Quantum corrections for a Bañados-Teitelboim-Zanelli black hole via the 2D reduced model

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The one-loop quantum corrections for a Bañados-Teitelboim-Zanelli (BTZ) black hole are considered using a dimensionally reduced 2D model. Two cases are analyzed: minimally coupled and conformally coupled 3D scalar matter. In the minimal case, Hartle-Hawking and Unruh vacuum states are defined and the corresponding semiclassical corrections of the geometry are found. The calculations are done for the conformal case too, in order to make a comparison with the exact results obtained previously for a spinless BTZ black hole. The exact corrections for an AdS_2 black hole for the 2D minimally coupled scalar field in Hartle-Hawking and Boulware states are found as a subcase.

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I. INTRODUCTION

For a long time it was believed that black hole solutions do not exist in three dimensions, and therefore the discovery of Bañados, Teitelboim, and Zanelli (BTZ) [1,2] came as a surprise. The BTZ black hole has many properties that the familiar four-dimensional (4D) black hole solutions do not possess. First, as a BTZ black hole can be obtained by identification of points in 3D anti–de Sitter (AdS) space it is locally AdS, which means that it has a constant (negative) curvature. Therefore its singularity is not a curvature singularity but a singularity in the causal structure. Also, the BTZ black hole is not asymptotically flat but, because of the identification mentioned, which breaks the symmetries of AdS space, the asymptotic region can be identified. The fact that the BTZ black hole is three dimensional simplifies many computations which can be done only approximately in four dimensions; e.g., the thermal Green function of the conformally coupled scalar field can be found. There are various interesting dimensional reductions from the BTZ black hole to two-dimensional configurations, too [3].

One of the most interesting questions in the analysis of black holes is the Hawking radiation. Considerable work has been done in the last few years in an effort to find 2D effective models that can describe the properties of 4D black holes and the corresponding radiated field. The main idea of this approach is to consider the effective action obtained by functional integration of the scalar field as a semiclassical correction to the gravitational action. Different variants of the 2D effective action are used in the literature, but in principle they describe the effects of the *s* modes of the scalar field to one-loop order. A similar approach has been used recently [4] for the BTZ black hole.

The purpose of this paper is twofold. Our first goal is to define the Unruh vacuum by means of a dimensionally reduced model. The definition of the Unruh vacuum still seems to be an open question for the BTZ black hole. We use an analogy with the Schwarzschild and Reisner-Nordström case, which offers the possibility of a straightforward definition of the Unruh state via nonsingularity of the energy-momentum tensor (EMT) on the future event horizon.

The other important point is to use the advantage of the low dimensionality of the BTZ solution in order to analyze the properties of the procedure of dimensional reduction. This problem is of great heuristic importance, as dimensional reduction is repeatedly done in different scenarios of string and brane theories, whereas the mechanism is fully understood only on the classical level. A study of the dimensional reduction from four to two dimensions in the case of the Schwarzschild black hole was done previously [5–8]. There are also some new ideas in the literature such as the dimensional-reduction anomaly [9]. However, the analysis is far from complete. In order to be able to compare with the results obtained for the 3D BTZ [10–12], we formulate a dimensionally reduced theory for conformally coupled matter. We define the Hartle-Hawking vacuum and calculate the back reaction effects.

Finally, for the sake of completeness, we discuss the most frequently used effective action, Polyakov-Liouville, for 2D minimally coupled scalars. As a dimensionally reduced spinless BTZ black hole is, in fact, a two-dimensional black hole with constant negative curvature, as a subcase we include a full discussion of the quantum corrections of the 2D AdS black hole.

The plan of the paper is the following. In Sec. II we introduce the general framework of the problem. Section III gives the analysis of the Unruh vacuum for the minimally coupled case, while conformal coupling is discussed in Sec. IV. Section V is devoted to the Polyakov-Liouville action and the 2D anti–de Sitter black hole.

II. GENERAL SETTING

We start with the three-dimensional gravitational action with negative cosmological constant $(-2\Lambda=-2l^{-2}<0)$ coupled to the scalar field f:

$$\Gamma_0^{(3)}=\frac{1}{16\pi G}\int d^3x\sqrt{-g^{(3)}}\left(R^{(3)}+\frac{2}{l^2}\right)-\frac{1}{16\pi G}\int d^3x\sqrt{-g^{(3)}}[(\nabla f)^2+\xi R^{(3)}f^2]. \qquad (1)$$

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The case $\xi=0$ describes the minimal coupling in 3D, while $\xi=\frac{1}{8}$ is the conformal coupling. This action admits the vacuum solution $f=0$. We consider the BTZ black hole solution which is locally AdS_3 space:

$$ds^2_{(3)}=-\left(\frac{r^2}{l^2}-lM\right)dt^2+Jl\,dt\,d\theta+r^2d\theta^2+\left(\frac{r^2}{l^2}-lM+\frac{J^2l^2}{4r^2}\right)^{-1}dr^2. \tag{2}$$

If we construct the metric reduced from Eq. (2) to a two-dimensional t,r hypersurface by the standard procedure [14], we obtain

$$ds^2=-g_{cl}dt^2+\frac{1}{g_{cl}}dr^2, \tag{3}$$

where the function $g_{cl}(r)$ is given by

$$g_{cl}(r)=\frac{r^2}{l^2}-lM+\frac{J^2l^2}{4r^2}=\frac{(r^2-r_+^2)(r^2-r_-^2)}{r^2l^2}. \tag{4}$$

As show in [2], the quantities M and J have the meaning of mass and angular momentum. The second equality in Eq. (4) holds when $Ml\geqslant J$; the case $Ml=J$ is the extremal BTZ black hole. One can see from the Penrose diagram that this space shows a great resemblance to the Reisner-Nordström black hole. The outer and inner horizons $r_\pm$ are given by

$$r_\pm^2=\frac{l^2}{2}(Ml\pm\sqrt{M^2l^2-J^2}). \tag{5}$$

Inversely,

$$M=\frac{r_+^2+r_-^2}{l^3},\qquad J=\frac{2r_+r_-}{l^2}. \tag{6}$$

Achucarro and Ortiz showed in [3] that the 2D metric (3) can be obtained from the dimensionally reduced action in the following way. Assume the axially symmetric metric ansatz

$$ds^2_{(3)}=g_{\mu\nu}dx^\mu dx^\nu+l^2\Phi^2(\alpha d\theta+A_\mu dx^\mu)^2, \tag{7}$$

where $g_{\mu\nu},\Phi$, and A_μ are the two-dimensional metric, dilaton, and U(1) gauge fields; all fields depend only on t,r. The constant α will be fixed later. The 3D scalar curvature for the ansatz (7) is

$$R^{(3)}=R-\frac{l^2\Phi^2}{4}F_{\mu\nu}F^{\mu\nu}-\frac{2\Box\Phi}{\Phi}, \tag{8}$$

where $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ and R denotes the curvature in 2D. Further, $\sqrt{-g^{(3)}}=\sqrt{-g}\,l\alpha\Phi$. Introducing the reduction formula (8) into the action (1) and integrating over the angular variable θ, we obtain the 2D action

$$\Gamma_0=\Gamma_g+\Gamma_m. \tag{9}$$

Its gravitational part Γ_g is, up to a total divergence, given by

$$\Gamma_g=\frac{l\alpha}{8G}\int d^2x\sqrt{-g}\Phi\left(R-\frac{l^2\Phi^2}{4}F^2+\frac{2}{l^2}\right), \tag{10}$$

while the part describing matter, Γ_m, is

$$\Gamma_m=-\frac{l\alpha}{8G}\int d^2x\sqrt{-g}\Phi\left[(\nabla f)^2+\xi f^2\left(R-\frac{l^2\Phi^2}{4}\times F^2-\frac{2\Box\Phi}{\Phi}\right)\right]. \tag{11}$$

In the following we will choose α such that $l\alpha/8G=1$. Also, instead of the dilaton field Φ we will use its logarithm: $\varphi=\log\Phi$.

In order to analyze the vacuum fluctuations of the scalar field f one has to integrate it out to first order in $\hbar$. Our approximation consists of the fact that we do the functional integration of f in the 2D action (11) and not in the full 3D action. We use the methods developed in [15]. The result that we obtain for the one-loop effective action is

$$\Gamma_1=\frac{1}{96\pi}\int d^2x\sqrt{-g}(12\xi-1)R\frac{1}{\Box}R+\frac{1}{8\pi}\int d^2x\sqrt{-g}\left[\left(\frac{1}{4}-2\xi\right)R\frac{1}{\Box}(\nabla\varphi)^2+\left(\frac{1}{2}-2\xi\right)R\varphi-\frac{\xi l^2}{4}R\frac{1}{\Box}e^{2\varphi}F^2\right]. \tag{12}$$

Note that the effective actions for 2D dilaton models are analyzed in various papers [15–17].

It is easier to use the local form of the action. The local form can be obtained by a suitable introduction of auxiliary fields [7,4]; however, it differs for the three cases we are going to discuss. Therefore, we proceed with the minimal case.

III. 3D MINIMAL COUPLING

For $\xi=0$, the effective action Γ_1 can be rewritten in the local form as

$$\Gamma_{1,min}=-\frac{1}{96\pi}\int d^2x\sqrt{-g}\left[2R\left(\psi-\frac{3}{2}\chi\right)+(\nabla\psi)^2-3(\nabla\psi)(\nabla\chi)-3(\nabla\varphi)^2\psi-6R\varphi\right], \tag{13}$$

where the auxiliary fields[1] ψ and χ satisfy the equations

$$\Box\psi=R, \tag{14}$$

$$\Box\chi=(\nabla\varphi)^2. \tag{15}$$

[1]Note that our auxiliary fields are not the same as those introduced in [4].

The full semiclassical action for the minimally coupled field is

$$\Gamma_{min}=\Gamma_g+\Gamma_{1,min}$$
$$=\int d^2x\sqrt{-g}e^{\varphi}\left(R+\frac{2}{l^2}-\frac{l^2}{4}e^{2\varphi}F_{\mu\nu}F^{\mu\nu}\right)$$
$$-\kappa\int d^2x\sqrt{-g}\left[2R\left(\psi-\frac{3}{2}\chi\right)+(\nabla\psi)^2\right.$$
$$\left.-3(\nabla\psi)(\nabla\chi)-3\psi(\nabla\varphi)^2-6R\varphi\right]. \tag{16}$$

We introduced the constant $\kappa=1/96\pi$, which is the perturbation parameter. The equations of motion obtained from the action (16) are (14), (15) and

$$\nabla_\mu(e^{3\varphi}F^{\mu\nu})=0, \tag{17}$$

$$R+\frac{2}{l^2}-\frac{3l^2}{4}e^{2\varphi}F^2=6\kappa e^{-\varphi}[-R+\nabla_\mu(\psi\nabla^\mu\varphi)], \tag{18}$$

$$g_{\alpha\beta}\Box\Phi-\nabla_\alpha\nabla_\beta\Phi-\Phi g_{\alpha\beta}\left(\frac{1}{l^2}-\frac{l^2}{8}\Phi^2F_{\mu\nu}F^{\mu\nu}\right)$$
$$-\frac{l^2}{2}\Phi^3F_{\beta\mu}F_\alpha{}^\mu$$
$$=T_{\alpha\beta}/2$$
$$=\kappa\left[\nabla_\alpha\psi\nabla_\beta\psi-\frac{3}{2}\nabla_\alpha\psi\nabla_\beta\chi-\frac{3}{2}\nabla_\alpha\chi\nabla_\beta\psi-3\psi\nabla_\alpha\varphi\nabla_\beta\varphi\right.$$
$$-2\nabla_\beta\nabla_\alpha\left(\psi-3\varphi-\frac{3}{2}\chi\right)-\frac{1}{2}g_{\alpha\beta}[(\nabla\psi)^2-3\nabla\psi\nabla\chi$$
$$\left.-3\psi(\nabla\varphi)^2]+2g_{\alpha\beta}\Box\left(\psi-3\varphi-\frac{3}{2}\chi\right)\right]. \tag{19}$$

$T_{\alpha\beta}$ denotes the energy-momentum tensor of the radiated matter:

$$T_{\mu\nu}=-\frac{2}{\sqrt{-g}}\frac{\delta\Gamma_1}{\delta g^{\mu\nu}}. \tag{20}$$

For $\kappa=0$ we obtain the classical vacuum equations of motion. The classical solution for Φ and $F_{\mu\nu}$ is

$$\Phi=e^{\varphi}=\frac{r}{l},\quad F^{\mu\nu}=\frac{\epsilon^{\mu\nu}}{\sqrt{-g}}\frac{Jl}{r^3}=E^{\mu\nu}\frac{Jl}{r^3}, \tag{21}$$

where $E^{\mu\nu}$ is the covariant antisymmetric tensor. The zeroth order solution for $g_{\mu\nu}$ is the BTZ metric (3). Note that $T_{\mu\nu}$ defined in Eq. (19), being of the first order in κ, is determined by the zeroth order solution for the fields ψ, χ, and φ.

The Hartle-Hawking vacuum state for the minimally coupled scalar field in the BTZ background was analyzed in the work of Medved and Kunstatter [4]. Here we will outline and rederive some of their results briefly for later comparison. In the Hartle-Hawking vacuum state all functions are independent of time. The solution of Eqs. (14), (15) is

$$\psi(r)=-\log g_{cl}(r)+Cr_*, \tag{22}$$

$$\chi(r)=\int\frac{dr}{g_{cl}(r)}\left(\int dr\frac{g_{cl}(r)}{r^2}\right)+Dr_*, \tag{23}$$

where r_*, the tortoise coordinate, is given for a nonextremal BTZ metric by

$$r_*=\int\frac{dr}{g_{cl}(r)}=\frac{l^2}{2(r_+^2-r_-^2)}\left(r_-\log\frac{r+r_-}{r-r_-}-r_+\log\frac{r+r_+}{r-r_+}\right). \tag{24}$$

The assumption that the energy-momentum tensor is regular on the outer horizon $r=r_+$ in the freely falling frame means that

$$T_{vv}<\infty,\quad \frac{T_{uv}}{g_{cl}}<\infty,\quad \frac{T_{uu}}{g_{cl}^2}<\infty\quad \text{for } r=r_+, \tag{25}$$

where the components of the EMT are given in the null u,v coordinates.[2] Using Eq. (25), for the constants C and D we obtain

$$C=2\frac{r_+^2-r_-^2}{l^2r_+},\quad D=-\frac{6r_+^2+2r_-^2}{3l^2r_+}. \tag{26}$$

Introducing these values, we get

$$\psi(r)=-\log\frac{(r+r_+)^2(r^2-r_-^2)}{r^2l^2}+\frac{r_-}{r_+}\log\frac{r+r_-}{r-r_-}, \tag{27}$$

$$\chi(r)=\frac{3r_+^2+r_-^2}{3(r_+^2-r_-^2)}\log\frac{(r+r_+)^2}{r^2-r_-^2}-\frac{(3r_+^2+r_-^2)r_-}{3(r_+^2-r_-^2)r_+}\log\frac{r+r_-}{r-r_-}$$
$$+\frac{1}{3}\log\frac{(r^2-r_-^2)^2}{rl^3}. \tag{28}$$

[2] In the rest of the text we will use three common choices of coordinates in parallel. These are Schwarzschild coordinates t,r, null coordinates u,v $(u=t-r_*, v=t+r_*)$, and Eddington-Finkelstein coordinates v,r.

The corresponding values of the EMT in the Hartle-Hawking vacuum are

$$\begin{aligned}T_{uu}=&\frac{\kappa}{2l^4r^6r_+^2}\Bigg\{[(r-r_+)^2-3r^6r_+^2-6r^5r_+(2r_+^2-r_-^2)\\&-r^4r_+^2(3r_+^2+2r_-^2)-2r^3r_+r_-^2(5r_+^2-3r_-^2)\\&-3r^2r_+^2r_-^2(2r_+^2-3r_-^2)+10rr_+^3r_-^4+5r_+^4r_-^4]\\&+3r_+^2(r^2-r_+^2)^2(r^2-r_-^2)^2\Bigg(\log\frac{(r+r_+)^2(r^2-r_-^2)}{r^2l^2}\\&-\frac{r_-}{r_+}\log\frac{r+r_-}{r-r_-}\Bigg)\Bigg\},\end{aligned}\tag{29}$$

$$\begin{aligned}T_{uv}=&\frac{\kappa}{2l^4r^6}(r^2-r_+^2)(r^2-r_-^2)[13r^4+3r^2(r_+^2+r_-^2)\\&-3r_+^2r_-^2],\end{aligned}\tag{30}$$

$$T_{vv}=T_{uu}.\tag{31}$$

The energy density of the radiation T_{tt} is

$$\begin{aligned}T_{tt}=&\frac{\kappa}{l^4r^6r_+}\Bigg[10r^8r_+-6r^7(r_+^2-r_-^2)+8r^6(r_+^3-3r_+r_-^2)\\&-6r^5(r_+^4-r_-^4)-r^4(6r_+^5-16r_+^3r_-^2+6r_+r_-^4)\\&+2r^3r_+^2r_-^2(r_+^2-r_-^2)+2r_+^5r_-^4+3(r^2-r_+^2)^2\\&\times(r^2-r_-^2)^2r_+\Bigg(\log\frac{(r+r_+)^2(r^2-r_-^2)}{r^2l^2}\\&-\frac{r_-}{r_+}\log\frac{r+r_-}{r-r_-}\Bigg)\Bigg].\end{aligned}\tag{32}$$

There is an important comment on the values of the energy density in the asymptotic region. One can see that T_{tt} diverges asymptotically ($r\to\infty$) as $r^2\log r$, a feature which is not present in the Schwarzschild case. But the Schwarzschild metric is asymptotically flat, while the BTZ metric has nonzero curvature and $g_{cl}(r)$ behaves as r^2 for $r\to\infty$. In order to better understand the properties of the BTZ metric, we transform the EMT to the locally flat coordinates t',r' at some distant fixed point (t,L). We get the asymptotics assuming that $r\sim L\to\infty$. The transformation of coordinates that we need is

$$t'=\sqrt{g_{cl}(L)}t,\qquad r'=\frac{1}{\sqrt{g_{cl}(L)}}r.\tag{33}$$

Asymptotically, $t'\sim(L/l)t\sim(r/l)t$ and

$$T_{t't'}\sim\frac{l^2}{r^2}T_{tt}\sim\frac{\kappa}{l^2}\left(10+6\log\frac{r}{l}\right),\tag{34}$$

so in the asymptotic region the local energy density diverges only logarithmically. This behavior of minimally coupled radiation in the BTZ background is rather unexpected, and we will see that the conformal coupling improves it.

Having fixed the components of the EMT, one can find the first correction of the metric, i.e., solve Eqs. (19) in the first order in κ. The one-loop corrected static ansatz for the metric is

$$ds^2=-g(r)e^{2k\omega(r)}dt^2+\frac{1}{g(r)}dr^2.\tag{35}$$

We take the function $g(r)$ in the form

$$g(r)=g_{cl}(r)-\kappa lm(r),\tag{36}$$

and then Eqs. (19) read, to first order,

$$2\frac{\kappa}{l}\omega'=T_{11}+\frac{T_{00}}{g_{cl}^2},\tag{37}$$

$$\kappa m'=\frac{T_{00}}{g_{cl}}.\tag{38}$$

Their solution is

$$\begin{aligned}m(r)=&\frac{4r^2-6(r_+^2+r_-^2)}{l^2r}+\frac{16r_-}{l^2}\log\frac{r+r_-}{r-r_-}\\&+\frac{3r^4-r_+^2r_-^2+3r^2(r_+^2+r_-^2)}{l^2r^3}\\&\times\left(\log\frac{(r+r_+)^2(r^2-r_-^2)}{r^2l^2}-\frac{r_-}{r_+}\log\frac{r+r_-}{r-r_-}\right),\end{aligned}\tag{39}$$

$$\omega(r)=F(r)-F(L),\tag{40}$$

where $F(r)$ is given by

$$\begin{aligned}F(r)=l\Bigg[&\frac{1}{r}+\frac{(r_+-3r_-)(r_++r_-)}{r_+(r_+-r_-)(r+r_-)}+\frac{(r_++3r_-)(r_+-r_-)}{r_+(r_++r_-)(r-r_-)}\\&-\frac{2(3r_+^2+r_-^2)}{(r+r_+)(r_+^2-r_-^2)}+\frac{32r_+r_-^2}{(r_+^2-r_-^2)^2}\log(r+r_+)\\&-\left(\frac{3r_-}{r_+r}-\frac{8r_-}{(r_++r_-)^2}\right)\log(r-r_-)\\&+\left(\frac{3r_-}{r_+r}-\frac{8r_-}{(r_+-r_-)^2}\right)\log(r+r_-)\\&-\frac{3}{r}\log\frac{(r+r_+)^2(r^2-r_-^2)}{r^2l^2}\Bigg].\end{aligned}\tag{41}$$

L is the integration constant. We have assumed that our system is in a 1D box of size L [5].

The first correction of the scalar curvature, $R=R_0+\kappa R_1$, with R_0,

$$R_0 = -g_{cl}'' = -\frac{2}{l^2} - 6\frac{r_+^2 r_-^2}{l^2 r^4}, \tag{42}$$

can be expressed in terms of m, ω as

$$R_1 = -3g_{cl}'\omega' + lm'' - 2g_{cl}\omega''. \tag{43}$$

It is regular on the horizon $r = r_+$. If one had not fixed C and D previously, the same values (26) would have been obtained assuming the regularity of R_1 on r_+. We find

$$R_1 = \frac{6}{lr_+ r^5}\Bigg[2r^3(r_+^2 - r_-^2) + 8r_+^3 r_-^2 - r_+(r^4 + r^2(r_+^2 + r_-^2) - 3r_+^2 r_-^2)\left(\log\frac{(r+r_+)^2(r^2 - r_-^2)}{r^2 l^2} - \frac{r_-}{r_+}\log\frac{r+r_-}{r-r_-}\right)\Bigg]. \tag{44}$$

The corrected value of the metric gives us the possibility of determining how the horizon of the black hole changes due to the back reaction of the Hawking radiation. The apparent horizon of the black hole (which in the static case coincides with the event horizon) is in 2D defined by the equation

$$g^{\mu\nu}\partial_\mu r_{AH}\partial_\nu r_{AH} = 0. \tag{45}$$

We define the corrected null coordinates $\overline{u}, \overline{v}$ for the general nonstatic metric by

$$ds^2 = -g(v,r)e^{2\kappa\omega(v,r)}dv^2 + 2e^{\kappa\omega(v,r)}dv\,dr = -\frac{1}{\mu}g(v,r)e^{2\kappa\omega(v,r)}d\overline{u}d\overline{v} \tag{46}$$

with $d\overline{v} = dv$, $d\overline{u} = \mu dv - [2\mu/g(v,r)]e^{-\kappa\omega}dr$ and μ the integration factor [8]. The condition (45) in $\overline{u}, \overline{v}$ coordinates is

$$\partial_{\overline{u}} r|_{r_{AH}} = 0, \quad \partial_{\overline{v}} r|_{r_{AH}} = 0, \tag{47}$$

or, equivalently,

$$e^{\kappa\omega}g(v,r)|_{r_{AH}} = 0. \tag{48}$$

If we write the apparent horizon as $r_{AH} = r_+ + \kappa r_1$, for the corrected value we get

$$r_{AH} = r_+ + k\frac{l^3 m(v, r_+) r_+}{2(r_+^2 - r_-^2)}. \tag{49}$$

In the static case of the Hartle-Hawking vacuum (49) the one-loop corrected value of the event horizon is

$$r_{AH} = r_+ + \kappa\frac{lr_+ r_-}{(r_+^2 - r_-^2)}\left(\frac{5r_+^2 - r_-^2}{r_+^2}\log\frac{r_+ + r_-}{r_+ - r_-} + \frac{3r_+^2 + r_-^2}{r_+ r_-}\log\frac{4(r_+^2 - r_-^2)}{l^2} - \frac{r_+^2 + 3r_-^2}{r_+ r_-}\right). \tag{50}$$

Having found ψ, χ, and the metric, one can easily calculate the corrections of the thermodynamical quantities temperature and entropy. Entropy is defined as [18]

$$S = -2\pi\epsilon_{\alpha\beta}\epsilon_{\gamma\delta}\frac{\partial\mathcal{L}}{\partial R_{\alpha\beta\gamma\delta}}\Bigg|_{r_{AH}} = 4\pi\left[\frac{r}{l} - \kappa\left(2\psi - 3\chi - 6\log\frac{r}{l}\right)\right]\Bigg|_{r_{AH}} \tag{51}$$

for the action (16). In the Hartle-Hawking state the corrected entropy is given by

$$S = 4\pi\Bigg[\frac{r_+}{l} + \kappa\Bigg(-\frac{r_+^2 + 3r_-^2}{r_+^2 - r_-^2} + \frac{5r_+^2 - r_-^2}{r_+^2 - r_-^2}\log\frac{4(r_+^2 - r_-^2)}{l^2} - 2\frac{r_+^2 + r_-^2}{r_+^2 - r_-^2}\log\frac{r_+^2 - r_-^2}{4r_+^2} + \log\frac{4r_+(r_+^2 - r_-^2)}{l^3} + 6\log\frac{r_+}{l}\Bigg)\Bigg], \tag{52}$$

while the temperature is

$$T_H = \frac{r_+^2 - r_-^2}{2\pi l^2 r_+}[1 - \kappa F(L)] - \kappa\left(\frac{r_-^4 + 9r_+^4 + 6r_+^2 r_-^2}{2\pi l r_+^2(r_+^2 - r_-^2)} - \frac{8r_-^2}{\pi l(r_+^2 - r_-^2)}\log\frac{16r_+^2}{l^2}\right). \tag{53}$$

We will now analyze the Unruh vacuum. The Unruh vacuum can be defined as a state of matter whose energy-momentum tensor is regular on the future event horizon. As is easily seen, the region $-\infty < t < \infty$, $r_+ \leqslant r < \infty$ of the t,r plane transforms into the interior of the triangle $v = -\infty$, $u = \infty$, $u = v$ in the u,v plane. The line $u = v$ is the time like boundary (asymptotic region) of the BTZ black hole; $u = \infty$ is the future event horizon while $v = -\infty$ is the past event horizon. In order to find the energy-momentum tensor we need to solve Eqs. (14), (15) in the general nonstatic case. Those equations can be transformed into a system of partial linear equations that is similar to the one obtained for the SSG model. For details we refer the reader to [8]. The general solution is

$$\psi(v,r) = -\log g_{cl}(r) + \mathcal{C}\left(r_* - \frac{v}{2}\right) + \mathcal{G}(v), \tag{54}$$

$$\chi(v,r) = \int\frac{dr}{g_{cl}(r)}\left(\int\frac{g_{cl}(r)}{r^2}dr\right) + \mathcal{D}\left(r_* - \frac{v}{2}\right) + \mathcal{H}(v), \tag{55}$$

where $\mathcal{C}, \mathcal{G}, \mathcal{D}, \mathcal{H}$ are arbitrary functions of their arguments. Note that the arguments in Eqs. (54), (55) are written in such a way that regularity on the future horizon $u \to \infty$, $v =$ const is equivalent to regularity on $r \to r_+ (r_* \to -\infty)$, as the values of v and its functions are constant on the future horizon.

While the value of T_{uv} is in the general case the same as Eq. (30), for T_{uu} and T_{vv} we obtain

$$T_{uu}=\frac{\kappa}{2l^4r^6}\Bigg\{-3r^8+2r^6(r_+^2+r_-^2)-3r^4(r_+^4-4r_+^2r_-^2+r_-^4)-6r^2r_+^2r_-^2(r_+^2+r_-^2)+5r_+^4r_-^4-3(r^2-r_+^2)^2(r^2-r_-^2)^2\times\left(\mathcal{G}+\mathcal{C}-\log\frac{(r^2-r_+^2)(r^2-r_-^2)}{r^2l^2}\right)-\mathcal{C}'l^2r^3[3r^4+3r^2(r_+^2+r_-^2)-r_+^2r_-^2]+l^4r^6(\mathcal{C}'^2-3\mathcal{C}'\mathcal{D}'-2\mathcal{C}''+3\mathcal{D}'')\Bigg\},\tag{56}$$

$$T_{vv}=\frac{\kappa}{2l^4r^6}\Bigg\{-3r^8+2r^6(r_+^2+r_-^2)-3r^4(r_+^4-4r_+^2r_-^2+r_-^4)-6r^2r_+^2r_-^2(r_+^2+r_-^2)+5r_+^4r_-^4-3(r^2-r_+^2)^2(r^2-r_-^2)^2\times\left(\mathcal{G}+\mathcal{C}-\log\frac{(r^2-r_+^2)(r^2-r_-^2)}{r^2l^2}\right)-2\mathcal{G}'l^2r^3[3r^4+3r^2\times(r_+^2+r_-^2)-r_+^2r_-^2]+4l^4r^6(\mathcal{G}'^2-3\mathcal{G}'\mathcal{H}'-2\mathcal{G}''+3\mathcal{H}'')\Bigg\}.\tag{57}$$

From these expressions one can see that, in order to enforce the regularity of T_{uu}/g_{cl}^2 on the future horizon, one needs to make the functions $\mathcal{C}$ and $\mathcal{D}$ linear in their arguments, $\mathcal{C}(x)=Cx$, $\mathcal{D}(x)=Dx$, with the Hartle-Hawking values of constants C,D given by Eq. (26). The functions $\mathcal{G},\mathcal{H}$ cannot be fixed in this manner. We will assume that $\mathcal{G}$ and $\mathcal{H}$ are also linear, which is in accordance with the constancy of luminosity of a black hole. We see then that the difference between outgoing and ingoing fluxes in the asymptotic region $r\to\infty(r_*\to 0)$ has the leading behavior

$$T_{uu}-T_{vv}\sim-\frac{3\kappa r}{2l^2}(C-2\mathcal{G}').\tag{58}$$

It is much smaller than the asymptotic value of the flux

$$T_{uu}\sim\frac{3\kappa r^2}{2l^4}\left(2\log r+\frac{C}{2}v-\mathcal{G}-1\right).\tag{59}$$

In fact, the asymptotic value of flux is not dominated by the function $\mathcal{G}(v)=\mathcal{G}(t+r_*)$ for $r_*\to 0$, although $\mathcal{G}$ fixes the luminosity of the black hole. The dominant term is the $r^2\log r$ term, and this term is the same for T_{uu} and T_{vv}. This is a rather peculiar characteristic of the BTZ black hole if we recall that for the Unruh vacuum usually the outgoing flux is constant while the ingoing flux vanishes (asymptotically).

One can verify that the given energy-momentum tensor really describes the Unruh vacuum because it is regular on the future horizon but divergent on the past event horizon $(v\to-\infty,u=\mathrm{const})$. Indeed, expressing the EMT (56), (57) in terms of r and u, a logarithmically divergent term for $u=\mathrm{const}$, $r_*\to-\infty$ appears independently of the choice of the functions $\mathcal{G}$ and $\mathcal{H}$, i.e., always except for $\mathcal{G}(v)=[(r_+^2-r_-^2)/l^2r_+]v$ which gives the time independence of the EMT and the Hartle-Hawking state.

Taking the above discussion into account, we see that the functions $\mathcal{G}$ and $\mathcal{H}$ cannot be fixed by the properties of the EMT. The simplest choice for the Unruh vacuum is $\mathcal{G}=\mathcal{H}=0$. Then

$$\psi(v,r)=-\frac{r_+^2-r_-^2}{l^2r_+}v-\left(\log\frac{(r+r_+)^2(r^2-r_-^2)}{r^2l^2}-\frac{r_-}{r_+}\log\frac{r+r_-}{r-r_-}\right),\tag{60}$$

$$\chi(v,r)=\frac{3r_+^2+r_-^2}{3l^2r_+}v+\frac{3r_+^2+r_-^2}{3(r_+^2-r_-^2)}\log\frac{(r+r_+)^2}{r^2-r_-^2}-\frac{(3r_+^2+r_-^2)r_-}{3(r_+^2-r_-^2)r_+}\log\frac{r+r_-}{r-r_-}+\frac{1}{3}\log\frac{(r^2-r_-^2)^2}{rl^3}.\tag{61}$$

The final expressions for $T_{\mu\nu}$ are

$$T_{uu}=\frac{\kappa}{2l^6r^6r_+}\Bigg[l^2(r-r_+)^2(-3r_+r^6-6(2r_+^2-r_-^2)\times r^5-r_+(3r_+^2+2r_-^2)r^4-2r_-^2(5r_+^2-3r_-^2)\times r^3-3r_+r_-^2(2r_+^2-3r_-^2)r^2+10r_+^2r_-^4r+5r_+^3r_-^4)+3(r^2-r_+^2)^2(r^2-r_-^2)^2(r_+^2-r_-^2)v+3l^2r_+(r^2-r_+^2)^2\times(r^2-r_-^2)^2\left(\log\frac{(r+r_+)^2(r^2-r_-^2)}{r^2l^2}-\frac{r_-}{r_+}\log\frac{r+r_-}{r-r_-}\right)\Bigg],\tag{62}$$

$$T_{vv}=\frac{\kappa}{2l^6r^6r_+}\Bigg[l^2r_+(-3r^8+2(r_+^2+r_-^2)r^6-3(r_+^4+r_-^4-4r_+^2r_-^2)r^4-6r_+^2r_-^2(r_+^2+r_-^2)r^2+5r_+^4r_-^4)+3(r^2-r_+^2)^2(r^2-r_-^2)^2(r_+^2-r_-^2)v+3l^2r_+(r^2-r_+^2)^2\times(r^2-r_-^2)^2\left(\log\frac{(r+r_+)^2(r^2-r_-^2)}{r^2l^2}-\frac{r_-}{r_+}\log\frac{r+r_-}{r-r_-}\right)\Bigg].\tag{63}$$

The same values of the energy-momentum tensor can be obtained by applying the procedure developed by Balbinot and Fabbri in [6].

Now we will find the corrected geometry. The one-loop ansatz for the metric is

$$ds^2=-g(v,r)e^{2\kappa\omega(v,r)}dv^2+2e^{\kappa\omega(v,r)}dv\,dr,\tag{64}$$

where $g(v,r)=g_{cl}(r)-\kappa l m(v,r)$. Putting this ansatz in Eq. (19) we get

$$\frac{\kappa}{l}\frac{\partial\omega}{\partial r}=\frac{T_{rr}}{2}, \tag{65}$$

$$-\kappa\frac{\partial m}{\partial r}=T_{rv}, \tag{66}$$

$$\kappa\frac{\partial m}{\partial v}=T_{vv}+g_{cl}T_{vr}. \tag{67}$$

Introducing the values (30), (62), (63) in the system of equations for $m(v,r)$ and $\omega(v,r)$ we obtain the one-loop correction for the metric:

$$\begin{aligned}m(v,r)=&-v\frac{r_+^2-r_-^2}{l^4r_+r^3}[-3r^4+8r_+r^3-3(r_+^2+r_-^2)\\&\times r^2+r_+^2r_-^2]+\frac{4r^2-6(r_+^2+r_-^2)}{l^2r}+16\frac{r_-}{l^2}\log\frac{r+r_-}{r-r_-}\\&+\frac{3r^4+3(r_+^2+r_-^2)r^2-r_+^2r_-^2}{l^2r^3}\\&\times\left(\log\frac{(r+r_+)^2(r^2-r_-^2)}{r^2l^2}-\frac{r_-}{r_+}\log\frac{r+r_-}{r-r_-}\right),\end{aligned} \tag{68}$$

$$\begin{aligned}\omega(v,r)=&\frac{l(3r_--r_+)(r_++r_-)}{r_+(r+r_-)(r_--r_+)}-\frac{l(r_--r_+)(r_++3r_-)}{r_+(r-r_-)(r_-+r_+)}\\&-\frac{2l(3r_+^2+r_-^2)}{(r+r_+)(r_+^2-r_-^2)}+\frac{8lr_-}{(r_+^2-r_-^2)^2}\\&\times\left((r_+^2+r_-^2)\log\frac{r-r_-}{r+r_-}+2r_+r_-\log\frac{(r+r_+)^2}{r^2-r_-^2}\right)\\&-\frac{3l}{r}\left(\log\frac{(r+r_+)^2(r^2-r_-^2)}{r^2l^2}-\frac{r_-}{r_+}\log\frac{r+r_-}{r-r_-}\right)\\&+\frac{l}{r}-3v\frac{r_+^2-r_-^2}{lr_+r}.\end{aligned} \tag{69}$$

The value of the apparent horizon is

$$\begin{aligned}r_{AH}=r_++\kappa\frac{l}{r_+(r_+^2-r_-^2)}&\left(r_+(3r_+^2+r_-^2)\log\frac{4(r_+^2-r_-^2)}{l^2}\right.\\&-r_-(r_-^2-5r_+^2)\log\frac{r_++r_-}{r_+-r_-}-r_+(r_+^2+3r_-^2)\\&\left.-\frac{v}{l^2}(r_+^2-r_-^2)^2\right),\end{aligned} \tag{70}$$

and the entropy for the Unruh state is given by

$$\begin{aligned}S=4\pi\Bigg[&\frac{r_+}{l}+k\Bigg(-\frac{r_+^2+3r_-^2}{r_+^2-r_-^2}+\frac{4r_+v}{l^2}\\&+\frac{5r_+^2-r_-^2}{r_+^2-r_-^2}\log\frac{4(r_+^2-r_-^2)}{l^2}+2\frac{r_+^2+r_-^2}{r_+^2-r_-^2}\log\frac{4r_+^2}{r_+^2-r_-^2}\\&+\log\frac{4r_+(r_+^2-r_-^2)}{l^3}+6\log\frac{r_+}{l}\Bigg)\Bigg].\end{aligned} \tag{71}$$

IV. 3D CONFORMAL COUPLING

We will now discuss the case of conformally coupled matter. The coupling constant for conformal coupling in three dimensions is $\xi=\frac{1}{8}$. The local form of the effective action (12) is given by

$$\begin{aligned}\Gamma_{1,conf}=\frac{\kappa}{2}\int d^2x\sqrt{-g}&\Bigg(R(2\psi+\chi)+(\nabla\psi)^2+(\nabla\psi)(\nabla\chi)\\&-\frac{3l^2}{4}\psi e^{2\varphi}F^2+6R\varphi\Bigg),\end{aligned} \tag{72}$$

so the full action reads

$$\begin{aligned}\Gamma_{conf}=&\Gamma_g+\Gamma_{1,conf}\\=&\int d^2x\sqrt{-g}e^{\varphi}\left(R+\frac{2}{l^2}-\frac{l^2}{4}e^{2\varphi}F_{\mu\nu}F^{\mu\nu}\right)\\&+\frac{\kappa}{2}\int d^2x\sqrt{-g}\Bigg(R(2\psi+\chi)+(\nabla\psi)^2+(\nabla\psi)(\nabla\chi)\\&-\frac{3l^2}{4}\psi e^{2\varphi}F^2+6R\varphi\Bigg).\end{aligned} \tag{73}$$

The equations that follow from the variational principle for Eq. (73) are

$$\Box\psi=R, \tag{74}$$

$$\Box\chi=-\frac{3l^2}{4}e^{2\varphi}F^2, \tag{75}$$

$$\nabla_\mu\left[\left(1+\frac{3}{2}\kappa\psi e^{-\varphi}\right)e^{3\varphi}F^{\mu\nu}\right]=0, \tag{76}$$

$$R+\frac{2}{l^2}-\frac{3l^2}{4}e^{2\varphi}F^2=-\kappa e^{-\varphi}\left(3R-\frac{3l^2}{4}\psi e^{2\varphi}F^2\right), \tag{77}$$

and

$$
\begin{aligned}
& g_{\alpha\beta}\Box\Phi-\nabla_\alpha\nabla_\beta\Phi-\Phi g_{\alpha\beta}\left(\frac{1}{l^2}-\frac{l^2}{8}\Phi^2F_{\mu\nu}F^{\mu\nu}\right)\\
&\quad -\frac{l^2}{2}\Phi^3F_{\mu\beta}F^{\mu}{}_{\alpha}\\
&\quad =T_{\alpha\beta}/2\\
&\quad =-\frac{\kappa}{2}\bigg[\nabla_\alpha\psi\nabla_\beta\psi+\frac{1}{2}\nabla_\alpha\psi\nabla_\beta\chi+\frac{1}{2}\nabla_\alpha\chi\nabla_\beta\psi\\
&\qquad -\frac{3l^2}{2}\psi e^{2\varphi}F_{\beta\nu}F_{\alpha}{}^{\nu}-\nabla_\beta\nabla_\alpha(2\psi+\chi+6\varphi)\\
&\qquad -\frac{1}{2}g_{\alpha\beta}\left((\nabla\psi)^2+\nabla\psi\nabla\chi-\frac{3l^2}{4}\psi e^{2\varphi}F^2\right)\\
&\qquad +g_{\alpha\beta}\Box(2\psi+\chi+6\varphi)\bigg].
\end{aligned}
\tag{78}
$$

We can again take the solution of Eq. (77) for the dilaton in the form $e^{\varphi}=r/l$, as this represents our choice of the radial coordinate. Then Eq. (76) can also be solved exactly:

$$
F^{\mu\nu}=E^{\mu\nu}e^{-3\varphi}\frac{J}{l^2}\left(1+3\kappa\frac{l\psi}{2r}\right)^{-1}.
\tag{79}
$$

We proceed with the static case. The zeroth order solution for ψ is

$$
\psi(r)=-\log g_{cl}(r)+Cr_*,
\tag{80}
$$

while for χ we get

$$
\chi(r)=\int\frac{dr}{g_{cl}(r)}\left(\int\frac{3J^2l^2}{2r^4}dr\right)+Dr_*.
\tag{81}
$$

Our goal is to solve Eq. (78) determining the back reaction on the metric, i.e., to extract the equations for the functions $m(r)$ and $\omega(r)$ from it. Let us note that, as can be seen from Eq. (79), in the conformal case the "electromagnetic field" $F_{\mu\nu}$ also has corrections of first order in κ. This means that in the original 3D metric the angular part has to be corrected, too. Technically, there are first order terms on both sides of Eq. (78). We collect all first order terms on the right hand side, and then the equations read

$$
2\frac{\kappa}{l}\omega'=T_{11}+\frac{T_{00}}{g_{cl}^2},
\tag{82}
$$

$$
\kappa m'=\frac{T_{00}}{g_{cl}}-\frac{3\kappa}{2}\frac{J^2l^2\psi}{r^4},
\tag{83}
$$

under the same ansatz (35) for $g_{\mu\nu}$ as before.

The procedure to determine the integration constants is the same as for the minimal coupling. The values of the constants in the Hartle-Hawking vacuum are

$$
C=2\frac{r_+^2-r_-^2}{l^2r_+},\qquad D=\frac{2r_-^2}{l^2r_+}.
\tag{84}
$$

For χ we get

$$
\begin{aligned}
\chi(r)=&\frac{r_+^2}{r_+^2-r_-^2}\log\frac{(r+r_-)(r-r_-)}{r^2}\\
&+\frac{r_-^3}{r_+(r_+^2-r_-^2)}\log\frac{(r+r_-)}{(r-r_-)}-\frac{2r_-^2}{r_+^2-r_-^2}\log\frac{r+r_+}{r},
\end{aligned}
\tag{85}
$$

while ψ is the same as in the 3D minimal case (and as it will be for the Polyakov-Liouville effective action). The energy-momentum tensor reads

$$
\begin{aligned}
T_{uu}&=T_{vv}\\
&=-\kappa\frac{(r-r_+)^2}{2l^4r^6}[3r^6+6r^5r_++r^4(3r_+^2-10r_-^2)\\
&\quad -20r^3r_+r_-^2+3r^2r_-^2(-3r_+^2+r_-^2)\\
&\quad +8rr_+r_-^4+4r_+^2r_-^4],
\end{aligned}
\tag{86}
$$

$$
\begin{aligned}
T_{uv}=&\kappa\frac{(r^2-r_+^2)(r^2-r_-^2)}{2l^4r^6}\bigg[r^4+3r^2(r_+^2+r_-^2)-12r_+^2r_-^2\\
&-3r_+^2r_-^2\left(\log\frac{(r+r_+)^2(r^2-r_-^2)}{r^2l^2}-\frac{r_-}{r_+}\log\frac{r+r_-}{r-r_-}\right)\bigg],
\end{aligned}
\tag{87}
$$

and the energy density

$$
\begin{aligned}
T_{tt}=&-\frac{\kappa}{l^4r^6}[2r^8-4r^6(2r_+^2+3r_-^2)\\
&+r^4(6r_+^4+38r_+^2r_-^2+6r_-^4)-2r^3r_+r_-^2(r_+^2-r_-^2)\\
&-24r^2r_+^2r_-^2(r_+^2+r_-^2)+16r_+^4r_-^4]\\
&-3\kappa\frac{(r^2-r_+^2)(r^2-r_-^2)}{l^4r^6}r_+^2r_-^2\bigg(\log\frac{(r+r_+)^2(r^2-r_-^2)}{r^2l^2}\\
&-\frac{r_-}{r_+}\log\frac{r+r_-}{r-r_-}\bigg).
\end{aligned}
\tag{88}
$$

We see that the asymptotic behavior of the EMT has improved now, as the leading term for $r\to\infty$ is $T_{tt}\sim-\kappa r^2/l^4$. This means that the energy density of radiation in the locally Minkowskian frame is constant. The solution for the functions m and ω is nonsingular on the horizon $r=r_+$:

$$
\begin{aligned}
m(r)=&\frac{1}{l^2r^3}\bigg[-2r^4-6r^2(r_+^2+r_-^2)\\
&+6r_+^2r_-^2-2r_-r^3\log\frac{r+r_-}{r-r_-}\\
&-r_+^2r_-^2\left(\log\frac{(r+r_+)^2(r^2-r_-^2)}{r^2l^2}-\frac{r_-}{r_+}\log\frac{r+r_-}{r-r_-}\right)\bigg],\\
\omega(r)=&F(r)-F(L),
\end{aligned}
\tag{89}
$$

where $F(r)$ is

$$F(r)=4\frac{l}{r}-\frac{l(r_+-2r_-)}{2(r+r_-)(r_+-r_-)}-\frac{l(r_++2r_-)}{2(r-r_-)(r_++r_-)}+\frac{lr_-^2}{(r+r_+)(r_+^2-r_-^2)}-\frac{2lr_+r_-^2}{(r_+^2-r_-^2)^2}\log\frac{(r+r_+)^2}{r^2-r_-^2}+\frac{lr_-(r_+^2+r_-^2)}{(r_+^2-r_-^2)^2}\log\frac{r+r_-}{r-r_-}. \tag{90}$$

For the first correction of the curvature we obtain

$$R_1=\frac{6}{lr^5}\left[r^4+3r_+^2r_-^2-2r_+^2r_-^2\left(\log\frac{(r+r_+)^2(r^2-r_-^2)}{r^2l^2}-\frac{r_-}{r_+}\log\frac{r+r_-}{r-r_-}\right)\right]. \tag{91}$$

Now we can compare our results with the results in the literature. Green functions for the BTZ black hole were calculated in [10–13]. The starting point of these calculations is the Green function for the scalar field in AdS_3 space. However, as anti–de Sitter space has a timelike infinity, it does not have a Cauchy surface. A prescription to fix the boundary conditions for the wave equation and define the orthonormal basis of eigenfunctions for quantization was developed by Avis, Isham, and Storey [20]. One conformally maps the AdS space into the half of the Einstein static universe (ESU) that is spatially compact and has a well defined Cauchy problem. The solutions for a conformally coupled scalar field in the ESU can be mapped back into the solutions for the conformally coupled scalar field in AdS, and hence from the basis of eigenfunctions in the ESU one inherits the basis in AdS. The use of the complete basis in the ESU gives the so-called transparent boundary conditions. Transparent boundary conditions have the property that the energy of the scalar field is not conserved. It is possible also to define two types of reflective boundary conditions (Dirichlet and Neumann) such that energy is conserved. The final step in the construction of Green functions for the BTZ black hole is to apply the method of images, which takes into account the identifications used to obtain BTZ space from AdS_3.

The Green function for the spinning BTZ black hole for transparent boundary conditions was obtained by Steif [12] and the back reaction to the metric discussed by Martinez and Zanelli [21]. We will not compare our results to those; transparent boundary conditions are not appropriate for description of the Hartle-Hawking state as they do not conserve energy. Lifschytz and Ortiz [10], and Shiraishi and Maki [11] found the Green function for reflective boundary conditions in the spinless case $J=0$ and the expectation value of the energy-momentum tensor. The components of the EMT have the relatively complicated form of infinite sum and nonpolynomial behavior, so it is not easy to compare them directly with Eqs. (86), (87) which are much simpler. Reference [10] showed that the energy density is positive for Dirichlet boundary conditions, while for Neumann boundary conditions it is not. We obtained $T_{tt}\sim-\kappa r^2/l^4$ for $r\to\infty$, or in the locally flat frame $T_{t't'}\sim-\kappa/l^2$. However, we know from the analysis of the Schwarzschild black hole that dimensional reduction can change the sign of the energy, as it takes into account only some of the modes of the scalar field. The EMT is regular for $r=r_+$ and singular as $r\to 0$ both in [10,11] and in our calculation.

Since the metric ansatz in [10] is not the same as the one we used (and it does not seem to be correct; see [21]), we will compare the corrections for the curvature. Reference [10] obtained that the curvature scalar R^2 diverges as $1/r^6$ near $r=0$. For $J=0$ in Eq. (91) we see that $R_1=6/lr$. However, this is only the correction of the two-dimensional piece of the curvature. In order to find the full three-dimensional correction, we should employ the reduction formula (8). Using the solutions written to the first order in κ,

$$\Phi=\frac{r}{l},\quad F^2=-2\frac{J^2l^2}{r^6}\left(1-3\kappa\frac{\psi l}{r}\right), \tag{92}$$

and

$$\Box\Phi=\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi)=\frac{1}{l}(g_{cl}'+\kappa g_{cl}\omega'-\kappa lm'), \tag{93}$$

we find the first correction:

$$R_1^{(3)}=\frac{8}{lr}+6\frac{Ml^2}{r^3}. \tag{94}$$

From this expression it can be seen that the 3D R^2 diverges as $1/r^6$ near $r=0$ in our calculation, too. Note that at zeroth order the reduction formula gives $R_0^{(3)}=-6/l^2$, but $R_0=-2/l^2-3J^2l^2/2r^4$. We will later use the fact that for $J=0$ the metric (4) describes the AdS_2 black hole.

Finally, we can compare the metric corrections which are given in [11] in the large mass limit. The function $\mu(r)$ used in [11] is proportional to our $m(r)$. It behaves as

$$\mu(r)\sim\frac{r_+}{r}-1,\quad \text{Neumann}, \tag{95}$$

$$\mu(r)\sim\left(\frac{r_+}{r}\right)^3-2\frac{r_+}{r}-1,\quad \text{Dirichlet}, \tag{96}$$

while our result for $m(r)$ is

$$m(r)=-2\frac{r^2+3r_+^2}{l^2r}. \tag{97}$$

If the limit $M\to\infty$ is understood as $r_+\gg r$, then the behavior of $m(r)$ is the same as Eq. (95) for Neumann boundary conditions [up to an integration constant which we, for the sake of simplicity, discarded in the expression (89) for m].

Coming back to the dimensionally reduced BTZ model, we want to make some additional remarks, skipping the details of the calculation. It is always interesting to give a particular analysis of the extremal black hole, and this was done in [4] for the case of minimally coupled matter. The conclusion of [4] was that the BTZ black hole behaves similarly to a dimensionally reduced Reisner-Nordström black hole [19]; namely, the EMT for minimally coupled matter in the Hartle-Hawking state of the extremal black hole is different from that in the limit $r_- \to r_+$ of nonextremal black holes. The differences show drastically on the event horizon: while the energy-momentum tensor for the extremal black hole is regular, in the nonextremal limit the EMT is singular.[3] Surprisingly, this is not so in the case of conformally coupled matter. If we, as before, assume the extremal form of the metric at the beginning of the analysis and impose regularity, we conclude that no choice of constants C,D exists such that the EMT is regular on the horizon. A similar conclusion holds for the curvature. Both quantities are the ''least divergent'' for $C=0$, which is exactly the nonextremal limit $r_- \to r_+$.

The other peculiar property of the conformal case is that we cannot define the Unruh vacuum.

V. 2D MINIMAL COUPLING

In this section we will consider the Polyakov-Liouville effective action. This action is obtained by functional integration of a scalar field coupled to gravity in two dimensions. The Polyakov-Liouville action is used very often for the exact or qualitative description of one-loop quantum effects of the scalar field and it has been widely discussed in the context of string theory and 2D dilaton gravity. It is given by

$$\Gamma_{1,PL} = -\frac{1}{96\pi}\int d^2x\sqrt{-g}R\frac{1}{\Box}R, \tag{98}$$

or in the local form

$$\Gamma_{1,PL} = -\kappa\int d^2x\sqrt{-g}[(\nabla\psi)^2 + 2R\psi]. \tag{99}$$

The auxiliary field ψ satisfies the equation $\Box\psi = R$. The energy-momentum tensor determined from Eq. (99) is

$$T_{\mu\nu} = 2\kappa\left(\nabla_\mu\psi\nabla_\nu\psi - 2\nabla_\mu\nabla_\nu\psi - \frac{1}{2}g_{\mu\nu}(\nabla\psi)^2 + 2g_{\mu\nu}\Box\psi\right). \tag{100}$$

We see that in the Polyakov-Liouville case the effective action looks much simpler than the actions $\Gamma_{min}, \Gamma_{conf}$ as it is expressed in terms of one auxiliary field.

Let us see what results we get for the Hartle-Hawking state now. The solution of the equation of motion for $\Gamma_{PL} = \Gamma_g + \Gamma_{1,PL}$ for static ψ is Eq. (22), with $C = 2(r_+^2 - r_-^2)/r_+^2 l$. The components of the energy-momentum tensor calculated from Eq. (100) are

$$T_{uv} = 2\kappa\frac{(r^2 - r_+^2)(r^2 - r_-^2)(r^4 + 3r_+^2 r_-^2)}{r^6 l^4} \tag{101}$$

and

$$T_{uu} = T_{vv} = -2\kappa\frac{(r^2 - r_+^2)^2 r_-^2[r^2(3r_+^2 - r_-^2) - 2r_+^2 r_-^2]}{r^6 l^4 r_+^2}. \tag{102}$$

The energy density is positive and has regular behavior in the asymptotic region

$$T_{tt} = 4\kappa\frac{(r^2 - r_+^2)[r^6 r_+^2 - r^4 r_-^2(4r_+^2 - r_-^2) + r^2 r_-^2 r_+^2(6r_+^2 + r_-^2) - 5r_+^2 r_-^2]}{r^6 l^4 r_+^2} \tag{103}$$

in the sense discussed at the end of Sec. III. The asymptotic value of the energy density in the locally flat frame is $4\kappa/l^2$. The corrections of the metric read

$$m(r) = \frac{2}{3r^2 r^6 r_+^2}\left(2r_-[3r^4 + 3r^2(r_+^2 + r_-^2) - 5r_+^2 r_-^2] + 3r^3(r_+^2 - r_-^2)^2\log\frac{r - r_-}{r + r_-}\right),$$

$$\omega(r) = l\frac{-2r^2(3r_+^2 + r_-^2) + 8r_+^2 r_-^2}{r r_+^2(r^2 - r_-^2)} + l\frac{3r_+^2 + r_-^2}{r_+^2 r_-}\log\frac{r + r_-}{r - r_-}, \tag{104}$$

while the first correction of the curvature vanishes, $R_1 = 0$.

The results given above are particularly interesting because they can be interpreted as the one-loop corrections for the AdS_2 black hole; namely, in the spinless case the action Γ_{PL} describes dilaton gravity with negative cosmological constant and with quantum corrections produced by a 2D minimally coupled scalar field. The classical part of this action is the Jackiw-Teitelboim 2D gravity model [22]. The classical solution has AdS_2 geometry

$$ds^2 = -\left(\frac{r^2}{l^2} - lM\right)dt^2 + \left(\frac{r^2}{l^2} - lM\right)^{-1}dr^2, \tag{105}$$

as for $J=0$ one has $r_- = 0$, $r_+ = \sqrt{lM^3}$. The curvature is $R_0 = -2/l^2$. $AdS_2 \times S^2$ appears as the near horizon geometry of the extremal Reisner-Nordström solution. Different vacuum states of the AdS_2 black hole were discussed by Spradlin and Strominger [23]. Fabbri, Navarro, and Navarro-

[3] By the regularity of the EMT on the horizon we always mean the regularity in the freely falling frame, as defined by Eq. (25).

Salas considered one-loop corrections for an evaporating AdS_2 black hole [24], again in connection with Reisner-Nordström geometry.

The line element (105) can be rewritten in the null form:

$$ds^2 = -\frac{lM}{\sinh^2[\sqrt{M/l}(v-u/2)]}du dv, \tag{106}$$

where $u = t - r_*$, $v = t + r_*$ as usual, and r_* is given by

$$r_* = -\sqrt{\frac{l}{M}}\text{arccoth}\frac{r}{\sqrt{l^3 M}}.$$

One can also introduce Kruskal coordinates:

$$U = -\sqrt{\frac{l}{M}}e^{-\sqrt{M/l}u}, \quad V = \sqrt{\frac{l}{M}}e^{\sqrt{M/l}v},$$

which are regular on the horizon r_+. In these coordinates the line element is

$$ds^2 = -\frac{4lM}{(1+MUV/l)^2}dU dV.$$

Equations (101)–(103) for $J=0$ and $C = 2r_+/l^2$ reduce to

$$T_{uv} = 2\kappa\frac{r^2 - r_+^2}{l^4}, \quad T_{uu} = T_{vv} = 0, \quad T_{tt} = 4\kappa\frac{r^2 - r_+^2}{l^4}, \tag{107}$$

and describe the EMT of the 2D black hole in the Hartle-Hawking state. This result can be obtained in another way. As is known, the Hartle-Hawking state is the conformal state $|UV\rangle$. It is easy to find the components of the EMT in this state using the transformation law of the EMT from the Boulware state $|uv\rangle$ to the Hartle-Hawking state $|UV\rangle$. On performing this transformation one easily recovers the previous result for the EMT.

Now we want to find the one-loop solution of this model. The equations of motion can be written in the following form:

$$R = -\frac{2}{l^2}, \tag{108}$$

$$g_{\alpha\beta}\Box\Phi - \nabla_\alpha\nabla_\beta\Phi - g_{\alpha\beta}\frac{\Phi}{l^2} = \frac{1}{2}T_{\alpha\beta}. \tag{109}$$

These equations can be solved not only perturbatively but exactly. If we assume that the one-loop metric is of the form (105), then we obtain the dilaton as

$$\Phi = \frac{r}{l} - 2\kappa \tag{110}$$

in the Hartle-Hawking state. It is interesting to note that in the Boulware state (where $C=0$) there is again an exact solution:

$$\Phi = \frac{r}{l} + \kappa\frac{r}{r_+}\log\frac{r+r_+}{r-r_+}. \tag{111}$$

The integration constants in previous formulas are chosen in agreement with the classical limit $\kappa \to 0$. We see that the one-loop corrected metric is the AdS_2 black hole again—quantum corrections neither change the character of the space nor produce the singularity at $r=0$.

VI. CONCLUSIONS

In this paper we treated the one-loop corrections of a dimensionally reduced BTZ black hole. We discussed three types of effective action which correspond to different couplings of scalar matter (3D minimal, 3D conformal, and 2D minimal couplings).

As a first case we analyzed the 3D minimally coupled scalar field. The Hartle-Hawking vacuum state of this model was obtained by Medved and Kunstatter in [4]. Although the Hartle-Hawking state can be defined straightforwardly and has a regular energy-momentum tensor on the event horizon for both extremal and nonextremal black holes, it has the unexpected property that the EMT diverges in the asymptotic region, e.g., $T_{uu} \to (3\kappa r^2/l^4)\log(r/l)$. This holds also in the locally Minkowskian frame, where the energy density diverges logarithmically. In this paper we defined the Unruh vacuum state, demanding the regularity of the EMT on the future event horizon. The Unruh vacuum for the BTZ black hole has unusual behavior, too. Both incoming and outgoing fluxes T_{vv} and T_{uu} tend to the same divergent value $(3\kappa r^2/l^4)\log(r/l)$ asymptotically. This value dominates the time-dependent term. Let us just recall that for the Schwarzschild black hole the ingoing flux T_{uu} asymptotically vanishes, while the outgoing flux tends to a constant.

We considered next 3D conformally coupled matter. In this case one obtains the result that the energy-momentum tensor in the Hartle-Hawking state is regular both on the event horizon and asymptotically. However, one cannot define a regular Hartle-Hawking vacuum for the extremal BTZ black hole as was possible for the minimal coupling. The EMT for the extremal black hole is singular on the horizon and equals the value obtained in the extremal limit of nonextremal black holes, $r_- \to r_+$.

We also compared the results of a 2D reduced model with the exact 3D results. The expressions for the 2D model are much simpler as they are approximate; it is not straightforward to compare them with the 3D expressions, which are usually in the form of infinite nonpolynomial series. Further, the values of EMT cannot be compared directly because they are defined as variations of Lagrangians in different dimensions. In any case, there is a relatively good agreement in some properties that we checked (the behavior of the curvature near $r=0$, behavior of the metric in the large-mass limit). Our results seem to correspond to Neumann boundary conditions of the scalar field.

Finally, in the spinless case our model reduces to the Jackiw-Teitelboim model. Adding the Polyakov-Liouville term as the effective action and fixing the corresponding EMT, we obtained the Hartle-Hawking and the Boulware vacuum states for AdS_2 black hole. In both cases the quantum corrections on the geometry vanish; the back reaction changes only the dilaton. The properties of these solutions will be analyzed in a future publication.

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