PHYSICAL REVIEW D, VOLUME 65, 064003

Dilatonic wormholes: Construction, operation, maintenance, and collapse to black holes

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(Received 18 October 2001; published 7 February 2002)

The CGHS two-dimensional dilaton gravity model is generalized to include a ghost Klein-Gordon field, i.e., with a negative gravitational coupling. This exotic radiation supports the existence of static traversible wormhole solutions, analogous to Morris-Thorne wormholes. Since the field equations are explicitly integrable, concrete examples can be given of various dynamic wormhole processes, as follows. (i) Static wormholes are constructed by irradiating an initially static black hole with the ghost field. (ii) The operation of a wormhole to transport matter or radiation between the two universes is described, including the back reaction on the wormhole, which is found to exhibit a type of neutral stability. (iii) It is shown how to maintain an operating wormhole in a static state, or return it to its original state, by turning up the ghost field. (iv) If the ghost field is turned off, either instantaneously or gradually, the wormhole collapses into a black hole.

DOI: 10.1103/PhysRevD.65.064003 PACS number(s): 04.70.Bw, 04.20.Cv, 04.50.+h

I. INTRODUCTION

The theoretical existence of space-time wormholes has intrigued experts and the public alike. Wheeler [1] speculated that quantum fluctuations in space-time topology can occur, so that the smooth space-time of classical Einstein gravity becomes, at the Planck scale, a continual foam of short-lived interconnections. If such a wormhole could be expanded to macroscopic size, it could provide a short cut between otherwise distant regions, like a bookworm tunneling between different pages of an atlas. Certainly all the standard stationary black-hole solutions have a wormhole ($R\times S^2$) spatial topology, describing two Alice-through-the-looking-glass universes joined by the famous Einstein-Rosen bridge [2], or wormhole throat. In such cases, the two universes are not causally connected, so that it is impossible to travel between them, any attempt leading instead into the black hole [3]. However, this is only just so; a light ray can be sent along the past boundary of one universe to the future boundary of the other, just escaping the black hole. Thus it is not difficult to imagine a small modification which would yield a traversible wormhole.

Morris and Thorne [4] popularized traversible wormholes as a respectable theoretical possibility, studying static, spherically symmetric cases in detail. The spatial topology is the same as in the black-hole cases, but the throat or minimal surface is preserved in time, so that observers can pass through it in either direction, traveling freely between the two universes. According to the Einstein equation, such a geometry requires matter which does not satisfy a classical positive-energy property, the weak energy condition. However, this condition can be violated quantum mechanically, e.g., in the Casimir effect, or in alternative gravity theories. Such negative-energy matter was dubbed exotic matter by Morris and Thorne. This provoked renewed interest in traversible wormholes, as reviewed by Visser [5]. The possible astrophysical existence of wormholes has been taken seriously enough for searches of observational data [6].

The Morris-Thorne framework begs development in at least two important respects, quite apart from generalizing beyond static, spherically symmetric cases. First, the exotic matter was not modeled, but simply assumed to exist in exactly the configuration needed to support the wormhole. Secondly, the back reaction of the transported matter on the wormhole was ignored. If the wormhole turns out to be unstable to such back reaction, it would be useless for actual transport. Many physicists' instinctive reaction is that negative energy, unbounded below, will lead to instability. Another practical question is: if the negative-energy source fails, i.e., the exotic matter is not maintained, what happens to the wormhole? Again, a pessimistic reaction is that the negative energy densities are likely to create naked singularities. An alternative prediction was that the wormhole would collapse to a black hole [7]. The same reference introduced a framework for studying dynamic wormholes, in which both wormholes and black holes are locally defined by the same geometrical objects, trapping horizons, with one key difference: black holes have achronal horizons and wormholes have temporal horizons, so that they are respectively one-way and two-way traversible. This also indicated that a wormhole could be constructed from a black hole using exotic matter.

To find definite answers to the above questions, it is necessary to specify the exotic matter model. Various studies have been based on alternative gravity theories or semiclassical quantum field theory, e.g., [8,9]. However, the difficulty of solving the field equations tends to obscure issues of principle. Here we propose a toy model, intended to describe the essential dynamics of a wormhole, but explicitly integrable, so that concrete answers to the above questions are easily found. A similar approach in the context of black-hole evaporation involved the Callan-Giddings-Harvey-Strominger (CGHS) two-dimensional dilaton gravity model [10], which was expected to share similar features with spherically symmetric black-hole evaporation by Hawking radiation. Certainly classical black-hole dynamics is qualitatively similar, for instance in regard to cosmic censorship,

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but such properties are much easier to prove in the toy model [11] than in the corresponding realistic model, the spherically symmetric Einstein-Klein-Gordon system [12]. Here we propose a simple generalization of the CGHS model to include a ghost Klein-Gordon field, i.e., with the gravitational coupling taking the opposite sign to normal. This is a specific model for the exotic matter, or more accurately exotic radiation, as we take the massless field for simplicity.

The article is organized as follows. Section II describes the model and how its general solution can be constructed from initial data. Section III reviews the static black-hole solution and introduces the static wormhole solution, each of which depend on one parameter, mass or horizon radius. Section IV describes what happens to an initially static wormhole when the ghost field supporting it is turned off and allowed to disperse. Section V shows how to construct a static wormhole by irradiating an initially static black hole with the ghost field. Section VI describes the operation of a wormhole for transport or signaling, using a normal Klein-Gordon field to model the matter or radiation, including the back reaction of the field on the wormhole. Section VII shows how to maintain the operating wormhole in a static state, or return it to its original state. Section VIII concludes.

II. DILATON GRAVITY

The CGHS two-dimensional dilaton gravity of Callan *et al.* [10] is generalized by the action

$$\int_S \mu\left[e^{-2\phi}(R+4(\nabla\phi)^2+4\lambda^2)-\frac{1}{2}(\nabla f)^2+\frac{1}{2}(\nabla g)^2\right] \tag{1}$$

where S is a 2-manifold, μ, R, and ∇ are the area form, Ricci scalar and covariant derivative of a Lorentz 2-metric on S, λ represents a negative cosmological constant, ϕ is a scalar dilaton field, f is a scalar field representing matter and g is a ghost scalar field with negative gravitational coupling. The last term is added to the CGHS action in order that g provides the negative energy densities needed to maintain a wormhole [4,5,7,13–16].

By choosing future-pointing null coordinates (x^+,x^-), the line element may be written as

$$ds^2=-2e^{2\rho}dx^+dx^- \tag{2}$$

where the conventional factor of 2 differs from that of earlier references [10,11]. One component of the field equations is

$$\partial_+\partial_-\phi=\partial_+\partial_-\rho \tag{3}$$

where $\partial_\pm=\partial/\partial x^\pm$, so the coordinate freedom $x^\pm\mapsto\hat{x}^\pm(x^\pm)$ can be used to take

$$\rho=\phi. \tag{4}$$

The remaining coordinate freedom is just

$$x^\pm\mapsto e^{\pm b}x^\pm+c^\pm \tag{5}$$

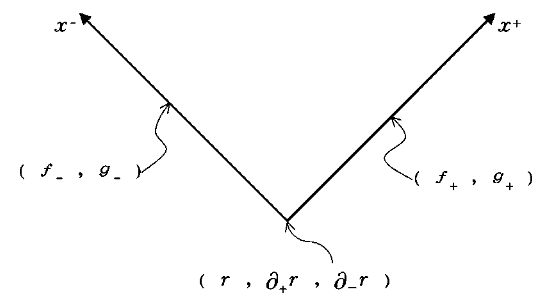

FIG. 1. Initial data, taking $x_0^\pm=0$.

where the constants $c^\pm$ fix the origin and the constant b refers to relative rescalings of $x^\pm$. It is convenient to transform the dilaton field ϕ to

$$r=2e^{-2\phi}. \tag{6}$$

Then the remaining field equations are the evolution equations

$$\partial_+\partial_-f=0 \tag{7}$$

$$\partial_+\partial_-g=0 \tag{8}$$

$$\partial_+\partial_-r=-4\lambda^2 \tag{9}$$

and the constraints

$$\partial_\pm\partial_\pm r=(\partial_\pm g)^2-(\partial_\pm f)^2. \tag{10}$$

The evolution equations (7)–(9) have the general solutions

$$f(x^+,x^-)=f_+(x^+)+f_-(x^-) \tag{11}$$

$$g(x^+,x^-)=g_+(x^+)+g_-(x^-) \tag{12}$$

$$r(x^+,x^-)=r_+(x^+)+r_-(x^-)-4\lambda^2x^+x^-. \tag{13}$$

The constraints (10) are preserved by the evolution equations in the $\partial_\mp$ directions, and so may be reduced to

$$\partial_\pm\partial_\pm r_\pm=G_\pm^2-F_\pm^2 \tag{14}$$

where the null derivatives

$$F_\pm=\partial_\pm f_\pm \tag{15}$$

$$G_\pm=\partial_\pm g_\pm \tag{16}$$

are convenient variables. The constraints are manifestly integrable for $r_\pm$ given initial data

$$(f_+,g_+)\quad\text{on}\quad x^-=x_0^- \tag{17}$$

$$(f_-,g_-)\quad\text{on}\quad x^+=x_0^+ \tag{18}$$

$$(r,\partial_+r,\partial_-r)\quad\text{at}\quad x^+=x_0^+,x^-=x_0^- \tag{19}$$

for constants $x_0^\pm$ (Fig. 1). Then the general procedure is to

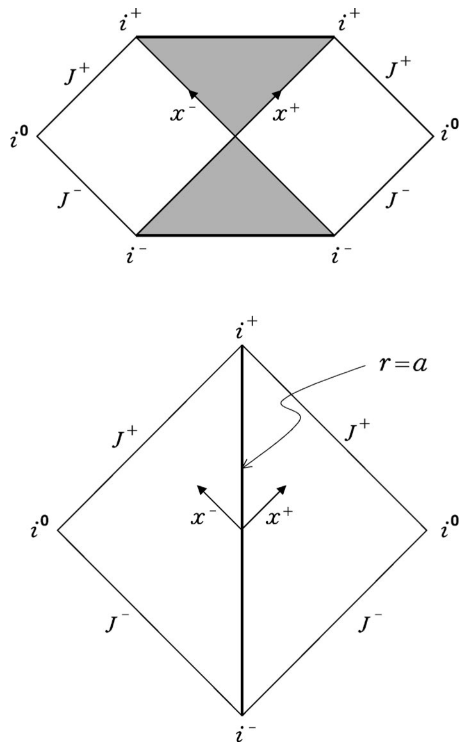

FIG. 2. Penrose conformal diagrams of (a) the static black hole and (b) the static wormhole. Both space-times are divided into two universes (unshaded regions), but observers can travel freely between them via the wormhole, whereas the black hole (upper shaded region) swallows any such attempt.

specify this initial data, integrate the constraints (14) for $r_\pm$, then the general solution follows as Eqs. (11)–(13).

Finally, we note from the vacuum constraints (10) that r is analogous to the areal radius in spherically symmetric Einstein gravity [11,17], correctly reproducing the expansion of a light wave in flat space-time, $\partial_\pm \partial_\pm r = 0$.

III. STATIC BLACK-HOLE AND WORMHOLE SOLUTIONS

In vacuum, $f = g = 0$, the general solution to the field equations is [10]

$$r = 2m - 4\lambda^2 x^+ x^- \tag{20}$$

where the origin has been fixed using Eq. (5). The constant m may be interpreted as the mass of the space-time, whose global structure has been described previously [11]. For $m > 0$ there is a static black hole, analogous to the Schwarzschild black hole [Fig. 2(a)]. The case $m=0$ is flat and the case $m<0$ contains an eternal naked singularity, analogous to the negative-mass Schwarzschild solution. Henceforth we take $m>0$. Note that the original reference [10] defined mass as λm, in which case λr would be analogous to radius, but for $\lambda > 0$ this makes no essential difference in the following.

Next, we consider the case that $f=0$ but $g \neq 0$, finding the solution

$$r = a + 2\lambda^2 (x^+ - x^-)^2 \tag{21}$$

$$g = 2\lambda (x^+ - x^-) \tag{22}$$

where the origin has again been fixed. A coordinate transformation $x^\pm = (t \pm z)/\sqrt{2}$ shows that the solution is static:

$$ds^2 = \frac{2(dz^2 - dt^2)}{a + 4\lambda^2 z^2}. \tag{23}$$

If $a>0$, this represents a traversible wormhole with analogous global structure to a Morris-Thorne wormhole, with a throat $r=a$ at $z=0$, joining two regions with $r>a$, a $z>0$ universe and a reflected $z<0$ universe [Fig. 2(b)]. If $a<0$, there is an eternal naked singularity $r=0$, while if $a=0$, the space-time has constant negative curvature, as can be seen by calculating the Ricci scalar [11]

$$R = r^{-1} \partial_+ r \partial_- r - \partial_+ \partial_- r. \tag{24}$$

Henceforth we take $a>0$.

In summary, the model naturally contains both static black holes and static traversible wormholes. Before proceeding, we note an important feature of both cases, the trapping horizons, defined by $\nabla r \cdot \nabla r = 0$, or equivalently $\partial_+ r = 0$ or $\partial_- r = 0$ [7,11,17,18]. In the black hole, they coincide with the event horizons $r = 2m$ at $x^- = 0$ and $x^+ = 0$ respectively. In the wormhole, there is a double trapping horizon, $\partial_+ r = \partial_- r = 0$, at the throat $r=a$. This illustrates how trapping horizons of different type may be used to locally define both black holes and wormholes [7]. Locating the trapping horizons, or equivalently the trapped regions where $\nabla r \cdot \nabla r < 0$, is a key feature of the following analysis of dynamic situations.

IV. WORMHOLE COLLAPSE

One can now study what happens to a static wormhole if its negative-energy source fails. We consider first that the supporting ghost field g is switched off suddenly from both sides of the wormhole, then that g is instead gradually reduced to zero.

A. Sudden collapse

We set the initial data so that there is a static wormhole, with g then switched off suddenly from both sides of the wormhole [Fig. 3(a)]:

$$G_\pm = \begin{cases} \pm 2\lambda, & x^\pm < 0, \\ 0, & x^\pm \geqslant 0. \end{cases} \tag{25}$$

Taking $f=0$, the constraints (14) are

$$\partial_\pm \partial_\pm r_\pm = \begin{cases} 4\lambda^2, & x^\pm < 0, \\ 0, & x^\pm \geqslant 0. \end{cases} \tag{26}$$

Integrating twice, we obtain

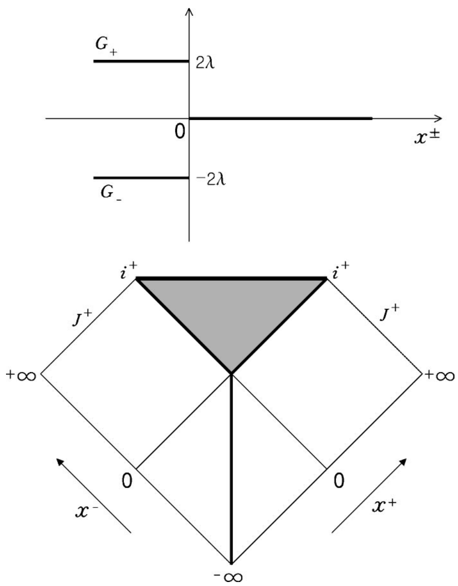

FIG. 3. (a) The ghost field g is switched off suddenly from both sides of the wormhole. (b) Conformal diagram: when the wormhole's negative-energy source fails suddenly at $x^\pm=0$, it immediately collapses into a black hole.

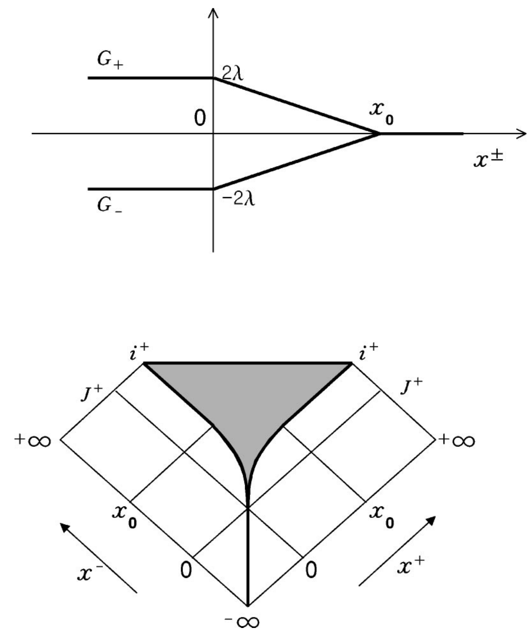

FIG. 4. (a) The ghost field g is gradually reduced to zero. (b) Conformal diagram: the wormhole throat bifurcates and the resulting non-static wormhole again eventually becomes a black hole. Shading in these diagrams indicates trapped regions, where $\partial_+ r \partial_- r > 0$.

$$r_\pm = \begin{cases} 2\lambda^2 (x^\pm)^2 + \dfrac{a}{2}, & x^\pm < 0, \\ \dfrac{a}{2}, & x^\pm \geqslant 0 \end{cases} \tag{27}$$

where the constants of integration are determined by continuity at $x^\pm=0$. The resulting solution from Eq. (13) is

$$r = \begin{cases} a + 2\lambda^2 (x^+ - x^-)^2, & x^+ < 0 \quad \text{and} \quad x^- < 0, \\ a + 2\lambda^2 x^+ (x^+ - 2x^-), & x^+ < 0 \quad \text{and} \quad x^- \geqslant 0, \\ a - 2\lambda^2 x^- (2x^+ - x^-), & x^+ \geqslant 0 \quad \text{and} \quad x^- < 0, \\ a - 4\lambda^2 x^+ x^-, & x^+ \geqslant 0 \quad \text{and} \quad x^- \geqslant 0. \end{cases} \tag{28}$$

One may recognize the solution in the final region as a static black hole (20) with mass $m=a/2$. The double trapping horizon of the static wormhole bifurcates into the event horizons of the black hole, all with $r=a=2m$ [Fig. 3(b)]. Thus the wormhole immediately collapses into a black hole.

The Schwarzschild-like relationship between mass and throat radius is no accident; there is a definition of active gravitational mass energy [11]

$$E = \frac{r}{2}\left(1 - \frac{\nabla r \cdot \nabla r}{4\lambda^2 r^2}\right) \tag{29}$$

which evaluates as $r/2$ on any trapping horizon. Thus a wormhole with a throat $r=a$ has an effective mass $a/2$. Here and elsewhere, it is useful to recall the analogy with spherically symmetric Einstein gravity, where r corresponds to areal radius and there is a similar definition of active gravitational mass energy [17].

B. Gradual collapse

Again starting with a static wormhole, we now reduce the ghost field gradually to zero in the simplest way, linearly [Fig. 4(a)]:

$$G_\pm = \begin{cases} \pm 2\lambda, & x^\pm < 0, \\ \mp \alpha x^\pm \pm 2\lambda, & 0 \leqslant x^\pm < x_0, \\ 0, & x_0 \leqslant x^\pm \end{cases} \tag{30}$$

where α is a constant and $x_0 = 2\lambda/\alpha$. Here $\alpha \to 0$ recovers the static wormhole and $\alpha \to \infty$ recovers sudden collapse. By similar calculations, again taking $f=0$,

$$\partial_\pm \partial_\pm r_\pm = \begin{cases} 4\lambda^2, & x^\pm < 0, \\ \alpha^2 x^{\pm 2} - 4\lambda \alpha x^\pm + 4\lambda^2, & 0 \leqslant x^\pm < x_0, \\ 0, & x_0 \leqslant x^\pm \end{cases} \tag{31}$$

integrate to

$$r_\pm = \begin{cases} 2\lambda^2 x^{\pm 2} + \dfrac{a}{2}, & x^\pm < 0, \\ \dfrac{\alpha^2}{12} x^{\pm 4} - \dfrac{2\alpha\lambda}{3} x^{\pm 3} + 2\lambda^2 x^{\pm 2} + \dfrac{a}{2}, & 0 \leqslant x^\pm < x_0, \\ \dfrac{8\lambda^3}{3\alpha} x^\pm - \dfrac{4\lambda^4}{3\alpha^2} + \dfrac{a}{2}, & x_0 \leqslant x^\pm. \end{cases} \tag{32}$$

and the solution

$$r=\begin{cases} a+2\lambda^2(x^+-x^-)^2, & x^+<0 \quad \text{and} \quad x^-<0\\ a+2\lambda^2(x^+-x^-)^2-\frac{2\alpha\lambda}{3}x^{-3}+\frac{\alpha^2}{12}x^{-4}, & x^+<0 \quad \text{and} \quad 0\leqslant x^-<x_0\\ a+2\lambda^2(x^+-x^-)^2-\frac{2\alpha\lambda}{3}x^{+3}+\frac{\alpha^2}{12}x^{+4}, & 0\leqslant x^+<x_0 \quad \text{and} \quad x^-<0\\ a-\frac{4\lambda^4}{3\alpha^2}-4\lambda^2x^+x^-+\frac{8\lambda^3}{3\alpha}x^-+2\lambda^2x^{+2}, & x^+<0 \quad \text{and} \quad x_0\leqslant x^-\\ a+2\lambda^2(x^+-x^-)^2-\frac{2\alpha\lambda}{3}(x^{+3}+x^{-3})+\frac{\alpha^2}{12}(x^{+4}+x^{-4}), & 0\leqslant x^+<x_0 \quad \text{and} \quad 0\leqslant x^-<x_0\\ a-\frac{4\lambda^4}{3\alpha^2}-4\lambda^2x^+x^-+\frac{8\lambda^3}{3\alpha}x^++2\lambda^2x^{-2}, & x_0\leqslant x^+ \quad \text{and} \quad x^-<0\\ a-\frac{4\lambda^4}{3\alpha^2}-4\lambda^2x^+x^-+\frac{8\lambda^3}{3\alpha}x^-+2\lambda^2x^{+2}-\frac{2\alpha\lambda}{3}x^{+3}+\frac{\alpha^2}{12}x^{+4}, & 0\leqslant x^+<x_0 \quad \text{and} \quad x_0\leqslant x^-\\ a-\frac{4\lambda^4}{3\alpha^2}-4\lambda^2x^+x^-+\frac{8\lambda^3}{3\alpha}x^++2\lambda^2x^{-2}-\frac{2\alpha\lambda}{3}x^{-3}+\frac{\alpha^2}{12}x^{-4}, & x_0\leqslant x^+ \quad \text{and} \quad 0\leqslant x^-<x_0\\ a-\frac{8\lambda^4}{9\alpha^2}-4\lambda^2\left(x^+-\frac{2\lambda}{3\alpha}\right)\left(x^--\frac{2\lambda}{3\alpha}\right), & x_0\leqslant x^+ \quad \text{and} \quad x_0\leqslant x^-. \end{cases} \tag{33}$$

The solution in the final region is again recognizable as a black hole (20), but with reduced mass and shifted by $2\lambda/3\alpha=x_0/3$ in $x^\pm$ compared with the sudden case. The trapping horizons $\partial_\pm r=0$ are located at

$$x^\mp=\begin{cases} x^\pm, & x^\pm<0,\\ x^\pm-\frac{1}{x_0}x^{\pm 2}+\frac{1}{3x_0^2}x^{\pm 3}, & 0\leqslant x^\pm<x_0,\\ \frac{x_0}{3}, & x_0\leqslant x^\pm. \end{cases} \tag{34}$$

The final expressions coincide with the locations of the event horizons of the black hole. The middle expressions are cubic curves which smoothly join the initial wormhole throat to the final event horizons [Fig. 4(b)]. Thus the double trapping horizon of the wormhole has again bifurcated, eventually forming a black hole. In between, the geometry may be characterized as a non-static wormhole, as observers may still traverse it for a certain time, after which the attempt leads only into the black hole.

V. WORMHOLE CONSTRUCTION

We next study how to convert a black hole into a traversible wormhole by irradiating it with the ghost field. This is not simply the time reverse of the wormhole collapses described above, which would represent the creation of a wormhole from a white hole, as one can see by inverting the conformal diagrams [Figs. 3(b),4(b)]. Instead, one needs to begin the irradiation at some positive value x_0 of the Kruskal-like coordinates $x^\pm$ [Fig. 2(a)]. Moreover, in order to close up a future trapped region by merging its trapping horizons, the negative energy densities required must be larger than those required to maintain the static wormhole which finally forms [7]. In other words, $|G_\pm|$ must first reach a maximum greater than 2λ. The simplest way to achieve this is to set initial data as double step functions [Fig. 5(a)]

$$G_\pm=\begin{cases} 0, & x^\pm<x_0,\\ \pm 2\beta\lambda, & x_0\leqslant x^\pm<x_1,\\ \pm 2\lambda, & x_1\leqslant x^\pm \end{cases} \tag{35}$$

where $\beta>1$ is a constant, to be determined by the requirement that the trapping horizons merge at $x^\pm=x_1$, for which we find $x_0=x_1(1-\beta^{-2})$. Then taking $f=0$, the constraints (14) are

$$\partial_\pm\partial_\pm r_\pm=\begin{cases} 0, & x^\pm<x_0,\\ 4\lambda^2\beta^2, & x_0\leqslant x^\pm<x_1,\\ 4\lambda^2, & x_1\leqslant x^\pm. \end{cases} \tag{36}$$

Integrating twice, assuming a black hole of mass m in the initial region, leads to

$$r_\pm=\begin{cases} m, & x^\pm<x_0,\\ 2\lambda^2\beta^2(x^\pm-x_0)^2+m, & x_0\leqslant x^\pm<x_1,\\ 2\lambda^2(x^{\pm 2}-x_0x_1)+m, & x_1\leqslant x^\pm \end{cases} \tag{37}$$

and

$$r=\begin{cases} 2m-4\lambda^2x^+x^-, & x^+<x_0 \quad \text{and} \quad x^-<x_0, \\ 2m-4\lambda^2x^+x^-+2\lambda^2\beta^2((x^+-x_0)^2+(x^--x_0)^2), & x_0\leqslant x^+<x_1 \quad \text{and} \quad x_0\leqslant x^-<x_1, \\ 2\lambda^2(x^+-x^-)^2+2m-4\lambda^2x_0x_1, & x_1\leqslant x^+ \quad \text{and} \quad x_1\leqslant x^- \end{cases} \tag{38}$$

where we have omitted the less relevant regions. One may recognize the solution in the final region as a static wormhole (21) with throat radius $2m-4\lambda^2x_0x_1$. Thus we require $2\lambda^2x_0x_1<m$. By choice of the parameters (x_0,x_1), we have constructed a static wormhole with any throat radius less than $2m$, the radius of the original black hole. The trapping horizons $\partial_\pm r=0$ are located at

$$x^{\mp}=\begin{cases} 0, & x^\pm<x_0, \\ \beta^2(x^\pm-x_0), & x_0\leqslant x^\pm<x_1, \\ x^\pm, & x_1\leqslant x^\pm \end{cases} \tag{39}$$

which are straight line segments. Thus the ghost radiation causes the trapping horizons of the initial black hole to shrink towards each other, eventually merging to form the throat of the final static wormhole [Fig. 5(b)]. The trapped region composing the black hole simply evaporates. This classically unexpected behavior is, of course, due to the negative energy densities. As in the wormhole collapse case, the trapping horizons can be smoothed off by taking smoother profiles for $G_\pm$, but the details are not particularly illuminating. In summary, static wormholes have been constructed by irradiating a black hole with ghost radiation.

One can also regard this as analogous to black-hole evaporation, with the ghost radiation modelling the ingoing negative-energy Hawking radiation, suggesting that the final state of black hole evaporation might be a stationary wormhole [7–9]. This would naturally resolve the information-loss puzzle, as there is no singularity in which information is lost; everything that fell into the black hole eventually reemerges.

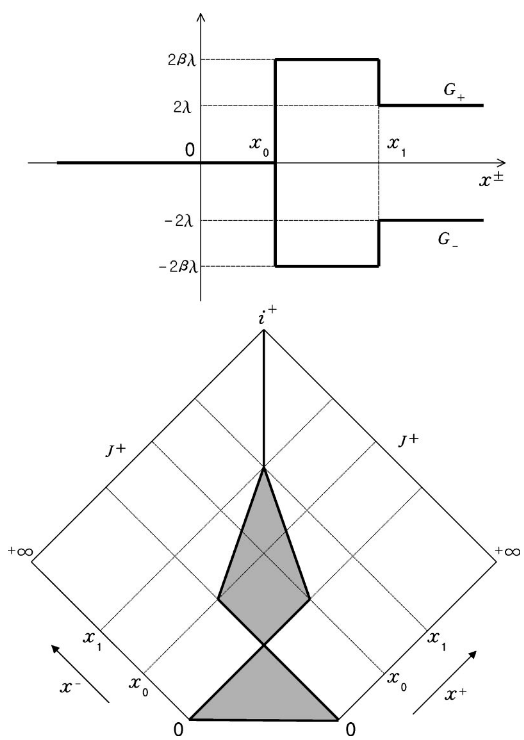

FIG. 5. (a) Irradiating a vacuum black hole with the ghost field g. (b) Conformal diagram: the initially static black hole, as in Fig. 2(a), becomes a dynamic wormhole, eventually reaching a static state as in Fig. 2(b). The black hole has been converted into a traversible wormhole.

VI. WORMHOLE OPERATION

If a wormhole is actually used to transport a parcel or person between the two universes, the transported matter will affect the wormhole by changing the gravitational field. In principle this occurs even if the wormhole is merely used for signalling. We will study the dynamical effects of such back reaction by using the field f to model the matter or radiation. The use of Klein-Gordon radiation rather than more realistic matter is for simplicity only; any source of mass energy would have more or less similar gravitational effects. In the current model, the constraints (14) show explicitly that increasing $F_\pm^2$ has an equivalent gravitational effect to reducing $G_\pm^2$, which we have already shown can cause collapse to

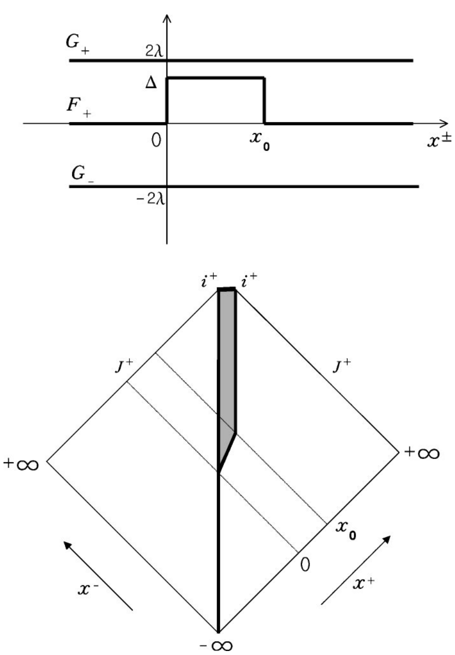

FIG. 6. (a) A step pulse of positive-energy radiation is beamed through the wormhole. (b) Conformal diagram: the wormhole becomes non-static but, for a small-energy pulse, remains traversible for a long time.

a black hole. Thus it is to be expected that too much transport would destroy the wormhole. A worse possibility is that the wormhole might be unstable to the slightest perturbation and start to collapse immediately. We investigate this below, giving the first concrete examples of wormhole operation including back reaction.

Again taking the simplest case, we consider a step pulse of positive-energy radiation [Fig. 6(a)]:

$$F_+=\begin{cases}0, & x^+<0,\\ \Delta, & 0\leqslant x^+<x_0,\\ 0, & x_0\leqslant x^+,\end{cases} \tag{40}$$

$$F_-=0$$

with $G_\pm=\pm 2\lambda$. The energy of the pulse may be defined as the change in the gravitational energy (29) due to the pulse, which evaluates at $x^-=0$ as

$$\epsilon=\frac{1}{4}(\Delta x_0)^2. \tag{41}$$

In the following, we assume that the energy of the pulse should not be too large, corresponding to the anticipated limit on how much mass energy can be sent through the wormhole without causing it to collapse into a black hole. Specifically we find $\epsilon<a/2$ to avoid an $r=0$ singularity in the middle region. Thus the pulse energy should be less than the effective mass of the wormhole. Inserting the conversion factor c^2/G from length to mass, a one-meter wormhole could transport over a hundred Earth masses. This is hardly a practical limitation, once we establish stability.

The constraints (14)

$$\partial_+\partial_+ r_+=\begin{cases}4\lambda^2, & x^+<0,\\ 4\lambda^2-\Delta^2, & 0\leqslant x^+<x_0,\\ 4\lambda^2, & x_0\leqslant x^+,\end{cases} \tag{42}$$

$$\partial_-\partial_- r_-=4\lambda^2$$

integrate to

$$r_+=\begin{cases}2\lambda^2 x^{+2}+\dfrac{a}{2}, & x^+<0,\\ 2\lambda^2 x^{+2}-\dfrac{\Delta^2}{2}x^{+2}+\dfrac{a}{2}, & 0\leqslant x^+<x_0,\\ 2\lambda^2 x^{+2}+\Delta^2 x_0\left(\dfrac{1}{2}x_0-x^+\right)+\dfrac{a}{2}, & x_0\leqslant x^+,\end{cases} \tag{43}$$

$$r_-=2\lambda^2 x^{-2}+\frac{a}{2}$$

and the solution is

$$r=\begin{cases}a+2\lambda^2(x^+-x^-)^2, & x^+<0,\\ a+2\lambda^2(x^+-x^-)^2-\dfrac{\Delta^2}{2}x^{+2}, & 0\leqslant x^+<x_0,\\ a+2\lambda^2(x^+-x^-)^2+\Delta^2 x_0\left(\dfrac{1}{2}x_0-x^+\right), & x_0\leqslant x^+.\end{cases} \tag{44}$$

The locations of the trapping horizons $\partial_+ r=0$ and $\partial_- r=0$ are given respectively by

$$x^-=\begin{cases}x^+, & x^+<0,\\ x^+-\dfrac{\Delta^2}{4\lambda^2}x^+, & 0\leqslant x^+<x_0,\\ x^+-\dfrac{\Delta^2}{4\lambda^2}x_0, & x_0\leqslant x^+,\end{cases} \tag{45}$$

$$x^+=x^-.$$

Thus the double trapping horizon of the initially static wormhole bifurcates when the radiation arrives [Fig. 6(b)]. After the pulse has passed, the two trapping horizons run parallel in the $x^\pm$ coordinates, forming a non-static traversible wormhole. If the pulse energy is small, $\epsilon\ll a$, the wormhole persists in an almost static state for a long time, $t\sim x_0 a/\epsilon$. Nev-

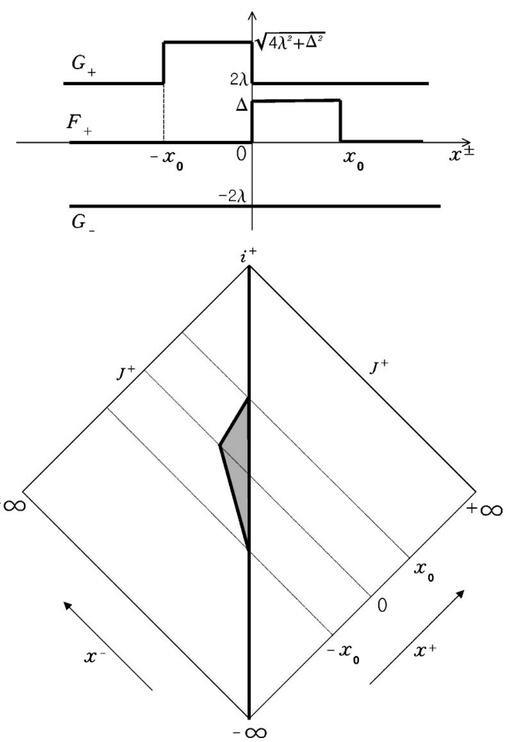

FIG. 7. (a) The step pulse of positive-energy radiation is balanced by a preceding pulse of negative-energy radiation. (b) Conformal diagram: the wormhole returns to its original static state.

ertheless, even for an arbitrarily weak pulse, eventually a spatial $r=0$ singularity develops, similar to that of the static black hole. Observers close enough to the singularity can no longer traverse the wormhole, so it constitutes a black hole with two event horizons. Thus the static wormhole exhibits a type of neutral stability, neither strictly stable in that it does not return to its initial state, nor strictly unstable in that there is no sudden runaway. Note that black holes are also neutrally stable in this sense; perturbing a black hole by dropping positive-energy matter into it increases the area of the trapping horizon finitely, by the first and second laws of black-hole dynamics [11,17,18].

VII. WORMHOLE MAINTENANCE

Keeping an operating wormhole viable indefinitely, defying its natural fate as a black hole, requires additional negative energy to balance the transported matter. The simplest way to maintain the wormhole is just to set initial data

$$G_\pm = \pm\sqrt{4\lambda^2+F_\pm^2} \tag{46}$$

so that the source terms in the constraints (14) cancel to the static wormhole values. Thus the wormhole remains static. The additional negative-energy radiation has balanced the positive-energy radiation, leaving the gravitational field unchanged.

Alternatively, the wormhole may be maintained by beaming in additional negative-energy radiation before or after the positive-energy pulse. We take the same step pulse in F_+ (40) and precede it with a compensating pulse in the ghost field [Fig. 7(a)]:

$$G_+ = \begin{cases} 2\lambda, & x^+ < -x_0, \\ \sqrt{4\lambda^2+\Delta^2}, & -x_0 \le x^+ < 0, \\ 2\lambda, & 0 \le x^+, \end{cases} \tag{47}$$

$$G_- = -2\lambda.$$

Again we require small pulse energy, $\epsilon < a/2$, to avoid a singularity. The constraints (14) become

$$\partial_+\partial_+ r_+ = \begin{cases} 4\lambda^2, & x^+ < -x_0, \\ 4\lambda^2+\Delta^2, & -x_0 \le x^+ < 0, \\ 4\lambda^2-\Delta^2, & 0 \le x^+ < x_0, \\ 4\lambda^2, & x_0 \le x^+, \end{cases} \tag{48}$$

$$\partial_-\partial_- r_- = 4\lambda^2$$

which integrate to

$$r_+ = \begin{cases} 2\lambda^2 x^{+2} + \dfrac{a}{2}, & x^+ < -x_0, \\ 2\lambda^2 x^{+2} + \Delta^2 x^+\left(\dfrac{1}{2}x^+ + x_0\right) - \dfrac{1}{2}\Delta^2 x_0^2 + \dfrac{a}{2}, & -x_0 \le x^+ < 0, \\ 2\lambda^2 x^{+2} - \Delta^2 x^+\left(\dfrac{1}{2}x^+ - x_0\right) - \dfrac{1}{2}\Delta^2 x_0^2 + \dfrac{a}{2}, & 0 \le x^+ < x_0, \\ 2\lambda^2 x^{+2} + \dfrac{a}{2}, & x_0 \le x^+, \end{cases} \tag{49}$$

$$r_- = 2\lambda^2 x^{-2} + \frac{a}{2}$$

and the solution follows as

$$r = \begin{cases} a + 2\lambda^2(x^+ - x^-)^2, & x^+ < -x_0, \\ a + 2\lambda^2(x^+ - x^-)^2 + \dfrac{1}{2}\Delta^2(x^{+2} + 2x_0 x^+ - x_0^2), & -x_0 \le x^+ < 0, \\ a + 2\lambda^2(x^+ - x^-)^2 - \dfrac{1}{2}\Delta^2(x^{+2} - 2x_0 x^+ + x_0^2), & 0 \le x^+ < x_0, \\ a + 2\lambda^2(x^+ - x^-)^2, & x_0 \le x^+. \end{cases} \tag{50}$$

The locations of the trapping horizons are

$$x^- = \begin{cases} x^+, & x^+ < -x_0, \\ x^+ + \dfrac{\Delta^2}{4\lambda^2}(x^+ + x_0), & -x_0 \leqslant x^+ < 0, \\ x^+ - \dfrac{\Delta^2}{4\lambda^2}(x^+ - x_0), & 0 \leqslant x^+ < x_0, \\ x^+, & x_0 \leqslant x^+, \end{cases} \tag{51}$$

$$x^+ = x^-$$

which again are straight line segments. The double horizon bifurcates when the ghost pulse arrives, temporarily opening up a trapped region, but the two horizons subsequently merge to form a static wormhole again [Fig. 7(b)]. This is not unexpected, since the energy of the ghost pulse, $\epsilon' = -\Delta^2 x_0^2/4$, balances that of the other pulse:

$$\epsilon + \epsilon' = 0. \tag{52}$$

In fact, the final state is identical to the initial state in this symmetric case. The wormhole can also be returned to a different static state, with different throat radius, by less symmetric double pulses. The examples show that there is no practical problem of fine-tuning the ghost field to keep the wormhole static. An almost static wormhole is still traversible and can be adjusted at any time to bring it closer to staticity, or to change its size. This is essentially due to the neutral stability.

VIII. CONCLUSION

Space-time wormholes remain in the realms of science fiction and theoretical physics. By the standards of either genre, they are not so far-fetched, differing from experimentally established physics by only one step, the existence of negative energy densities in sufficient concentrations. Here we have not addressed this issue but, assuming a positive answer, have investigated the behavior of the resulting wormholes, evolving dynamically according to field equations. In particular, we have found detailed answers to the following practical questions. (i) How can one construct a traversible wormhole? By bathing a black hole in exotic radiation. (ii) Is an operating wormhole stable under the back reaction of the transported matter? In this case, neutrally stable. (iii) How can a wormhole be maintained indefinitely for transport or signaling? By balance of positive and negative energy. (iv) What happens if the negative-energy source fails? The wormhole collapses into a black hole.

This was mostly predicted by a general theory of wormhole dynamics [7], but here we have given concrete examples, by virtue of the exact solubility of the field equations of two-dimensional dilaton gravity. Despite the supposedly unphysical nature of a ghost Klein-Gordon field, fears about instability, runaway processes and naked singularities proved to be unfounded. More realistic situations may differ in some respects, such as wormhole stability, which may be affected by backscattering and will presumably depend on the exotic matter model. However, the same principles and methods should apply, such as energy balance and tracking of trapping horizons. Indeed, apart from the inclusion of exotic matter or radiation, the methods are the same as those used to analyze black-hole dynamics [17,18]. In particular, the explicit examples of dynamic interconversion of black holes and wormholes should assuage objections that they are fundamentally different objects. Rather, wormholes and black holes have similar physics and a unified theory.

ACKNOWLEDGMENTS

S.A.H. was supported by Korea Research Foundation grant KRF-2001-015-DP0095, S-W.K. by Korea Research Foundation grant KRF-2000-041-D00128 and H.L. by KOSEF grant R01-2000-00015.

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