Brane localization of gravity in higher derivative theory

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We consider a class of higher order corrections in the form of Euler densities of arbitrary rank n to the standard gravity action in D dimensions. We have previously shown that this class of corrections allows for domain wall solutions despite the presence of higher powers of the curvature. In the present paper we explicitly solve the linearized equation of motion for gravity fluctuations around the domain wall background and show that there always exist one massless state (graviton) propagating on the wall and a continuous tower of massive states propagating in the bulk.

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It may very well be that the theory of gravity as we know it today is only an effective theory and the usual Einstein-Hilbert action should be supplemented with corrections involving higher powers of the curvature tensor. This point of view is supported for example by string theory or the presence of the conformal anomalies in all quantum field theories coupled to gravity.

In this paper we are interested in corrections of a special type—Euler densities of arbitrary order n (n being a power of the curvature tensor) [1] in arbitrary space-time dimension D. It was shown in [2] that in the presence of an arbitrary number of Euler densities in the Lagrangian, there always exist domain wall solutions. The order n should be less than or equal to $D/2$ where D is the dimension of space-time ($D=2n$ although formally the total derivative gives part of the conformal anomaly). Euler densities appear for example in the α' expansion of the string theory effective action [3]. The Euler density of order $n=2$ (equal in this case to the Gauss-Bonnet combination) has been discussed in the presence of branes in many papers, some of which are [4–14].

In the present paper we analyze the linearized equations of motion for the fluctuations of the metric around the domain wall solutions in the theory with an arbitrary number of Euler densities. It is a generalization of the original idea [15] and the analysis performed for the Gauss-Bonnet $n=2$ case [4] (the Gauss-Bonnet term for intersecting domain walls was recently discussed in [14]). The fluctuations are assumed to be gravitonlike i.e. transverse and traceless.

It turns out that in spite of the presence of higher powers of curvature the picture is very similar to the usual Randall-Sundrum scenario [15]. There exists one normalizable massless bound state and a continuous tower of massive states with a small amplitude on the wall. The strategy adopted to calculate the equations of motions is based on the explicit formulas for the Euler densities derived in our previous paper [2].

The metric of the domain wall is conformally flat (and this fact was extensively used in [2]) but this is no longer the case when we add a fluctuation to the metric. Therefore calculating the equations of motion for the fluctuation requires calculating a first order correction (linear in the Weyl tensor) to the formulas in [2]. When all the contributions are added, the resulting equation of motion turns out to be almost identical to the lowest order case, the only difference being in the actual value of the coefficients of the equations. It is extremely important to notice that the graviton (normalizable massless mode confined to the wall) exists independently of the number of dimensions and the presence of Euler densities of higher order. It is quite amazing that the picture in the presence of Euler densities is almost identical to the lowest order scenario [15].

The Euler densities in D dimensions are defined (in the form notation) as

$$\boldsymbol{I}^{(n)}=\frac{1}{(D-2n)!}\epsilon_{a_1a_2\cdots a_D}R^{a_1a_2}\wedge\cdots\wedge R^{a_{2n-1}a_{2n}}\wedge e^{a_{2n+1}}\wedge\cdots\wedge e^{a_D}. \tag{1}$$

(For $D=2n$ they are topological invariants and formally total derivatives but a careful regularization shows that one cannot discard them either in the action or in the equations of motion since they correspond to the conformal anomaly.)

We will consider models in which D-dimensional gravitational interactions are described by the sum of such Euler densities and in which there is a $(D-2)$-brane (a domain wall). The action is the sum of the bulk and brane contributions:

$$S=S_{\text{bulk}}+S_{\text{brane}},$$

$$S_{\text{bulk}}=\int d^Dx\sqrt{-g}\sum_{n=0}^{n_{max}}\kappa_nI^{(n)}, \tag{2}$$

$$S_{\text{brane}}=\int d^{D-1}x\sqrt{-\tilde{g}}(-\lambda+\lambda_1\tilde{R}+\cdots).$$

The metric on the brane is given by $\tilde{g}_{\mu\nu}(x^\rho)=g_{\mu\nu}(x^\rho,y=0)$ where $y=x^D$ and $\mu,\nu,\ldots=1,\ldots,D-1$ while $M,N,\ldots=1,\ldots,D$. In the brane action we write explicitly

only the lowest order terms. The first two terms of the bulk action are known from conventional gravity. The one with $n=0$ corresponds to the cosmological constant: $I^{(0)}=1$, $\kappa_0=-\Lambda$. The one with $n=1$ is the usual Hilbert-Einstein term, $I^{(1)}=R$ with the coefficient $\kappa_1=(2\kappa^2)^{-1}$. The maximal number of higher order terms is $n_{max}\leqslant[D/2]$ as discussed in [2].

We assume in the following that $\lambda_1=0$; $\lambda_1\neq 0$ would give an additional contribution $\lambda_1 m^2 h_m(0)/2$ to the right hand side of Eq. (36) discussed later so it would affect only the massive modes.

Let us start with the bulk equations of motion. They can be obtained from the variation of the vielbein in the bulk action [for $n\leqslant(D-1)/2$]

$$\sum_n \frac{\kappa_n(D-2n)}{(D-2n)!}\epsilon_{a_1a_2\cdots a_{D-1}a}R^{a_1a_2}\wedge\cdots\wedge R^{a_{2n-1}a_{2n}} \times\wedge e^{a_{2n+1}}\wedge\cdots\wedge e^{a_{D-1}}=0. \tag{3}$$

We can write the curvature two-form as

$$R^{ab}=C^{ab}+\frac{1}{D-2}(e^a\wedge K^b-e^b\wedge K^a) \tag{4}$$

where C^{ab} is a two-form composed of the Weyl tensor C_{MNRS} while K^a is a one-form which will play an important role in our calculations and which is defined (for invertible vielbeins e^a_M) as $K^a=K_{MN}e^{Ma}dx^N=K^a{}_b e^b$ with

$$K_{MN}=R_{MN}-\frac{1}{2(D-1)}g_{MN}R. \tag{5}$$

The main purpose of this paper is to analyze the localization of the effective brane gravity in higher order theories proposed in our previous paper [2]. In order to do this we will look for solutions of the linearized equations of motion (3) for fluctuations of the metric:

$$g_{MN}=g^0_{MN}+\epsilon h_{MN} \tag{6}$$

for which the background metric g^0_{MN} is of the domain wall type that was proven in [2] to satisfy the equations of motion (3). The line element of this domain wall background is equal to

$$ds^2=e^{-2f(y)}\eta_{\mu\nu}dx^\mu dx^\nu+dy^2 \tag{7}$$

with the warp factor function given by $f(y)=\sigma|y|$. The fluctuation h_{MN} is assumed to propagate along the brane and be transverse and traceless:

$$h_{DM}=0,\quad \eta^{\mu\nu}h_{\mu\nu}=k^\mu h_{\mu\nu}=0. \tag{8}$$

We decompose the general fluctuation into modes with definite mass [from the $(D-1)$-dimensional point of view]

$$h_{\mu\nu}(x^\sigma,y)=\sum\!\!\!\!\!\!\int e^{ik_\sigma x^\sigma}h_{m\mu\nu}(y) \tag{9}$$

where k is a $(D-1)$-dimensional momentum satisfying $k^2=-m^2$ and the sum is for discrete modes while the integral is for continuous modes [in the above formula and in the next one the factor $\exp(ik_\sigma x^\sigma)$ represents the sum of independent *real* fluctuations with a given mass]. Let us now concentrate on one mode $h_{m\mu\nu}(y)$. Calculating the curvature tensor for the metric

$$g_{\mu\nu}(x^\sigma,y)=g^0_{\mu\nu}(x^\sigma,y)+\epsilon e^{ik_\sigma x^\sigma}h_{m\mu\nu}(y),$$
$$g_{DD}(x^\sigma,y)=g^0_{DD}(x^\sigma,y), \tag{10}$$

the equation of motion (3) will be expanded to first order in ϵ. For the background metric (7) the Weyl tensor vanishes and therefore C^{ab} is already of order ϵ:

$$C^{ab}=\epsilon(C_1)^{ab}+\mathcal{O}(\epsilon^2). \tag{11}$$

Thus, only terms up to the first order in $(C_1)^{ab}$ should be kept in Eq. (3). There is a second contribution to the linearized equations of motion for $h_{\mu\nu}$ coming from the correction to K^a,

$$K^a=(K_0)^a+\epsilon(K_1)^a+\mathcal{O}(\epsilon^2), \tag{12}$$

and in the product (3) for this contribution also only the first power of $(K_1)^a$ should be kept.

Using the above ϵ expansion we can write the equation of motion (3) up to terms linear in ϵ in the form

$$0=e\sum_n \kappa_n\frac{2^n(D-1-n)!}{(D-2)^n(D-1-2n)!}[(H^{(n)}_0)^N_M+\epsilon(H^{(n)}_1)^N_M +\epsilon(G^{(n)}_1)^N_M] \tag{13}$$

where we introduced the following tensors (note that $H^{(n)}_0$ have different normalization from the corresponding tensors $H^{(n)}$ of Ref. [2]):

$$(H^{(n)}_0)^N_M=(\delta^N_M\delta^{N_1}_{M_1}\delta^{N_2}_{M_2}\cdots\delta^{N_n}_{M_n}\pm\text{perm.}) \times(K_0)^{M_1}_{N_1}(K_0)^{M_2}_{N_2}\cdots(K_0)^{M_n}_{N_n}, \tag{14}$$

$$(H^{(n)}_1)^N_M=n(\delta^N_M\delta^{N_1}_{M_1}\delta^{N_2}_{M_2}\cdots\delta^{N_n}_{M_n}\pm\text{perm.}) \times(K_1)^{M_1}_{N_1}(K_0)^{M_2}_{N_2}\cdots(K_0)^{M_n}_{N_n}, \tag{15}$$

$$(G^{(n)}_1)^N_M=\frac{n(D-2)}{4(D-1-n)}(\delta^N_M\delta^{N_0}_{M_0}\delta^{N_1}_{M_1}\cdots\delta^{N_n}_{M_n}\pm\text{perm.}) \times(C_1)^{M_0M_1}_{N_0N_1}(K_0)^{M_2}_{N_2}\cdots(K_0)^{M_n}_{N_n}. \tag{16}$$

In [2] it was shown that the lowest order (ϵ^0) term vanishes for the domain wall metric (7) with the warp factor σ determined in terms of the coupling constants κ_n (this domain wall is flat when some relation among the κ_n is satisfied).

Let us start the discussion of corrections of order ϵ from $H_1^{(n)}$. From Eq. (15) one can get the recurrence relations for $H_1^{(n)}$ (valid not only for the domain wall but for all background metrics):

$$(H_1^{(n)})_M^N = n[\,\delta_M^N (K_1)_P^Q (H_0^{(n-1)})_Q^P - (K_1)_M^P (H_0^{(n-1)})_P^N - (K_0)_M^P (H_1^{(n-1)})_P^N]. \tag{17}$$

To proceed further let us specialize to the transverse and traceless fluctuations of the metric (6) in the domain wall background (7). In this case the tensor K_1 satisfies the following conditions:

$$(K_1)_\mu^\mu = 0, \quad (K_1)_\mu^D = (K_1)_D^\mu = (K_1)_D^D = 0. \tag{18}$$

The tensor K_0 has been calculated in [2],

$$(K_0^{(n)})_\mu^\nu = -\delta_\mu^\nu \frac{D-2}{2}\left(\frac{\partial f}{\partial y}\right)^2,$$

$$(K_0^{(n)})_D^D = -\frac{D-2}{2}\left(\frac{\partial f}{\partial y}\right)^2 + \frac{\partial^2 f}{\partial y^2}, \tag{19}$$

while $H_0^{(n)}$ differ only by normalization from $H^{(n)}$ defined in [2]:

$$(H_0^{(n)})_\mu^\nu = \delta_\mu^\nu (-1)^n \left(\frac{D-2}{2}\right)^n \frac{(D-1)!}{(D-1-n)!} \times \left(\frac{\partial f}{\partial y}\right)^{2n-2}\left[\left(\frac{\partial f}{\partial y}\right)^2 - \frac{2n}{D-1}\left(\frac{\partial^2 f}{\partial y^2}\right)\right],$$

$$(H_0^{(n)})_D^D = (-1)^n \left(\frac{D-2}{2}\right)^n \frac{(D-1)!}{(D-1-n)!}\left(\frac{\partial f}{\partial y}\right)^{2n}. \tag{20}$$

Substituting Eqs. (19) and (20) into formula (17) and using the conditions (18) we get the following expression for $H_1^{(n)}$:

$$(H_1^{(n)})_\mu^\nu = (K_1)_\mu^\nu \sum_{k=1}^{n} (-1)^k \frac{n!}{(n-k)!(D-1)^k} \times [(K_0)_\rho^\rho]^{k-1} (H_0^{(n-k)})_\sigma^\sigma \tag{21}$$

with all other components vanishing. It explicitly reads

$$(H_1^{(n)})_\mu^\nu = (K_1)_\mu^\nu (-1)^n \left(\frac{D-2}{2}\right)^{n-1} \frac{n(D-2)!}{(D-1-n)!} \times \left(\frac{\partial f}{\partial y}\right)^{2n-4}\left[\left(\frac{\partial f}{\partial y}\right)^2 - \frac{2(n-1)}{D-2}\left(\frac{\partial^2 f}{\partial y^2}\right)\right]. \tag{22}$$

Let us now turn to $G_1^{(n)}$. For a general background metric g_{MN}^0 the expression obtained by taking into account all permutations in Eq. (16) is very complicated. The result contains different combinations of all $H_0^{(k)}$ with $k<n$ and it is not possible to write it as a simple recurrence analogous to Eq. (17) valid for $H_1^{(n)}$. Therefore, we will present explicit formulas for $G_1^{(n)}$ only for the domain wall background (7). In this case, using the symmetry properties of the Weyl tensor C and the fact that $(K_0)_\mu^\nu$ and $(H_0^{(k)})_\mu^\nu$ are proportional to δ_μ^ν, we get

$$(G_1^{(n)})_\mu^\nu = (C_1)_{\mu D}^{\nu D} \frac{n!(D-2)}{D-1-n} \times \sum_{k=1}^{n} \frac{k(-1)^k}{(n-k)!(D-1)^k}[(K_0)_\rho^\rho]^{k-1}[(D-1) \times (H_0^{(n-k)})_D^D - (H_0^{(n-k)})_\rho^\rho]. \tag{23}$$

Substituting the explicit formulas for K_0 and $H_0^{(k)}$ (19,20) it can be rewritten in the following form:

$$(G_1^{(n)})_\mu^\nu = (C_1)_{\mu D}^{\nu D}(-1)^n \left(\frac{D-2}{2}\right)^{n-1} \times \frac{2n(D-2)!}{(D-1-n)!}\,\frac{(n-1)}{D-3}\left(\frac{\partial^2 f}{\partial y^2}\right)\left(\frac{\partial f}{\partial y}\right)^{2n-4}. \tag{24}$$

We see that the tensors $H_1^{(n)}$ and $G_1^{(n)}$ are proportional to K_1 and C_1, respectively, which can be found using the decomposition of the curvature tensor [given by Eq. (4)]:

$$(K_1)_\mu^\nu = \left[-\frac{1}{2}\frac{\partial^2}{\partial y^2} + \frac{D-5}{2}\left(\frac{\partial f}{\partial y}\right)\frac{\partial}{\partial y} + (D-3)\left(\frac{\partial f}{\partial y}\right)^2 - \left(\frac{\partial^2 f}{\partial y^2}\right) - \frac{1}{2}m^2 e^{2f}\right] h_{m\mu}^{\ \ \nu}(y), \tag{25}$$

$$(C_1)_{\mu D}^{\nu D} = \frac{D-3}{D-2}\left[-\frac{1}{2}\frac{\partial^2}{\partial y^2} - \frac{3}{2}\left(\frac{\partial f}{\partial y}\right)\frac{\partial}{\partial y} - \left(\frac{\partial f}{\partial y}\right)^2 - \left(\frac{\partial^2 f}{\partial y^2}\right) - \frac{1}{2(D-3)}m^2 e^{2f}\right] h_{m\mu}^{\ \ \nu}(y). \tag{26}$$

In both these tensors the operators acting on $h_{m\mu}^{\ \ \nu}(y)$ do not depend on the Lorentz indices μ,ν so we will drop these indices from now on.

Now we are ready to get the equation of motion for the fluctuation $h_m(y)$. We rewrite Eq. (13) using the explicit forms of $H_1^{(n)}, G_1^{(n)}$, K_1, and C_1, Eqs. (22), (24)–(26). The part of this equation linear in ϵ gives

$$0=\sum_n^{n_{max}}\kappa_n(-1)^n\frac{2n(D-3)!}{(D-1-2n)!}\Bigg[-\frac{1}{2}(f')^{2n-2}\frac{\partial^2}{\partial y^2}+\frac{D-5}{2}(f')^{2n-1}\frac{\partial}{\partial y}-(n-1)(f')^{2n-3}(f'')\frac{\partial}{\partial y}+(D-3)\times(f')^{2n}-(2n-1)(f')^{2n-2}(f'')-\frac{1}{2}m^2e^{2f}(f')^{2n-2}+\frac{n-1}{D-3}m^2e^{2f}(f')^{2n-4}(f'')\Bigg]h_m(y). \tag{27}$$

At this point we can use the explicit form of the warp factor $f(y)=\sigma|y|$. One should be careful when performing this substitution because the first derivative of $f(y)$ is not continuous at $y=0$ and the second derivative of $f(y)$ is proportional to the Dirac delta at $y=0$. Similar care is needed when calculating terms containing derivatives of the fluctuation h_m because the solutions to the above equation are also functions of $|y|$. Regularizing the Dirac delta function one can find the following equalities:[1]

$$(f')^{2k}(f'')=\frac{2}{2k+1}\sigma^{2k+1}\delta(y), \tag{28}$$

$$(f')^{2k}\frac{\partial^2}{\partial y^2}h_m(y)=\sigma^{2k}h_m''(|y|)+\frac{2}{2k+1}\sigma^{2k}\delta(y)h_m'(0^+), \tag{29}$$

$$(f')^{2k-1}(f'')\frac{\partial}{\partial y}h_m(y)=\frac{2}{2k+1}\sigma^{2k}\delta(y)h_m'(0^+), \tag{30}$$

where

$$h_m'(0^+)=\lim_{y\to 0^+}\frac{\partial}{\partial y}h_m(y). \tag{31}$$

Making these substitutions in Eq. (27) we find the following bulk ($y\neq 0$) equation of motion for h_m:

$$0=\sum_n^{m_{max}}\kappa_n(-1)^n\frac{n(D-3)!}{(D-1-2n)!}\sigma^{2n-2}\times[-h_m''+(D-5)\sigma h_m'+2(D-3)\sigma^2h_m-m^2e^{2\sigma|y|}h_m]. \tag{32}$$

It is rather amazing that the expression in the square brackets does not depend on n. Therefore for arbitrary κ_n and n_{max} the bulk equation of motion for h_m reduces to

$$0=-h_m''+(D-5)\sigma h_m'+2(D-3)\sigma^2h_m-m^2e^{2\sigma|y|}h_m. \tag{33}$$

Its solution for $m=0$ is equal to

$$h_0(y)=A_0e^{-2\sigma|y|}+B_0e^{(D-3)\sigma|y|}v \tag{34}$$

while for $m\neq 0$ it can be written using the Bessel functions of order $(D-1)/2$:

$$h_m(y)=e^{\{[(D-5)/2]\sigma|y|\}}\left[A_mJ_{(D-1)/2}\left(\frac{m}{\sigma}e^{\sigma|y|}\right)+B_mY_{(D-1)/2}\left(\frac{m}{\sigma}e^{\sigma|y|}\right)\right]. \tag{35}$$

For each m only one of the above combinations of the solutions of the bulk equation (32) is the solution of the full equation (27). In order to identify this combination we have to take into account the part of the equation of motion (27) proportional to $\delta(y)$. It reads

$$0=\sum_n^{n_{max}}\kappa_n(-1)^n\frac{n(D-3)!}{(D-1-2n)!}\sigma^{2n-1}\left[-\frac{h_m'(0^+)}{\sigma}-2h_m(0)+\frac{2(n-1)}{(D-3)(2n-3)}\frac{m^2}{\sigma^2}h_m(0)\right]. \tag{36}$$

This equation is equivalent to the following boundary condition at $y=0$:

$$\frac{h_m'(0^+)}{h_m(0)}=-2\sigma\left(1-\frac{m^2}{\sigma^2}\frac{1}{D-3}\frac{\sum_n\kappa_n(-1)^n[n/(D-1-2n)!][(n-1)/(2n-3)]\sigma^{2n}}{\sum_n\kappa_n(-1)^n[n/(D-1-2n)!]\sigma^{2n}}\right). \tag{37}$$

[1]The necessary denominators of $2k+1$ have not been taken into account for example by the authors of Ref. [4] (who discuss the case $D=5$), leading to wrong equations of motion.

For $m \neq 0$ it is quite complicated and depends on κ_n but in principle can be used to fix the ratio of the coefficients A_m/B_m for the massive modes.

The situation is much simpler for $m=0$ because in this case the above boundary condition does not depend on κ_n and simplifies to

$$\frac{h_0'(0^+)}{h_0(0)} = -2\sigma. \tag{38}$$

Applying this condition to Eq. (34) we find that for arbitrary space-time dimension, D, and for arbitrary strength of the higher order interactions (given by the coefficients κ_n) the massless solution of the equations of motion always exists and is given by

$$h_0(y) = \exp(-2\sigma|y|). \tag{39}$$

We would like to interpret these solutions as massless, normalizable 4-dimensional gravitons. It turns out to be possible, but some care is needed. Let us start with the normalizability issue. To check whether the above solutions (massless and massive) are normalizable we have to choose an appropriate integration measure. Such a measure can be determined by the requirement that the kinetic operator for the gravitons should be self-adjoint [which is not satisfied in the case of Eq. (32)]. To find the proper operator it is helpful to change the variable y and to rescale the fluctuation h_m:

$$h_m(y) = (1+\sigma|z|)^{(D-6)/2}\hat{h}_m(z) \tag{40}$$

where

$$\sigma z = \mathrm{sgn}(y)(e^{\sigma|y|}-1). \tag{41}$$

The equation of motion then reads

$$-\frac{d^2}{dz^2}h_m(z) + \frac{D(D-2)\sigma^2}{4(1+\sigma|z|)^2}h_m(z) = m^2 h_m(z) \tag{42}$$

and is explicitly self-adjoint with a flat measure (which we can take equal to 1).

The properly normalized massless solution is given by

$$\hat{h}_0(z) = \sqrt{\frac{(D-3)\sigma}{2}}(1+\sigma|z|)^{-(D-2)/2} \tag{43}$$

and is normalizable (for $D>3$) while the massive modes are given by the formula

$$\hat{h}_m(z) = \sqrt{\frac{m}{\sigma}+m|z|}\left[A_m J_{(D-1)/2}\left(\frac{m}{\sigma}+m|z|\right) + B_m\left(\frac{m}{\sigma}\right)^{D-3} Y_{(D-1)/2}\left(\frac{m}{\sigma}+m|z|\right)\right]. \tag{44}$$

Massive modes are asymptotically (for large $|z|$) plane waves and therefore for infinite range of z not normalizable.

We showed that the massless mode is always normalizable but this is not enough to interpret it as a graviton. The reason is the following. The bulk equation of motion (27) contains an overall factor depending on κ_n and on the warp factor σ:

$$\sum_n^{n_{max}} \kappa_n(-1)^{(n-1)}\frac{n(D-3)!}{(D-1-2n)!}\sigma^{2n-2}. \tag{45}$$

Although the value of this factor is not important for the solution, we have to remember that the sign of this factor is related to the sign of the kinetic energy term for the fluctuations in the effective Lagrangian. The wrong sign of the kinetic energy indicates instability of the assumed background. Thus, our domain wall solution can be stable only if the parameters κ_n are such that the sum in Eq. (45) is positive. Using the rescaled couplings (introduced in [2])

$$p_n = (-1)^{n-1}\kappa_n\frac{(D-1)!}{(D-1-2n)!} \tag{46}$$

the necessary condition for the stability of the domain wall solutions can therefore be written in the form

$$\sum_{n=1}^{n_{max}} n p_n \sigma^{2n-2} > 0. \tag{47}$$

It is interesting to compare the above condition with the bulk and boundary equations [2] that must be satisfied by the warp factor, σ, of the background domain wall metric (7):

$$\sum_{n=1}^{n_{max}} p_n \sigma^{2n} = -\Lambda, \tag{48}$$

$$\sum_{n=1}^{n_{max}} \frac{n}{2n-1} p_n \sigma^{2n-1} = \frac{D-1}{4}\lambda. \tag{49}$$

In our previous paper [2] we considered the possibility of domain wall solutions without the bulk or/and the brane cosmological constants. Such solutions are acceptable only if the new condition (47) is satisfied and this must be checked for any specific model described by a set of the coefficients κ_n. It is not easy to discuss the consequences of Eq. (47) in general but one can make the following observation.

Let us assume that we insist on $\lambda=0$—the ''self-supporting'' brane solution without any matter on the brane (the bulk cosmological constant may be different from 0). Let us define a polynomial

$$P(x) = \sum_{n=1}^{n_{max}} \frac{n p_n}{2n-1} x^{2n-1}. \tag{50}$$

The condition for vanishing λ reads $P(\sigma)=0$ while the condition for the correct sign of the kinetic terms is $P'(\sigma)>0$. From the positivity of the gravitational constant $\kappa_1 = (2\kappa^2)^{-1}$ it follows that $P'(0)>0$. This means of course that the trivial domain wall with $\sigma=0$ (i.e. the Minkowski space) has the correct sign of the graviton kinetic term. This means also that the first nontrivial domain wall (the one with the smallest positive σ) with $\lambda=0$ is unstable. But not all

solutions with $\lambda=0$ must be unstable. The number of different solutions with $\lambda=0$ and $\sigma\neq 0$ is equal to the number of positive zeros of $P(x)$ which is at most $(n_{max}-1)$. If $P(x)$ has only first order zeros than ''every second'' solution with $\lambda=0$ can be stable because such a function changes its derivative when moving from one zero point to the next one. Thus, a nontrivial domain wall with vanishing brane cosmological constant is possible if $n_{max}\geqslant 3$ and this can be satisfied for space-time dimension $D\geqslant 6$.

A similar analysis can be performed for domain walls with vanishing bulk cosmological constant, $\Lambda=0$—the only difference is to use the polynomial $\Sigma_{n=1}^{n_{max}} p_n x^{2n}$ instead of that defined in Eq. (50).

In the case of both cosmological constants vanishing it is not difficult to see that for $n_{max}\geqslant 3$ there exists a range of values of $p_1,\ldots,p_{n_{max}}$ for which all three conditions (47)–(49) can be simultaneously satisfied with $\lambda=\Lambda=0$ and $\sigma\neq 0$. Thus, stable solutions with vanishing brane and/or bulk cosmological constant are possible if the space-time is at least 6 dimensional.

In conclusion, we considered a class of models with higher order gravity corrections in the form of the Euler densities with arbitrary power n of the curvature tensor in arbitrary space-time dimension D. The fluctuations around the domain wall type solutions (found in [2]) were shown to have a similar spectrum as in the lowest order case $(n=1)$—the bulk equation of motion rather miraculously turned out to depend only on D and not on n. The boundary condition at the wall for massive modes has some n dependence. There exists one normalizable massless mode and a continuum of massive modes (without the energy gap). The solutions for all D are almost the same as in the original Randall-Sundrum model with the Hilbert-Einstein action [15] (apart from some numerical factors), and the discussion about the applicability of Newton's law and the effective number of dimensions can be carried over from [15] to the general case discussed in this paper virtually unchanged.

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