Cosmological structure problem of the ekpyrotic scenario

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We address the perturbation power spectrum generated in the recently proposed ekpyrotic scenario by Khoury *et al.* The issue has been raised recently by Lyth who used the conventional method based on a conserved variable in the large-scale limit, and derived different results from Khoury *et al.* The calculation is straightforward in the uniform-curvature gauge where the generated blue spectrum with a suppressed amplitude survives as the final spectrum. Whereas, although the metric fluctuations become unimportant and a scale-invariant spectrum is generated in the zero-shear gauge, the mode does not survive the bounce, but has the same final result. Therefore, an exponential potential leads to a power-law expansion or contraction $a \propto |t|^p$, and the power p dictates the final power spectra of both the scalar and tensor structures. If $p \ll 1$ as one realization of the ekpyrotic scenario suggests, the results are $n_S - 1 \simeq 2 \simeq n_T$ and the amplitude of the scalar perturbation is suppressed relative to that of the gravitational wave by a factor $\sqrt{p}/2$. Both results confirm Lyth's. An observation is made on the constraint on the dynamics of the seed generating stage from the requirement of a scale-invariant spectrum.

I. INTRODUCTION

The ekpyrotic universe scenario based on colliding branes embedded in an extra-dimensional bulk has been suggested recently by Khoury *et al.* in [1]. Perhaps because of its ambitious plan to explain the origin of the hot big bang, and also of its plan to generate the scale-invariant (Harrison-Zeldovich [2]) spectrum without resorting to the inflation-type accelerating stage [3], it has been under close examination [4]. In particular, a quite different scalar spectrum, including both the amplitude and the slope, was derived by Lyth in [5] which is supposed to be fatally threatening the scenario as a viable addition to the early universe models; see also [6,7] for recent additional arguments against [1,3].

If the embedded 3 space literally passes through the singularity during the collision phase of the branes, thus making the $(3+1)$-dimensional equations obsolete, probably we do not have a handle on how to calculate the generated spectra from the scenario. It is suggested that the ekpyrotic scenarios [1,3,8] in fact go through a singularity in the viewpoint of our three space, and in such a case we anticipate serious problems associated with the breakdown of the basic equations we are using. In this sense, to address the structural seed generation mechanism properly in the ekpyrotic scenario, it is likely that we need to handle the perturbation analyses in the context of the higher dimension which is at the moment an unsettled issue. If we *assume*, however, that the $(3+1)$-dimensional effective field theoretical description works, and the linear perturbation theory holds during the bouncing stage, we can apply some well known tools of the cosmological perturbations developed especially over the past two decades. The current controversy about the scalar spectrum is in this narrow context [1,3,5,6] which we will also accept in the following. The possibility of breakdown of the linear perturbation theory as the scale factor approaches singularity was pointed out by Lyth in [7]. If we agree, however, that we can handle the situation using the linear pertur-

bation based on the $3+1$ spacetime effective theory with nonsingular bounce, as we will show below, the results are already well known in the literature which are used to make correct estimations [5,6].

An exponential type potential leads to a power-law expansion [9]

$$a \propto |t|^p, \quad V = -\frac{p(1-3p)}{8\pi G} e^{-\sqrt{16\pi G/p}\,\phi}, \quad (1)$$

which includes the contraction as well. With $p \ll 1$ this potential is an example of the ekpyrotic scenario considered in [1,3]. Assuming (i) both the scalar and the tensor perturbations were generated from quantum fluctuations (of the field and the metric) during such a power-law era, and were pushed outside horizon, the analytic forms of the spectra based on the vacuum expectation values are known in the literature, see Eqs. (47),(48) in [10] for a summary. Under the simplest vacuum state, we have

$$\mathcal{P}_{\hat{\varphi}_v}^{1/2} = \sqrt{4\pi G p}\, \frac{|H|}{2\pi} 2^{-1/(1-p)} \left|\frac{1-p}{p}\right|^{-p/(1-p)} \frac{\Gamma\left(\frac{1-3p}{2(1-p)}\right)}{\Gamma(3/2)}$$

$$\times [k/(a|H|)]^{1/(1-p)}, \quad (2)$$

$$\mathcal{P}_{\hat{C}_{\alpha\beta}}^{1/2} = (2/\sqrt{p})\mathcal{P}_{\hat{\varphi}_v}^{1/2}, \quad (3)$$

where $C_{\alpha\beta}$ is the tensor-type metric fluctuations, and $\varphi_v \equiv \varphi - (aH/k)v$ introduced by Lukash in [11] is a gauge-invariant combination which is proportional to the perturbed three-space curvature (φ) in the comoving gauge ($v \equiv 0$); φ_v is also introduced as ϕ_m in [12], and is the same as $\mathcal{R}$ in [10]. For our notation, see [13]. Overhats indicate the spectra based on the vacuum expectation value of the quantum fluctuations of the field and the metric. Now, assuming (ii) the scale stays outside horizon while there are transitions (like

the inflation to the radiation dominated eras, and radiation to the matter dominated eras), due to the conservation property of the growing solutions of both φ_v and $C_{\alpha\beta}$ [see Eqs. (20), (24)] we can identify the above spectra $\mathcal{P}_{\hat{\varphi}_v}$ and $\mathcal{P}_{\hat{C}_{\alpha\beta}}$ imprinted during the quantum generation stage just after the horizon crossing with the classical power spectra $\mathcal{P}_{\varphi_v}$ and $\mathcal{P}_{C_{\alpha\beta}}$ based on on the spatial averages at the second horizon crossing epoch. Therefore, spectral indices for the scalar (S) and the tensor (T) structures, $n_S - 1 \equiv d\ln\mathcal{P}_{\varphi_v}/d\ln k$ and $n_T \equiv d\ln\mathcal{P}_{C_{\alpha\beta}}/d\ln k$, become

$$n_S - 1 = 2/(1-p) = n_T. \qquad (4)$$

Thus, in the power-law inflation limit with large p we have the scale-invariant spectra $n_S - 1 \simeq 0 \simeq n_T$.

In the ekpyrotic scenario, although there is no acceleration phase before the radiation dominated big bang stage, the two assumptions (i), (ii) above apply as well; the growing solutions of φ_v and $C_{\alpha\beta}$ are conserved as long as we have the large-scale conditions (see later) are met, and we will see that these conditions are satisfied during the transition from the collapsing to the expanding phases in the ekpyrotic scenario. Thus, now with $p \ll 1$ we have $n_S - 1 \simeq 2 \simeq n_T$ which differs from the result claimed in [1,3] for n_S. Besides the wrong spectral slope, Eq. (3) shows that the amplitude of scalar structure is suppressed relative to the one of gravitational wave, which probably means that the scalar perturbation should be negligible as pointed out in [5].

We still observe the different opinions maintained in the literature: (i) the final scale-invariant spectrum from the scenario [1,3], (ii) the final blue spectrum with negligible amplitude when the bounce is nonsingular [5,6], and (iii) the breakdown of the linear theory in the singular bounce [7]. The issues are manifold involving different gauge conditions, different matching conditions used, and others. In the following we will address the issues involved in the differences and will indicate how the analyses consistently support (ii) or (iii) instead of (i). Since there is no controversy over the tensor spectrum, we will concentrate on the scalar spectrum.

II. QUANTUM GENERATION: TWO GAUGES

The different opinions between [1,3] and [5,6] can be partly traced to different generated spectra before the model makes the bounce. In a naive calculation ignoring the metric fluctuations, [1] showed a scale-invariant spectrum. In a rigorous calculation (based on $\delta\phi$ in the uniform-curvature gauge), [5], however, showed that including the metric fluctuation is important resulting in a blue spectrum with suppressed amplitude. Then, [3] showed that, in fact, in the zero-shear gauge the metric fluctuation becomes negligible, thus confirming the original scale-invariant spectrum; see below. Different results from different gauge conditions in the large-scale limit are not surprising because the metric fluctuations often dominate in that scale. Although the final observable result should be the same, the intermediate steps could depend on the gauge conditions we choose for the analyses. In the case of the uniform-curvature gauge the later evolution is

simple as explained below Eq. (3), and the generated spectrum simply survives as the final spectrum, whereas the analyses in the zero-shear gauge is somewhat intricated which we will explain in the following and the next section.

In order to clarify the situation, in this section we present the generated scalar perturbation during the quantum generation stage in the two gauge conditions. We introduce

$$\varphi_{\delta\phi} \equiv \varphi - \frac{H}{\dot{\phi}}\delta\phi \equiv -\frac{H}{\dot{\phi}}\delta\phi_\varphi, \qquad (5)$$

which are just different definitions of a gauge-invariant combination of φ and $\delta\phi$; $\varphi_{\delta\phi}$ can be interpreted as the φ in the uniform-field gauge ($\delta\phi = 0$), and $\delta\phi_\varphi$ is the $\delta\phi$ in the uniform-curvature gauge ($\varphi = 0$); as $v = (k/a)\delta\phi/\dot{\phi}$ for the field [14], we have $\varphi_{\delta\phi} = \varphi_v$. The above relation is powerful in analyzing the classical evolution and the quantum generation of scalar perturbation, see [13] for a summary. Whereas, as we will explain later, using $\delta\phi_\chi \equiv \delta\phi - \dot{\phi}\chi$, which is $\delta\phi$ in the zero-shear gauge ($\chi \equiv 0$), the analysis becomes somewhat involved eventually leading to the same final result.

The equation for the perturbed scalar field is simplest when viewed in the uniform-curvature gauge [15]

$$\delta\ddot{\phi}_\varphi + 3H\delta\dot{\phi}_\varphi + (k^2/a^2 + V_{,\phi\phi})\delta\phi_\varphi$$
$$+ 2\frac{\dot{H}}{H}\left(3H - \frac{\dot{H}}{H} + 2\frac{\ddot{\phi}}{\dot{\phi}}\right)\delta\phi_\varphi = 0, \qquad (6)$$

where the terms in the second line come from the metric perturbations, compare Eqs. (7),(A9) in [15]; other gauge conditions cause more complicated contributions from the metric [16]. Calling the metric term a metric back reaction could be misleading because the perturbed field excites or accompanies the metric fluctuations simultaneously. Keeping the contribution from the metric is necessary and makes the equation consistent and even simpler in the sense that we have a general large-scale solution, see Eq. (20). When the background is supported by a near exponential expansion the whole term from the metric, and $V_{,\phi\phi}$ separately, nearly vanish; this explains why the original derivation of the inflationary spectra in [17,14] was successful even without fully considering the metric perturbations. However, situation could be different in other cases like the power-law expansion (contraction as well) where the ekpyrotic scenario based on exponential potential is one example. In the power-law expansion in Eq. (1) the metric term *cancels* with $V_{,\phi\phi}$ exactly, see Eq. (22) in [15]. It happens that for $p \ll 1$ the $V_{,\phi\phi}$ term without the metric term gives a contribution which can be translated to the $n_S \simeq 1$ scalar power spectrum; this was pointed out in [5]. However, this term should be cancelled exactly by the metric term. With this metric effect taken into account we end up with a massless free scalar field equation which can be translated to $n_S \simeq 3$ generated spectrum. As explained below Eq. (3) this generated spectrum simply survives the later evolution, see next section.

In the zero-shear gauge the equation for $\delta\phi_\chi$ was derived in Eq. (27) of [15]:

$$
\delta\ddot{\phi}_\chi + \left[3H + \frac{8\pi G(\ddot{\phi}+H\dot{\phi})}{\dot{H}+k^2/a^2}\dot{\phi}\right]\delta\dot{\phi}_\chi + \left[\frac{k^2}{a^2}+V_{,\phi\phi}+4\dot{H}\right.
$$

$$
\left. -\frac{8\pi G(\ddot{\phi}+H\dot{\phi})}{\dot{H}+k^2/a^2}\ddot{\phi}\right]\delta\phi_\chi = 0, \tag{7}
$$

which looks quite complicated compared with Eq. (6). In the small-scale limit, Eqs. (6),(7) both reduce to the massless and free scalar field equation. In the large-scale limit, we have

$$
\delta\ddot{\phi}_\chi + 3H\delta\dot{\phi}_\chi + V_{,\phi\phi}\delta\phi_\chi + 4\dot{H}\delta\phi_\chi = 0. \tag{8}
$$

Thus, in this limit, the only metric contribution which is the last term becomes simple. As we have $V_{,\phi\phi}=-2(1-3p)/t^2$ and $4\dot{H}=-p/t^2$, for $p\simeq0$ the metric contribution becomes negligible compared with $V_{,\phi\phi}$; this is in contrast with the uniform-curvature gauge case where the metric term cancels exactly with the $V_{,\phi\phi}$ term. The authors of [3] pointed out that the analysis in this gauge, which is rigorous, is the same as the one based on naive calculation ignoring the metric perturbations presented in [1]. Thus, the generated spectrum for $\delta\phi_\chi$ is scale-invariant, whereas the one for $\delta\phi_\varphi$ is blue. Later, we will show how these two apparently different results lead to the same final observable spectrum in the expanding phase.

Now, we present a rigorous derivation of the generated perturbation in the two gauges. Using Mukhanov's notation in [18] we have, see also Eqs. (46),(47),(68)–(70) in [16],

$$
u = -\frac{4\pi Gz}{k^2}\left(\frac{v}{z}\right)', \qquad v = \frac{1}{4\pi Gz}(zu)', \tag{9}
$$

$$
v'' + \left[k^2-\frac{z''}{z}\right]v = 0, \qquad u'' + \left[k^2-\frac{(1/z)''}{1/z}\right]u = 0, \tag{10}
$$

$$
v \equiv a\delta\phi_\varphi, \qquad u \equiv -\varphi_\chi/\dot{\phi}, \qquad z \equiv a\dot{\phi}/H, \tag{11}
$$

where $\varphi_\chi \equiv \varphi-H\chi$, and a prime indicates a time derivative based on η where $dt\equiv ad\eta$. We have

$$
\delta\phi_\chi = \delta\phi_\varphi + (\dot{\phi}/H)\varphi_\chi, \tag{12}
$$

which follows from Eq. (5) evaluated in the zero-shear gauge. In the power-law case, using Eqs. (7),(8) of [3] we have ($p\neq1$)

$$
\frac{z''}{z} = -\frac{p(1-2p)}{(1-p)^2}\frac{1}{\eta^2}, \qquad \frac{(1/z)''}{1/z} = \frac{p}{(1-p)^2}\frac{1}{\eta^2}. \tag{13}
$$

Notice that Eqs. (10),(13) lead to Bessel equations for v and u with different orders. Using the quantization based on the action formulation of Mukhanov in [18], we have the mode function solutions

$$
\delta\phi_{\varphi k} = \frac{\sqrt{\pi|\eta|}}{2a}[c_1(k)H^{(1)}_{\nu_v}(k|\eta|)+c_2(k)H^{(2)}_{\nu_v}(k|\eta|)],
$$

$$
\varphi_{\chi k} = \frac{|H|\sqrt{\pi|\eta|}}{2k\sqrt{2p}M_{pl}}[c_1(k)H^{(1)}_{\nu_u}(k|\eta|)
$$

$$
+c_2(k)H^{(2)}_{\nu_u}(k|\eta|)],
$$

$$
\nu_v = \frac{3p-1}{2(p-1)}, \qquad \nu_u = \frac{p+1}{2(p-1)}. \tag{14}
$$

The quantization condition implies $|c_2|^2-|c_1|^2=\pm1$ where the sign corresponds to the sign of η; the positive frequency Minkowski space mode in the small-scale limit corresponds to $c_1=0$ for positive η and $c_2=0$ for negative η. The other variables $\varphi_{\delta\phi}$ and $\delta\phi_\chi$ can be recovered using Eqs. (5),(12). One can check that the solutions in Eqs. (14),(12) are consistent with Eq. (9).

The power-spectrum from vacuum quantum fluctuation is related to the mode function solution as $\mathcal{P}_{\delta\hat{\phi}_\varphi}=k^3/(2\pi^2)|\delta\phi_{\varphi k}|^2$ and similarly for $\mathcal{P}_{\delta\hat{\phi}_\chi}$, [15]. Thus, in the large-scale limit we can derive [for $\nu=0$ we have an additional $2\ln(k|\eta|)$ factor]

$$
\mathcal{P}^{1/2}_{\delta\hat{\phi}_\varphi} = \frac{|H|}{2\pi}\left|\frac{1-p}{p}\right|\frac{\Gamma(\nu_v)}{\Gamma(3/2)}\left(\frac{k|\eta|}{2}\right)^{1/(1-p)}
$$
$$
\times|c_2-c_1|, \qquad \nu_v \geq 0, \tag{15}
$$

$$
= \frac{|H|}{2\pi}\left|\frac{1-p}{p}\right|\frac{\Gamma(-\nu_v)}{\Gamma(3/2)}\left(\frac{k|\eta|}{2}\right)^{(2-3p)/(1-p)}
$$
$$
\times|c_2 e^{i\nu_v\pi}-c_1 e^{-i\nu_v\pi}|, \qquad \nu_v \leq 0, \tag{16}
$$

$$
\mathcal{P}^{1/2}_{\delta\hat{\phi}_\chi} = \frac{|H|}{2\pi}\frac{1}{2}\frac{\Gamma(\nu_u)}{\Gamma(3/2)}\left(\frac{k|\eta|}{2}\right)^{1/(1-p)}
$$
$$
\times|c_2-c_1|, \qquad \nu_u \geq 0, \tag{17}
$$

$$
= \frac{|H|}{2\pi}\frac{1}{2|p|}\frac{\Gamma(-\nu_u)}{\Gamma(3/2)}\left(\frac{k|\eta|}{2}\right)^{-p/(1-p)}
$$
$$
\times|c_2 e^{i\nu_u\pi}-c_1 e^{-i\nu_u\pi}|, \qquad \nu_u \leq 0. \tag{18}
$$

The spectral index of the generated fluctuation is $n_{\delta\phi_\varphi}-1=d\ln\mathcal{P}_{\delta\hat{\phi}_\varphi}/(d\ln k)$ and similarly for $n_{\delta\phi_\chi}$. By taking the vacuum state to match with the positive frequency Minkowski space mode, thus $|c_2-c_1|=1$, etc., Eqs. (15),(5) give results in Eqs. (2),(4), whereas Eq. (18) is the one derived in Eq. (17) of [3] for $p\simeq0$. Both results are consistent, except that what we need for later surviving spectrum is the one for $\delta\phi_\varphi$, which is directly related to $\varphi_{\delta\phi}$, and not the one for $\delta\phi_\chi$ which does *not* survive, see next section. That is, for $p\simeq0$, as we follow the later evolution we can show that $n_S=n_{\varphi_{\delta\phi}}=n_{\delta\phi_\varphi}\neq n_{\delta\phi_\chi}$. One important fact to notice is that the spectra in Eqs. (15),(17) are time independent, whereas the ones in Eqs. (16),(18) are time dependent.

III. CLASSICAL EVOLUTION AND FINAL SPECTRUM

Since the final scalar spectrum could be directly related to the large angular scale CMB anisotropies the final result should be physical, and the methods (involving the gauges, matching conditions, etc.) used to get the results should not affect the final results. In this section, assuming the linear perturbation theory is valid for scales we are interested in, we present the evolution of the perturbation generated during the collapsing phase as the background model goes through (smooth and nonsingular) bounce into the expanding phase.

In the large-scale limit, thus ignoring k^2 terms, Eq. (10) have the general solutions [19,13]

$$\varphi_\chi(k,t) = \frac{4\pi G\mu a^2}{k^2 - 3K}\delta_v(k,t)$$

$$= 4\pi G C(k)\frac{H}{a}\int^t \frac{a(\mu+P)}{H^2}dt + \frac{H}{a}d(k),$$
$$(19)$$

$$\varphi_v(k,t) = C(k) - d(k)\frac{k^2}{4\pi G}\int^t \frac{dt}{a^3 Q},$$
$$(20)$$

$$\varphi_\delta(k,t) = \varphi_v + \frac{1}{12\pi G(\mu+P)}\frac{k^2}{a^2}\varphi_\chi,$$
$$(21)$$

where $Q = (\mu+P)/c_s^2 H^2$ with $c_s^2 \equiv \dot{P}/\dot{\mu}$ for the fluid, and $Q = \dot{\phi}^2/H^2$ for the field; for notations, see[1]. C and d are spatially dependent two integration constants. We call the term with coefficient C the C-mode and the other the d-mode. The C-mode is relatively growing and the d-mode is decaying in an expanding phase, whereas the C-mode is relatively decaying and the d-mode can grow in a contracting phase. We emphasize the general character of these solutions which are valid considering generally time varying equation of state $P(\mu)$ or potential $V(\phi)$. Above results are valid for $K=0$; for more general forms applicable to general K and Λ, see [13]. Notice that the d-modes of φ_v and φ_δ are already higher order in the large-scale expansion compared with the one of φ_χ.

As the large-scale power spectra in Eqs. (15),(17) are time-independent, these can be identified with the constant C-modes of φ_v and φ_χ in Eqs. (20),(19); in our power-law background we have $\delta\phi_\varphi \propto \varphi_v$ and $\delta\phi_\chi \propto \varphi_\chi$ which follow from Eqs. (5),(12). Thus, for $p \gg 1$, Eqs. (15),(17) both give the same final scale-invariant spectra in Eq. (4). Whereas, notice the time dependences of Eqs. (16),(18) which are proportional to $|t|/a^3$ and $|H|/a$. These can be identified with the d-modes of φ_v and φ_χ in Eqs. (20),(19); the latter one

was correctly pointed out in [6], and a scale-invariant case with $p = \frac{2}{3}$ for φ_v corresponds to the d-mode, thus not interesting. For the ekpyrotic scenario with $p \ll 1$ we have Eq. (15) for $\delta\phi_\varphi$ thus φ_v, and Eq. (18) for $\delta\phi_\chi$ thus φ_χ. Hence, while the power spectrum for φ_v contributes to the C-mode, the one for φ_χ contributes to the d-mode. As will be further explained later, despite its apparent growth in time the d-mode generated in a contracting phase is not interesting because it will affect only the d-mode which is a real decayng mode in the later expanding phase.

In the case of $p < 1$, since $k/a|H| \propto |\eta| \propto |t|^{1-p}$ becomes small[2] as we approach the bouncing epoch, the large-scale general solutions in Eqs. (19)–(21) are well valid considering time-varying $P(\mu)$ or $V(\phi)$ including sudden jumps. Since the solution is valid considering general time-varying equation of state or potential, the vanishing potential near bounce in [3] will *not affect* the final result as long as the scale remains in the large scale, see later.

As we already have the general solutions the matching conditions are not necessary for the case which is supposed to be an approximation. However, since [3] employed some *ad hoc* matching conditions to make the scale-invariant spectrum of φ_χ to survive as the dominant mode in the expanding phase, in the following we will explain how the *proper* matching conditions lead to a consistent result with the one based on the general solutions in Eqs. (19)–(21).

In [21] two gauge-invariant joining variables were derived which are continuous at the transition accompanying a discontinuous change in pressure assuming perfect fluids. These are

$$\varphi_\chi, \varphi_\delta.$$
$$(22)$$

These are shown to be continuous for general K and Λ in arbitrary scale. Instead of φ_χ we can use δ_v in Eq. (19) as the continuous variable as well. The transitions between scalar fields, and between the fluid and the field are treated separately, see below Eq. (15) of [21]. For the background, a and $\dot{a}$ should be continuous at the transition. Consider two phases *I* and *II* with different equation of states, making a transition at t_1. In the large-scale limit by matching φ_χ and φ_δ in Eqs. (19)–(21) we can see that to the leading order in the large-scale expansion we have

$$C_{II} = C_I,$$

$$d_{II} = d_I + 4\pi G C_I\left[\int^{t_1}\frac{a(\mu+P)}{H^2}dt\bigg|_I\right.$$
$$\left. - \int^{t_1}\frac{a(\mu+P)}{H^2}dt\bigg|_{II}\right].$$
$$(23)$$

This is consistent with the result in Eq. (17) of [21]. Thus, to the leading order in the large-scale expansion the C-mode of

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[1] $\delta_v \equiv \delta + 3(aH/k)(1+w)v$ is the same as the density perturbation (δ) in the comoving gauge; $w \equiv P/\mu$, P the pressure and μ the energy density, $\delta \equiv \delta\mu/\mu$. $\varphi_\delta \equiv \varphi + \delta/[3(1+w)]$ is the same as φ in the uniform-density gauge ($\delta \equiv 0$). In Bardeen's notation [12] we have $\varphi_\chi = \Phi_H$ and $\delta_v \equiv \epsilon_m$. φ_δ is the same as ζ introduced by Bardeen in [20], and for $K=0=\Lambda$ it becomes the one introduced in [14]. Notice that ζ used in [3,6] is our $-\varphi_v$.

[2] In order to tell the large-scale nature, what we have to compare is the z''/z and $z(1/z)''$ terms relative to k^2 term in Eq. (10).

φ_v remains the same, whereas the d-mode of φ_χ is affected by the transition and also the previous history of the d- and C-modes [21]. Thus, the evolution of the C-mode does not depend on the intermediate stages while the perturbations are in the large scale. While in the super-horizon scale the effect of the entropic term of the scalar field is negligible, thus the same conclusions apply to the case including the field as well.

These results from the joining method coincide with the general large-scale conservation solutions in Eqs. (19)–(21) which are valid for the time varying equation of state $P(\mu)$ or potential $V(\phi)$. Thus, our joining method simply confirms that by using the *proper* joining variables we can recover the correct results. We note that the results based on the integral solutions or the joining methods are *not sensitive* to whether the background is expanding or collapsing. Only condition required is the large-scale condition where we could ignore the k^2-term in the perturbation equation. Analyses made in [6] confirm our general conclusions above in the specific situation of the ekpyrotic scenario.

We note that in the collapsing background, the d-modes in Eq. (19) grows in time; it is called the growing solution in [3,6]. We have shown that the leading order scale-invariant spectrum of $\delta\phi_\chi$ generated in the collapsing phase with $p \ll 1$ should be identified as the d-mode. Despite its apparent growth in time we are not interested in this d-mode based on the following reasons. First, Eq. (19) is a general solution valid for time-varying equation of state or potential, thus independently of whether the time-dependence is growing or decaying the d-mode remains as the d-mode which decays in the eventual expanding phase. Secondly, by using the sudden jump approximation we have shown that the d-mode does *not* influence the C-mode which is the proper relatively growing mode in expanding phase. Thus, d-mode is uninteresting afterall: it dies away in a few Hubble expansion as the model enters the expanding phase.

In [3] the authors *proposed* to use the non-divergent variables at the transition as the joining variable, and arrived at matching the two coefficients of a variable δ_v, see below Eq. (43) of [3]. In a single component situation we need two matching conditions, and we should use the matching conditions on two independent variables, not on the two coefficients of one variable. The matching conditions *proposed* in [3] are *different* from the ones in Eq. (22) and we doubt their validity.

Near the bounce of ekpyrotic scenario the potential is restored so that it nearly vanishes (see [3]). This gives a situation with $a \propto |t|^{1/3}$ which can be simulated by a $w=1$ ideal fluid. As long as the seed fluctuations were generated during $p \ll 1$ contracting phase and the scale remains in the large-scale during the bounce we have shown that the changing potential does not affect the C-mode fluctuation. Thus, our conclusion about the final spectrum is not affected by such a change in the potential.

In the phase with effectively $w=1$ equation of state, the d-modes of φ_v and φ_δ in Eqs. (20),(21) diverge logarithmically, whereas φ_χ in Eq. (19) diverges as $|t|^{-4/3}$. In this case the d-mode of δ_v is finite *due to* multiplication of a kine-

matic compensating factor $1/(\mu a^2)$ in Eq. (19). The authors of [3] paid particular attention to the regular behavior of δ_v in such a background, and claimed their matching conditions based on it. The density perturbation in perturbed spacetime has no particular significance compared with the metric (or curvature) perturbations, and its time (in)dependence is due purely to multiplication of a time dependent background factor and gauge dependent; in a similar context the kinematic nature of the growth of the super-horizon scale density perturbation in the expanding phase was emphasized in [12]. A complete set of solutions in six different fundamental gauge conditions are derived in the Table 4 of [22], where we find that the d-modes of curvature perturbation in most of the gauge condition diverge logarithmically in time. Although the d-mode of φ_χ diverges more strongly, Bardeen [12] has noted that "this overstates the physical strength of the singularity." The singular divergences occur if we reach a singularity at the bouncing epoch with vanishing scale-factor a. In passing we note that the corresponding large-scale general solution of the gravitational wave is [13]

$$C_\beta^\alpha(k,t) = c_\beta^\alpha(k) - d_\beta^\alpha(k) \int^t \frac{dt}{a^3}. \quad (24)$$

In the $w=1$ background the d_β^α-mode also diverges logarithmically in the same manner as $\varphi_v, \varphi_\delta$, etc. As the curvature perturbation in most gauge conditions and the gravitational wave show logarithmic divergences, we believe the big-crunch singularity with $w=1$ equation of state is still unstable; for $-\frac{1}{3} < w < 1$ (or $-1 < w < 1$ for the gravitational wave) the perturbations are more singular, see Table 2 of [22]. This conclusion *differs* from the ones in [3,23].

IV. CONCLUSION

Our analyses and results are based on two important assumptions: (i) the contracting and the expanding phases are smoothly (a and $\dot{a}$ are continuous) connected by a nonsingular bounce, and (ii) the linear theory is valid. The generated spectrum during collapsing phase with $p \simeq 0$ shows blue spectrum for $\delta\phi_\varphi$ which is identified as the C-mode. The authors of [3] find that $\delta\phi_\chi$ has a scale-invariant spectrum, but [6] and we have shown that it should be identified as the d-mode which shows apparent growth in the contracting phase, but decays as the model enters the expanding phase, thus uninteresting; the matching conditions also show that C-mode in expanding phase is not affected by any d-mode in previous history as long as the large-scale condition is met. Although this may sound strange (because the growing solution in contracting phase is feeded into the decaying one in expanding phase), it is actually apparent in the general large-scale solutions in Eqs. (19)–(21). More detailed analyses using some phenemenological bouncing models based on classical gravity confirm these results [24].

However, as the ekpyrotic scenario encounters a singular ($a=0$) bounce [1,3,8] we are not sure whether the above analyses based on classical gravity can survive such a bounce. This does not mean that the other case suggested in [3], that through the bounce the d-mode in contracting phase

is switched into the C-mode in the expanding phase, is plausible at all; notice that C and d are coefficients of the two independent solutions. Especially, we note that the matching conditions used in [3] are *ad hoc* and *inconsistent* with the known matching conditions in the literature. [7] pointed out that as model approaches the singular epoch the d-mode fluctuation grows large enough that the linear theory could break down. If the bouncing universe is at all possible in future string theory context as conjectured in [8], and if it involves the singular transition, the fate of perturbations should be handled in the context of *that* string theory. Thus, our *conclusion* is that either the final spectrum is blue with suppressed amplitude or the issue should be handled in the future string theory context with a concrete mechanism for the bounce. In either case we find no supporting argument to accept the final scale-invariant spectrum based on the analyses made in [1,3].

V. DISCUSSIONS

As in the pre-big-bang scenario which also gives very blue spectra $n_S - 1 \simeq 3 \simeq n_T$ [25,26], in order to become a viable model to explain the large-scale structures and the cosmic microwave background radiation anisotropy the ekpyrotic scenario should resort to the other mechanism which is unknown at the moment; perhaps one can find suitable parameter space in the isocurvature modes by considering multicomponents as in the pre-big-bang scenario [27]. In contrast with the pre-big-bang scenario where the amplitudes of the scalar and tensor structures are comparable, see Eq. (42) in [26], since the scalar structure in the ekpyrotic scenario is suppressed relative to the tensor one, the isocurvature possibility to generate the observed structures is more plausible, except that pure isocurvature modes are unfavored by the large-scale structure and the cosmic microwave background anisotropy observations [28].

Before we have the fully considered perturbations both in the brane and the bulk, and the concrete mechanism for the bounce, the results based on the effective field theory should be regarded as *preliminary* ones. This is an unsettled issue at the moment and whether the resulting spectra from full consideration could be scale-invariant is far from clear. Similar anticipation is made about whether more complete consideration of the quantum corrections (which is actually required as we approach the transition epoch) can make the pre-big-bang scenario a less blue and eventually scale-invariant spectra; there is a signature in the right direction, but not enough at the moment [29].

Based on the above results, we can make the following observation. Assuming power-law expansion or contraction $a \propto |t|^p$ during the seed generating stage from quantum fluctuations, the observational requirement of the scale-invariant spectrum for the scalar structure requires $p \gg 1$, thus $-1 < w \ll -\frac{1}{3}$. Thus, for an expanding phase we need accelerated expansion, whereas for a contracting phase we need a damped collapse. During the damped collapse, however, we have $k/a|H|$ becoming large as we approach the bouncing epoch $t \to -0$, thus violating the large-scale condition we used; this can introduce a scale dependent damping in the final spectra. As we have mentioned, in an undamped contraction with $p < 1$ we have the large-scale condition well met during the transition, but the resulting spectrum is not scale-invariant.

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