$N=1$ supergravity chaotic inflation in the braneworld scenario

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We study an $N=1$ supergravity chaotic inflationary model in the context of the braneworld scenario. It is shown that successful inflation and reheating consistent with phenomenological constraints can be achieved via the new terms in the Friedmann equation arising from brane physics, provided the five-dimensional Planck scale satisfies $M_5 \lesssim 3.6\times 10^{15}$ GeV. Interestingly, the model satisfies observational bounds with sub-Planckian field values, implying that chaotic inflation on the brane is free from the well known difficulties associated with the presence of higher order nonrenormalizable terms in the superpotential. A lower bound on M_5 is obtained from the requirement that the reheating temperature is higher than the temperature of the electroweak phase transition, $M_5 \gtrsim 1.6\times 10^{13}$ GeV.

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I. INTRODUCTION

Chaotic inflationary models [1] stand out for their simplicity and fairly natural initial conditions for the onset of inflation. These features are particularly appealing in the context of supergravity and superstring theories, where the natural scale for fields is the Planck scale. In the context of $N=1$ supergravity, however, realizations of chaotic inflation in minimal and in $SU(1,1)$ theories are somewhat special so as to ensure sufficient inflation [2,3]. Furthermore, chaotic inflation requires super-Planckian field values both to ensure sufficient inflation and the correct amount of cosmic microwave background (CMB) anisotropies, in which case higher order nonrenormalizable terms in the superpotential would completely dominate the dynamics since no well motivated symmetry is known to prevent them. Assuming that this problem can be circumvented somehow, one finds that, in order to accommodate in a satisfactory way the bounds on the reheating temperature and energy density fluctuations, a two-scale chaotic inflationary sector is required [4], in contrast with the situation found in $N=1$ supergravity new inflationary type models, where a single scale suffices [5,6].

In this work we show that specific features of the braneworld scenario allow for the abovementioned difficulties with higher order nonrenormalizable terms to be quite naturally avoided and that current observational contraints can be accounted for with a single scale at the superpotential level.

Higher dimensional superstring motivated cosmological solutions suggest that matter fields that are related to open string modes lie on a lower dimensional brane while gravity propagates in the bulk [7]. It is striking that in these scenarios extra dimensions are not restricted to be small [8] and that the fundamental D-dimensional scale, M_D, where $D=4+d$, can be considerably smaller than the four-dimensional Planck scale. In this work we shall consider the $D=5$ case. If one assumes that Einstein equations with a negative cosmological constant hold (an anti–de Sitter space is required) in D dimensions and that matter fields are confined to the 3-brane then the four-dimensional Einstein equation is given by [9]

$$G_{\mu\nu}=-\Lambda g_{\mu\nu}+\frac{8\pi}{M_P^2}T_{\mu\nu}+\left(\frac{8\pi}{M_5^3}\right)^2 S_{\mu\nu}-E_{\mu\nu}, \tag{1}$$

where $T_{\mu\nu}$ is the energy momentum on the brane, $S_{\mu\nu}$ is a tensor that contains contributions that are quadratic in $T_{\mu\nu}$, and $E_{\mu\nu}$ corresponds to the projection of the five-dimensional Weyl tensor on the 3-brane (physically, for a perfect fluid, it is associated with nonlocal contributions to the pressure and energy flux). The four-dimensional cosmological constant is related to the five-dimensional cosmological constant and the 3-brane tension λ as

$$\Lambda=\frac{4\pi}{M_5^3}\left(\Lambda_5+\frac{4\pi}{3M_5^3}\lambda^2\right), \tag{2}$$

while the Planck scale is given by

$$M_P=\sqrt{\frac{3}{4\pi}}\frac{M_5^3}{\sqrt{\lambda}}. \tag{3}$$

In a cosmological setting, where the 3-brane resembles our Universe and the metric projected onto the brane is an homogeneous and isotropic flat Robertson-Walker metric, the generalized Friedmann equation has the following form [10]:

$$H^2=\frac{\Lambda}{3}+\left(\frac{8\pi}{3M_P^2}\right)\rho+\left(\frac{4\pi}{3M_5^3}\right)^2\rho^2+\frac{\epsilon}{a^4}, \tag{4}$$

where ϵ is an integration constant arising from $E_{\mu\nu}$. During inflation, the last term in Eq. (4), the "dark radiation" term, rapidly vanishes and will be disregarded hereafter. Moreover, observations require that the cosmological constant is negligible in the early Universe, meaning that Λ_5 and the brane tension were fine tuned, $\Lambda_5 \simeq -4\pi\lambda^2/3M_5^3$. The Friedmann equation can then be written in the following way:

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$$H^2=\frac{8\pi}{3M_P^2}\rho\left[1+\frac{\rho}{2\lambda}\right]. \tag{5}$$

Notice that the new term in ρ^2 is dominant at high energies, compared to $\lambda^{1/4}$, but quickly decays at lower energies. Requiring the new term to be subdominant during nucleosynthesis implies that $\lambda \gtrsim (1\ \text{MeV})^4$ and, therefore [11],

$$M_5 \gtrsim \left(\frac{1\ \text{MeV}}{M_P}\right)^{2/3} M_P = 10\ \text{TeV}. \tag{6}$$

A much more stringent bound can be obtained in models where the fifth dimension is infinite as it yields an extra contribution $M_5^6\lambda^{-2}r^{-2}$ to Newton's gravitational force, which should be, of course, small beyond scales $r \gtrsim 1$ mm (see, e.g., Ref. [12] and references therein). From Eq. (3) it follows that $M_5 > 10^5$ TeV. We shall see in the ensuing discussion that cosmological considerations intrinsic to supersymetric cosmology can set even more stringent bounds on M_5.

In what follows we shall discuss the effect of the new term in the Friedmann equation on the slow roll-over parameters and show that it allows for a single scale chaotic inflationary model, in the context of $N=1$ supergravity. Moreover, we study how parameters in the superpotential and λ are affected by constraints on the magnitude of the energy density perturbations required to explain the anisotropies in the cosmic microwave background (CMB) radiation observed by Cosmic Background Explorer (COBE) as well as on the reheating temperature. In fact, in supergravity cosmological models one has to ensure that the reheating temperature does not exceed $T_{RH} \lesssim 2.5\times 10^8 (100\ \text{GeV}/m_{3/2})$ GeV [13], in order not to generate an overabundance of gravitinos and the ensuing photodissociation of light elements at nucleosynthesis. In the context of superstring cosmology, inflationary models have to face further problems such as the fate of the dilaton and moduli fields, the so-called postmodern Polonyi problem [14].

We also show that the observational quantities arising from the model originate in sub-Planckian field regimes and, hence, higher dimensional nonrenormalizable operators in the superpotential are not relevant.

II. THE MODEL

We shall assume that the inflaton is the scalar component of a gauge singlet superfield Φ in the hidden sector of the theory. We start by splitting the superpotential in a supersymmetry-breaking, a gauge, and an inflationary sector:

$$W=P+G+I. \tag{7}$$

The $N=1$ supergravity theory describing the interaction of gauge singlet fields is specified by the Kähler function, in terms of which the scalar potential is given by

$$V=\frac{1}{4}e^K[G_a(K^{-1})^a_b G^b-3|W|^2], \tag{8}$$

where $G_a = K_a W + W_a$, and the indices a,b denote derivatives with respect to the chiral superfields Φ. We consider the minimal choice for the Kähler function in $N=1$ supergravity:

$$K(\Phi,\Phi^\dagger)=\Phi^\dagger\Phi, \tag{9}$$

corresponding to canonical kinetic energy for the scalar fields. With this choice, the scalar potential for the inflaton field ϕ is obtained from the superpotential $I(\Phi)$ as

$$V(\phi)=e^{|\phi|^2/M^2}\left[\left|\frac{\partial I}{\partial \Phi}+\frac{\Phi^\star I}{M^2}\right|^2-3\frac{|I|^2}{M^2}\right]_{\Phi=\phi}, \tag{10}$$

where $M=M_P/\sqrt{8\pi}$.

Requiring the cosmological constant to vanish and that supersymmetry remains unbroken at the minimum of the potential, $\Phi=\Phi_o$, leads to the following constraints on the superpotential:

$$I(\Phi_o)=\frac{\partial I}{\partial \Phi}(\Phi_o)=0. \tag{11}$$

Consider the simplest form for the superpotential I which satisfies the above conditions

$$I(\Phi)=\mu(\Phi-\Phi_0)^2, \tag{12}$$

where μ is a mass parameter that determines the energy scale for inflation. Hereafter, we set $\Phi_0=0$. The relevant part of the inflaton potential (along the real ϕ direction) is then given by

$$V(\phi)=\mu^2\left(4\phi^2+\frac{\phi^4}{M^2}+\frac{\phi^6}{M^4}\right)e^{\phi^2/M^2}. \tag{13}$$

Consistency between the slow-roll approximation and the full evolution equations requires that there are constraints on the slope and curvature of the potential. One can define two slow-roll parameters [15]

$$\epsilon \equiv \frac{M_P^2}{16\pi}\left(\frac{V'}{V}\right)^2\frac{1+V/\lambda}{(2+V/\lambda)^2}, \tag{14}$$

$$\eta \equiv \frac{M_P^2}{8\pi}\left(\frac{V''}{V}\right)\frac{1}{1+V/2\lambda}. \tag{15}$$

Notice that both parameters are suppressed by an extra factor λ/V at high energies and that at low energies, $V \ll \lambda$, they reduce to the standard form. The end of inflation will take place for a field value ϕ_f, such that

$$\max\{\epsilon(\phi_f),|\eta(\phi_f)|\}=1. \tag{16}$$

The number of e-folds during inflation is given by $N=\int_{t_i}^{t_f}H dt$, which becomes [15]

$$N \simeq -\frac{8\pi}{M_P^2}\int_{\phi_i}^{\phi_f}\frac{V}{V'}\left[1+\frac{V}{2\lambda}\right]d\phi \tag{17}$$

in the slow-roll approximation. We see that as a result of the modified Friedmann equation at high energies, the expansion rate is increased by a factor $V/2\lambda$, allowing for a smaller initial inflaton field value $\phi_{\rm i}$ for a given number of e-folds, which is crucial for achieving the goal of obtaining sufficient inflation with sub-Planckian field values in our model.

III. CONSTRAINTS FROM REHEATING

After inflation, the field ϕ releases its energy via the coupling to fields in the other sectors in Eq. (7), thus reheating the Universe. Since the inflaton is hidden from the other sectors of the theory, it couples to lighter fields with gravitational strength of the order μ/M. At minimum, the inflaton field has a mass

$$m_{\phi}=2\sqrt{2}\mu, \tag{18}$$

leading to a decay width

$$\Gamma_{\phi}\simeq\frac{m_{\phi}}{(2\pi)^3}\left(\frac{\mu}{M}\right)^3, \tag{19}$$

and a reheating temperature

$$T_{RH}\simeq\left(\frac{30}{\pi^2 g_{RH}}\right)^{1/4}\sqrt{M\Gamma}$$

$$\simeq\frac{2}{\pi^2}\left(\sqrt{\frac{15}{g_{RH}}}\frac{\mu^3}{M}\right)^{1/2}, \tag{20}$$

where g_{RH} is the number of degrees of freedom at T_{RH}.

As already mentioned, a quite severe upper bound on T_{RH} comes from the requirement that gravitinos are not abundantly regenerated in the postinflationary reheating epoch. Indeed, once regenerated beyond a certain density, stable thermal gravitinos would dominate the energy density of the Universe or, if they decay, have disruptive effects on nucleosynthesis, causing light element photodissociation and distortions in the CMB. Avoiding these difficulties implies in the following bounds [7]:

$$T_{RH}\lesssim 2\times 10^9,\;\; 6\times 10^9 \;\;\text{GeV} \tag{21}$$

for $m_{3/2}=1,\;10$ TeV. For our model, demanding that T_{RH} be less than 2×10^9 GeV leads, for $g_{RH}=150$, to a limit on parameter μ

$$\mu\lesssim 3.7\times 10^{-6}M, \tag{22}$$

which coincides with the bound obtained in Ref. [4] for $SU(1,1)$ supergravity since, in either case, reheating is naturally controlled by the lowest order quadratic term in the superpotential.

We have checked that the above bound also ensures that gravitino production via inflaton decay is sufficiently suppressed, as in [4,5]. Naturally, one could contemplate other more specifical reheating mechanisms. Parametric resonance [16] seems to be unimportant given that $m_{\phi}\gg\Gamma_{\phi}$ and the absence of bilinear-type couplings in supergravity. On the other hand, anharmonic terms coupling the inflaton with other chiral superfields can be an alternative route for reheating [17]. In the braneworld scenario, gravitational particle production has been discussed for an exponentially decaying inflaton potential [18].

IV. CONSTRAINTS FROM CMB ANISOTROPIES

The amplitude of scalar perturbations is given by [15]

$$A_s^2\simeq\left(\frac{512\pi}{75M_P^6}\right)\frac{V^3}{V'^2}\left[1+\frac{V}{2\lambda}\right]^3\Bigg|_{k=aH}, \tag{23}$$

where the right-hand side should be evaluated as the comoving scale equals the Hubble radius during inflation, $k=aH$. Thus the amplitude of scalar perturbations is increased relative to the standard result at a fixed value of ϕ for a given potential.

In order to obtain the value of ϕ when scales corresponding to large-angle CMB anisotropies, as observed by COBE, left the Hubble radius during inflation, we take $N_{\star}\approx 55$ and $\phi_i=\phi_{\star}$ in Eq. (17). Combining with Eq. (23) and using the fact that the observed value from COBE is $A_s=2\times 10^{-5}$, we obtain, after a numerical analysis, a further (stronger) constraint on μ

$$\mu\lesssim 2\times 10^{-8}M, \tag{24}$$

which, in turn, implies an upper bound on M_5, namely $M_5\lesssim 3\times 10^{-4}M_p$.

The scale dependence of the perturbations is described by the spectral tilt [15]

$$n_s-1\equiv\frac{d\ln A_s^2}{d\ln k}\simeq -6\epsilon+2\eta, \tag{25}$$

where the slow-roll parameters are given in Eqs. (14) and (15). Notice that, as $V/\lambda\to\infty$, the spectral index is driven towards the Harrison-Zel'dovich spectrum, $n_s\to 1$. For our model, Eq. (13), we obtain

$$n_s\simeq 0.95 \quad \text{for} \quad \mu=2\times 10^{-8}M. \tag{26}$$

The ratio between the amplitude of tensor and scalar perturbations is given by [19]

$$\frac{A_t^2}{A_s^2}\simeq\frac{3M_P^2}{16\pi}\left(\frac{V'}{V}\right)^2\frac{2\lambda}{V}. \tag{27}$$

We get, for our model,

$$r\simeq 4\pi\frac{A_T^2}{A_s^2}\simeq 0.22 \quad \text{for} \quad \mu=2\times 10^{-8}, \tag{28}$$

which is consistent with current upper limits, $r<0.4$ (see, e.g., [20]).

For consistency, it should be verified that enough inflation can occur. Indeed, for $\mu=2\times 10^{-8}$, we get $N=65$ for $\phi_i=0.094M_P$ and a huge amount of e-foldings can be obtained for larger values of ϕ_i, e.g., $N=9.1\times 10^6$ for $\phi_i=0.5M_P$.

TABLE I. Relevant physical quantities, for different values of μ, for the simple $N=1$ supergravity model of Eq. (13).

μ/M	$\phi_\star/M_P$	M_5/M_P	n_s	r	T_{RH} (GeV)
2×10^{-8}	0.09	3×10^{-4}	0.949	0.21	7.7×10^5
10^{-8}	0.035	1.2×10^{-4}	0.953	0.18	2.7×10^5
5×10^{-9}	0.018	6×10^{-5}	0.955	0.17	9.6×10^4
10^{-9}	0.0035	1.2×10^{-5}	0.955	0.17	8.6×10^3
10^{-10}	0.00035	1.2×10^{-6}	0.955	0.17	272

For smaller values of μ (see Table I), we get smaller values of $\phi_\star$ and M_5, but n_s and r change very little. The number of e-folds of inflation increases for the same initial value of ϕ and ϕ_f, calculated according to Eq. (16); for instance, for $\mu=10^{-10}M_P$, $\phi_i=0.094M_P$ and $\phi_f=9\times10^{-5}M_P$, we get $N=3\times10^{11}$, about five orders of magnitude greater than the value obtained for the case $\mu=2\times10^{-8}M$ (see above). Notice that the values of n_s and r exhibited in Table I are consistent with latest CMB data from DASI [20], BOOMERANG [21], and MAXIMA [22]. However, for $\mu\lesssim1\times10^{-10}M$, the reheating temperature becomes smaller than the typical value for the temperature of the electroweak phase transition, $T_{EW}\simeq300$ GeV. Thus, $\mu\lesssim1\times10^{-10}M$ implies a premature breaking of the electroweak phase transition, from which we can extract a new bound on the five-dimensional mass scale (see Table I)

$$M_5\gtrsim1.3\times10^{-6}M_P. \tag{29}$$

Naturally, as field values become smaller, the approximation which consists of expanding the potential in ϕ/M and takes just the first (quadratic) term becomes better; and it is easy to obtain analytic expressions for the relevant quantities as it is reduced to the case analyzed in Ref. [15].

V. CONCLUSIONS

Hence, we see that it is possible to sucessfully implement chaotic inflation in a simple $N=1$ supergravity model within the braneworld scenario, without the need to fine tune the parameters of the potential. In fact, a single mass parameter μ is needed, as in $N=1$ supergravity new inflationary models [5,6].

An upper bound on μ, $\mu\lesssim3.7\times10^{-6}M$, is obtained from the requirement that sufficiently few gravitinos are regenerated in the postinflationary reheating epoch. However, a more severe upper bound is obtained from requiring adequate density fluctuations (both slope and amplitude) as observed by COBE, namely, $\mu\lesssim2\times10^{-8}M$ implying for the five-dimensional mass scale that $M_5\lesssim3\times10^{-4}M_P$. Requiring that the reheating temperature be greater than the typical temperature of the electroweak phase transition, $T_{EW}\simeq300$ GeV, leads to upper bounds on these quantities, namely, $\mu\gtrsim1.06\times10^{-10}M$ and $M_5\gtrsim1.3\times10^{-6}M_P$.

Finally, we have shown that, remarkably, successful chaotic inflation can be achieved with sub-Planckian field values, thus avoiding the need to invoke hypothetic symmetries that would prevent the presence of higher order nonrenormalizable terms in the superpotential. This last feature is important to prevent an overproduction of tensor perturbations in the CMB, a problem that can be particularly acute in chaotic inflation scenarios [23], and that, ultimately due to the quadratic energy density term that appears in the Friedmann equation in the context of the braneworld scenario, is absent in our model.

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