Symmetry and inflation

Luis P. Chimento*
Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires Ciudad Universitaria, Pabellón I, 1428 Buenos Aires, Argentina
and Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Av. Rivadavía 1917, 1033 Buenos Aires, Argentina
(Received 9 November 2001; published 5 March 2002)

We find the group of symmetry transformations under which the Einstein equations for the spatially flat Friedmann-Robertson-Walker universe are form invariant. They relate the energy density and the pressure of the fluid to the expansion rate. We show that inflation can be obtained from nonaccelerated scenarios by a symmetry transformation. We derive the transformation rule for the spectrum and spectral index of the curvature perturbations. Finally, the group is extended to investigate inflation in the anisotropic Bianchi type-I spacetime and the brane-world cosmology.

DOI: 10.1103/PhysRevD.65.063517 PACS number(s): 98.80.Cq, 98.80.Hw

I. INTRODUCTION

There exist several interesting physical problems where the Einstein field equations for homogeneous, isotropic, and spatially flat cosmological models [1–4] and a Bianchi type-I metric [5] with a variety of matter sources can be linearized and solved by writing them in invariant form [6]. The variational problem for the distribution function that maximizes the generalized Fisher information is solved in the same way [7]. In all these cases explicit use of the nonlocal transformation group has been made.

In this paper we focus our attention on the group of transformations leading to accelerated expansion scenarios. In many of these scenarios the effective potential energy density of a scalar field is responsible for an epoch of accelerated inflationary expansion [8]. In particular so-called assisted inflation [9] will be investigated in detail. It was introduced in the context of standard Friedmann cosmology where the cumulative effects of multiple scalar fields with an exponential potential give rise to inflation, provided that these fields interact only through the geometry. This kind of potential arises in various higher-dimensional supergravity [10] and superstring [11] models [12–15]. We will see that the cooperative effects of adding energy density into the Friedmann equation lead to inflation. So the configurations of one and several scalar fields are related by a simple symmetry transformation of the corresponding Einstein-Klein-Gordon (EKG) equations. In this sense they can be considered as equivalent cosmological models. Assisted inflation has been mainly studied in the case of power-law solutions ($a \propto t^p$) for the spatially flat Friedmann-Robertson-Walker (FRW) cosmology, which can be shown [9] to be its late-time attractor. Recently, this scenario was discussed in FRW and Bianchi spacetimes using the general exact solution in [16], with a baryotropic perfect fluid in [17], and in the context of warm inflation in [18]. However, by appealing to the symmetry group of the EKG equations, we will investigate inflation without using any particular solution.

New developments in superstring and M theory [19,20] suggest that our three-dimensional universe is described by a brane embedded in a higher-dimensional space [21]. There, the standard model interactions are confined to a $(1+3)$-dimensional hypersurface, the *brane*, while the gravitational field may propagate through the *bulk*. The realization of this model introduces important features that distinguish brane cosmology from the standard scenario. In fact the Friedmann equation is modified at very high energies, acquiring an extra quadratic term in the density [22,23]. This term generally makes it easier to obtain an inflationary scenario in the early universe, by contributing extra friction to the scalar field equation of motion. In [24,25] a kind of assisted inflation mechanism was implemented in the brane-world scenario, associating it with the quadratic term. We think that it will be interesting to continue one step further by considering the brane assisted inflation as emerging from symmetry transformations of the Einstein equations on the brane.

The symmetry transformations that preserve the form of the Einstein equations introduce an alternative concept of equivalence between different physical problems. We may say that several cosmological models are equivalent when the corresponding dynamical equations are form invariant under the action of that group. Thus, it turns out to be of great interest to investigate the consequences of this group from the physical and mathematical points of view.

The paper is organized as follows. In Sec. II we introduce the cosmological symmetry group (CSG) of the Einstein equations in FRW spacetime. In Sec. III we investigate the consequences of this CSG on the inflation in FRW spacetime. In addition, we generalize previous studies on multiscalar field cosmologies driven by an exponential potential. In Sec. IV we examine the anisotropic inhomogeneous Bianchi type-I model. Section V is devoted to studying the link between the accelerated expansion scenario and the CSG, in brane cosmology. Finally, in Sec. VI the conclusions are stated.

II. FORM INVARIANCE SYMMETRY IN FLAT FRIEDMANN-ROBERTSON-WALKER SPACETIMES

We write the Einstein equations in the flat FRW spacetime:

*Email address: chimento@df.uba.ar

$$ds^2=-dt^2+a^2(t)[dx^2+dy^2+dz^2], \tag{1}$$

with a perfect fluid

$$3H^2=\rho, \tag{2}$$

$$\dot{\rho}+3H(\rho+p)=0, \tag{3}$$

where ρ is the energy density, p the pressure, and $H=\dot{a}/a$. For a different perfect fluid with energy density $\overline{\rho}$ and pressure $\overline{p}$ the above equations take the form

$$3\overline{H}^2=\overline{\rho}, \tag{4}$$

$$\dot{\overline{\rho}}+3\overline{H}(\overline{\rho}+\overline{p})=0. \tag{5}$$

By "invariant form" we mean that the system of equations (4),(5) transforms into Eqs. (2),(3) under the symmetry transformations

$$\overline{\rho}=\overline{\rho}(\rho), \tag{6}$$

$$\overline{H}=\left(\frac{\overline{\rho}}{\rho}\right)^{1/2}H, \tag{7}$$

$$\overline{p}=-\overline{\rho}+\left(\frac{\rho}{\overline{\rho}}\right)^{1/2}(\rho+p)\frac{d\overline{\rho}}{d\rho}, \tag{8}$$

where $\overline{\rho}=\overline{\rho}(\rho)$ is an invertible function. Hence the FRW equations for a perfect fluid have form invariance symmetry. The symmetry transformations (6)–(8) define a Lie group for any function $\overline{\rho}(\rho)$, which can be used to solve the FRW equations and get accelerated expansion scenarios, as will be seen in the following sections. Equation (7) expresses the invariant relation $\overline{\Omega}=\Omega$ for the density parameter.

Now, it is interesting to investigate the transformation properties of the relevant physical parameters. For instance, the expansion of the universe in power-law cosmologies $a(t)\propto t^{\alpha}$ is completely described by the Hubble parameter and the deceleration parameter. In these models $q(t)=-H^{-2}(\ddot{a}/a)$ transforms as

$$\overline{q}+1=\left(\frac{\rho}{\overline{\rho}}\right)^{3/2}\frac{d\overline{\rho}}{d\rho}(q+1) \tag{9}$$

under the symmetry transformations (6)–(8). If we consider perfect fluids with equations of state $p=(\gamma-1)\rho$ and $\overline{p}=(\overline{\gamma}-1)\overline{\rho}$, respectively, then the baryotropic indices γ and $\overline{\gamma}$ transform as

$$\overline{\gamma}=\left(\frac{\rho}{\overline{\rho}}\right)^{3/2}\frac{d\overline{\rho}}{d\rho}\gamma \tag{10}$$

under the symmetry transformations (6)–(8). In addition, using Eqs. (9) and (10), we get a form invariant relation $(\overline{q}+1)/\overline{\gamma}=(q+1)/\gamma$ between the deceleration parameter and the baryotropic index.

Inflationary solutions occur when $\ddot{a}>0$; this means that the expansion is dominated by a gravitationally repulsive stress that violates the strong energy condition, so that $\rho+3p<0$. Imposing this condition on Eq. (9) we obtain $d\overline{\rho}^{(-1/2)}/d\rho^{-1/2}<1/(q+1)$, which for a nonaccelerated cosmological model with $q\approx\text{const}>0$ gives $\overline{\rho}>(q+1)^2\rho$. Such a model, with $\overline{q}<0$, is accelerated.

In the particular case where $\overline{\gamma}$ is constant, from Eq. (10) we get

$$a^{3\overline{\gamma}/2}=-\frac{\overline{\gamma}\rho_0^{1/2}}{2}\int\frac{d\rho}{\gamma\rho^{3/2}} \tag{11}$$

where ρ_0 is a constant. This shows that a cosmological model containing a perfect fluid with $\gamma=\gamma(\rho)$ can be mapped into one with constant adiabatic index, permitting us to solve a complicated FRW model in terms of an easier one.

III. SCALAR FIELD CASE

The problem of a homogeneous scalar field ϕ driven by an exponential potential $V(\phi)=V_0\mathrm{e}^{-k\phi}$ minimally coupled to gravity in the flat FRW spacetime is formulated by the system of equations

$$3H^2=\frac{1}{2}\dot{\phi}^2+V, \tag{12}$$

$$\ddot{\phi}+3H\dot{\phi}+V'=0, \tag{13}$$

where V_0 and k are constants. Potentials of this type are of interest because they may be considered an approximation to more complex potentials. In fact, in higher-dimensional superstring theories, the scalar field is like one of the matter fields that contribute to the action and effective potential of the theory. Loop expansion [11], or expansion in the number of interacting particles [26], of the action leads to a perturbative expression of the potential which is a summation of exponential terms [13,14]. There is another important reason behind choosing exponential potentials, because there exists a late-time attractor solution for all the participating fields [9]. In this kind of model the EKG equations have power-law solutions

$$a\propto t^{2/k^2}, \tag{14}$$

so that they inflate at all times when $k^2<2$.

Let us now assume that n homogeneous scalar fields ϕ_i do not interact directly, but are driven by a sum of n exponential potentials $V_i=V_{0i}e^{-k_i\phi_i}$. From now on we consider a simplified problem in which the constants k_i and V_{0i} satisfy $k_1=k_2=\cdots=k_n=k$ and $V_{01}=V_{02}=\cdots=V_{0n}=V_0$. As the asymptotic evolution of all scalar fields tends to a common limit, the special scalar field configuration in which all scalar fields are equal is a late-time attractor. We may take $\phi_1 = \phi_2 = \cdots = \phi_n \equiv \phi$, so that $V_1 = V_2 = \cdots = V_n \equiv V = V_0 e^{-k\phi}$ and Eqs. (12), (13) become

$$3H^2 = n\left[\frac{1}{2}\dot{\phi}^2 + V\right], \tag{15}$$

$$\ddot{\phi} + 3H\dot{\phi} - kV = 0. \tag{16}$$

They have the power-law solutions

$$a_n \propto t^{2n/k^2}, \tag{17}$$

which inflate at all times when $k^2 < 2n$. Thus, the fields cooperate to make inflation more likely in the so-called assisted inflation [9]. In [16] it was shown that the general solution of the EKG equations (15) and (16) has the same kind of behavior in the large time limit.

Rewriting Eqs. (15) and (16) in terms of the rules $\bar{\dot{\phi}} \to \sqrt{n}\dot{\phi}$, $\bar{V} \to nV$, $\bar{k} \to k/\sqrt{n}$, and $\bar{H} \to H$ they become the EKG equations for the field $\bar{\phi}$ and the expansion rate $\bar{H}$. Therefore, we obtain $\bar{\rho}(\bar{\phi}) = n^2 \rho(\phi)$, where $\bar{\rho}$ and ρ are the energy densities corresponding to the fields $\bar{\phi}$ (*n-field configuration*) and ϕ (*one-field configuration*), respectively. Then, the sets of equations (12),(13) and (15),(16) become the sets of equations (2),(3) and (4),(5), respectively. In this identification the energy-momentum tensors of the scalar fields ϕ and $\bar{\phi}$ have been written in the perfect fluid form

$$\rho(\phi) = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad \bar{\rho}(\bar{\phi}) = \frac{1}{2}\bar{\dot{\phi}}^2 + \bar{V}(\bar{\phi}); \tag{18}$$

$$p(\phi) = \frac{1}{2}\dot{\phi}^2 - V(\phi), \quad \bar{p}(\bar{\phi}) = \frac{1}{2}\bar{\dot{\phi}}^2 - \bar{V}(\bar{\phi}). \tag{19}$$

Using the fact that the EKG equations can be written in invariant form, we will assume the transformation

$$\bar{\rho} = n^2 \rho \tag{20}$$

with $n > 1$ in order to obtain cooperative effects, in Eqs. (7) and (8). Then we get

$$\bar{H} = nH \tag{21}$$

and

$$\bar{p} = -n^2 \rho + n\dot{\phi}^2. \tag{22}$$

For an arbitrary potential the former gives the transformation rule for the scale factor $\bar{a} = a^n$, which is equivalent to the relation between Eqs. (14) and (17), while the latter shows that the pressure becomes negative for increasing n. We conclude that the symmetry transformation (20)–(22) relates a scalar field configuration characterized by ρ, p, and H to another field configuration characterized by $\bar{\rho}$, $\bar{p}$, and $\bar{H}$. In this one inflation is more likely for any potential whenever $n > 1$. When the potential has the exponential form and n represents the number of scalar fields we get the usual assisted inflation. Here, it is obtained by imposing the form invariance of the EKG equations.

We complete this section by giving the transformation rule of the remaining parameters [see Eqs. (9),(10)]:

$$\bar{q} = -1 + \frac{1}{n}(1+q), \tag{23}$$

$$\bar{\gamma} = \frac{\gamma}{n}. \tag{24}$$

This shows that an expanding universe with a positive deceleration parameter transforms into an accelerated one if n is taken large enough.

A. Density fluctuations

The contributions of the density fluctuations differ significantly in different inflationary universe models. In this context, it seems interesting to derive the spectrum and spectral index for the perturbations that would be created during the periods of inflation and investigate their transformation properties when the CSG is acting. For one-scalar field models, the spectrum of the curvature perturbation reads [27]

$$P_{\mathcal{R}} = \left(\frac{H}{2\pi}\frac{\partial N}{\partial \phi}\right)^2, \tag{25}$$

where $N = \int H\,dt$ is the number of e-foldings of inflationary expansion remaining. Thus we have

$$P_{\mathcal{R}}(\tilde{k}) = \left(\frac{H}{2\pi}\right)^2 \frac{H^2}{\dot{\phi}^2}\Bigg]_{aH=\tilde{k}}, \tag{26}$$

where H and $\dot{\phi}$ have to be evaluated at the time when the perturbation with the wave number of interest $\tilde{k}$ leaves the Hubble scale during inflation. Also, in this case, the spectral index $n_{\mathcal{R}}(\tilde{k})$ defined as

$$n_{\mathcal{R}}(\tilde{k}) = 1 + \frac{d \ln P_{\mathcal{R}}}{d \ln \tilde{k}} \tag{27}$$

is given by [27]

$$n_{\mathcal{R}} - 1 = 6\frac{\dot{H}}{H^2} + 2\frac{V''}{V}. \tag{28}$$

To investigate the behavior of Eqs. (26) and (28) under the symmetry transformation (20) and give an explicit result, we choose an exponential potential, because it produces late-time attractor solutions and permits us to calculate the density perturbations generated by the scalar fields exactly. In this interesting case, the availability of exact solutions for the scale factor allows us to find that

$$\bar{P}_{\mathcal{R}}(\tilde{k}) = n^3 P_{\mathcal{R}}(\tilde{k}) \tag{29}$$

and

$$1-\overline{n}_{\mathcal{R}}=\frac{1}{n}(1-n_{\mathcal{R}}). \tag{30}$$

The first expression shows that cumulative effects in the energy density increases n^3 times the spectrum of the curvature perturbation corresponding to a single scalar field. The second expression shows that the spectrum generated by the spectral index is closer to scale-invariance when n is large. Note that the spectral index transforms in the same way as the deceleration parameter (23).

For other potentials the density perturbation calculation and its transformation properties must be performed separately for each case.

B. Two interacting perfect fluid interpretation of the scalar field

For the two-fluid model we assume that the energy-momentum tensor T_{ik} splits into two perfect fluid parts,

$$T_{ik}=\overset{1}{T}_{ik}+\overset{2}{T}_{ik}, \tag{31}$$

with $T_{ik}^{(1,2)}=\rho_{(1,2)}u_iu_k+p_{(1,2)}h_{ik}$, where $\rho_{(1,2)}$ and $p_{(1,2)}$ are the energy density and the equilibrium pressure of species 1 and 2, respectively. For simplicity we assume that both components share the same four-velocity u^i. The quantity h_{ik} is the projection tensor $h_{ik}=g_{ik}+u_iu_k$. Interactions between the fluid components amount to the mutual exchange of energy and momentum. Consequently, there will be no local energy-momentum conservation for the subsystems separately. Only the energy-momentum tensor of the system as a whole is conserved.

Let us consider the equation of state $p_{(1,2)}=(\gamma_{(1,2)}-1)\rho_{(1,2)}$ for the two fluids, where $\gamma_{(1,2)}$ is the baryotropic index of species 1 and 2, respectively. Because of the additivity of the stress-energy tensor it makes sense to introduce an effective perfect fluid description with equation of state $p=(\gamma-1)\rho$ where $p=p_1+p_2$, $\rho=\rho_1+\rho_2$, and

$$\gamma=\frac{\gamma_1\rho_1+\gamma_2\rho_2}{\rho_1+\rho_2} \tag{32}$$

is the overall (i.e., effective) baryotropic index. For this effective perfect fluid the dynamical equations are identical to Eqs. (2) and (3).

In general, the energy densities ρ_1 and ρ_2 change under the symmetry transformation $\overline{\rho}=\overline{\rho}(\rho)$. It is important to stress that the transformed fluids should have the same characteristics as the original ones. For this reason we will suppose that the adiabatic indices γ_1 and γ_2 defining their properties are not affected by the transformation. Then they are form invariant quantities so that the final result is

$$\overline{\rho}_1=\frac{1}{\gamma_2-\gamma_1}\left[\gamma_2-\gamma\left(\frac{\rho}{\overline{\rho}}\right)^{3/2}\frac{d\overline{\rho}}{d\rho}\right]\overline{\rho}, \tag{33}$$

$$\overline{\rho}_2=-\frac{1}{\gamma_2-\gamma_1}\left[\gamma_1-\gamma\left(\frac{\rho}{\overline{\rho}}\right)^{3/2}\frac{d\overline{\rho}}{d\rho}\right]\overline{\rho}. \tag{34}$$

Because of the additivity of the stress-energy tensor we will describe the scalar field in terms of two interacting fluids, namely, $\rho_1=\dot{\phi}^2/2$ and $\rho_2=V$, with equations of state

$$p_1=\rho_1, \qquad p_2=-\rho_2, \tag{35}$$

meaning that $\gamma_1=2$ (stiff matter) and $\gamma_2=0$ (vacuum energy). Due to the interactions between the two fluid components there will be no local energy-momentum conservation for each fluid separately. Only the energy-momentum tensor of the system as a whole is conserved. This is equivalent to the Klein-Gordon equation (13) and we may consider the effective perfect fluid description expressed by Eq. (32). In this case the effective baryotropic index is given by

$$\gamma=\frac{2\rho_1}{\rho}=-\frac{2\dot{H}}{3H^2}. \tag{36}$$

Using the interacting perfect fluids picture, Eqs. (35),(36) may be identified with Eqs. (31),(32). Then the symmetry transformation (20) induces a change of $\rho_1=\dot{\phi}^2/2$ and $\rho_2=V$ given by Eqs. (33) and (34):

$$\overline{\dot{\phi}^2}=n\dot{\phi}^2, \tag{37}$$

$$\overline{V}=n^2\left(\frac{1}{2}\dot{\phi}^2+V\right)-\frac{n}{2}\dot{\phi}^2. \tag{38}$$

Also, from Eqs. (7) and (36), we obtain $\overline{H}=nH$ and $\overline{\gamma}=\gamma/n$, so that for large n we are going to an accelerated expansion scenario. In the particular case where the transformed potential is proportional to the original one, $\overline{V}\propto V$, and n represents the number of present fields, the potential has an exponential form [28] and we get the usual assisted inflation.

IV. BIANCHI TYPE-I COSMOLOGY

Now, we assume that the two perfect fluid components do not exchange energy and momentum. Therefore, there will be local energy-momentum conservation for the fluids separately. In this case, Einstein's equations

$$3H^2=\rho_1+\rho_2, \tag{39}$$

$$\dot{\rho}_1+3H(\rho_1+p_1)=0, \tag{40}$$

$$\dot{\rho}_2+3H(\rho_2+p_2)=0 \tag{41}$$

are form invariant under the symmetry transformations

$$\overline{\rho}_1=\overline{\rho}_1(\rho_1,\rho_2), \qquad \overline{\rho}_2=\overline{\rho}_2(\rho_1,\rho_2), \tag{42}$$

$$\overline{H}=\left(\frac{\overline{\rho}}{\rho}\right)^{1/2}H, \tag{43}$$

$$\overline{p_1} = -\overline{\rho_1} + \left(\frac{\rho}{\overline{\rho}}\right)^{1/2}\left[(\rho_1+p_1)\frac{\partial\overline{\rho_1}}{\partial\rho_1} + (\rho_2+p_2)\frac{\partial\overline{\rho_1}}{\partial\rho_2}\right], \qquad (44)$$

$$\overline{p_2} = -\overline{\rho_2} + \left(\frac{\rho}{\overline{\rho}}\right)^{1/2}\left[(\rho_1+p_1)\frac{\partial\overline{\rho_2}}{\partial\rho_1} + (\rho_2+p_2)\frac{\partial\overline{\rho_2}}{\partial\rho_2}\right], \qquad (45)$$

where $\rho=\rho_1+\rho_2$ and $\overline{\rho}=\overline{\rho_1}+\overline{\rho_2}$. On the other hand, the deceleration parameter and the adiabatic indices transform as

$$\overline{q}+1 = \frac{\rho^{3/2}}{(\rho+p)\overline{\rho}^{3/2}}\left[(\rho_1+p_1)\frac{\partial\overline{\rho}}{\partial\rho_1} + (\rho_2+p_2)\frac{\partial\overline{\rho}}{\partial\rho_2}\right](q+1), \qquad (46)$$

$$\overline{\gamma_1} = \frac{1}{\overline{\rho_1}}\left(\frac{\rho}{\overline{\rho}}\right)^{1/2}\left[\gamma_1\rho_1\frac{\partial\overline{\rho_1}}{\partial\rho_1} + \gamma_2\rho_2\frac{\partial\overline{\rho_1}}{\partial\rho_2}\right], \qquad (47)$$

$$\overline{\gamma_2} = \frac{1}{\overline{\rho_2}}\left(\frac{\rho}{\overline{\rho}}\right)^{1/2}\left[\gamma_1\rho_1\frac{\partial\overline{\rho_2}}{\partial\rho_1} + \gamma_2\rho_2\frac{\partial\overline{\rho_2}}{\partial\rho_2}\right], \qquad (48)$$

where $p=p_1+p_2$.

Assuming the simple symmetry transformations

$$\overline{\rho_1}=n^2\rho_1, \qquad \overline{\rho_2}=n^2\rho_2 \qquad (49)$$

in Eqs. (43)–(48) we get $\overline{H}=nH$, $\overline{p_1}=-n^2\rho_1+n(\rho_1+p_1)$, $\overline{p_2}=-n^2\rho_2+n(\rho_2+p_2)$, $\overline{q}=-1+(q+1)/n$, $\overline{\gamma_1}=\gamma_1/n$, and $\overline{\gamma_2}=\gamma_2/n$. Then, for large n, we get an accelerated expanding scenario.

We will apply these results to investigate the connections between inflation and the CSG in Bianchi type-I spacetime. To this end, we use the analytical technique presented in [29], where the equivalence was shown between a FRW double field cosmology and a Bianchi type-I spacetime filled with a self-interacting scalar field, treating the shear as a massless scalar field. There, the inflation in the Bianchi type-I spacetime was closely related to inflation in the flat FRW spacetime. In the usual synchronous form the metric of the locally rotationally symmetric spacetime is

$$ds^2=-dt^2+a_1^2(t)\,dx^2+a_2^2(t)(dy^2+dz^2). \qquad (50)$$

The expansion of this anisotropic model containing a perfect fluid source with pressure p_1 and energy density ρ_1 is governed by Eqs. (39)–(41) [30], where $p_2=\rho_2$ (stiff matter), $3H=H_1+2H_2$ is the rate of volume expansion, $\sigma=\rho_2^{1/2}=(1/\sqrt{3})[H_1-H_2]$ is the shear scalar $\sigma=\frac{1}{2}\sigma_{\mu\nu}\sigma^{\mu\nu}$, $\sigma_{\mu\nu}$ is the shear tensor, $H_1=\dot{a}_1/a_1$, and $H_2=\dot{a}_2/a_2$. Also, we have

$$a_1=a\,e^{2\psi/\sqrt{6}}, \qquad a_2=a\,e^{-\psi/\sqrt{6}}, \qquad (51)$$

where a is the scale factor of the FRW model given by Eqs. (39)–(41) and ψ is a new free scalar field

$$\psi(t)=\sqrt{2}\int\sigma(t)\,dt, \qquad (52)$$

defined in terms of the shear scalar.

If the energy density ρ_1 and the pressure p_1 of the perfect fluid source are identified with a scalar field ϕ through the expressions (18),(19), then we can check whether this model inflates for a generic self-interacting potential. To this end, we will look at the sign of the deceleration parameter $q=-\theta^{-2}(3\dot{\theta}+\theta^2)$, where $\theta=u^a_{;a}$ is the expansion and $\dot{\theta}=\theta_{,a}u^a$, u^a being the four-velocity of the cosmic fluid. Since we are dealing with comoving coordinates $u^a=(1,0,0,0)$, q transforms as

$$\overline{q}=-1+\frac{3}{n}\,\frac{\dot{\phi}^2+2\sigma^2}{\dot{\phi}^2+2V+2\sigma^2} \qquad (53)$$

under the symmetry transformation (20). So, when the energy density of the system increases, inflation will be more likely $[\overline{q}=-1+(q+1)n^{-1}]$. This result extends the one of [16] where, using an exponential potential, it was shown that the cumulative phenomena of noninteracting scalar fields cooperate to "assist" the inflation in a Bianchi type-I model.

V. BRANE ASSISTED INFLATION IN FRIEDMANN-ROBERTSON-WALKER SPACETIME

The Friedmann equation in brane cosmology is modified by an extra quadratic term in the energy density ρ. At very high energies the quadratic term will indeed drastically change the evolution of the scalar field and may be important in the early stage of the universe. In particular it contributes to increasing the friction in the scalar field equation. So it is expected that the implications of this modification will be significant for the standard model of cosmology, and in particular for the inflationary paradigm.

The modified Friedmann equation for a perfect fluid obeying the standard conservation equation (3) in our brane world is given by

$$3H^2=\rho\left(1+\frac{3}{\lambda^2}\rho\right), \qquad (54)$$

where the contributions from bulk gravitons and higher-dimensional cosmological constant have been set to zero [31]. We are using units such that $c=8\pi G=1$. The brane tension λ defines the scale below which the quadratic correction may be neglected and the standard FRW evolution is recovered.

The conservation equation (3) for a perfect fluid and Eq. (54) are form invariant under the symmetry transformations

$$\overline{H}=\sqrt{\frac{\overline{\rho}(1+(3/\lambda^2)\overline{\rho})}{\rho(1+(3/\lambda^2)\rho)}}H, \qquad (55)$$

$$\overline{p}=-\overline{\rho}+\sqrt{\frac{\rho(1+(3/\lambda^2)\rho)}{\overline{\rho}(1+(3/\lambda^2)\overline{\rho})}}(\rho+p)\frac{d\overline{\rho}}{d\rho}. \qquad (56)$$

These induce the following transformation law for the deceleration parameter:

$$\bar{q}+1=\left(\frac{\rho}{\bar{\rho}}\right)^{3/2}\frac{1+\frac{6}{\lambda^2}\bar{\rho}}{1+\frac{6}{\lambda^2}\rho}\left[\frac{1+\frac{3}{\lambda^2}\rho}{1+\frac{3}{\lambda^2}\bar{\rho}}\right]^{3/2}\frac{d\bar{\rho}}{d\rho}(q+1). \tag{57}$$

From these equations we may investigate the behavior of the transformed parameters at very high energies where the quadratic term will dominate the evolution of the perfect fluid. In this case, Eq. (54) is approximated by $H\simeq\rho/\lambda$ and Eqs. (55)–(57) give $\bar{H}\simeq nH(\bar{a}\simeq a^n)$, $\bar{p}\simeq -n\rho+(\rho+p)$, and $\bar{q}\simeq -1+(q+1)/n$ for the symmetry transformation $\bar{\rho}=n\rho$. Comparing this transformation with Eq. (20), we see that the quadratic term makes it easier to obtain assisted inflation in the early universe. When n represents n identical noninteracting scalar fields driven by n identical potentials V, then $\bar{\dot{\phi}}^2\simeq\dot{\phi}^2$ and $\bar{V}\simeq n\rho_\phi-\dot{\phi}^2/2$. Thus, the combined action of the fields induces a symmetry transformation affecting the expansion rate of the universe, $\bar{H}=nH$. This means that the more scalar fields, the quicker is the expansion of the universe. We see again that cumulative effects of the energy density lead to assisted inflation and give rise to the CSG of the Einstein equations on the brane.

VI. CONCLUSIONS

We have found a symmetry transformation (6)–(8) under which the Einstein equations in FRW cosmology are form invariant. This symmetry group relates the expansion rate (*geometrical variable*) with the energy density and pressure of the perfect fluid (*source variables*). Hence, the cooperative effects of adding energy density into the Friedmann equation give rise to inflation. On the contrary, less energy density in the Friedmann equation tends to hinder the inflationary process. Therefore, using Eqs. (6)–(8) it is possible to relate a nonaccelerated scenario to an inflationary scenario by a symmetry transformation.

We have studied the effects of the appearance of more than one scalar field in FRW spacetimes and have shown that assisted inflation, mainly developed with exponential potential and power-law solutions, can be seen as an application of a symmetry transformation between the configurations of one-scalar field and n-scalar fields. In the same way, making use of the CSG, we have obtained assisted inflation for generic potentials in Bianchi type-I spacetime and FRW braneworld cosmology.

Finally, we conclude that it is very interesting to study these kinds of symmetry transformation and their associated symmetry groups, which have received little attention up to now. We shall continue exploring this subject in future papers.

ACKNOWLEDGMENTS

This work was supported by the University of Buenos Aires under Project X223. We thank Alejandro S. Jakubi for useful discussions of this work.

[1] L.P. Chimento and A.S. Jakubi, Class. Quantum Grav. **14**, 1811 (1997).
[2] L.P. Chimento, A.S. Jakubi, V. Méndez, and R. Maartens, Class. Quantum Grav. **14**, 3363 (1997).
[3] L.P. Chimento, Class. Quantum Grav. **15**, 965 (1998).
[4] M. Reuter and C. Wetterich, Phys. Lett. B **188**, 38 (1987).
[5] J.M. Aguirregabiria and L.P. Chimento, Class. Quantum Grav. **13**, 3197 (1996).
[6] L.P. Chimento, J. Math. Phys. **38**, 2565 (1997).
[7] L.P. Chimento, F. Pennini, and A. Plastino, Phys. Rev. E **62**, 7462 (2000).
[8] L.F. Abbot and S.Y. Pi, *Inflationary Cosmology* (World Scientific, Singapore, 1986).
[9] A.R. Liddle, A. Mazumdar, and F.E. Schunck, Phys. Rev. D **58**, 061301(R) (1998).
[10] A. Salam and E. Sezgin, Phys. Lett. **147B**, 47 (1984).
[11] E.S. Fradkin and A.A. Tseytlin, Phys. Lett. **158B**, 316 (1985).
[12] B.A. Campell, A. Linde, and K.A. Olive, Nucl. Phys. **B355**, 146 (1991).
[13] M. Ozer and M.O. Taha, Phys. Rev. D **45**, 997 (1992).
[14] R. Easther, Class. Quantum Grav. **10**, 2203 (1993).
[15] R. Easther, K. Maeda, and D. Wands, Phys. Rev. D **53**, 4247 (1996).
[16] J.M. Aguirregabiria, A. Chamorro, L.P. Chimento, and N. Zuccalá, Phys. Rev. D **62**, 084029 (2000).
[17] A.A. Coley and R.J. van den Hoogen Phys. Rev. D **62**, 023517 (2000).
[18] L.P. Chimento, D. Pavón, and A.S. Jakubi (unpublished).
[19] E. Witten, Nucl. Phys. **B443**, 85 (1995).
[20] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).
[21] V. Rubakov and M.E. Shaposhnikov, Phys. Lett. **159B**, 22 (1985).
[22] P. Binétruy, C. Deffayet, and D. Langlois, Nucl. Phys. **B565**, 269 (2000); P. Binétruy, C. Deffayet, U. Ellwanger, and D. Langlois, Phys. Lett. B **477**, 285 (2000).
[23] T. Shiromizu, K. Maeda, and M. Sasaki, Phys. Rev. D **62**, 024012 (2000).
[24] A.S. Majumdar, Phys. Rev. D **64**, 083503 (2001).
[25] A. Mazundar, S. Panda, and A. Pérez-Lorenzana, Nucl. Phys. **B614**, 101 (2001).
[26] D.J. Gross and J.H. Sloan, Nucl. Phys. **B291**, 41 (1987).
[27] M. Sasaki and E.D. Stewart, Prog. Theor. Phys. **95**, 71 (1996).
[28] J.D. Barrow and P. Saich, Class. Quantum Grav. **10**, 279 (1993).
[29] J.E. Lidsey, Class. Quantum Grav. **9**, 1239 (1992).
[30] J.D. Barrow, Phys. Lett. B **187**, 12 (1987).
[31] R. Maartens, D. Wands, B.A. Bassett, and I.P.C. Heard, Phys. Rev. D **62**, 041301(R) (2000).