f_B and f_{B_s} from QCD sum rules

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The decay constants of the pseudoscalar mesons B and B_s are evaluated from QCD sum rules for the pseudoscalar two-point function. Recently calculated perturbative three-loop QCD corrections are incorporated into the sum rule. An analysis in terms of the bottom quark pole mass turns out to be unreliable due to large higher order radiative corrections. On the contrary, in the $\overline{\mathrm{MS}}$ scheme the higher order corrections are under good theoretical control and a reliable determination of f_B and f_{B_s} becomes feasible. Including variations of all input parameters within reasonable ranges, our final results for the pseudoscalar meson decay constants are $f_B = 210 \pm 19$ MeV and $f_{B_s} = 244 \pm 21$ MeV. Employing additional information on the product $\sqrt{B_B} f_B$ from global fits to the unitarity triangle, we are in a position to also extract the B-meson B parameter $B_B = 1.26 \pm 0.45$. Our results are quite compatible with analogous determinations of the above quantities in lattice QCD.

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I. INTRODUCTION

Experimental effort in recent years has provided us with a wealth of new information on the decays of bottom hadrons. To achieve a good understanding of these data, also the impact of the strong interactions has to be controlled quantitatively. This requires the accurate calculation of hadronic matrix elements involving B hadrons. Generally, hadronic matrix elements contain contributions from low energies and thus nonperturbative methods should be employed for their evaluation. Current approaches include lattice QCD, QCD sum rules and the heavy quark effective theory (HQET). In this work, we shall consider a calculation of the simplest type of hadronic matrix elements, namely the pseudoscalar B- and B_s-meson decay constants f_B and f_{B_s} in the framework of QCD sum rules [1–4].

The pseudoscalar decay constants parametrize B-meson matrix elements of the axial-vector current with the corresponding quantum numbers and are defined by

$$\langle 0|(\bar{q}\gamma_\mu\gamma_5 b)(0)|B(p)\rangle = i f_B p_\mu ,$$
$$\langle 0|(\bar{s}\gamma_\mu\gamma_5 b)(0)|B_s(p)\rangle = i f_{B_s} p_\mu . \tag{1}$$

Throughout this work we assume isospin symmetry and q can denote an up or down quark. Weak interactions induce the leptonic decay of the B meson. For example, f_B then appears in the decay width of the process $b\bar{u}\to l\bar{\nu}_l$ which takes the form

$$\Gamma(B^-\to l^-\bar{\nu}_l) = \frac{G_F^2}{8\pi}|V_{ub}|^2 f_B^2 m_l^2 m_B\left(1-\frac{m_l^2}{m_B^2}\right), \tag{2}$$

completely analogous to the corresponding decay of the light pseudoscalar mesons. Despite the suppression by the small factors m_l^2 and $|V_{ub}|^2$, there is some hope that the leptonic decay $B\to l\bar{\nu}_l$ can be measured at the B factories within the next years. Once f_B is assumed to be known, this would provide a very clean determination of $|V_{ub}|$. In any case, f_B is an important quantity for it also enters more complicated hadronic matrix elements of B mesons like form factors or matrix elements of four-quark operators.

The calculation of heavy meson decay constants in QCD has a rather long history. For charmed mesons, they were first considered in [5,6], whereas the extraction of f_B from QCD sum rules was investigated in [7–16]. The first determination of f_B [7] dates back already twenty years. Nevertheless, due to recent theoretical progress, we find it legitimate to reconsider this problem. Very recently, the perturbative three-loop order α_s^2 correction to the correlation function with one heavy and one massless quark has been calculated [17,18] for the first time. It turns out that in the pole mass scheme, which was used for most previous analyses, due to renormalon problems [19], the perturbative expansion is far from converging. However, taking the quark mass in the modified minimal subtraction ($\overline{\mathrm{MS}}$) scheme [20], a very reasonable behavior of the higher orders is obtained and a reliable determination of f_B becomes feasible [66].

The starting point for the sum rule analysis is the two-point function $\Psi(p^2)$ of two hadronic currents.

$$\Psi(p^2) \equiv i\int dx\, e^{ipx}\langle\Omega|T\{j_5(x)j_5(0)^\dagger\}|\Omega\rangle, \tag{3}$$

where Ω denotes the physical vacuum and $j_5(x)$ will be the divergence of the axial vector current,

$$j_5(x) = (M+m):\bar{q}(x)i\gamma_5 Q(x):, \tag{4}$$

with M and m being the masses of $Q(x)$ and $q(x)$. In the following, $Q(x)$ denotes the heavy quark which later will be

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specified to be the bottom quark, whereas $q(x)$ can be one of the light quarks up, down or strange. Note that the current $j_5(x)$ is a renormalization invariant operator. In the case of $(\overline{u}b)$ the corresponding matrix element is given by

$$(m_b+m_u)\langle 0|(\overline{u}i\gamma_5 b)(0)|B\rangle = f_B m_B^2, \tag{5}$$

where m_B is the B-meson mass.

Up to a subtraction polynomial which depends on the large p^2 behavior, $\Psi(p^2)$ satisfies a dispersion relation (for the precise conditions see [21]):

$$\Psi(p^2)=\int_0^\infty \frac{\rho(s)}{(s-p^2-i0)}ds+\text{subtractions}, \tag{6}$$

where $\rho(s)$ is defined to be the spectral function $\rho(s) \equiv \mathrm{Im}\,\Psi(s+i0)/\pi$. To suppress contributions in the dispersion integral coming from higher excited states, it is further convenient to apply a Borel (inverse Laplace) transformation to Eq. (6) which leads to [67]

$$u\mathcal{B}_u\Psi(p^2)\equiv u\hat{\Psi}(u)=\int_0^\infty e^{-s/u}\rho(s)ds. \tag{7}$$

$\mathcal{B}_u$ is the Borel operator and the subtraction polynomial has been removed by the Borel transformation. As we shall discuss in detail below, the left-hand side of this equation is calculable in renormalization group improved perturbation theory in the framework of the operator product expansion, if the Borel parameter u can be chosen sufficiently large.

Under the *crucial* assumption of quark-hadron duality, the right-hand side of Eq. (7) can be evaluated in a hadron-based picture, still maintaining the equality, and thereby relating hadronic quantities like masses and decay widths to the fundamental standard model parameters. Generally, however, from experiments the phenomenological spectral function $\rho_{ph}(s)$ is only known from threshold up to some energy s_0. Above this value, we shall use the theoretical expression $\rho_{th}(s)$ also for the right-hand side. In the case of the B mesons, we approximate the phenomenological spectral function by the pole of the lowest lying hadronic state plus the theoretical spectral function above the threshold s_0,

$$\rho_{ph}(s)=m_B^4 f_B^2\delta(s-m_B^2)+\theta(s-s_0)\rho_{th}(s). \tag{8}$$

This is legitimate if s_0 is large enough so that perturbation theory is applicable. The central equation of our sum-rule analysis for f_B then takes the form

$$m_B^4 f_B^2=\int_0^{s_0} e^{(m_B^2-s)/u}\rho_{th}(s)ds. \tag{9}$$

Besides the sum rule of Eq. (9), in our numerical analysis we shall also utilize a second sum rule which arises from differentiating Eq. (7) with respect to $1/u$:

$$\begin{aligned}-\frac{d}{d(1/u)}[u\hat{\Psi}(u)] &= \int_0^\infty s e^{-s/u}\rho(s)ds \\ &= m_B^6 f_B^2 e^{-m_B^2/u} \\ &\quad + \int_{s_0}^\infty s e^{-s/u}\rho_{th}(s)ds. \end{aligned} \tag{10}$$

Taking the ratio of the sum rules of Eqs. (10) and (9), the decay constant drops out, and, as far as the phenomenological side is concerned, we end up with a sum rule which only depends on the heavy meson mass m_B. In our numerical analysis, this additional sum rule will be used to fix the continuum threshold s_0 from the experimental value of m_B. The resulting s_0 is then used in the f_B sum rule of Eq. (9).

In Sec. II, we give the expressions for the perturbative pseudoscalar spectral function up to the next-next-to-leading order in the strong coupling, and in Sec. III, the nonperturbative condensate contributions are summarized. Section IV contains our numerical analysis of the sum rules. Finally, in Sec. V, we compare our results to previous determinations of f_B in the literature and we present an estimate of the hadronic B parameter in the B-meson system B_B.

II. PERTURBATIVE SPECTRAL FUNCTION

In perturbation theory, the pseudoscalar spectral function has an expansion in powers of the strong coupling constant,

$$\rho(s)=\rho^{(0)}(s)+\rho^{(1)}(s)a(\mu_a)+\rho^{(2)}(s)a(\mu_a)^2+\cdots, \tag{11}$$

with $a\equiv\alpha_s/\pi$. The leading order term $\rho^{(0)}(s)$ results from a calculation of the bare quark-antiquark loop and is given by

$$\rho^{(0)}(s)=\frac{N_c}{8\pi^2}(M+m)^2 s\left(1-\frac{M^2}{s}\right)^2. \tag{12}$$

For the moment, we have only kept the small quark mass m in the global factor $(M+m)^2$ and have set it to zero in the subleading contributions. Higher order corrections in m up to order m^4 will be discussed further below.

Our expressions for the spectral function always implicitly contain a θ-function which specifies the starting point of the cut in the correlator $\Psi(s)$. Although generally, we prefer to utilize the $\overline{\mathrm{MS}}$ mass, in order to have a scale independent starting point of the cut, in this case we chose the pole mass M_{pole}. Modulo higher order corrections, it is always possible to rewrite the mass in the logarithms which produce the cut in terms of the pole mass such that the θ-function takes the form $\theta(s-M_{\mathrm{pole}}^2)$.

The order α_s correction for the two-point function $\Psi(s)$ was for the first time correctly calculated in Ref. [22], keeping complete analytical dependencies in both masses M and m. Further details on the calculation can also be found in Ref. [23]. From these results it is a simple matter to obtain the corresponding imaginary part:

$$\rho^{(1)}(s)=\frac{N_c}{16\pi^2}C_F(M+m)^2 s(1-x)\Bigg\{(1-x)[4L_2(x)+2\ln x \times\ln(1-x)-(5-2x)\ln(1-x)]+(1-2x) \times(3-x)\ln x+3(1-3x)\ln\frac{\mu_m^2}{M^2}+\frac{1}{2}(17-33x)\Bigg\}, \tag{13}$$

where $x\equiv M^2/s$ and $L_2(x)$ is the dilogarithmic function [24]. The explicit form of the first order correction is sensitive to the definition of the quark mass at the leading order. Equation (13) corresponds to running quark masses in the $\overline{\mathrm{MS}}$ scheme, $M(\mu_m)$ and $m(\mu_m)$, evaluated at the scale μ_m.

The term proportional to $\ln\mu_m^2/M^2$ cancels the scale dependence of the mass at the leading order, reflecting the fact that $\rho(s)$ is a physical quantity, i.e., independent of the renormalisation scale and scheme. Transforming the quark mass into the pole mass scheme [68], the resulting expression becomes scale independent and of course agrees with Eq. (4) of [8]. As shall be discussed in more detail below, however, the perturbative corrections to f_B in the pole mass scheme turn out to be rather large and we refrain from performing a numerical analysis of the sum rule in this scheme. Therefore, our expressions for the spectral function will only be presented in the $\overline{\mathrm{MS}}$ scheme.

The three-loop, order α_s^2 correction $\rho^{(2)}(s)$ has only been calculated very recently by Chetyrkin and Steinhauser [17,18] for the case of one heavy and one massless quark. A completely analytical computation of the second order two-point function is currently not feasible. However, one can construct a seminumerical approximation for $\rho^{(2)}(s)$ by using Padé approximations together with conformal mappings into a suitable kinematical variable [25,26]. The input used in this procedure is the knowledge of eight moments for the correlator for large momentum $x\to 0$, seven moments for small momentum $x\to\infty$, and partial information on the threshold behavior $x\to 1$. In our analysis, we have made use of the program $Rvs.m$ which contains the required expressions for $\rho^{(2)}(s)$ and was kindly provided to the public by the authors of [17,18].

In Refs. [17,18], the pseudoscalar spectral function $\rho(s)$ has been calculated in the pole mass scheme. Thus we still have to add to $\rho^{(2)}(s)$ the contributions which result from rewriting the pole mass in terms of the $\overline{\mathrm{MS}}$ mass. The two contributions $\Delta_1\rho^{(2)}$ and $\Delta_2\rho^{(2)}$ which arise from the leading and first order contributions, respectively, are given by

$$\Delta_1\rho^{(2)}(s)=\frac{N_c}{8\pi^2}(M+m)^2 s[(3-20x+21x^2)r_m^{(1)^2} -2(1-x)(1-3x)r_m^{(2)}], \tag{14}$$

$$\Delta_2\rho^{(2)}(s)=-\frac{N_c}{8\pi^2}C_F(M+m)^2 s r_m^{(1)}\Bigg\{(1-x)(1-3x) \times[4L_2(x)+2\ln x\ln(1-x)]-(1-x) \times(7-21x+8x^2)\ln(1-x)+(3-22x +29x^2-8x^3)\ln x+\frac{1}{2}(1-x)(15-31x)\Bigg\}. \tag{15}$$

Explicit expressions for the coefficients $r_m^{(1)}$ and $r_m^{(2)}$ can be found in Appendix B. Furthermore, in Refs. [17,18] the renormalization scale of the coupling μ_a was set to M_{pole}. Since in our numerical analysis we plan to vary the scale μ_a independently from μ_m, the contribution which results from reexpressing $a(M)$ in terms of $a(\mu_a)$ in the two-loop part needs to be included as well.

Close to threshold, in the pole mass scheme, the pseudoscalar spectral function behaves as $v^2(\alpha_s\ln v)^k$ where $v\equiv(1-x)/(1+x)$ at any order k in perturbation theory. This behavior, however, does not persist in the $\overline{\mathrm{MS}}$ scheme, where for each order, an additional factor of $1/v$ is obtained, such that the order α_s^2 correction goes like a constant for $v\to 0$. Nevertheless, as we will see in more detail below, numerically the corrections for the integrated spectral function show a much better convergence than in the pole mass scheme.

Let us now come to a discussion of the corrections in the small mass m. At the leading order in the strong coupling and up to order m^4, they can, for example, be found in Ref. [27]:

$$\rho_m^{(0)}(s)=\frac{N_c}{8\pi^2}(M+m)^2\Bigg\{2(1-x)Mm-2m^2 -2\frac{(1+x)}{(1-x)}\frac{Mm^3}{s}+\frac{(1-2x-x^2)}{(1-x)^2}\frac{m^4}{s}\Bigg\}. \tag{16}$$

The somewhat bulky expressions for the first order α_s correction can be obtained by expanding the results of [22,23] in terms of m and have been relegated to Appendix C. Numerically, the size of the order α_s corrections increases with increasing order in the expansion in m. However, even for the case of B_s the mass corrections in m_s become negligible before the perturbative expansion for these corrections breaks down.

In the process of performing the expansion of the results of [22,23] in terms of m, it is found that starting with order m^3 logarithmic terms of the form $\ln m$ appear in the expansion. They are of infrared origin, and in the framework of the operator product expansion it should be possible to absorb them by a suitable definition into the higher dimensional operator corrections, the vacuum condensates. If the operator product expansion is performed in terms of non-normal ordered, minimally subtracted condensates rather than the more commonly used normal ordered ones, the mass logarithms indeed disappear [27–29].

III. CONDENSATE CONTRIBUTIONS

In the following, we summarize the contributions to the two-point function coming from higher dimensional operators which arise in the framework of the operator product expansion and parametrize the appearance of nonperturbative physics, if the energy approaches the confinement region. Here, we decided to present directly the integrated quantity $u\hat{\Psi}(u)$ because the spectral functions corresponding to the condensates contain δ-distribution contributions.

The leading order expression for the dimension-three quark condensate is known since the first works on the pseudoscalar heavy-light system [8]:

$$u\hat{\Psi}_{\bar{q}q}^{(0)}(u) = -(M+m)^2 M\langle\bar{q}q\rangle e^{-M^2/u}\left[1-\left(1+\frac{M^2}{u}\right)\frac{m}{2M} + \frac{M^2 m^2}{2u^2}\right]. \tag{17}$$

To estimate higher order mass corrections in our numerical analysis, we have included the corresponding expansion up to order m^2 [27]. From the mass logarithms of the perturbative order α_s and m^3 correction, it is a straightforward matter to also deduce the first order correction to the quark condensate since the mass logarithms must cancel once the quark condensate is expressed in terms of the non-normal ordered condensate [27–29]. We were not able to find the following result in the literature and assume that it is new:

$$u\hat{\Psi}_{\bar{q}q}^{(1)}(u) = \frac{3}{2}C_F a(M+m)^2 M\langle\bar{q}q\rangle\left\{\Gamma\left(0,\frac{M^2}{u}\right) - \left[1+\left(1-\frac{M^2}{u}\right)\left(\ln\frac{\mu_m^2}{M^2}+\frac{4}{3}\right)\right]e^{-M^2/u}\right\}, \tag{18}$$

where $\Gamma(n,z)$ is the incomplete Γ function. Again, the term $\ln \mu_m^2/M^2$ cancels the scale dependence of the mass at the leading order.

The next contribution in the operator product expansion is the dimension-four gluon condensate. Although its influence on the heavy-light sum rule turns out to be very small, we have nevertheless included it in the analysis. The corresponding expression for the Borel transformed correlator is given by

$$u\hat{\Psi}_{FF}^{(0)}(u) = \frac{1}{12}(M+m)^2\langle aFF\rangle e^{-M^2/u}. \tag{19}$$

In some earlier works on the pseudoscalar sum rule this contribution appears with a wrong sign [8,9,15], although of course this has negligible influence on the numerical results.

The last condensate contribution that we consider in this work is the dimension-five mixed quark-gluon condensate which still has some influence on the sum rule since it is enhanced by the heavy quark mass. Again here the result is well known from the literature and we just cite it for the convenience of the reader:

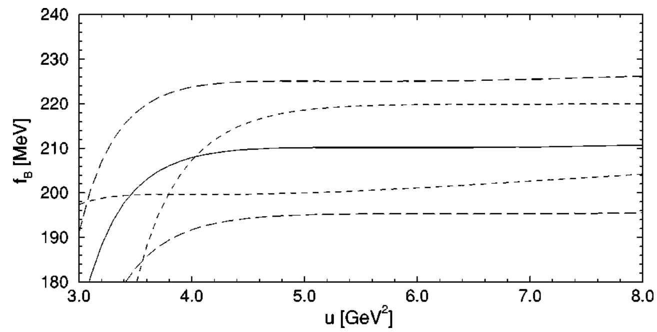

FIG. 1. f_B as a function of the Borel parameter u for different sets of input parameters. Solid line: central values of Table I; long-dashed line: $m_b(m_b)=4.16$ GeV (upper line), $m_b(m_b)=4.26$ GeV (lower line); dashed line: $\mu_m=3$ GeV (lower line), $\mu_m=6$ GeV (upper line).

$$u\hat{\Psi}_{\bar{q}Fq}^{(0)}(u) = -(M+m)^2\frac{M\langle g_s\bar{q}\sigma Fq\rangle}{2u}\left(1-\frac{M^2}{2u}\right)e^{-M^2/u}. \tag{20}$$

We have checked explicitly that the contribution of the next-higher dimensional operator, the four-quark condensate, is extremely small, and thus have neglected all higher dimensional operators. The corresponding results for the condensate contributions to the sum rule of Eq. (10) can be calculated straightforwardly by differentiating the above expressions with respect to $1/u$.

IV. NUMERICAL ANALYSIS

In our numerical analysis of the pseudoscalar heavy-light sum rule, we shall mainly discuss the values of our input parameters, their errors, and the impact of those errors on the values of f_B and f_{B_s}. To begin, however, let us investigate the behavior of the perturbative expansion.

As was already mentioned above, in the pole mass scheme the first two order α_s and α_s^2 corrections to $\hat{\Psi}(u)$ are of similar size than the leading term, thus not showing any sign of convergence. For central values of our input parameters and a typical value $u=5\ \mathrm{GeV}^2$, the first order correction amounts to 78% and the second order to 85% of the leading term. To be consistent with the perturbative result for $\rho(s)$, we have used $m_b^{\mathrm{pole}}=4.82$ GeV, which results from relation (B5) up to order α_s^2. Because of the large corrections, we shall not pursue an analysis in the pole mass scheme any further. On the contrary, in the $\overline{\mathrm{MS}}$ scheme for $\mu_m=\mu_a=m_b$ and $u=5\ \mathrm{GeV}^2$, the first and second order corrections are 11% and 2% of the leading term, respectively, while at $\mu_m\approx 4.5$ GeV the second order term vanishes entirely. Hence, in the $\overline{\mathrm{MS}}$ scheme the perturbative expansion converges rather well and is under good control.

In Figs. 1 and 2, as the solid lines we display the leptonic decay constants f_B and f_{B_s}, for central values of all input

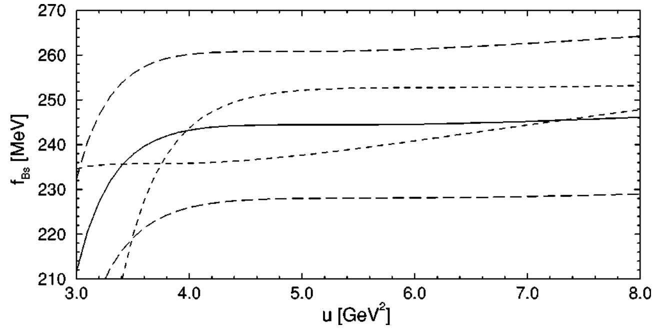

FIG. 2. f_{B_s} as a function of the Borel parameter u for different sets of input parameters. Solid line: central values of Table II; long-dashed line: $m_b(m_b)=4.16$ GeV (upper line), $m_b(m_b)=4.26$ GeV (lower line); dashed line: $\mu_m=3$ GeV (lower line), $\mu_m=6$ GeV (upper line).

parameters which have been collected in Tables I and II, as a function of the Borel variable u. For $u\lesssim 4\ \mathrm{GeV}^2$ the power corrections become comparable to the perturbative term, whereas for $u\gtrsim 6\ \mathrm{GeV}^2$ the continuum contribution gets as important as the phenomenological part. Thus a reliable sum rule analysis should be possible in the range roughly given by $4\ \mathrm{GeV}^2\lesssim u\lesssim 6\ \mathrm{GeV}^2$. In this region we extract our central results $f_B=210$ MeV and $f_{B_s}=244$ MeV.

As an additional input parameter the continuum threshold s_0 is required. This parameter can be determined from the ratio of the sum rules of Eqs. (10) and (9), which only depends on the heavy meson mass. To this end, for a certain set of input parameters, s_0 is tuned such as to reproduce the Particle Data Group values for m_B and m_{B_s} [30] in the stability region (a minimum in this case) of the ratio of sum rules. In Tables I and II, we also present the resulting values for s_0 and the corresponding location u_0 of the minimum of the m_B sum rule. For central values of all input parameters, we obtain $s_0=33.6\ \mathrm{GeV}^2$ and $u_0=5.6\ \mathrm{GeV}^2$ for the B meson, as well as $s_0=35.5\ \mathrm{GeV}^2$ and $u_0=5.1\ \mathrm{GeV}^2$ for the B_s meson. In Fig. 3, we show the resulting m_B and m_{B_s} as a function of u for central input parameters. As can be seen from this figure, in the stability region, the sum rule reproduces the physical heavy meson masses which are indicated as horizontal lines. Our results for f_B and f_{B_s} are then extracted at u_0, around which also the sum rules for the decay constants are most stable and display an inflection point.

TABLE I. Values for all input parameters, continuum thresholds s_0 [GeV^2], points of maximal stability u_0 [GeV^2], and corresponding uncertainties for f_B (MeV).

Parameter	Value	s_0	u_0	Δf_B
$m_b(m_b)$	4.21 ± 0.05 GeV	33.1 34.2	6.1 5.2	∓15
μ_m	$3.0-6.0$ GeV	33.5 34.4	6.8 4.0	±10
μ_a	$3.0-6.0$ GeV	34.2 33.1	5.1 6.2	$+2$ -1
$\langle\bar{q}q\rangle(2\ \mathrm{GeV})$	$-(267\pm17\ \mathrm{MeV})^3$	33.9 33.3	5.7 5.5	±6
$\mathcal{O}(\alpha_s^2)$	$2\times\mathcal{O}(\alpha_s^2)$ no $\mathcal{O}(\alpha_s^2)$			±2
$\alpha_s(M_Z)$	0.1185 ± 0.0020			±1
$\langle aFF\rangle$	$0.024\pm0.012\ \mathrm{GeV}^4$			±1
m_0^2	$0.8\pm0.2\ \mathrm{GeV}^2$			∓1

TABLE II. Values for all input parameters, continuum thresholds s_0 [GeV^2], points of maximal stability u_0 [GeV^2], and corresponding uncertainties for f_{B_s} [MeV].

Parameter	Value	s_0	u_0	Δf_{B_s}
$m_b(m_b)$	4.21 ± 0.05 GeV	34.8 36.4	5.4 4.8	∓16
μ_m	$3.0-6.0$ GeV	35.2 37.2	6.2 3.6	$+8$ -9
μ_a	$3.0-6.0$ GeV	36.2 34.9	4.7 5.5	$+1$
$\langle\bar{s}s\rangle/\langle\bar{q}q\rangle$	0.8 ± 0.3	35.9 35.2	5.3 4.7	±8
$\langle\bar{q}q\rangle(2\ \mathrm{GeV})$	$-(267\pm17\ \mathrm{MeV})^3$	35.7 35.3	5.2 4.9	$+5$ -4
$m_s(2\ \mathrm{GeV})$	100 ± 15 MeV			±2
$\mathcal{O}(\alpha_s^2)$	$2\times\mathcal{O}(\alpha_s^2)$ no $\mathcal{O}(\alpha_s^2)$			±3
$\alpha_s(M_Z)$	0.1185 ± 0.0020			±1
$\langle aFF\rangle$	$0.024\pm0.012\ \mathrm{GeV}^4$			±1
m_0^2	$0.8\pm0.2\ \mathrm{GeV}^2$			∓1

The dominant source of uncertainty for the decay constants is the error on the bottom quark mass m_b. For this value we have taken an average over recent determinations [31–39] which results in $m_b(m_b)=4.21\pm0.05$ GeV. The error on m_b has been chosen such that all individual results are included within one standard deviation. The corresponding variations of f_B and f_{B_s} are displayed as the long-dashed lines in Figs. 1 and 2, where the upper line corresponds to a lower value of m_b and the lower line to a larger m_b. The impact of the variation of m_b on the error of f_B and f_{B_s} has been quantified in Tables I and II.

Another important source of uncertainty is the renormalization scale μ_m. We have decided to vary μ_m in the range

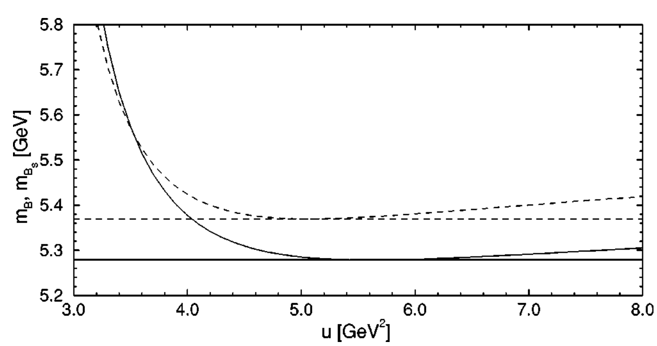

FIG. 3. m_B (solid line) and m_{B_s} (dashed line) as a function of the Borel parameter u for central input parameters. The horizontal lines indicate the corresponding experimental values for these quantities.

3–6 GeV, with a central value $\mu_m = m_b$. If μ_m is smaller than about 3 GeV, the perturbative corrections become too large and the expansion unreliable. As the dashed lines in Figs. 1 and 2, we then show the corresponding results for $\mu_m = 3$ GeV (lower line) and $\mu_m = 6$ GeV (upper line). The uncertainties for f_B and f_{B_s} which result from μ_m are again listed in Tables I and II. To indicate the influence of even lower scales, let us briefly discuss the case $\mu_m = 2.5$ GeV. Here, we find $s_0 = 38.4\ \mathrm{GeV}^2$ being rather large, as well as $u_0 = 2.9\ \mathrm{GeV}^2$ which is very small. At such a low u_0, the perturbative and operator product expansions are not very reliable. Nevertheless, the value for f_B extracted at u_0 turns out surprisingly close to our central result, such that the error estimate of Table I is more conservative. The variation of μ_a, on the other hand, only has a minor impact on the error of f_B and f_{B_s} and is also given in Tables I and II.

The present uncertainties in the remaining QCD parameters α_s, the strange quark mass m_s and the condensate parameters have much less influence on the errors of f_B and f_{B_s}. Thus let us be more brief with the discussion of these quantities. The current value of $\alpha_s(M_Z)$ by the Particle Data Group, $\alpha_s(M_Z) = 0.1185 \pm 0.0020$ [30], has been used, whereas our choice for the strange mass $m_s(2\ \mathrm{GeV}) = 100 \pm 15$ MeV is obtained from two very recent analyses of scalar and pseudoscalar QCD sum rules [40,41]. The resulting m_s is compatible to the determination from hadronic τ decays, as well as lattice QCD results [42–44]. Besides the variation of $\alpha_s(M_Z)$, in order to estimate the influence of higher order corrections, we have either removed or doubled the known $\mathcal{O}(\alpha_s^2)$ correction. The resulting uncertainty for the decay constants, however, turns out to be small.

Our value for the quark condensate has been extracted from the Gell-Mann–Oakes–Renner relation [45] with current values for the up- and down-quark masses [41]. The ratio $\langle \bar{s}s \rangle / \langle \bar{q}q \rangle$ has been chosen such as to include results from Refs. [3,46–49,69]. The mixed quark-gluon condensate is parametrized by $\langle g_s \bar{q} \sigma F q \rangle = m_0^2 \langle \bar{q}q \rangle$ with m_0^2 being determined in Ref. [50], and finally, for the gluon condensate we take a generous range which includes previous values found in the literature. All uncertainties for f_B and f_{B_s} resulting from these parameters are also listed in Tables I and II. Where entries for s_0 and u_0 are missing, we have used the values corresponding to central input parameters.

Adding all errors for the various input parameters in quadrature, our final results for the B and B_s meson leptonic decay constants are

$$f_B = 210 \pm 19\ \ \mathrm{MeV}, \quad f_{B_s} = 244 \pm 21\ \mathrm{MeV}. \tag{21}$$

In the next section, we shall compare these values with previous QCD sum rule and lattice QCD determinations.

V. CONCLUSIONS

The only truly nonperturbative method to compute hadronic matrix elements is QCD on a space-time lattice and thus it is very interesting to compare our findings to the corresponding results in lattice gauge theory. For the leptonic heavy meson decay constants, they have been compiled in a recent review article by Bernard [51,70]. Taking into account dynamical sea quark effects and estimating the corresponding uncertainties, his world averages read

$$f_B = 200 \pm 30\ \mathrm{MeV}, \quad \frac{f_{B_s}}{f_B} = 1.16 \pm 0.04. \tag{22}$$

The lattice value for f_B is in good agreement with our result of Eq. (21), and also our ratio $f_{B_s}/f_B = 1.16$ turns out to be perfectly consistent with Eq. (22). Nevertheless, due to sizable discretisation errors on the lattice, in our opinion, at present the QCD sum rule determination of the decay constants is more precise.

We now come to a comparison with recent QCD sum rule results for f_B and f_{B_s}. The status of sum rule calculations of f_B in the pole mass scheme has been summarized in the review article [14] with the result $f_B = 180 \pm 30$ MeV. Although roughly 15% lower, within the errors this result is compatible with our value (21). However, due to the large perturbative corrections in the pole mass scheme, and the strong dependence on the bottom quark mass which in [14] was taken to be $m_b^{\mathrm{pole}} = 4.7 \pm 0.1$ GeV, the theoretical error is not controlled reliably. Let us remark that the order α_s^3 correction in the relation between $\overline{\mathrm{MS}}$ mass and pole mass alone gives a shift of m_b^{pole} by roughly 200 MeV [52–54]. Typical results for f_{B_s} turn out to be about 35 MeV higher than f_B [14], so that the difference between f_{B_s} and f_B is in agreement to our result. Our result for f_B is also completely compatible with the very recent analysis of Ref. [16], which was performed in the framework of HQET and resulted in $f_B = 206 \pm 20$ MeV, suffering however from the problems of the pole mass discussed above.

After submission of our work, an independent analysis of the heavy-light meson sum rules by Narison [55] was published, which also employs the heavy quark mass in the $\overline{\mathrm{MS}}$ scheme. For the convenience of the reader, even though Ref. [55] appeared later, we comment on this analysis. The main difference to our analysis lies in the fact that in Ref. [55] the bottom quark mass is extracted from the sum rule for m_B, with the result $m_b(m_b) = 4.05 \pm 0.06$ GeV. We have checked that for this value of m_b one needs $s_0 = 37.5\ \mathrm{GeV}^2$ to reproduce m_B, and finds a stability region around $u_0 = 4.3\ \mathrm{GeV}^2$. Inserting these parameters into the f_B sum rule, we obtain $f_B = 270$ MeV, in conflict to our result (21). We are able to reproduce the value quoted by Narison, $f_B = 205$ MeV, at $u = 2.7\ \mathrm{GeV}^2$, which roughly corresponds to his preferred $\tau \equiv 1/u$ value. Around this u, however, the f_B sum rule is unstable, casting doubts on the procedure of also demanding stability in the continuum threshold s_0, besides the u stability. Furthermore, the rather low value of m_b compared to our world average presented above, as well as the very high value of s_0, indicate that the pseudoscalar sum rule is not a good place to determine m_b. On the other hand, our investigation demonstrates that perfectly compatible results are obtained with a more standard value for m_b. The ratio f_{B_s}/f_B has been calculated by the same author in [56] with the result $f_{B_s}/f_B=1.16\pm0.05$, in agreement to our findings.

The heavy-meson decay constant f_B plays an important role in the mixing of neutral B^0 and $\overline{B}^0$ mesons. The relevant hadronic matrix element can be expressed as [57]

$$\langle\overline{B}^0|\hat{Q}_{\Delta B=2}|B^0\rangle=\frac{8}{3}B_Bf_B^2m_B^2, \tag{23}$$

where $\hat{Q}_{\Delta B=2}$ is the scale invariant four-quark operator which mediates B^0-$\overline{B}^0$ mixing and B_B is the corresponding scale invariant B parameter which parametrizes the deviation of the matrix element from the factorization approximation. In the factorization approximation, by definition we would have $B_B=1$. The combination $\sqrt{B_B}f_B$ can be extracted from an analysis of experimental data on B^0-$\overline{B}^0$ mixing together with additional inputs which determine the matrix elements of the quark mixing or Cabibbo-Kobayashi-Maskawa matrix. A very recent analysis then yields $\sqrt{B_B}f_B=236\pm35\,\mathrm{MeV}$ [58,59]. Taking together our result for f_B and the quoted value for $\sqrt{B_B}f_B$, we are in a position to give an estimate of the scale invariant B-parameter B_B, which reads

$$B_B=1.26\pm0.45. \tag{24}$$

For simplicity we have assumed Gaussian errors in both input quantities. The result again is in very good agreement to corresponding determinations of B_B on the lattice which gave $B_B=1.30\pm0.12\pm0.13$ [51], although our error in this case is bigger.

To conclude, in this work we have presented a QCD sum rule determination of the leptonic heavy-meson decay constants f_B and f_{B_s}. Due to large perturbative higher order corrections, an analysis in terms of the bottom quark pole mass appeared unreliable. On the contrary, employing the heavy quark mass in the $\overline{\mathrm{MS}}$ scheme, up to order α_s^2 the perturbative expansion displays good convergence and a reliable determination of f_B and f_{B_s} turned out possible. Our central results have been presented in Eq. (21), where the dominant uncertainty arose from the present error in the bottom quark mass $m_b(m_b)$. Taking into account independent information on $\sqrt{B_B}f_B$ from B^0-$\overline{B}^0$ mixing, we were also in a position to give an estimate on the B-meson B-parameter B_B in Eq. (24). All our results are in very good agreement to lattice QCD determinations of the same quantities. Further improvements of our results will only be possible if the dominant theoretical uncertainties could be reduced. This would require a more precise value of the bottom mass, and a reduction of the renormalization scale dependence, requiring the next perturbative order α_s^3 correction, which at present seems to be out of reach.

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APPENDIX A: THE BOREL TRANSFORM

The Borel operator $\mathcal{B}_u$ is defined by ($s\equiv-p^2$)

$$\mathcal{B}_u\equiv\lim_{\substack{s,n\to\infty\\ s/n=u}}\frac{(-s)^n}{(n-1)!}\frac{\partial^n}{(\partial s)^n}. \tag{A1}$$

The Borel transformation is an inverse Laplace transform [60]. If we set

$$\hat{f}(u)\equiv\mathcal{B}_u[f(s)],\quad \text{then}\quad f(s)=\int_0^\infty\frac{1}{u}\hat{f}(u)e^{-s/u}du. \tag{A2}$$

In this work we just need the following Borel transform:

$$\mathcal{B}_u\left[\frac{1}{(x+s)^\alpha}\right]=\frac{1}{u^\alpha\Gamma(\alpha)}e^{-x/u}. \tag{A3}$$

Cases in which logarithms appear can be treated by first evaluating the spectral function and then calculating the dispersion integral of Eq. (7).

APPENDIX B: RENORMALIZATION GROUP FUNCTIONS

For the definition of the renormalization group functions we follow the notation of Pascual and Tarrach [61], except that we define the β function such that β_1 is positive. The expansions of $\beta(a)$ and $\gamma(a)$ take the form

$$\beta(a)=-\beta_1a-\beta_2a^2-\beta_3a^3-\cdots, \tag{B1}$$

$$\gamma(a)=\gamma_1a+\gamma_2a^2+\gamma_3a^3+\cdots, \tag{B2}$$

with

$$\beta_1=\frac{1}{6}[11C_A-4Tn_f],$$

$$\beta_2=\frac{1}{12}[17C_A^2-10C_ATn_f-6C_FTn_f], \tag{B3}$$

and

$$\gamma_1=\frac{3}{2}C_F,$$

$$\gamma_2=\frac{C_F}{48}[97C_A+9C_F-20Tn_f]. \tag{B4}$$

The relation between pole and running $\overline{\mathrm{MS}}$ mass is given by

$$M(\mu_m)=M_{\rm pole}[1+a(\mu_a)r_m^{(1)}(\mu_m)+a(\mu_a)^2 r_m^{(2)}(\mu_a,\mu_m)], \tag{B5}$$

where

$$r_m^{(1)}=r_{m,0}^{(1)}-\gamma_1\ln\frac{\mu_m}{M(\mu_m)}, \tag{B6}$$

$$\begin{aligned} r_m^{(2)}=&r_{m,0}^{(2)}-[\gamma_2+(\gamma_1-\beta_1)r_{m,0}^{(1)}]\ln\frac{\mu_m}{M(\mu_m)}\\ &+\frac{\gamma_1}{2}(\gamma_1-\beta_1)\ln^2\frac{\mu_m}{M(\mu_m)}\\ &-\left[\gamma_1+\beta_1\ln\frac{\mu_m}{\mu_a}\right]r_m^{(1)}. \end{aligned}\tag{B7}$$

The coefficients of the logarithms can be calculated from the renormalization group [26] and the constant coefficients $r_{m,0}^{(1)}$ and $r_{m,0}^{(2)}$ are found to be [62,63]

$$r_{m,0}^{(1)}=-C_F, \tag{B8}$$

$$\begin{aligned} r_{m,0}^{(2)}=&C_F^2\left(\frac{7}{128}-\frac{15}{8}\zeta(2)-\frac{3}{4}\zeta(3)+3\zeta(2)\ln 2\right)\\ &+C_AC_F\left(-\frac{1111}{384}+\frac{1}{2}\zeta(2)+\frac{3}{8}\zeta(3)-\frac{3}{2}\zeta(2)\ln 2\right)\\ &+C_FT\left(\frac{3}{4}-\frac{3}{2}\zeta(2)\right)+C_FTn_f\left(\frac{71}{96}+\frac{1}{2}\zeta(2)\right). \end{aligned}\tag{B9}$$

APPENDIX C: MASS CORRECTIONS AT ORDER α_s

Below, we present the order α_s mass corrections to the pseudoscalar spectral function which arise from expanding the results by [22,23] up to order m^4, after the higher dimensional operators have been expressed in terms of non-normal ordered condensates:

$$\begin{aligned} \rho_m^{(1)}(s)=&\frac{N_c}{8\pi^2}C_F(M+m)^2Mm\Big\{(1-x)[4L_2(x)+2\ln x\ln(1-x)-2(4-x)\ln(1-x)]+2(3-5x+x^2)\\ &\times\ln x+3(2-3x)\ln\frac{\mu_m^2}{M^2}+2(7-9x)\Big\}, \end{aligned}\tag{C1}$$

$$\begin{aligned} \rho_{m^2}^{(1)}(s)=&-\frac{N_c}{8\pi^2}C_F(M+m)^2m^2\Big\{(1-x)[4L_2(x)+2\ln x\ln(1-x)]-(2+x)(4-x)\ln(1-x)+(6+2x-x^2)\ln x\\ &+6\ln\frac{\mu_m^2}{M^2}+(8-3x)\Big\}, \end{aligned}\tag{C2}$$

$$\begin{aligned} \rho_{m^3}^{(1)}(s)=&-\frac{N_c}{8\pi^2}C_F(M+m)^2\frac{Mm^3}{s}\Big\{4L_2(x)+2\ln x\ln(1-x)-2\frac{(7+7x-2x^2)}{(1-x)}\ln(1-x)+2\frac{(6+7x-2x^2)}{(1-x)}\ln x\\ &+6\frac{(2-x^2)}{(1-x)^2}\ln\frac{\mu_m^2}{M^2}+\frac{(9+8x-9x^2)}{(1-x)^2}\Big\}, \end{aligned}\tag{C3}$$

$$\begin{aligned} \rho_{m^4}^{(1)}(s)=&\frac{N_c}{8\pi^2}C_F(M+m)^2\frac{m^4}{s}\Big\{2L_2(x)+\ln x\ln(1-x)-\frac{(13-24x-27x^2+2x^3)}{2(1-x)^2}\ln(1-x)\\ &+\frac{(12-22x-27x^2+2x^3)}{2(1-x)^2}\ln x+3\frac{(4-12x+x^2+3x^3)}{2(1-x)^3}\ln\frac{\mu_m^2}{M^2}+\frac{(6-64x+15x^2+11x^3)}{4(1-x)^3}\Big\}. \end{aligned}\tag{C4}$$

[1] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979); **B147**, 448 (1979); **B147**, 519 (1979).
[2] L. J. Reinders, H. R. Rubinstein, and S. Yazaki, Phys. Rep. **127**, 1 (1985).
[3] S. Narison, *QCD Spectral Sum Rules*, World Scientific Lecture Notes in Physics Vol. 26 (World Scientific, Singapore, 1989).
[4] *Vacuum Structure and QCD Sum Rules*, edited by M. A. Shifman, Current Physics: Sources and Comments Vol. 10 (North-Holland, Amsterdam, 1992).
[5] V. A. Novikov, M. A. Shifman, A. I. Vainshtein, M. B. Voloshin, and V. I. Zakharov, Phys. Rep. **41**, 1 (1978).
[6] V. A. Novikov, M. A. Shifman, A. I. Vainshtein, M. B. Voloshin, and V. I. Zakharov in ''West Lafayette 1978, Proceedings, Neutrinos '78,'' West Lafayette, 1978, C278-c288.
[7] L. J. Reinders, H. R. Rubinstein, and S. Yazaki, Phys. Lett. **104B**, 305 (1981).

[8] T. M. Aliev and V. L. Eletskiĭ, Sov. J. Nucl. Phys. **38**, 936 (1983).
[9] S. Narison, Phys. Lett. B **198**, 104 (1987).
[10] L. J. Reinders, Phys. Rev. D **38**, 947 (1988).
[11] C. A. Dominguez and N. Paver, Phys. Lett. B **269**, 169 (1991).
[12] E. Bagan, P. Ball, V. M. Braun, and H. G. Dosch, Phys. Lett. B **278**, 457 (1992).
[13] M. Neubert, Phys. Rev. D **45**, 2451 (1992).
[14] A. Khodjamirian and R. Rückl, in *Heavy Flavours II*, edited by A. J. Buras and M. Lindner (World Scientific, Singapore, 1998), hep-ph/9801443.
[15] S. Narison, Nucl. Phys. B (Proc. Suppl.) **74**, 304 (1999).
[16] A. A. Penin and M. Steinhauser, Phys. Rev. D (to be published), DESY-01-112 (2001), hep-ph/0108110.
[17] K. G. Chetyrkin and M. Steinhauser, Phys. Lett. B **502**, 104 (2001).
[18] K. G. Chetyrkin and M. Steinhauser, Eur. Phys. J. C **21**, 319 (2001).
[19] M. Beneke and V. M. Braun, Nucl. Phys. **B426**, 301 (1994).
[20] W. A. Bardeen, A. J. Buras, D. W. Duke, and T. Muta, Phys. Rev. D **18**, 3998 (1978).
[21] N. N. Bogolyubov, B. V. Medvedev, and M. K. Polivanov, *Theory of Dispersion Relations* (Moscow State Technical Publishing House, Moscow, 1958).
[22] D. J. Broadhurst, Phys. Lett. **101B**, 423 (1981).
[23] S. C. Generalis, J. Phys. G **16**, 785 (1990).
[24] L. Lewin, *Polylogarithms and Associated Functions* (North-Holland, Amsterdam, 1981).
[25] D. J. Broadhurst, J. Fleischer, and O. V. Tarasov, Z. Phys. C **60**, 287 (1993).
[26] K. G. Chetyrkin, J. H. Kühn, and M. Steinhauser, Nucl. Phys. **B482**, 213 (1996).
[27] M. Jamin and M. Münz, Z. Phys. C **60**, 569 (1993).
[28] V. P. Spiridonov and K. G. Chetyrkin, Sov. J. Nucl. Phys. **47**, 522 (1988).
[29] K. G. Chetyrkin, C. A. Dominguez, D. Pirjol, and K. Schilcher, Phys. Rev. D **51**, 5090 (1995).
[30] Particle Data Group, D. E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).
[31] M. Jamin and A. Pich, Nucl. Phys. **B507**, 334 (1997); Nucl. Phys. B (Proc. Suppl.) **74**, 300 (1999).
[32] J. H. Kühn, A. A. Penin, and A. A. Pivovarov, Nucl. Phys. **B534**, 356 (1998).
[33] K. Melnikov and A. Yelkhovsky, Phys. Rev. D **59**, 114009 (1999).
[34] A. A. Penin and A. A. Pivovarov, Nucl. Phys. **B549**, 217 (1999).
[35] M. Beneke and A. Signer, Phys. Lett. B **471**, 233 (1999).
[36] A. H. Hoang, Phys. Rev. D **61**, 034005 (2000).
[37] V. Gimenez, L. Giusti, G. Martinelli, and F. Rapuano, J. High Energy Phys. **03**, 018 (2000).
[38] A. Pineda, J. High Energy Phys. **06**, 022 (2001).
[39] J. H. Kühn and M. Steinhauser, Nucl. Phys. B **619**, 588 (2001).
[40] K. Maltman and J. Kambor, ZU-TH-26-01 (2001), hep-ph/0108227.
[41] M. Jamin, J. A. Oller, and A. Pich, IFIC/01-52 (2001).
[42] R. Gupta and K. Maltman, LAUR-00-5684 (2001), hep-ph/0101132.
[43] S. Chen *et al.*, Eur. Phys. J. C **22**, 31 (2001).
[44] T. Kaneko, plenary talk presented at Lattice 2001, Berlin (2001).
[45] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).
[46] H. G. Dosch, M. Jamin, and S. Narison, Phys. Lett. B **220**, 251 (1989).
[47] K. Langfeld and M. Schaden, Phys. Lett. B **272**, 100 (1991).
[48] M. Jamin and M. Münz, Z. Phys. C **66**, 633 (1995).
[49] C. A. Domingnez, A. Ramlakan, and K. Schilcher, Phys. Lett. B **511**, 59 (2001).
[50] A. A. Ovchinnikov and A. A. Pivovarov, Sov. J. Nucl. Phys. **48**, 721 (1988).
[51] C. Bernard, Nucl. Phys. B (Proc. Suppl.) **94**, 159 (2001).
[52] K. G. Chetyrkin and M. Steinhauser, Phys. Rev. Lett. **83**, 4001 (1999).
[53] K. G. Chetyrkin and M. Steinhauser, Nucl. Phys. **B573**, 617 (2000).
[54] K. Melnikov and T. van Ritbergen, Phys. Lett. B **482**, 99 (2000).
[55] S. Narison, Phys. Lett. B **520**, 115 (2001).
[56] S. Narison, Phys. Lett. B **322**, 247 (1994).
[57] A. J. Buras, M. Jamin, and P. H. Weisz, Nucl. Phys. **B347**, 491 (1990).
[58] A. Höcker, H. Lacker, S. Laplace, and F. Le Diberder, Eur. Phys. J. C **21**, 225 (2001).
[59] A. Höcker (private communications), slac.stanford.edu/~hoecker/ckmfitter/allresults/all.ps
[60] D. V. Widder, *The Laplace Transform* (Princton University Press, Princeton, NJ, 1946).
[61] P. Pascual and R. Tarrach, Lecture Notes in Physics Vol. 194 (1984), p. 1.
[62] R. Tarrach, Nucl. Phys. **B183**, 384 (1981).
[63] N. Gray, D. J. Broadhurst, W. Grafe, and K. Schilcher, Z. Phys. C **48**, 673 (1990).
[64] G. Amoros, J. Bijnens, and P. Talavera, Nucl. Phys. **B602**, 87 (2001).
[65] C. T. Sachrajda, talk presented at Lepton Photon 2001, hep-ph/0110304.
[66] After completion of this work, we became aware of an independent analysis on the same subject [16], where also the new order α_s^2 corrections are included, however employing the framework of HQET.
[67] All relevant formulas for the Borel transformation are collected in Appendix A.
[68] Explicit expressions for the relation between pole and $\overline{\mathrm{MS}}$ mass are collected in Appendix B.
[69] We have not taken into account the very recent result $\langle\bar{s}s\rangle/\langle\bar{q}q\rangle = 1.7$ [64], obtained in the framework of chiral perturbation theory (χPT), which would lead to $f_{B_s} = 270$ MeV. In χPT the value of the quark condensate depends on the subtraction procedure employed, and it is not clear how these results relate to $\langle\bar{q}q\rangle$ in the $\overline{\mathrm{MS}}$ scheme. The large value obtained in [64] can almost be excluded on the basis of our f_B, together with independent lattice results for the ratio f_{B_s}/f_B.
[70] See also Ref. [65].