$

where $\omega_A^{\hat{B}\hat{C}}$ is the spin connection expressed in a standard way through derivatives of the vielbein and $\sigma_{\hat{B}\hat{C}} = \frac{1}{4} [\Gamma_{\hat{B}}, \Gamma_{\hat{C}}]$ is generator of local Lorentz rotations.

The $U(1)$ gauge group of electromagnetism is a one-parametric subgroup of the isometry group $\mathcal{G}$ generated by a Killing vector field $\xi_U^\alpha(y)$. Taking a coordinate $y^1 = \theta$ along the integral curves of $\xi_U^\alpha(y)$ we can write the metric (4) as

$$g_{AB} = \begin{pmatrix} f(y^a)\eta_{\mu\nu} + \varphi A_\mu A_\nu & \varphi A_\mu & 0 \\ \varphi A_\mu & \varphi & 0 \\ 0 & 0 & g_{ab} \end{pmatrix} \qquad (8)$$

where $\varphi = \varphi(y^a), a = 2, \ldots, n$, since the metric coefficients do not depend on θ.

Taking the coordinate vielbein for the metric (8) we have

$$\Gamma^A D_A \Psi = \frac{1}{\sqrt{f}} \Gamma^{\hat{\mu}} (\partial_\mu - A_\mu \partial_\theta) \Psi + \frac{1}{\sqrt{\varphi}} \Gamma^{\hat{\theta}} \partial_\theta \Psi + \Gamma^a \left(D_a + \frac{f_{,a}}{f} \right.$$

$$\left. + \frac{\varphi_{,a}}{4\varphi} \right) \Psi + \frac{\sqrt{\varphi}}{8f} F_{\mu\nu} \Gamma^{\hat{\mu}} \Gamma^{\hat{\nu}} \Gamma^{\hat{\theta}} \Psi \qquad (9)$$

where D_a is the covariant derivative with respect to the metric g_{ab}. We take Ψ of the form

$$\Psi = e^{iq\theta} \psi(x) \chi(y^a) \qquad (10)$$

where q is an integer. ψ is a four-dimensional spinor which satisfies the massless Dirac equation

$$i\Gamma^{\hat{\mu}} \partial_\mu \psi = 0 \qquad (11)$$

and χ is a n-dimensional spinor which obeys

$$i\Gamma^a \left(D_a + \frac{f_{,a}}{f} + \frac{\varphi_{,a}}{4\varphi} \right) \chi - \frac{q}{\sqrt{\varphi}} \Gamma^{\hat{\theta}} \chi = 0. \qquad (12)$$

For such configurations the action (6) reduces to

$$S = \int d^n y f^2 \sqrt{\varphi |g_{ab}|} |\chi|^2 \int d^4x \, \bar{\psi} \left[\frac{i}{\sqrt{f}} \Gamma^{\hat{\mu}} (\partial_\mu - iqA_\mu) \right.$$

$$\left. + \frac{i\sqrt{\varphi}}{4f} F_{\mu\nu} \sigma^{\hat{\mu}\hat{\nu}} \Gamma^{\hat{\theta}} \right] \psi. \qquad (13)$$

We can take $\Gamma^{\hat{\mu}} = \gamma^\mu, \Gamma^{\hat{\theta}} = \gamma^5$ where γ^μ, γ^5 are conventional four-dimensional gamma matrices. The $(4+n)$-dimensional spinor Ψ must be normalized with respect to the norm

$$\langle \Psi, \Psi \rangle = \int d^4x \, d^n y \sqrt{-g} \, \bar{\Psi} \Gamma^0 \Psi, \qquad (14)$$

which yields the following normalization condition for the n-dimensional spinor χ:

$$\int d^n y f^{3/2} \sqrt{\varphi |g_{ab}|} |\chi|^2 = 1. \qquad (15)$$

From Eq. (13) we see that the nonminimal coupling of the fermion field to A_μ has the form

$$S_{int} = R_\chi \int d^4x \, \bar{\psi} F_{\mu\nu} \sigma^{\mu\nu} \gamma^5 \psi \qquad (16)$$

with

$$R_\chi = \frac{1}{4} \int d^n y f \varphi \sqrt{|g_{ab}|} |\chi|^2. \qquad (17)$$

Note that the strength of the nonminimal coupling depends on the higher-dimensional profile χ of the fermion zero mode and, therefore, it can be different for different fermions. In models with large or infinite extra dimensions R_χ characterizes the scale of localization of fermions.

In the absence of $F_{\mu\nu}$, the action (13) is invariant under chiral rotations

$$P\psi = e^{-i\alpha\gamma^5}\psi \qquad (18)$$

where α is an arbitrary angle. Until we provide a particular mechanism by which the fermion field ψ gets a mass, we have no definite prescription for fixing α. For example, in the five dimensional Kaluza-Klein theory the term proportional to $\Gamma^{\hat\theta}\partial_\theta\Psi$ in Eq. (9) can be interpreted as a mass term after a $\alpha=\pi/2$ chiral rotation (18). The anomalous term transforms as

$$P^\dagger(i\gamma^0\sigma^{\mu\nu}\gamma^5)P = \gamma^0(\sin 2\alpha + i\gamma^5\cos 2\alpha)\sigma^{\mu\nu} \qquad (19)$$

under the chiral rotation (18). Thus, in general, the anomalous coupling (16) is

$$\mathcal{S}_{int} = e\int d^4x\, F_{\mu\nu}\bar\psi(R_\chi\sin 2\alpha + iR_\chi\cos 2\alpha\gamma^5)\sigma^{\mu\nu}\psi, \qquad (20)$$

where e is the electron charge (we have adopted the usual normalization for the electromagnetic field). The parameters A and B (2) are respectively

$$A = eR_\chi\sin 2\alpha \qquad (21)$$

$$B = eR_\chi\cos 2\alpha. $$

As mentioned previously, this term gives a contribution to the anomalous magnetic moment of the leptons as well as to their electric dipole moments. In old fashion Kaluza-Klein theory, the nonminimal coupling (20) is weak, but it can be sizable in the brane world models where the localized gauge fields are Kaluza-Klein gauge fields associated to rotations around the brane [16,17].

The contributions of this nonminimal coupling to the anomalous magnetic moment of a fermion and to its electric dipole moment can be deduced directly from Eq. (20). One gets

$$a_\psi^{KK} = R_\chi m_\psi\sin 2\alpha \qquad (22)$$

for the anomalous magnetic moment of the fermion and

$$d_\psi^{KK} = \frac{e}{2}R_\chi\cos 2\alpha \qquad (23)$$

for the electric dipole moment the fermion. The mass of the fermion is denoted by m_ψ.

We shall first discuss the electric dipole moment. The electric dipole moment of the electron and of the muon have been measured very precisely [18]

$$d_e^{exp} < (0.18\pm 0.12\pm 0.10)\times 10^{-26}e\ \text{cm}, \qquad (24)$$

$$d_\mu^{exp} < (3.7\pm 3.4)\times 10^{-19}e\ \text{cm}. \qquad (25)$$

The same precision has not been achieved for the τ-lepton:

$$d_\tau^{exp} < 3.1\times 10^{-16}e\ \text{cm}. \qquad (26)$$

Using the experimental data, one gets

$$R_e\cos 2\alpha < 4.6\times 10^{-14}\ \text{GeV}^{-1} \qquad (27)$$

for the electron,

$$R_\mu\cos 2\alpha < 9.4\times 10^{-6}\ \text{GeV}^{-1} \qquad (28)$$

for the muon and

$$R_\tau\cos 2\alpha < 7.9\times 10^{-3}\ \text{GeV}^{-1} \qquad (29)$$

for the τ-lepton. Note that R_χ (17), $\chi = e,\mu,\tau$, can take different values for different fermions. The best limit is obtained for the electron electric dipole moment.

We now consider their anomalous magnetic moments. Let us first consider the electron. The value of the anomalous magnetic moment of the electron predicted by the standard model is strongly dependent on the value of the fine-structure constant [19]. Typically one gets

$$\Delta a_e = a_e(\text{exp}) - a_e(\text{SM}) = 34(33.5)\times 10^{-12}, \qquad (30)$$

taking the fine-structure constant from the quantum Hall effect measurement. Thus we get

$$R_e\sin 2\alpha < \frac{\Delta a_e}{m_e} = 6.7\times 10^{-8}\ \text{GeV}^{-1}. \qquad (31)$$

In the case of the muon, we can interpret the observed deviation (1) as an effect of the nonminimal coupling. One obtains the following constraint for the product $R_\mu\sin 2\alpha$:

$$R_\mu\sin 2\alpha \lesssim \frac{\Delta a_\mu}{m_\mu} = 4\times 10^{-8}\ \text{GeV}^{-1}. \qquad (32)$$

Besides the magnitude of the deviation from the standard model prediction for the anomalous magnetic moment, its sign is of crucial importance. This sign depends on the angle α (18) which is determined by the fermion mass generating mechanism. Therefore, one gets a constraint on model building $\alpha\in[0,\pi/2]$. One cannot obtain constraints from the anomalous magnetic moment of the τ-lepton.

From the above estimates we see that the magnitude R_χ (16) of the nonminimal coupling of fermions to the electromagnetic field is less than or of the order of 10^{-21} cm. Remember, that R_χ characterizes the higher-dimensional profile of a fermion. For example, in the models with large extra dimensions it is an estimate of the size of the region where the fermions are localized ("thickness" of the brane).

We would like to point out that a term (2) arises also for neutrinos. Thus, neutrinos are expected to have a magnetic moment. Using the available limit [18] for the magnetic moment of an electron-type neutrino, one gets

$$R_\nu\sin 2\alpha < 3\times 10^{-7}\ \text{GeV}^{-1}. \qquad (33)$$

A similar constraint is obtained for the μ-type neutrino.

More stringent limits can be obtained from astrophysical considerations [21].

It is important to note that in models with large extra dimensions, there are extra contributions to the anomalous magnetic moment which come from Kaluza-Klein excitations of bulk fields (e.g., bulk gravitons) [6,20] which can result in effects of the same order of magnitude as Eq. (1).

---

[1] Muon g−Collaboration, H. N. Brown *et al.*, Phys. Rev. Lett. **86**, 2227 (2001).

[2] A. Czarnecki and W. J. Marciano, Phys. Rev. D **64**, 013014 (2001); J. Calmet, S. Narison, M. Perrottet, and E. de Rafael, Rev. Mod. Phys. **49**, 21 (1977); for an update see S. Narison, Phys. Lett. B **513**, 53 (2001).

[3] F. J. Yndurain, hep-ph/0102312.

[4] K. Lane, hep-ph/0102131; P. Das, S. Kumar Rai, and S. Raychaudhuri, hep-ph/0102242; Z. Xiong and J. M. Yang, Phys. Lett. B **508**, 295 (2001); X. Calmet, H. Fritzsch, and D. Holtmannsspotter, Phys. Rev. D **64**, 037701 (2001); C. Yue, Q. Xu, and G. Liu, J. Phys. G **27**, 1807 (2001); Y. Dai, C. Huang, and A. Zhang, hep-ph/0103317.

[5] L. Everett, G. L. Kane, S. Rigolin, and L. Wang, Phys. Rev. Lett. **86**, 3480 (2001); J. L. Feng and K. T. Matchev, *ibid.* **86**, 3480 (2001); E. A. Baltz and P. Gondolo, *ibid.* **86**, 5004 (2001); S. Komine, T. Moroi, and M. Yamaguchi, Phys. Lett. B **506**, 93 (2001); J. Hisano and K. Tobe, *ibid.* **510**, 197 (2001); T. Ibrahim, U. Chattopadhyay, and P. Nath, Phys. Rev. D **64**, 016010 (2001); J. Ellis, D. V. Nanopoulos, and K. A. Olive, Phys. Lett. B **508**, 65 (2001); R. Arnowitt, B. Dutta, B. Hu, and Y. Santoso, *ibid.* **505**, 177 (2001); K. Choi, K. Hwang, S. K. Kang, K. Y. Lee, and W. Y. Song, Phys. Rev. D **64**, 055001 (2001); S. P. Martin and J. D. Wells, *ibid.* **64**, 035003 (2001); T. Ibrahim, U. Chattopadhyay, and P. Nath, *ibid.* **64**, 016010 (2001); J. E. Kim, B. Kyae, and H. M. Lee, Phys. Lett. B **520**, 298 (2001); S. Komine, T. Moroi, and M. Yamaguchi, *ibid.* **507**, 224 (2001); D. F. Carvalho, J. Ellis, M. E. Gomez, and S. Lola, *ibid.* **515**, 323 (2001); H. Baer, C. Balazs, J. Ferrandis, and X. Tata, Phys. Rev. D **64**, 035004 (2001); S. Baek, T. Goto, Y. Okada, and K. Okumura, *ibid.* **64**, 095001 (2001); C. Chen and C. Q. Geng, Phys. Lett. B **511**, 77 (2001); K. Enqvist, E. Gabrielli, and K. Huitu, *ibid.* **512**, 107 (2001); D. G. Cerdeno, E. Gabrielli, S. Khalil, C. Munoz, and E. Torrente-Lujan, Phys. Rev. D **64**, 093012 (2001); A. Arhrib and S. Baek, hep-ph/0104225; F. Richard, hep-ph/0104106; K. Cheung, C. Chou, and O. C. Kong, Phys. Rev. D **64**, 111301(R) (2001).

[6] S. C. Park and H. S. Song, Phys. Lett. B **506**, 99 (2001); C. S. Kim, J. D. Kim, and J. Song, *ibid.* **511**, 251 (2001); K. Agashe, N. G. Deshpande, and G. H. Wu, *ibid.* **511**, 85 (2001).

[7] E. Ma and M. Raidal, Phys. Rev. Lett. **87**, 011802 (2001); **87**, 159901(E) (2001); Z. Xing, Phys. Rev. D **64**, 017304 (2001); M. Raidal, Phys. Lett. B **508**, 51 (2001).

[8] D. Chakraverty, D. Choudhury, and A. Datta, Phys. Lett. B **506**, 103 (2001); K. Cheung, Phys. Rev. D **64**, 033001 (2001); D. A. Dicus, H. He, and J. N. Ng, Phys. Rev. Lett. **87**, 111803 (2001).

[9] T. Huang, Z. H. Lin, L. Y. Shan, and X. Zhang, Phys. Rev. D **64**, 071301(R) (2001); D. Choudhury, B. Mukhopadhyaya, and S. Rakshit, Phys. Lett. B **507**, 219 (2001); S. N. Gninenko and N. V. Krasnikov, *ibid.* **513**, 119 (2001).

[10] T. W. Kephart and H. Pas, hep-ph/0102243; A. Dedes and H. E. Haber, J. High Energy Phys. **05**, 006 (2001); S. Rajpoot, hep-ph/0103069; M. B. Einhorn and J. Wudka, Phys. Rev. Lett. **87**, 071805 (2001); U. Mahanta, Phys. Lett. B **511**, 235 (2001); R. A. Diaz, R. Martinez, and J. A. Rodriguez, Phys. Rev. D **64**, 033004 (2001); C. A. Pires and P. S. Silva, *ibid.* **64**, 117701 (2001); E. O. Iltan, hep-ph/0103105; M. Krawczyk, hep-ph/0103223; S. Baek, P. Ko, and H. S. Lee, Phys. Rev. D **65**, 035004 (2002); N. A. Ky and H. N. Long, hep-ph/0103247; Y. Wu and Y. Zhou, Phys. Rev. D **64**, 115018 (2001); S. Baek, N. G. Deshpande, X. G. He, and P. Ko, *ibid.* **64**, 055006 (2001).

[11] W. E. Thirring, Acta Phys. Austriaca, Suppl. **9**, 256 (1972); A. Hosoya, K. Ishikawa, Y. Ohkuwa, and K. Yamagishi, Phys. Lett. **134B**, 44 (1984); R. O. Weber, Class. Quantum Grav. **4**, 453 (1987); A. O. Barut and T. Gornitz, Found. Phys. **15**, 433 (1985).

[12] M. J. Duff, B. E. Nilsson, and C. N. Pope, Phys. Rep. **130**, 1 (1986).

[13] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Rev. D **59**, 086004 (1999).

[14] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999); **83**, 4690 (1999).

[15] E. Witten, Nucl. Phys. **B186**, 412 (1981).

[16] A. Neronov, Phys. Rev. D **64**, 044018 (2001).

[17] A. Neronov, Phys. Rev. D **65**, 044004 (2002).

[18] Particle Data Group, D. E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).

[19] V. W. Hughes and T. Kinoshita, Rev. Mod. Phys. **71**, S133 (1999).

[20] M. L. Graesser, Phys. Rev. D **61**, 074019 (2000); H. Davoudiasl, J. L. Hewett, and T. G. Rizzo, Phys. Lett. B **493**, 135 (2000); R. Casadio, A. Gruppuso, and G. Venturi, *ibid.* **495**, 378 (2000); G. C. McLaughlin and J. N. Ng, *ibid.* **493**, 88 (2000).

[21] G. G. Raffelt, Phys. Rep. **320**, 319 (1999).