Phenomenological study of lepton mass matrix textures

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(Received 31 October 2001; published 20 February 2002)

The three active light neutrinos are used to explain neutrino oscillations. The inherently bilarge mixing neutrino mass matrix and the Fritzsch-type, bismall mixing charged lepton mass matrix are assumed. By requiring a maximal ν_μ-ν_τ mixing for the atmospheric neutrino problem and a mass-squared difference appropriate for the almost maximal mixing solution to the solar neutrino problem, the following quantities are predicted: ν_e-ν_μ mixing, V_{e3}, CP violation in neutrino oscillations, and the effective electron-neutrino mass relevant to neutrinoless double beta decays.

DOI: 10.1103/PhysRevD.65.057303 PACS number(s): 14.60.Pq,12.15.Ff

Understanding the fermion mass pattern is a great challenge in elementary particle physics. Lacking a standard theory for flavor physics, a phenomenological ansatz might be very helpful [1]. In view of the recent observation about neutrino oscillations [2], this paper studies the lepton sector. The masses of charged leptons have been known experimentally quite well [3]. They are expected to have a similar origin as quarks which have small mixings among three generations.

The small neutrino masses indicated by experiments can be naturally understood by the seesaw mechanism [4]. However, the observations have shown increasing evidence that leptonic mixings are bimaximal, or almost bimaximal among the three generations. Such a mixing scenario was then considered in various ways [5–7].

This paper starts from the flavor eigenstates of both charged leptons and neutrinos. We assume that the charged lepton mass matrix is of the Fritzsch type [8]: namely,

$$\mathcal{M}_l=\begin{pmatrix} 0 & ae^{i\alpha} & 0 \\ ae^{-i\alpha} & 0 & be^{i\beta} \\ 0 & be^{-i\beta} & c \end{pmatrix}, \tag{1}$$

where $c\gg b\gg a>0$ and $a<b^2/c$. The neutrino mass matrix is of the inherently bilarge mixing type [5]:

$$\mathcal{M}_\nu=\begin{pmatrix} \epsilon & m_1 & m_2 \\ m_1 & \epsilon & 0 \\ m_2 & 0 & \epsilon \end{pmatrix}, \tag{2}$$

where $m_1\sim m_2\gg\epsilon>0$. Note that m_1, m_2, and ϵ are always real in the above form of $\mathcal{M}_\nu$. These two matrices are of simplicity in the analysis, and the parameters in them are uniquely fixed. Although Eq. (2) will be speculated upon further in this paper, we still have no definite principles for these two matrices [Eqs. (1) and (2)]. Some more theoretical works for the bimaximal leptonic mixing were considered in Refs. [5–7].

The mass matrix Eq. (1) gives

$$\begin{aligned} a&=\left(\frac{m_e m_\mu m_\tau}{m_e-m_\mu+m_\tau}\right)^{1/2}, \\ b&=\left(m_\mu m_\tau+m_\mu m_e-m_e m_\tau-\frac{m_e m_\mu m_\tau}{m_e-m_\mu+m_\tau}\right)^{1/2}, \\ c&=m_e-m_\mu+m_\tau. \end{aligned} \tag{3}$$

Equation (2) gives neutrino masses,

$$\begin{aligned} m_{\nu_1}&=-\sqrt{m_1^2+m_2^2}+\epsilon, \\ m_{\nu_2}&=\sqrt{m_1^2+m_2^2}+\epsilon, \\ m_{\nu_3}&=\epsilon. \end{aligned} \tag{4}$$

Charged leptons provide bismall mixing among the three generations, whereas neutrinos provide bilarge mixing. The diagonalization of $\mathcal{M}_l$ is made by the following unitary matrix [9]:

$$U_l=\begin{pmatrix} U_{11}^l & U_{12}^l & U_{13}^l \\ U_{21}^l e^{-i\alpha} & U_{22}^l e^{-i\alpha} & U_{23}^l e^{-i\alpha} \\ U_{31}^l e^{-i(\alpha+\beta)} & U_{32}^l e^{-i(\alpha+\beta)} & U_{33}^l e^{-i(\alpha+\beta)} \end{pmatrix}, \tag{5}$$

where

$$U_{11}^l=\left[1+\left(\frac{m_e}{a}\right)^2+\left(\frac{b}{a}\frac{m_e}{m_\tau-m_\mu}\right)^2\right]^{-1/2}, \tag{6}$$

$$U_{22}^l=\left[1+\left(\frac{a}{m_\mu}\right)^2+\left(\frac{b}{m_\tau+m_e}\right)^2\right]^{-1/2},$$

$$U_{33}^l=\left[1+\left(\frac{m_\mu-m_e}{b}\right)^2+\left(\frac{a}{b}\frac{m_\mu-m_e}{m_\tau}\right)^2\right]^{-1/2},$$

$$U_{12}^l=-\frac{a}{m_\mu}U_{22}^l,$$

$$U^l_{13}=\frac{a}{b}\,\frac{m_\mu-m_e}{m_\tau}U^l_{33},$$

$$U^l_{23}=\frac{m_\mu-m_e}{b}U^l_{33},$$

$$U^l_{21}=\frac{m_e}{a}U^l_{11},$$

$$U^l_{31}=-\frac{b}{a}\,\frac{m_e}{m_\tau-m_\mu}U^l_{11},$$

$$U^l_{32}=-\frac{b}{m_\mu+m_e}U^l_{22}.$$

$\mathcal{M}_\nu$ is diagonalized by

$$U_\nu=\begin{pmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0\\ \frac{\sin\theta}{\sqrt{2}} & \frac{\sin\theta}{\sqrt{2}} & -\cos\theta\\ \frac{\cos\theta}{\sqrt{2}} & \frac{\cos\theta}{\sqrt{2}} & \sin\theta\end{pmatrix}, \tag{7}$$

where $\sin\theta=m_1/\sqrt{m_1^2+m_2^2}$. Note that U_ν is independent of ϵ. The physical lepton mixing is given by

$$V=U_l^\dagger U_\nu. \tag{8}$$

It is the combination of the large mixing from U_ν and the small mixing from U_l that gives the maximal mixing of ν_μ-ν_τ. In our scenario, $\cos\theta$ deviates from $\pm 1/\sqrt{2}$ remarkably. This is because the (23) component of V is mainly composed of $\cos\theta$ and $U^l_{23}\sim\sqrt{m_\mu/m_\tau}\sim 0.3$, which is not negligible. On the other hand, the matrix U_ν itself will give a maximal mixing in the ν_e-ν_μ oscillation, because the charged lepton contribution to V_{12} is only about $\sqrt{m_e/m_\mu}\sim 0.01$.

Let us discuss the numerical results. The quantity $\sqrt{m_1^2+m_2^2}$ is taken to be 0.05 eV as indicated by the atmospheric neutrino problem. By requiring the maximal ν_μ-ν_τ mixing, we obtain

$$m_1\simeq 4.3\times 10^{-2}\ \text{eV},\quad m_2\simeq 2.5\times 10^{-2}\ \text{eV}. \tag{9}$$

The solar neutrino problem is solved by the energy independent solution [10] which needs

$$|\epsilon|\simeq 10^{-3}\text{–}10^{-4}\ \text{eV}\quad \text{or}\quad 10^{-6}\text{–}10^{-8}\ \text{eV}. \tag{10}$$

The ν_e-ν_μ mixing deviates from the maximal one slightly. With the above results, we get

$$\sin^2 2\theta_{e\mu}\simeq 0.99. \tag{11}$$

The ν_e-ν_τ mixing is predicted as

$$|V_{e3}|\simeq 0.049. \tag{12}$$

The CP violation in the neutrino oscillations is determined by the rephasing-invariant parameter J [11],

$$\mathrm{Im}(V_{i\lambda}V_{j\rho}V^*_{i\rho}V^*_{j\lambda})=J\sum_{k,\delta}\epsilon_{ijk}\epsilon_{\lambda\rho\delta}. \tag{13}$$

In our case, Eqs. (5)–(8) give

$$\begin{aligned}J=&\frac{U^l_{12}}{2}(-U^{l2}_{11}+U^{l2}_{21}\sin^2\theta+U^{l2}_{31}\cos^2\theta+U^l_{21}U^l_{31}\sin 2\theta\cos\beta)[U^l_{22}\sin\theta\sin\alpha+U^l_{32}\cos\theta\sin(\alpha+\beta)]\\ &-\frac{U^l_{11}}{2}(-U^{l2}_{12}+U^{l2}_{22}\sin^2\theta+U^{l2}_{32}\cos^2\theta+U^l_{22}U^l_{32}\sin 2\theta\cos\beta)[U^l_{21}\sin\theta\sin\alpha+U^l_{31}\cos\theta\sin(\alpha+\beta)]\\ &\simeq\frac{1}{4}\sqrt{\frac{m_e}{m_\mu}}\cos\theta\left\{\sin 2\theta\sin\alpha-2\sqrt{\frac{m_\mu}{m_\tau}}[\sin(\alpha+\beta)-2\sin^2\theta\sin\alpha\cos\beta]\right\}.\end{aligned} \tag{14}$$

Numerically, choosing $\alpha=\beta=\pi/2$, we can get $J\simeq 0.008$; choosing $\alpha=0$ and $\beta=\pi/2$, $J\simeq 0.004$.

The neutrinoless double beta decay experiments will measure the effective electron-neutrino mass

$$\langle m_{\nu_e}\rangle\equiv\left|\sum_\lambda V^2_{e\lambda}m_{\nu_\lambda}\right| \tag{15}$$

which, by keeping ϵ terms to the leading order, in our case is

$$\begin{aligned}\langle m_{\nu_e}\rangle=&2\sqrt{m_1^2+m_2^2}U^l_{11}[(U^l_{21}\sin\theta+U^l_{31}\cos\theta\cos\beta)^2\\ &+(U^l_{31}\cos\theta\sin\beta)^2]^{1/2}-\epsilon U^{l2}_{11}\\ &\simeq 2\sqrt{m_1^2+m_2^2}\sqrt{\frac{m_e}{m_\mu}}\sin\theta-\epsilon\simeq 0.006\ \text{eV}.\end{aligned} \tag{16}$$

Experiments in the near future will check the reality of the lepton mass matrices studied in this paper. In addition to

SNO, Borexino and KamLAND will check the result of Eq. (11) for the ν_e-ν_μ mixing [12]. The long baseline neutrino experiments [13] and neutrino factories will measure V_{e3} and CP violation in neutrino oscillations. GENIUS is able to test the $\langle m_{\nu_e}\rangle$ given in Eq. (16).

Finally let us look at the underlying reasons of the neutrino mass matrix in Eq. (2). These Majorana masses are thought to be generated by the seesaw mechanism. It is natural to assume that the Dirac neutrino mass matrix has a similar form as that of charged leptons,

$$\mathcal{M}_{\mathcal{D}}=\begin{pmatrix} 0 & \tilde{a} & 0 \\ \tilde{a} & 0 & \tilde{b} \\ 0 & \tilde{b} & \tilde{c} \end{pmatrix}, \tag{17}$$

where the possible phases are not considered because $\mathcal{M}_\nu$ of Eq. (2) is real and what we are looking at is magnitudes of right-handed neutrino masses. In this case, the texture of Eq. (2) requires the following form of the right-handed neutrino mass matrix:

$$\mathcal{M}_{\mathcal{R}}=\frac{1}{\epsilon}\begin{pmatrix} \tilde{a}^2\cos^2\theta & -\tilde{a}\tilde{b}\sin\theta\cos\theta & -\tilde{a}\cos\theta(\tilde{c}\sin\theta-\tilde{b}\cos\theta) \\ -\tilde{a}\tilde{b}\sin\theta\cos\theta & \tilde{b}^2\sin^2\theta & \tilde{b}\sin\theta(\tilde{c}\sin\theta-\tilde{b}\cos\theta) \\ -\tilde{a}\cos\theta(\tilde{c}\sin\theta-\tilde{b}\cos\theta) & \tilde{b}\sin\theta(\tilde{c}\sin\theta-\tilde{b}\cos\theta) & (\tilde{c}\sin\theta-\tilde{b}\cos\theta)^2 \end{pmatrix}$$
$$+\frac{\tilde{a}}{\sqrt{m_1^2+m_2^2}}\begin{pmatrix} 0 & \tilde{a}\sin\theta & 0 \\ \tilde{a}\sin\theta & 2\tilde{b}\cos\theta & \tilde{c}\cos\theta+\tilde{b}\sin\theta \\ 0 & \tilde{c}\cos\theta+\tilde{b}\sin\theta & 0 \end{pmatrix}. \tag{18}$$

Note that in the above equation, the first matrix is the leading one. But it is of rank one. Only with the second matrix, which is a perturbation to the first, is $\mathcal{M}_R$ nonsingular. In the right-handed neutrino spectrum, there is a heavy one with mass around 10^{15}–10^{16} GeV, and there are two relatively light neutrinos which are about two orders smaller than the first if we take $\epsilon\sim 10^{-4}$ eV. It seems that the form of $\mathcal{M}_R$ needs some tuning in order to keep the form of the texture assumed in Eq. (2). We wonder if there is a natural way to produce it, for instance from some flavor symmetry.

We would like to thank Zhi-Zhong Xing for helpful discussions and comments. C.L. was supported in part by the National Natural Science Foundation of China with Grant No. 10047005.

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