Stochastic production of kink-antikink pairs in the presence of an oscillating background

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We numerically investigate the production of kink-antikink pairs in a $(1+1)$-dimensional ϕ^4 field theory subject to white noise and periodic driving. The twin effects of noise and periodic driving acting in conjunction lead to considerable enhancement in the kink density compared to the thermal equilibrium value, for low dissipation coefficients and for a specific range of frequencies of the oscillating background. The dependence of the kink density on the temperature of the heat bath, the amplitude of the oscillating background and value of the dissipation coefficient is also investigated. An interesting feature of our result is that kink-antikink production occurs even though the system always remains in the broken symmetry phase.

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I. INTRODUCTION

During the past decade, the evolution of quantum and classical fields out of equilibrium has received a lot of attention. The interest generated in this subject matter is motivated by many physical scenarios in the early universe as well as in condensed matter systems, where nonequilibrium phenomenon plays a crucial role. Of particular relevance is the formation and evolution of a quark-gluon plasma, the formation of topological defects, out-of-equilibrium phase transition dynamics and its impact on domain formation and growth. The nonequilibrium evolution of fields is also a crucial input in understanding how the current baryon asymmetry of the universe was generated.

The formation of topological defects [1] is a generic feature of most symmetry breaking phase transitions which give rise to a vacuum manifold with a nontrivial topology. The most widely discussed mechanism of topological defect formation during phase transitions was first put forward by Kibble in a seminal paper twenty-five years ago [2]. A crucial feature of this mechanism is that it depends solely on the topology of the vacuum manifold and on the space-time dimension, but not on the details of the field dynamics, which makes it universally applicable across a diverse range of energy scales. This also allows many of the predictions of defect formation in the early universe to be tested in the laboratory using condensed matter systems such as superfluid helium and liquid crystals [3]. Recent experiments with liquid crystals have spectacularly demonstrated the validity of Kibble mechanism in first order phase transitions by testing universal aspects of the predictions [4,5]. A complete understanding of defect formation in systems (such as superfluid helium-IV) undergoing a second order phase transitions is still lacking [6].

Dynamics can and does play a crucial role in topological defect formation. It has been demonstrated [7] that the density of defects in first order transitions depends on the velocity of the bubble wall which is determined by the dissipation coefficient of the medium. Moreover, the dynamics of bubble collisions in a first order transition has been shown to lead to a new mechanism of defect formation stemming from the flipping of the order parameter field through the zero of the field. For the case of a spontaneously broken global U(1) field theory in $2+1$ dimensions, this results in the *discontinuous* change in phase of the field by π, leading to the formation of a vortex-antivortex pair. (In $3+1$ dimensions, a string loop is produced.) This mechanism first observed in systems where the symmetry is spontaneously as well as explicitly broken [8] was later shown to be generically valid and investigated in detail [9].

In second order phase transitions, the phenomenon of critical slowing down [10] (freezing of field dynamics near the critical point) is crucial in determining the initial density of defects. The dependence of defect density on the quench rate of a second order transition has also been extensively investigated [11–13].

Until recently, conventional wisdom suggested that topological defect formation could occur only during phase transitions. However, it has been recently shown [14] that topological defects can also be produced by the flipping mechanism [9] under conditions in which the system always remains in the broken symmetry phase without undergoing any phase transition. The production of vortex-antivortex pairs was found to be induced by field oscillations brought about by the coupling of the U(1) scalar field to a periodically oscillating background [14]. The flipping of the order-parameter field in localized regions resulted in the formation of vortex-antivortex pairs, for a certain range of resonant frequencies of the periodically oscillating background.

It is then natural to ask how the defect densities would be affected in the presence of noise. After all, most physical systems in nature are not closed but are continually involved in exchanging energy with a heat bath [15,16]. The heat bath often represents other degrees of freedom which interact with the system in accordance with the fluctuation-dissipation theorem. In this paper we address this issue by studying the formation of kink-antikink pairs in $(1+1)$ dimensions.

We should also mention that a Langevin dynamics approach has also been used to study bubble dynamics in a noisy background [17], long lived oscillating field configu-

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rations (oscillons) in a thermal bath [18] and to obtain the nucleation rates of bubble formation in a first order phase transition using both additive and multiplicative noise [19].

The investigation of kink-antikink production has been carried out in a variety of contexts. The nonequilibrium dynamics of kink formation was investigated [20] for a damped scalar field theory undergoing a symmetry breaking phase transition with the aim of understanding how the defect densities and correlation lengths depend upon the choice of dissipation coefficient and initial conditions. The regime of validity of the linear and Hartree approximation was also elucidated and it was shown that nonlinear effects play a crucial role in determining the initial defect densities after a quench. The thermal production of kinks has been investigated both analytically and numerically. Scalapino, Sears and Ferrell used the transfer integral technique to calculate the exact partition function and correlation functions for a dilute gas of kink-antikink pairs [21]. Krumhansl and Schreiffer (KS) later showed that in the low temperature limit where the dilute gas approximation is valid, the kink contribution to the partition fuction can be factored out and identified with a tunneling term [22]. Currie *et al.* [23] further generalized the results of KS by taking into account the interaction of the kinks with phonons. They also studied kink production in the sine-Gordon model in addition to the real scalar field model. Numerical investigations to check the theoretical predictions like the thermal nucleation rate and kink life-time have also been carried out [24,25]. More recently, Alexander, Habib and Kovner [26] investigated thermal production of kinks both analytically and numerically by using a Langevin equation with additive white noise to model the effect of thermal fluctuations. They were able to identify the temperature regime below which the dilute gas (WKB) approximation is valid. By introducing a double-Gaussian approximation they were able to obtain an excellent agreement between their theoretical prediction of thermal kink densities and their numerical results. Langevin simulations in the intermediate temperature range were recently carried out by Gleiser and Muller [27], where they also pointed out the important issue of lattice spacing dependence of results in simulations of stochastic field equations. The thermal nucleation of interacting kink-antikink pairs in the sine-Gordon model has been investigated by Büttiker and Landauer, in the overdamped limit [28]. The nucleation rate in the overdamped limit was found to be proportional to the square of the equilibrium density [28]; however some studies [29] also suggested that the nucleation rate is proportional to the cube of the equilibrium density of kinks. The resolution of this controversy depends on unambiguously ascertaining whether the kink lifetime is inversely proportional to the equilibrium kink density [30] or the square of the equilibrium kink density [29]. Recent work for both sine-Gordon [30] and ϕ^4 models [31], involving extensive numerical simulations of the nucleation and annihilation dynamics of thermal kinks and antikinks, have clarified that the nucleation rate is proportional to the square of the equilibrium density. In all these papers, the effect of an oscillating background driving the kink-producing field was not considered. Such a situation has been discussed by Marchesoni *et al.* [32] for an overdamped field theory.

In this paper we focus on numerically investigating the formation of kinks in a $(1+1)$-dimensional spontaneously broken *relativistic* (underdamped) scalar field theory coupled to an oscillating background and subject to white noise.[1] The coupling of the field to the oscillating background and thermal noise induces large amplitude field oscillations for a certain range of frequencies, thus enabling the field to cross over the potential barrier resulting in the nucleation of kink-antikink pairs. We find that kink-antikink pairs are produced in spite of the fact that the system both initially and for all subsequent times, remains in the broken symmetry phase. This has important consequences for topological defect production [14] since it implies that topological defects need not be produced only during a phase transition, as was believed earlier. Moreover, kink-antikink production is found to depend sensitively on the dissipation coefficient and the effect becomes considerably suppressed for high dissipation coefficients.

The physical situation we discuss is of relevance to both early universe as well as condensed matter physics. During reheating after inflation, the inflaton field starts oscillating about its vacuum value and would act like a driving force to any other scalar field to which it is coupled. The oscillations eventually die out due to particle production caused by transfer of energy from the oscillating inflaton to the quanta of fields to which it is coupled, leading to reheating of the universe and a transition from matter dominated to the radiation dominated phase. The process of (p)reheating of the universe after inflation has been the subject of intensive study during the last decade during which a new theory of reheating (due to inflaton decay via explosive soft particle production) was developed and studied in detail [34]. The effect of noise on the growth of fluctuations has also been studied [35] in the context of reheating after inflation. Domain wall [36] and cosmic string [37] production during (p)reheating has also been investigated. The main premise of these papers was that (p)reheating could result in nonthermal symmetry restoration [38] after inflation. Topological defects would then be produced during the subsequent symmetry breaking brought about by rescattering effects and/or cooling due to expansion, in the usual manner (i.e. via Kibble mechanism). However, as pointed out recently [14], topological defects can form even *without* the system undergoing any thermal or nonthermal phase transition; simply because of large amplitude oscillations of the defect producing field, induced by its coupling to a spatially homogeneous, oscillating, inflaton field.

In condensed matter systems, the oscillating background can be thought of as an external oscillating influence such as temperature, pressure or even an electric or magnetic field coupled to the system. A system subject to noise and periodic

[1]Strictly speaking there is no phase transition in $d \leqslant 2$ spatial dimensions, for systems in equilibrium [33]. However, in our case, the system is always evolving out-of-equilibrium and so the Mermin-Wagner theorem does not apply.

driving has also been extensively studied in nonlinear dynamics, in the context of stochastic resonance [39]. The field theory system considered here, shares the characteristic features of systems undergoing stochastic resonance. However, the phenomenon we observe, i.e. enhancement in kink-antikink densities due to the twin effects of noise and periodic driving, is distinct from stochastic resonance, as will be discussed later.

The main purpose of this work is to investigate the effect of noise and an oscillating background on the density of kink-antikink pairs. The presence of the oscillating background ensures that the system is always out-of-equilibrium and so the defect production in this case is distinct from the thermal production of kink-antikink pairs [21–27,32]. The main result of this work is the observed enhancement of kink-antikink density, compared to the thermal equilibrium value, as a result of the twin effects of noise and coupling of the field with an oscillating background.

The paper is organized as follows. In Sec. II, we describe our model and the numerical algorithm we use to solve the Langevin equation. There we also discuss some of the issues related to the lattice-spacing dependence of the results of Langevin simulations. The results of our numerical simulations are described in Sec. III. The dependence of kink-antikink density on parameters such as bath temperature, amplitude of oscillation of the background homogeneous field and the dissipation coefficient are investigated. The range of frequencies for which defect production occurs is also obtained. We end with a brief summary and discussion of our results in Sec. IV.

II. THE MODEL AND NUMERICAL TECHNIQUES

The Lagrangian density for a spontaneously broken real scalar field theory in $1+1$ dimensions is

$$\mathcal{L}=\frac{1}{2}(\partial_\mu\varphi)(\partial^\mu\varphi)-\frac{\lambda}{4}(\varphi^2-\varphi_0^2)^2-\frac{1}{2}g^2\chi^2\varphi^2 \tag{1}$$

where φ is a real scalar field coupled to a spatially homogeneous, oscillating background field χ given by $\chi=\chi_0\sin(\omega t)$ and g^2 is the coupling parameter. In the absence of the coupling term ($g^2=0$), the above Lagrangian density leads to the field equation

$$\partial^2\varphi/\partial t^2-\nabla^2\varphi+\lambda\varphi(\varphi^2-\varphi_0^2)=0, \tag{2}$$

whose static solution admits extended but localized topological structures called kinks and antikinks

$$\varphi(x)_\pm=\varphi_0\tanh\left[\sqrt{\frac{\lambda}{2}}\phi_0(x\pm x_0)\right] \tag{3}$$

where the $\pm$ signs correspond to kink and anti-kink located at x_0 and $-x_0$ respectively. It is often convenient to work with dimensionless quantities obtained by scaling the variables as follows:

$$\varphi\to\phi=\frac{\varphi}{\varphi_0},\quad \chi\to\frac{\chi}{\varphi_0},$$

$$x\to\sqrt{\lambda}\varphi_0 x,\quad t\to\sqrt{\lambda}\varphi_0 t,$$

$$g^2\to\frac{g^2}{\lambda} \tag{4}$$

$$T\to\frac{T}{\sqrt{\lambda}\varphi_0^3},$$

$$\eta\to\frac{\eta}{\sqrt{\lambda}\varphi_0}.$$

In terms of the above dimensionless quantities, the kink solution [Eq. (3)] is now given by $\phi(x)_\pm=\tanh[(x\pm x_0)/\sqrt{2}]$ and the kink mass is easily calculated to be $M_k=\sqrt{8/9}$.

To take into account the effect of interaction of the system with a background thermal bath, it is customary to use a stochastic description in which the system interacts with a thermal bath at a given temperature T. The presence of a thermal bath with which the system interacts can be easily motivated. Most realistic physical scenarios involve a system exchanging energy with a heat bath which may either represent external thermalized degrees of freedom or, in some cases, certain degrees of freedom, typically the hard modes, of the system, which thermalize on a shorter time scale compared with the rest of the system [15,16]. These can therefore be considered as an environment with which the system, consisting of the nonthermal degrees of freedom (those with longer thermalization time scales; usually the soft modes in a field theory), exchange energy. This interaction would eventually drive the system to thermal equilibrium (in the absence of the oscillating background field). In this description, the Langevin field equation, in terms of scaled dimensionless variables, takes the form

$$\frac{\partial^2\phi}{\partial t^2}+\eta\frac{\partial\phi}{\partial t}-\frac{\partial^2\phi}{\partial x^2}+\phi(\phi^2-1)+g^2\chi^2\phi=\xi(x,t) \tag{5}$$

where the dimensionless damping coefficient η and the Gaussian (delta correlated) white noise term $\xi(x,t)$ are related by the fluctuation dissipation theorem

$$\langle\xi(x,t)\xi(x',t')\rangle=2T\eta\delta(x-x')\delta(t-t') \tag{6}$$

to ensure the equilibration of the system in the absence of the coupling term. T is the rescaled, dimensionless temperature of the heat bath.

When the coupling term is small, i.e. for $g^2\chi_0^2\ll 1$, its presence does not affect the above form of the kink-antikink solutions. The noise and the perturbation term cause the center of mass of the kink or antikink to become a random variable [40]. The small coupling term also acts as a positive mass term and is responsible for shifting the vacuum states towards zero. The critical value of $g^2\chi_0^2$ for which the broken symmetry is restored is $(g^2\chi_0^2)_c=1$. To ensure that the system always remains in the broken phase, one must have

$$g^2\chi_0^2 \ll 1. \tag{7}$$

Since the coupling term in the Lagrangian is quadratic in ϕ, the shape of the effective potential remains unchanged. The oscillating background field has the effect of modulating the barrier height and the position of the two disconnected vacuum states of the potential. During one period of oscillation of the background field, the two degenerate vacuum states of ϕ change its value from a minimum of $\pm\sqrt{1-g^2\chi_0^2}$ for $t=(2n+1)(\pi/2\omega)$; $n=0,\pm1,\pm2,\ldots$ (corresponding to the minimum barrier height separating the two disconnected vacua) to a maximum of ±1 for $t=n(\pi/\omega)$; $n=0,\pm1,\pm2,\ldots$ (corresponding to the maximum barrier height).

In order to study the production of kink-antikink pairs, due to the effects of noise and periodic driving, we solve Eq. (5) numerically using a stochastic second order staggered leapfrog algorithm with periodic boundary conditions. Such an algorithm has been previously employed by Gleiser and collaborators [25,27,41] to study thermal production of kink-antikink pairs as well as to investigate ways of matching Langevin lattice simulation results with continuum field theories. We briefly outline the algorithm used and then discuss some of the subtleties associated with lattice simulations of Langevin equations.

The discretized version of Eq. (5) using the second order stochastic staggered leapfrog algorithm can be cast as

$$\phi_{i,n+1}=\frac{2\phi_{i,n}-\left(1-\frac{\eta\Delta t}{2}\right)\phi_{i,n-1}+(\Delta t)^2\left(\nabla^2\phi-V'(\phi_{i,n})+\sqrt{\frac{2T\eta}{\Delta t\Delta x}}u_{i,n}\right)}{1+\frac{\eta\Delta t}{2}} \tag{8}$$

where "i" and and "n" are the spatial and temporal lattice indices respectively. $V'(\phi)$ is the derivative of the potential with respect to ϕ and $u_{i,n}$ is a Gaussian random number with zero mean and unit variance, i.e. $\langle u_{i,n}\rangle=0,\langle u_{i,n}u_{i,m}\rangle=\delta_{m,n}$ (numerically generated by a Box-Mueller algorithm [42]); with $u_{i,n}$ being related to $\xi_{i,n}$ by the relation

$$\xi_{i,n}=\sqrt{\frac{2T\eta}{\Delta t\Delta x}}u_{i,n}\,. \tag{9}$$

The presence of a lattice introduces natural UV and IR momentum cutoffs in the theory since the smallest and the largest momentum modes that can be probed by the simulation are proportional to inverse lattice size $(2\pi/L)$ and inverse lattice spacing $(\pi/\Delta x)$ respectively. Here $L=N\Delta x$ is the lattice size, N being the total number of lattice points. Finite-size effects can be ruled out by using a large lattice size. However, computational constraints prevent the choice of an arbitrarily small lattice spacing. Also in our case, the presence of an external time scale specified by the frequency of the oscillating background field requires

$$\Delta t \ll \omega^{-1}. \tag{10}$$

The choice of lattice spacing is also crucial especially in Langevin simulations as has been pointed out earlier [27,41,43]. The results of Langevin simulation turn out to be lattice-spacing dependent unless appropriate counterterms are introduced in the effective potential. There is some ambiguity about the manner in which the results scale with Δx. A perturbative counterterm linear in Δx when added to the potential, was found [27] to be adequate enough to remove the lattice spacing dependence of the Langevin simulation results. However, it has been argued [43] that a perturbative method is often inadequate in obtaining the correct manner in which the simulation results scale with Δx. In particular, the one-loop perturbative procedure [27] does not give any corrections for the free theory. Using a nonperturbative approach based on an exact solution of the thermal partition function, Bettencourt *et al.* [43] were able to show[2] that the convergence of the results with lattice spacing scales as $(\Delta x)^2$ and not as Δx as the perturbative one-loop calculation [27] suggested. In both papers, the systems considered were in thermal equilibrium and this aspect facilitated the calculation of appropriate counterterms, perturbatively [27], and through an exact computation of the thermal partition function [43].

In our case however, the presence of the coupling (of the field) with the oscillating background and the low dissipation coefficient (which makes transfer of energy between the heat bath and the system inefficient) prevents the thermalization of the system even for large times for which the simulations were run. Adding counterterms calculated for systems in thermal equilibrium is therefore not helpful in removing the lattice spacing dependence of the results. At this stage, the issue of removal of lattice spacing dependence of Langevin simulations for nonequilibrium systems remains unresolved. We hope to address this problem in a future publication.

We have carried out our simulations with the spatial lattice spacing $\Delta x=0.4$, the temporal lattice spacing $\Delta t=0.01$. The physical lattice size (L) was kept fixed at 6553.6 which corresponds to $N=16384$ lattice points for $\Delta x=0.4$. We have also set $g^2=1$ in all our simulations so that the condition (7) reduces to

[2]In this method, the nature of the counterterm depends on the time-stepping algorithm used for evolving the Langevin equation. A Euler differencing scheme was used in [43].

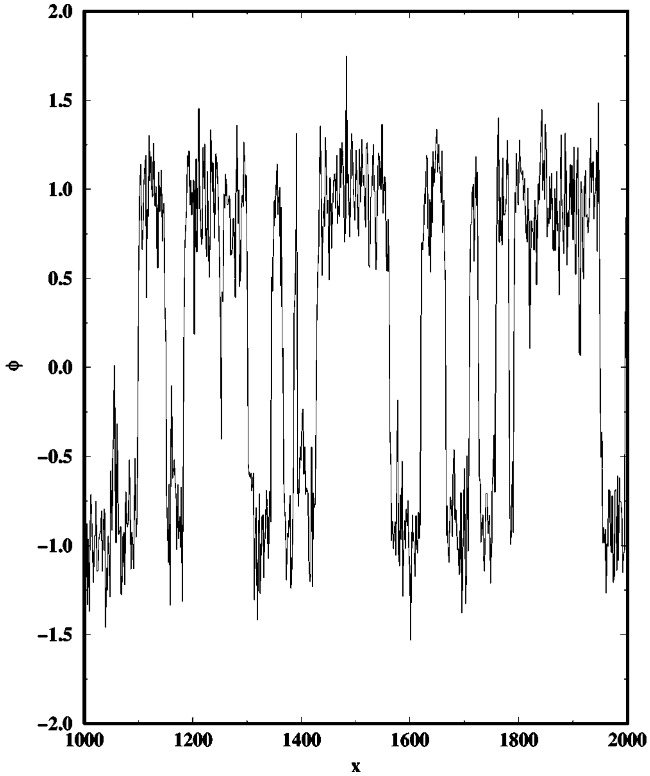

FIG. 1. The field configuration at $t=3000$, for a portion of the lattice showing kink-antikink pairs. The parameter values are $T=0.12, \eta=0.01, \chi_0=0.34, \omega=1.10$.

$$\chi_0^2 \leqslant 1. \tag{11}$$

The issue of how to identify kinks in a Langevin lattice simulation has attracted much controversy [26,27], mainly because of the fact that at high temperatures, it becomes extremely difficult to distinguish between kinks, phonons and large amplitude, nonperturbative and non-topological fluctuations. It is fair to say that an unambiguous technique for counting kinks on the lattice remains to be discovered. In this paper, we follow the technique employed in [26] where a zero crossing of the order parameter field is counted as a kink only if there are no other zero crossings for one kink width to its left and right. The kink width in terms scaled and dimensionless units is $2\sqrt{2}$ which corresponds to approximately 8 lattice units, for $\Delta x=0.4$. The total number of kinks and antikinks is just twice the number of kinks counted because kinks and antikinks are always produced in pairs. Since in these simulations, we are interested only in the low temperature regime, where kinks are easily identifiable as shown in Fig. 1, this method provides a fairly accurate way of counting kinks.

III. NUMERICAL RESULTS

We are now in a position to describe the results of our numerical simulations. The initial conditions of our simulations correspond to the situation in which the field over the entire lattice, undergoes small amplitude fluctuations about the positive of the two degenerate vacuum states. The amplitude of fluctuations are of $\mathcal{O}(10^{-3})$ and incapable of taking the field over the potential barrier. We have checked that our results are independent of initial conditions and remain unchanged even if we choose the initial field configuration to be spatially homogeneous over the entire lattice with its value corresponding to either one of the two degenerate vacuum states. As evident, from our choice of initial conditions, there are no kinks or antikinks present initially. The amplitude of oscillation of the background field is chosen to be $\leqslant 0.38$ so that the condition (11) is satisfied and this alone is incapable of inducing the field to climb over the potential barrier in the absence of coupling to the environment.

In the absence of the oscillating background ($g^2=0$), thermal fluctuations can make localized portions of the field flip over the potential barrier, resulting in the formation of a kink-antikink pair [26,27]. However, the density of pairs produced depends on the strength of thermal fluctuations given by $D=T\eta$ which is a measure of the amount of energy transferred from the heat bath to the system. The thermal nucleation of kink-antikink pairs is suppressed by the Boltzmann factor $e^{-M_k/T}$. Earlier studies [26,27] were carried out with the noise strength $D \geqslant \mathcal{O}(10^{-1})$ since their main focus was on thermal equilibrium production of kinks. The phenomenon we observe occurs at low noise strengths in underdamped systems evolving out of equilibrium. In our simulations we restrict the damping coefficient $\eta<0.05$. The temperature of the heat bath is taken to be $\mathcal{O}(10^{-1})$, so the noise strength $D \sim 10^{-3}$. For such low noise strengths, the production of kinks by thermal fluctuations is few and far between. However, in the presence of noise and nonvanishing coupling g^2, a dramatic enhancement in kink production is observed for a certain range of frequencies of the oscillating background field.

A periodically modulated nonlinear system like the one described by Eq. (5) is expected to exhibit resonant behavior for a certain range of frequencies of the modulator. The presence of noise acting in conjunction with the periodic modulation induces large amplitude fluctuations in the field enabling it (in localized regions) to cross over the potential barrier. This results in the formation of kink-antikink pairs in a manner similar to the production of vortex-antivortex pairs discussed recently [14]. The theoretical analysis of this phenomenon is extremely complicated not only because of the nonlinear nature of a system with infinite degrees of freedom, but also because one has to deal with stochastic partial differential equations. We have therefore decided to take recourse to numerical simulations to investigate this phenomenon. The quantities of importance are the mean field given by $\langle \phi(x,t)\rangle \equiv \overline{\phi}(t) = (1/L)\int_0^L \phi(x,t)dx$, the fluctuation defined as $\delta\phi(t) = \sqrt{\langle \phi(x,t)^2\rangle - \langle \phi(x,t)\rangle^2}$ and the density of kink-antikink pairs $n(t)=2N_k(t)/L$; where $N_k(t)$ is the total number of kinks at a given time, counted in the manner described earlier.

To identify the resonant frequency regime, it is important to realize that too large an oscillation frequency would cause the effective potential to change in a time scale which is much smaller than the destabilization time scale of the field (from its initial state), as a result of which the field would not

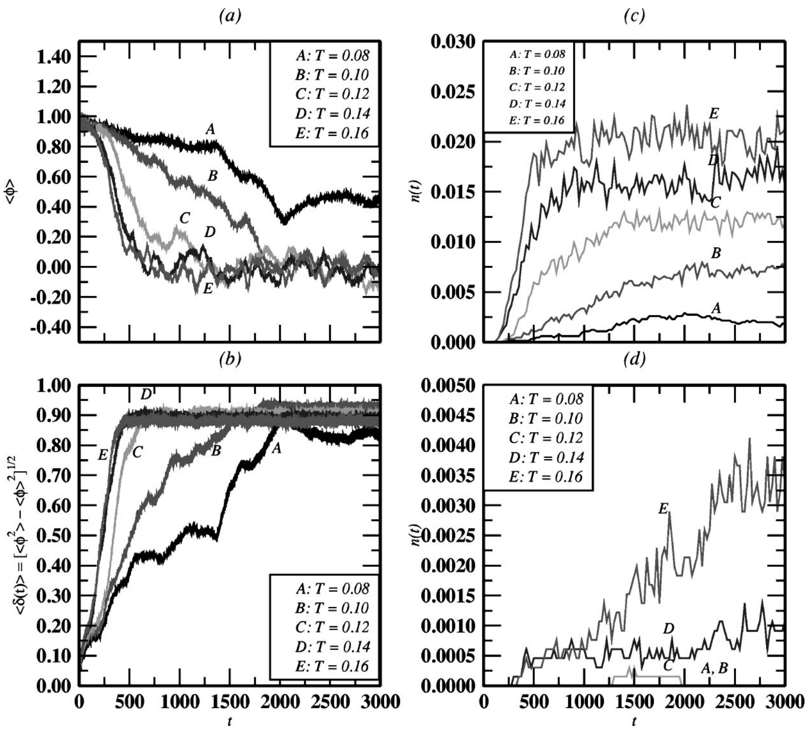

FIG. 2. The time evolution of the (a) mean field, (b) fluctuations, (c) kink-antikink density, (d) kink-antikink density in the absence of the oscillating background ($g^2=0$); for five different bath temperatures. The plots labeled A,B,C,D,E correspond to temperatures $T=0.08,0.1,0.12,0.14,0.16$ respectively. Other parameter values which are kept fixed are $\eta=0.01$, $\omega=1.10$, $\chi_0=0.34$. Note the difference in scales on the y axis of (c) and (d).

feel the change in the shape of the potential. On the other hand, too small an oscillation time scale would result in the destabilized field having sufficient time to relax to the vacuum state of the changing effective potential (apart from fluctuations due to the presence of noise). With these considerations in mind we find that the range of frequency required to induce resonant amplification of field amplitude leading to the enhanced production of kinks is $0.3\leqslant\omega\leqslant2.5$.

We first give results for the variation of the mean field, the fluctuation and the kink density with change in the temperature (T) of the heat bath. Figure 2 shows the plots (for a single noise realization) with fixed ω, χ_0, η and T varying from $0.08-0.16$ in steps of 0.02. The mean field value starting from its initial value around 1 decreases to zero and eventually starts oscillating about zero. On the other hand, the fluctuation grows exponentially in a short time scale to its asymptotic value and remains nearly constant thereafter. The kink density increases with temperature as expected, the mean field is also found to decay more quickly to zero for higher temperatures and the fluctuation also grows more steeply signifying an increase in the growth exponent. The time around which the fluctuation plateaus out is also the time around which the average kink-antikink density becomes nearly constant implying that kinks and antikinks are nucleated and annihilated at nearly the same rate. The kink-antikink density for the same set of temperatures but with the coupling $g^2=0$ (i.e. in the absence of the oscillating background) is shown in Fig. 2(d). A comparison between Fig. 2(c) and Fig. 2(d) clearly shows considerable (by at least an order of magnitude) enhancement in the kink density when *both* noise and periodic driving is present. This is especially evident for low temperatures. The presence of the oscillating background as well as the low dissipation coefficient prevents the thermalization of the system for time scales up to which the simulations were run.

The decay of the mean field value to zero is *not* an indication of symmetry restoration (a common misconception existing in the literature [36]) but is indicative of the formation of a large number of kink-antikink pairs. To establish this unambiguously we plotted the probability distribution of the field over the entire lattice by appropriately binning the field values. Figure 3(a) shows the initial probability distribution of the field for the entire lattice. In view, of the choice of initial conditions, the sharp peak in the probability distribution about $\phi=1$ is easily understandable. Figure 3(b) shows the field probability distribution at a later time ($t=3000$) after the fluctuation has flattened out and the average kink density has become nearly constant. The fact that the probability distribution is still peaked about the nonvanishing vacuum expectation values clearly implies that the symmetry remains broken. In contrast to Fig. 3(a) two distinct peaks of nearly the same height about $\phi\simeq\pm1$ are now observed. This can be easily explained by the fact that the presence of a large number of kink-antikink pairs causes the fraction of the field around $\phi\sim1$ to be nearly the same as the fraction around $\phi\sim-1$. The generic form of the probability distribution function depicted in Fig. 3(b) is observed till the end of the simulation, which allows us to conclude that kink-antikink pairs are produced even though the system *always* remains in the broken phase (just as in [14]). This is contrary to the situation discussed in the context of topological defect production during inflationary (p)reheating [36,37]. There, the fluctuations grow large enough to restore symmetry and defects are produced in the conventional manner when the symmetry is subsequently broken due to mode-scattering effects and/or expansion of the universe.

Figure 4 shows the mean field value, fluctuation and kink density obtained after averaging over 100 different noise re-

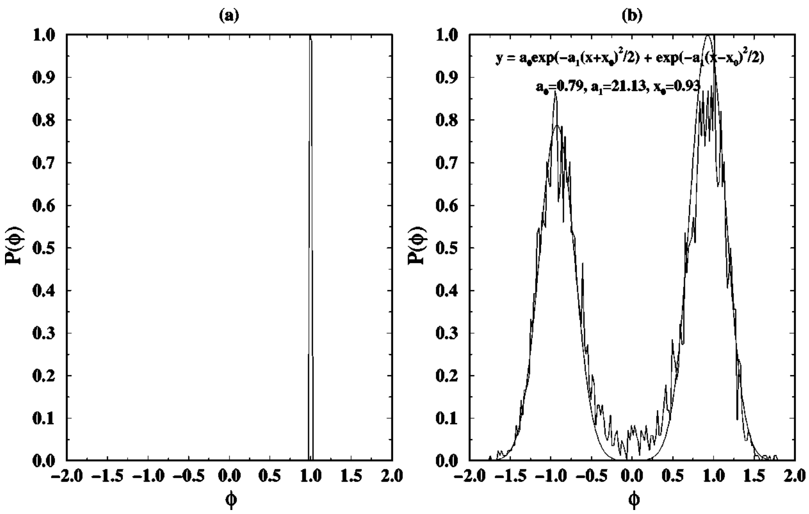

FIG. 3. (a) Probability distribution of Φ at (a) $t=0$ and (b) $t=3000$. The solid line in (b) is an asymmetric double-Gaussian fit of the data given by the function $a_0\exp[-(a_1/2)(x+x_0)^2]+\exp[-(a_1/2)(x-x_0)^2]$, where $a_0=0.79, a_1=21.13, x_0=0.94$. The large twin peaks around $\Phi=\pm 1$, even at late times, is a clear indication that the symmetry remains spontaneously broken.

alizations. The upper and lower curves correspond to the $\pm 1\sigma$ standard deviation from the noise averaged middle curve. The generic features observed in Fig. 2 are also seen here. At late time, the $\pm 1\sigma$ error becomes quite small as evident from the fluctuation plots in Fig. 4(a).

The dependence of the kink density on the amplitude of oscillation χ_0 is shown in Fig. 5. As mentioned earlier, the coupling term acts like a positive mass term and this dictates the choice of χ_0 in accordance to constraint (11). An increase in χ_0 does lead to an increase in defect density as is evident from Fig. 5. However, since our main interest lies in studying defect production dynamics for small amplitudes of the oscillating background, such that the coupling to the oscillating background by itself is incapable of exciting kink production; we restrict the amplitude to $\chi_0 \leqslant 0.38$.

The dependence of the kink density on the dissipation coefficient is depicted in Fig. 6. We emphasize that our results are valid only for very low dissipation coefficients. For large dissipation coefficients, the field oscillations are considerably suppressed leading to a suppression in kink densities as evident from the plots of Fig. 6.

The kink densities are also crucially dependent on the choice of frequency of the oscillating background field. We have found that large amplitude field oscillations are induced, leading to kink-antikink production, only for frequencies lying in the range $0.3 \leqslant \omega \leqslant 2.5$. For frequencies beyond this window, no significant enhancement of kink densities compared to their thermal equilibrium value is observed. As is evident from Fig. 7, there exists an optimum value of frequency (all other parameters remaining fixed) for which kink density is maximized. This optimum value also depends on the temperature of the heat bath (T), damping coefficient (η) and the amplitude of oscillations (χ_0).

IV. DISCUSSIONS AND CONCLUSION

In this paper, we have investigated a novel phenomenon in a $(1+1)$-dimensional field theory admitting topological solitons called kinks. We investigated the production of kink-antikink pairs when the system is subject to the twin effects of noise and periodic driving via its coupling to an oscillating but spatially homogeneous background field. For a certain range of frequencies of the oscillating background, there occurs considerable enhancement in the densities of kinks compared to their thermal equilibrium values. We also studied the effect on kink density of parameters such as the damping coefficient, the temperature of the heat bath and the amplitude of oscillations of χ. Our results are particularly

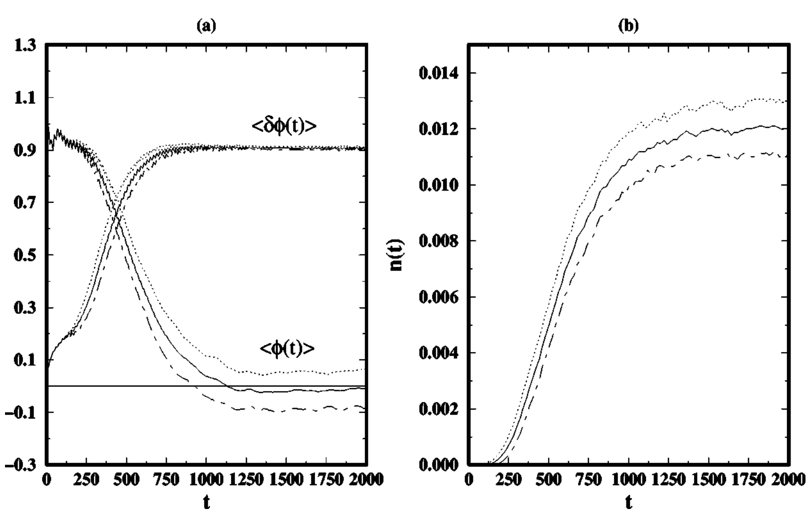

FIG. 4. Time evolution of the noise averaged values of the (a) mean field and fluctuations, (b) kink-antikink density, together with the $\pm 1\sigma$ error. The solid line indicates the noise-averaged mean while the dashed lines above and below the mean correspond to $+1\sigma$ and -1σ deviation from the noise averaged value. Noise average has been carried out over 100 different noise realizations.

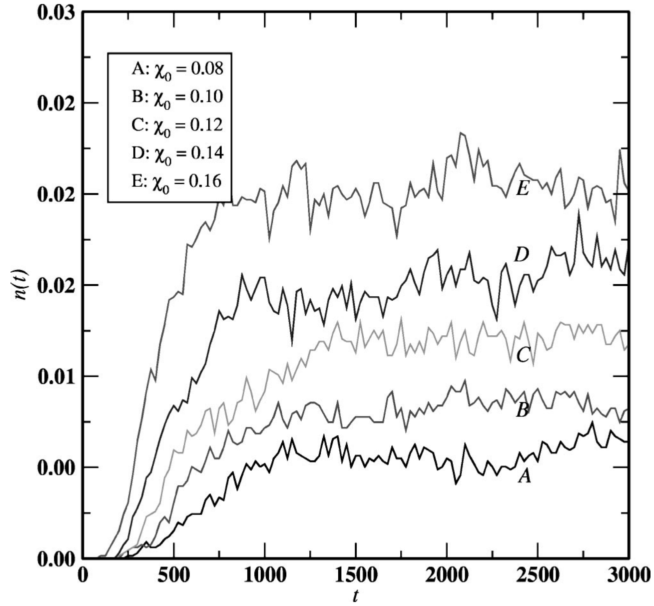

FIG. 5. The variation of the kink-antikink density with the amplitude χ_0 of the oscillating background field. Other parameter values are kept fixed at $T=0.12, \eta=0.01, \omega=1.10$.

sensitive to the value of the dissipation coefficient and the enhancement in kink density compared to their thermal equilibrium value is observed only for low dissipation coefficients. Kink-antikink pair production is observed even though the system remains in the broken phase throughout the course of the simulations. This further demonstrates that topological defects need not be produced only during symmetry breaking phase transitions, as pointed out earlier [14].

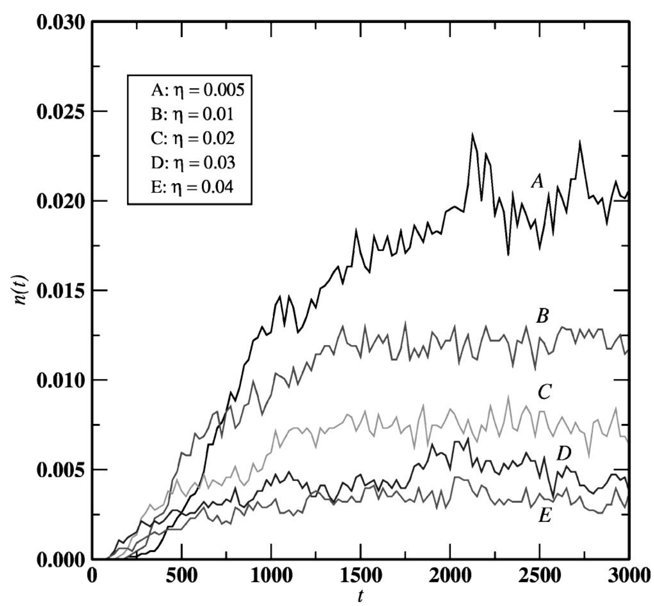

FIG. 6. The variation of the kink-antikink density with the dissipation coefficient η. Other parameter values are kept fixed at $T=0.12, \chi_0=0.34, \omega=1.10$.

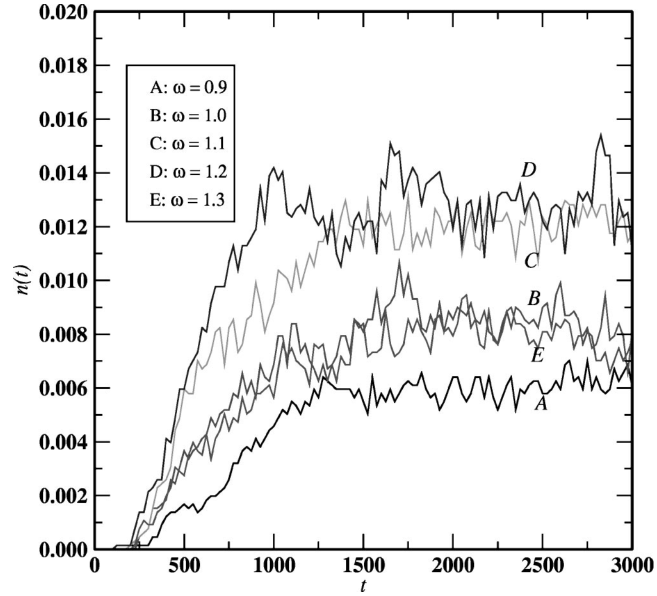

FIG. 7. The variation of the kink-antikink density with the frequency ω of the oscillating background. Other parameter values are kept fixed at $T=0.12, \chi_0=0.34, \eta=0.01$.

At this stage it is tempting to compare our results with the intriguing phenomenon of stochastic resonance (SR) which has been extensively investigated in the literature on nonlinear dynamical systems [39]. The characteristic feature of SR is that an increase in the noise strength can sometimes lead to more coherent behavior when the nonlinear dynamical system is also subject to a periodic driving force. In particular, by tuning the noise strength, a significant improvement in the signal-to-noise ratio (also manifest through peaks in the noise averaged power spectrum) is achieved. The study of stochastic resonance for spatially extended systems has been carried out for Ginzburg-Landau type field theories, albeit restricted to the over-damped regime [32,44]. There it was found that an appropriate choice of the frequency of the periodic driving obtained by matching the thermal activation time scale (given by the inverse of Kramers rate) to half the period of the modulating background, can result in periodically synchronized behavior of the mean field about $\phi=0$ (see Fig. 2 of Ref. [44]). In our case however, no synchronization of the mean field is observed, rather it is found to decay to zero from its initial value in one of the vacuum states, and thereafter keep on oscillating erratically about the zero field value. As has been demonstrated (see Fig. 3), this behavior can be attributed to kink-antikink production which occurs in spite of the fact that the system remains in the broken phase. This comparison makes it clear that the phenomenon we have discussed is quite distinct from that of stochastic resonance. The investigation of the phenomenon of stochastic resonance in underdamped system is currently in progress [45].

There is much that needs to be investigated. Apart from the study of the phenomenon of SR in underdamped systems, a theoretical understanding of the phenomenon discussed here is required. In particular, an analytical derivation of the frequency window required for enhanced kink-antikink production would be both interesting and useful. Moreover, the issue of lattice spacing dependence of results of nonequilibrium dynamical systems requires clarification. Also, a study of kink production in the sine-Gordon model coupled to an oscillating background would be interesting. We plan to address these issues in a future work.

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