Macroscopic effects in cold magnetized nucleons and electrons with anomalous magnetic moments

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(Received 25 June 2001; published 22 January 2002)

A model of a degenerate neutron gas in chemical equilibrium with a background of degenerate electrons and protons in a constant uniform ultrastrong magnetic field is applied to describe the state of matter in the cores of strongly magnetized neutron stars. Expressions for the thermodynamic quantities are obtained including the anomalous magnetic moments of the fermions. It is shown that (1) the inclusion of the anomalous magnetic moments of charged fermions leads to nonperiodic magnetic oscillations of their thermodynamic quantities in strong magnetic fields, (2) the total stress energy tensor relevant for neutron star structure must include contributions from both the magnetized matter and the magnetic field and as a result the total pressure produced is anisotropic, and (3) complete spin polarization of neutrons occurring in superstrong magnetic fields must lead to an increase in the degeneracy pressure compared with the zero field case at the same neutron densities. It is hoped that the results obtained will have applications for the structure in neutron stars with ultrastrong frozen-in magnetic fields.

DOI: 10.1103/PhysRevD.65.056001 PACS number(s): 04.40.Dg, 05.30.Fk, 13.40.Em, 97.60.Jd

Study of a relativistic electron gas in a strong magnetic field was stimulated by the discovery of magnetic fields of the order of $B \geqslant 10^{13}$ G at the neutron star surface [1–3]. Such a magnetic field "frozen in" a neutron star may become much stronger in its central domain. Gravitational collapse of macroscopic magnetized bodies, composed of neutrons in a strong magnetic field B, may lead to extremely magnetized neutron stars, or magnetars [4,5]. Their magnetic fields at the star surface are estimated to be of the order of $B \geqslant 10^{15}$ G [6,7] and the magnetic induction at the star core may go up to 10^{18} G [8]. The magnetic induction B needed to affect neutron star structure directly was estimated in [9] to be $B \sim 2\times 10^{18}(M/1.4M_{\odot})(R/10\text{ km})^{-2}$ G, where M and R are, respectively, the neutron star mass and radius.

Recently, many works have been concerned with the effect of strong magnetic fields on elementary processes occurring at the star core. The behavior of a relativistic nucleon and electron gas in a constant strong magnetic field was studied in [10]. The equation of state and the magnetization of relativistic fermions in strong uniform magnetic fields including the anomalous magnetic moments of the fermions were partly discussed in [11–14]. The effect of strong magnetic fields on dense neutron-star matter was studied in [15,16] in the mean field approximation. However, contributions from the magnetic field energy density and pressure were not included [15,16] while the anisotropic effect of uniform magnetic fields either was not given enough attention [11,12] or was considered incorrectly [13,14].

As is known [17,18] the chemical equilibrium in a degenerate gas of neutrons (n), protons (p), and electrons (e) must take into account the direct URCA processes $p+e \to n+\nu$, $n \to p+e+\overline{\nu}$ (ν and $\overline{\nu}$ denote the neutrino and antineutrino, respectively) in which the total number of baryons $n_b = n_p + n_n$ is conserved and the electroneutrality condition ($n_p = n_e$) is satisfied. Since n_b is conserved, the total energy density $\mathcal{E}$ depends only on n_n and the approximate equilibrium concentration is maintained through the equation $\mu_n = \mu_p + \mu_e$ among the particles' chemical potentials. Neutrinos are assumed to escape from the star so its chemical potential is taken as zero. We recall the magnitudes of the so-called critical magnetic fields: $B_0 = m_e^2/|e| = 4.4\times 10^{13}$ G (m_e and e are the rest mass and electric charge of the electron), and $B_0^* = m_p^2/e = 3.4\times 10^6 B_0$ (m_p and e are the rest mass and electric charge of the proton). We use units where $\hbar = c = 1$.

We study the effect of strong magnetic fields on the thermodynamic properties (the energy density, pressure, and magnetization) of degenerate nucleons and electrons (*npe* gas) with the inclusion of the anomalous magnetic moments (AMM's) of the fermions.

As compared to the earlier work cited, here we present new results. These results prove the following. (1) The inclusion of the anomalous magnetic moments of the charged fermions leads to nonperiodic magnetic oscillations of their thermodynamic quantities in strong magnetic fields. The period of the nonperiodic oscillations is a B-dependent quantity, unlike the oscillation period of the so-called van Alphen–de Haas oscillations. (2) The total stress energy tensor (i.e., the total energy and pressure) relevant for neutron-star structure must include contributions from both the magnetized matter and the magnetic field. The total pressure produced is anisotropic, having a smaller value along than perpendicular to the magnetic field; the outcome could be a gravitational collapse of the magnetized star along the magnetic field. The magnetic field contribution begins to dominate the magnetized matter pressure, for the neutron densities of interest at $B > 3\times 10^{-2} B_0^*$. (3) The complete spin polarization of neutrons occurring in ultrastrong magnetic fields must lead to a significant increase in the pressure compared with the zero field case at the same neutron densities. (4) There is a reason to discuss the appearance of spontaneous magnetization in cold neutron-star matter when exchange effects between neutrons are included.

The energy spectra for fermions in a constant uniform magnetic field with the inclusion of the AMM's are given by

$$E_{ns} = \sqrt{p^2 + [\sqrt{|e|B(2n+1-s\,\mathrm{sgn}\,e) + m_{e,p}^2} - sM_{e,p}B]^2} \tag{1}$$

(for charged fermions [19]) where p is the component of the momentum along the magnetic field $\mathbf{B}=(0,0,B)$, e is the electric charge of the fermion, $n=0,1,2,...$ enumerates the Landau levels, and

$$E=\sqrt{p^2+(\sqrt{p_\perp^2+m_n^2}+sM_nB)^2} \tag{2}$$

(for a neutron [20]) where p and $p_\perp$ are parallel and perpendicular components (to the magnetic field $\mathbf{B}$) of the momentum; $m_{e,p,n}$ are the fermion masses, $s=\pm 1$ are the spin projections along the magnetic field axis, and $M_{e,p,n}$ are the anomalous magnetic moments of the fermions. It will also be noted that neutrons with anomalous magnetic moments can interact with electromagnetic fields through nonminimal coupling.

If $M_{e,p}=0$ then all the energy levels described by Eq. (1), except the level with $n=0$, $s=-1$ (for $\operatorname{sgn} e<0$) and the level with $n=0$, $s=+1$ (for $\operatorname{sgn} e>0$) are doubly degenerate: the levels with n, $s=+1$ and $n+1$, $s=-1$ (for $\operatorname{sgn} e<0$) and the levels with n, $s=-1$ and $n+1$, $s=+1$ (for $\operatorname{sgn} e>0$) coincide. The state with $n=0$, $s=-1$ (for $\operatorname{sgn} e<0$) and with $n=0$, $s=+1$ (for $\operatorname{sgn} e>0$) is the single ground state; in this state the electron (proton) spin projection onto the magnetic field direction may take only the value $s=-1$ $(s=+1)$. The spacing between the Landau levels in the magnetic fields B_0 for electrons and B_0^* for protons becomes equal to the rest energy of these particles.

If $M_{e,p}\neq 0$ then the ground states are single states with $n=0$, $s=-1$ for $\operatorname{sgn} e<0$ and with $n=0$, $s=1$ for $\operatorname{sgn} e>0$. Equations (1) and (2) show clearly that every energy level splits up into two spin- (and $M_{e,p,n}$)-dependent levels when the interaction of the anomalous magnetic moments with the magnetic field is included.

In strong magnetic fields $B\gg B_0$, contributions from the anomalous magnetic moments of the nucleons must necessarily be considered. Indeed, experimentally, the magnitudes of the nucleon AMM's are $M_p=(g_p/2-1)M_N$ for the proton and $M_n=g_nM_N/2$ for the neutron, where M_N is the nuclear magneton and $g_p=5.58$ and $g_n=-3.82$ are the Landé g factors for the proton and neutron, respectively. Since the anomalous magnetic moments of the nucleons are of the order of M_N, their coupling energies $|M_n,M_p|B\cong 70(B/B_0^*)$ MeV become significant for $B\gg B_0$ and can lead to changes in the chemical equilibrium condition and in the nucleon Fermi energies. For the Fermi energies of interest (from a few MeV to tens of MeV), it is clear that significant changes can occur when $B>10^{-2}B_0^*$.

For electrons, this is far from the case. To see the reason we should remember that the anomalous magnetic moment of an electron has a dynamical origin (due to the so-called radiative corrections) [21] and its magnitude is very small (in weak magnetic fields) [21] $M_e=-(g_e/2-1)M_B\equiv -(e^2/2\pi)M_B$, where M_B is the Bohr magneton. In weak magnetic fields $|M_e|$ is much less than M_B. It becomes a vanishing function of the magnetic field for $B\gg B_0$ [22]. Therefore, contributions from the electron AMM are very small for the range $B_0^*\gg B\gg B_0$.

At zero temperature and in the presence of a constant uniform magnetic field B, the energy density of charged fermions, $\mathcal{E}_{e,p}$, is defined by

$$\mathcal{E}_{e,p}=\frac{eB}{2\pi^2}\sum_{s=1,-1}\sum_{0}^{n_{\max}}\int_0^{p_{e,p}^f}dp \times\sqrt{p^2+(\sqrt{2|eB|n+m_{e,p}^2}-sM_{e,p}B)^2}. \tag{3}$$

Here, $p_{e,p}^f$ is the Fermi momentum for a level with given n and s. After integrating Eq. (3) in p, we obtain

$$\mathcal{E}_{e,p}=\frac{e^2B(B_0,B_0^*)}{4\pi^2}\sum_{s=\pm 1}\sum_{n=0}^{n_{\max}}\Bigg[z_n(e,p)\sqrt{1+z_n^2(e,p)+b_{e,p}n} +(\sqrt{1+b_{e,p}}\mp sM_{e,p}B/m_{e,p})^2 \times\ln\frac{z_n(e,p)+\sqrt{1+z_n^2(e,p)+b_{e,p}n}}{\sqrt{1+b_{e,p}n}}\Bigg], \tag{4}$$

in which

$$z_n^2(e,p)=\frac{\mu_{e,p}^2-(\sqrt{2|eB|n+m_{e,p}^2}+sM_{e,p}B)^2}{m_{e,p}^2},$$

$$b_{e,p}\equiv\frac{2|eB|}{m_{e,p}^2}, \tag{5}$$

$$n_{\max}=\left[\frac{(\mu_{e,p}\mp sM_{e,p}B)^2-m_{e,p}^2}{2|eB|}\right]. \tag{6}$$

The pressure of the charged fermions in constant uniform magnetic fields, $P_{e,p}$, is defined by

$$P_{e,p}=\frac{eB}{2\pi^2}\sum_{s=1,-1}\sum_{0}^{n_{\max}}\int_0^{p_{e,p}^f}dp \times\frac{p^2}{\sqrt{p^2+(\sqrt{2|eB|n+m_{e,p}^2}+sM_{e,p}B)^2}}. \tag{7}$$

Integrating this, we obtain

$$P_{e,p}=\frac{e^2B(B_0,B_0^*)}{4\pi^2}\sum_{s=\pm 1}\sum_{n=1}^{n_{\max}}\Bigg[z_n(e,p)\sqrt{1+z_n^2(e,p)+b_{e,p}n} -(\sqrt{1+b_{e,p}n}\mp sM_{e,p}B/m_{e,p})^2 \times\ln\frac{z_n(e,p)+\sqrt{1+z_n^2(e,p)+b_{e,p}n}}{\sqrt{1+b_{e,p}n}}\Bigg]. \tag{8}$$

The magnetization of the gas is

$$\mathcal{M}_{e,p}=-\partial\mathcal{E}_{e,p}/\partial B.$$

Note that any physical quantity $(\mathcal{E}_{e,p},P_{e,p},\mathcal{M}_{e,p})$ as a function of magnetic induction may be separated into monotonic and oscillating parts. The oscillating term describes the so-called Landau oscillations [23]. In order to find the oscillating part, say $\mathcal{E}_{e,p}$, it is helpful to insert in Eq. (4) a $\theta(x)$ function of argument $x \equiv z_n^2(e,p)$ with

$$z_n^2(e,p) \simeq \frac{\mu_{e,p}^2 - 2|eB|n - m_{e,p}^2 - 2sm_{e,p}M_{e,p}B}{m_{e,p}^2},$$

and to expand the sum over n from 0 to ∞. Then, the oscillating functions may be found by using for the summation in n the Poisson summation formula [23]

$$\sum_{n=0}^{\infty} f(n) = \int_0^{\infty} f(x)dx + f(0)/2 + 2\sum_{k=1}^{\infty} \int_0^{\infty} f(x)\cos(2\pi kx)dx. \tag{9}$$

Finally, one can get the leading oscillating term of the energy density in the form

$$\mathcal{E}_{\text{osc}} \simeq \frac{\mu_{e,p}}{8\pi^{5/2}} \sum_{s=\pm 1} \sum_{k=1}^{\infty} \left(\frac{|eB|}{\pi k}\right)^{3/2} \frac{\cos(2\pi k\omega - \pi/4)}{k\omega}, \tag{10}$$

where $\omega = (\mu_{e,p}^2 - m_{e,p}^{*2})/2|eB|$, $m_{e,p}^* \cong m_{e,p} \pm M_{e,p}B$.

We see that the energy density of charged fermions (as well as the pressure and magnetization) oscillates with the frequencies $\omega_\pm = \mu_{e,p}^2 - (m_{e,p} \pm M_{e,p}B)^2$; the oscillation periods over the variable $(2|eB|)^{-1}$ are B-dependent quantities. So the magnetic oscillations of thermodynamical quantities describing charged fermions with anomalous magnetic moments are not strictly periodic (see also [24]).

At zero temperature the energy density of neutrons in the presence of a constant uniform magnetic field, $\mathcal{E}_n$, is

$$\mathcal{E}_n = \frac{1}{2\pi^2} \sum_{s=1,-1} \int_0^{p_\perp^f} \int_0^{p^f} dp_\perp dp\, p_\perp \times \sqrt{p^2 + (\sqrt{p_\perp^2 + m_n^2} + sM_nB)^2} \tag{11}$$

where

$$p^f = \sqrt{\mu_n^2 - (\sqrt{p_\perp^2 + m_n^2} + sM_nB)^2}$$

and

$$p_\perp^f = \sqrt{(\mu_n - sM_nB)^2 - m_n^2}.$$

After integrating Eq. (11), we obtain

$$\begin{aligned}\mathcal{E}_n = \frac{1}{16\pi^2} \sum_{s=1,-1} \Bigg\{ & \mu_n(\mu_n^2 - m_{n*}^2)^{3/2} + \mu_n^3 \sqrt{\mu_n^2 - m_{n*}^2} - \frac{m_{n*}^4}{2} \ln \frac{\mu_n + \sqrt{\mu_n^2 - m_{n*}^2}}{\mu_n - \sqrt{\mu_n^2 - m_{n*}^2}} - \frac{sM_nB}{3} \\ & \times \Bigg[-5m_{n*}\mu_n \sqrt{\mu_n^2 - m_{n*}^2} + 2m_{n*}^3 \\ & \times \ln \frac{\mu_n + \sqrt{\mu_n^2 - m_{n*}^2}}{\mu_n - \sqrt{\mu_n^2 - m_{n*}^2}} + \mu_n^3 \arcsin \frac{\sqrt{\mu_n^2 - m_{n*}^2}}{\mu_n} \Bigg] \Bigg\},\end{aligned} \tag{12}$$

where $m_{n*} = m_n + sM_nB$.

The pressure of neutrons in constant magnetic fields defined by $P = -(\partial \mathcal{E}_n V/\partial V)_{S=0}$ (V and S are the volume and entropy of the neutron gas) is

$$\begin{aligned}P_n = \frac{1}{8\pi^2} \sum_{s=1,-1} \Bigg\{ & \left(\frac{\mu_n^3}{3} - \frac{5\mu_n m_{n*}^2}{6}\right) \sqrt{\mu_n^2 - m_{n*}^2} \\ & + \frac{m_{n*}^4}{4} \ln \frac{\mu_n + \sqrt{\mu_n^2 - m_{n*}^2}}{\mu_n - \sqrt{\mu_n^2 - m_{n*}^2}} + sM_nB \Bigg[\left(\frac{m_{n*}^3}{2\mu_n} - \frac{25m_{n*}\mu_n}{18}\right) \sqrt{\mu_n^2 - m_{n*}^2} + \frac{m_{n*}(\mu_n^2 - m_{n*}^2)}{18\mu_n} \\ & + \frac{m_{n*}^3}{3} \ln \frac{\mu_n + \sqrt{\mu_n^2 - m_{n*}^2}}{\sqrt{\mu_n^2 - m_{n*}^2}} \\ & + \frac{\mu_n m_{n*}^2}{6} \arcsin \frac{\sqrt{\mu_n^2 - m_{n*}^2}}{\mu_n} \Bigg] \Bigg\}.\end{aligned} \tag{13}$$

The exact expression for the magnetization of neutrons has the form

$$\begin{aligned}\mathcal{M}_n = \frac{1}{4\pi^2} \sum_{s=1,-1} sM_n \Bigg[& \mu_n \sqrt{\mu_n^2 - m_{n*}^2} \left(\frac{7m_n}{6} + sM_nB\right) \\ & + \frac{4}{3} m_{n*}^3 \ln \frac{\mu_n + \sqrt{\mu_n^2 - m_{n*}^2}}{\mu_n - \sqrt{\mu_n^2 - m_{n*}^2}} + \mu_n^3 \arcsin \frac{\sqrt{\mu_n^2 - m_{n*}^2}}{\mu_n} \Bigg]\end{aligned} \tag{14}$$

and its value in the two limits is given by

$$\mathcal{M}_n \simeq \frac{9e^2B}{2\pi^2} \left(\frac{n_n}{n_{0n}}\right)^{1/3}, \quad n_n \ll n_{0n},$$

$$\mathcal{M} \simeq \frac{9e^2B}{2\pi^2} \left(\frac{n_n}{n_{0n}}\right)^{2/3}, \quad n_n \gg n_{0n} = \frac{m_n^3}{3\pi^2}. \tag{15}$$

The monotonic parts of the magnetization of the protons are described by Eq. (15) in the corresponding limits where we must replace n_n, n_{0n}, M_n by n_p, n_{0p}, M_p. As a result, the numerical factor 9 and the ratio n_n/n_{0n} in Eq. (15) must be replaced by 13 and n_p/n_{0p}, respectively.

The oscillating parts of the energy density and the pressure are small with respect to their monotonic ones. In contrast, the absolute value of the oscillating part of the magnetization is much greater than the monotonic one at a given chemical potential. Indeed, since

$$\mathcal{M}_{\rm osc} \sim \mathcal{M}_{\rm mon}(\mu/MB)^{1/2}, \tag{16}$$

where μ is the chemical potential and M is the magnetic moment of charged fermions, for the oscillating part of the magnetization of protons one can get

$$\mathcal{M}_p \cong \frac{13e^2B}{2\pi^2}\left(\frac{n_p}{n_{0n}}\right)^{2/3}\left(\frac{B_0^*}{B}\right)^{1/2}, \quad n_p \ll n_{0n},$$

$$\mathcal{M}_p \cong \frac{13e^2B}{2\pi^2}\left(\frac{n_p}{n_{0n}}\right)^{5/6}\left(\frac{B_0^*}{B}\right)^{1/2}, \quad n_p \gg n_{0n}. \tag{17}$$

At $n_p \ll n_{0n}$, the oscillating part of the magnetization mainly contributes to the macroscopic magnetic strength ($\mathbf{H}$), which is defined by [25]

$$\mathbf{H} = \mathbf{B} - 4\pi\boldsymbol{\mathcal{M}}(\mathbf{B}). \tag{18}$$

Consider the total stress tensor of the matter and the magnetic field. The energy-momentum tensor of the magnetic field has the following components [26]:

$$T^{00} = -T^{33} = T^{11} = T^{22} = (B - 4\pi\mathcal{M})B/8\pi. \tag{19}$$

Extra terms, in addition to the usual ones proportional to B^2, are introduced into the structure of the pressure from the magnetic field.

The neutron fraction $n_n/(n_n+n_p)$ at the central densities of neutron stars depending upon both the density and the magnetic field is supposed to be 0.9–0.8 so the contribution from neutrons typically dominates. The leading terms of the total stress tensor of the matter and the magnetic field can therefore be written as

$$T_{11} = T_{22} = (\gamma-1)\mathcal{E}_\gamma + (1/2-\gamma)\mathcal{M}B + B^2/8\pi,$$

$$T_{33} = (\gamma-1)\mathcal{E}_\gamma + (3/2-\gamma)\mathcal{M}B - B^2/8\pi, \tag{20}$$

with

$$\mathcal{E}_{\gamma=5/3} \cong \frac{3m_n^4}{24\pi^2}\left(\frac{n_n}{n_{0n}}\right)^{5/3}, \quad \mathcal{E}_{\gamma=4/3} \cong \frac{3m_n^4}{45\pi^4(3\pi^2)^{2/3}}\left(\frac{n_n}{n_{0n}}\right)^{4/3}. \tag{21}$$

Here $\gamma = 5/3, 4/3$; $\mathcal{E}_{\gamma=5/3,4/3}$ and $\mathcal{M}B$ are the energy (at $B = 0$) and magnetic energy densities (the magnetic pressure) of neutrons at $n_n \ll n_{0n}$ and $n_n \gg n_{0n}$, respectively.

When $n_n \ll n_{0n}$ the magnetic pressure of neutrons has the form

$$\mathcal{M}B \simeq 2\times 10^4 \mathcal{E}_{\gamma=5/3}\left(\frac{B}{B_0^*}\right)^2\left(\frac{n_{0n}}{n_n}\right)^{4/3}. \tag{22}$$

The total stress tensor may contain the sum of the several species involved, that is, $\mathcal{E} = \Sigma_i\mathcal{E}_i$, and $\mathcal{M} = \Sigma_i\mathcal{M}_i$, $i = n,p$.

It follows from Eq. (22) that the magnetic field contributions dominate the magnetic pressure of the matter at the neutron densities of most interest (at $n_n \leqslant n_{0n}$) while the field contributions can dominate the matter pressure only for $B > 10^{-3}B_0^*$ at nuclear densities, and for $B > 3\times 10^{-2}B_0^*$ at the central densities of neutron stars (at $n_n \sim 0.1n_{0n}$). The magnetic field contributions must therefore be included when the neutron star is in mechanical equilibrium under the balance of the magnetic field, magnetized neutrons, and gravitational pressures. The total stress tensor is anisotropic, having a smaller value along (due to the $-B^2/8\pi$ term) than transverse the magnetic field. So the outcome could be a collapse of the star along the magnetic field but not a transverse collapse (compare with the conclusions made in [13,14]).

Thus, the neutron star with $B \sim 3\times 10^{-2}B_0^*$ and $n_n \simeq 0.1n_{0n}$ could become an oblate spheroid but the existence of such ultrastrong magnetic fields is not likely.

Electrons and protons in the presence of constant uniform magnetic fields can become completely spin polarized even though the interactions of the anomalous magnetic moments of the fermions with the field are not included [22]. This happens when only one (lowest) Landau level is occupied by the fermions [22,10]:

$$\frac{\sqrt{2}n_i}{3n_{0n}}(B_0^*/B)^{3/2} < 1, \quad i = e,p.$$

Neutrons may also become spin (up) polarized in constant uniform ultrastrong magnetic fields (see [11]) but only if the interaction of the anomalous magnetic moment with the magnetic field is included. In order to estimate the magnitude of the magnetic field required to induce this effect for the neutrons, we need to determine the number density of quantum states (NDQS) in the presence of the magnetic field. This number is given by

$$n_n = \frac{1}{2\pi^2}\sum_{s=1,-1}\left\{\frac{(\mu_n^2 - m_{n*}^2)^{3/2}}{3} + \frac{sM_nB}{2}\left[m_{n*}\sqrt{\mu_n^2 - m_{n*}^2} - \mu_n^2 \arcsin\frac{\sqrt{\mu_n^2 - m_{n*}^2}}{\mu_n}\right]\right\}. \tag{23}$$

Note that at $B = 0$ the NDQS is equal to the number density of neutrons, but they differ in the presence of magnetic fields since the neutron energy now must include a term describing the interaction of the anomalous magnetic moment with the magnetic field. With increasing B, the fraction of spin (up) polarized neutrons increases. This leads to a corresponding increase in the degeneracy pressure. It is clear that complete spin polarization of neutrons may occur at the magnitude of B when the NDQS of neutrons with $s = 1$ becomes equal to n_n, and from Eq. (23) we obtain for B, $B_n \cong 0.6(\mu_n^2-m_n^2)/2|M_n|$. At $B=B_n$ the NDQS with spin $s=1$ becomes equal to n_n and the increase of the thermodynamical quantities is halted since the neutrons needed to further fill the quantum states with $s=1$ are absent. When the magnetic induction approaches B_n the pressure of neutrons is increased by almost 1.5 times over the zero field case at the same neutron densities. This increases the maximum mass of the star relative to the field-free value.

It should be noted that Eq. (18) has a nonzero solution $(B\neq 0)$ even if $\mathbf{H}\to\mathbf{0}$. Indeed, substituting Eq. (15) into Eq. (18), we obtain

$$B=\frac{18e^2B}{\pi}\left(\frac{n_n}{n_{0n}}\right)^{2/3}$$

which implies that spontaneous magnetization may occur at

$$\left(\frac{n_n}{n_{0n}}\right)^{2/3}=\frac{\pi}{18e^2}\cong 24. \tag{24}$$

Spontaneous magnetization is likely to occur if we assume that the exchange interaction exists between neutrons. To discuss the problem, let us consider a quantum system of $N=n_nV$ neutrons in the volume V and introduce

$$N_+=N(1+\eta)/2, \quad N_-=N(1-\eta)/2 \tag{25}$$

for the neutron concentrations with the spin up (N_+) and down (N_-). In terms of N_+ and N_- the energy of N neutrons in the magnetic field $\mathbf{B}$ at $n_n \ll n_{0n}$ may be written in the form

$$E_n=\mathcal{E}_0[(1+\eta)^{5/3}+(1-\eta)^{5/3}]-NM_nB\eta, \tag{26}$$

where $\mathcal{E}_0=3N\mu_n/10$, and μ_n is the chemical potential of neutrons at $B=0$.

We assume that the exchange interaction between neutrons may be described in analogy with the exchange interaction of conduction electrons [27], that is, the exchange energy of neutrons with parallel spins is $-J<0$, and that with antiparallel spins is zero. Then, the energy (26) must be written as

$$E_n(J)=\mathcal{E}_0[(1+\eta)^{5/3}+(1-\eta)^{5/3}]-JN^2(1+\eta^2)/4 -NM_nB\eta. \tag{27}$$

The minimum of Eq. (27) with respect to η is determined by

$$\frac{dE_n(J)}{d\eta}=\tfrac{5}{3}\mathcal{E}_0[(1+\eta)^{2/3}-(1-\eta)^{2/3}-a\eta-NM_nB]=0. \tag{28}$$

Here

$$a=3JN^2/10\mathcal{E}_0\equiv JN/\mu_n. \tag{29}$$

If $a<4/3$ then it follows from Eq. (28) under the assumption $\eta\ll 1$ that Eq. (28) has the only real root $(\eta_{\rm eq}=0)$, and the magnetization has the form

$$\mathcal{M}_n=-\frac{\partial E_n(J)}{\partial B}=\frac{6NM_n^2B}{4\mu_n-3JN}. \tag{30}$$

However, when $a>4/3$ then Eq. (28) at $B=0$ has other real solutions. At $a\cong 4/3$ these real roots are $\eta_{\rm eq}=\pm 3\sqrt{3(3a/4-1)}$. In the case $a>4/3$ the energy at $B=0$ and $\eta=0$ turns out to be unstable and the energy in a ferromagnetic state $(\eta\neq 0)$ will be less than that in a paramagnetic state [27].

Unfortunately, the constant J, having the meaning of the exchange energy of neutrons, is unknown. We shall suppose that the magnitude of J may roughly be estimated in analogy with the electronic exchange energy for bound states. The latter may be written as $J_{ee}\sim 10^{-2}e^2/b$, where b is a constant. Then, by similar arguments for J, one can suppose that it has the form

$$J\sim \epsilon g^2\frac{\exp(-b_n/l)}{a}\times\text{energy}\sim \epsilon g^2(n_n)^{1/3} \times\exp[-c(n_{0n}/n_n)^{1/3}]\times\text{energy},$$

where ϵ is a small numerical constant, g is the strong interaction constant, c is a numerical constant of the order of unity, $b_n\sim(n_n)^{-1/3}$ is the characteristic distance between neutrons, and l is the Compton length of the π meson. Finally, the condition $a>4/3$ may easily be written as

$$N>(2/\epsilon g^2)(n_n/n_{0n})^{1/3}\exp[c(n_{0n}/n_n)^{1/3}].$$

This discussion implies that new mechanisms for the creation of ultrastrong magnetic fields frozen in neutron stars should be considered.

In this paper it has been shown that (1) nonperiodic magnetic oscillations must appear of all the thermodynamical quantities of charged fermions with AMM's in magnetic fields; (2) the total pressure produced by the magnetized matter and the magnetic field is anisotropic and the outcome could be a gravitational collapse of the magnetized star along the magnetic field; (3) the complete spin polarization of neutrons in ultrastrong magnetic fields must lead to an increase in the degeneracy pressure of neutrons and as a result this may increase the maximum mass of the star relative to the field-free value; (4) there is a reason to discuss the appearance of spontaneous magnetization in a cold neutron-star matter when exchange effects between neutrons are included.

Thus, a description of neutron-star matter in the presence of ultrastrong magnetic fields must necessarily include the effects due to the nucleon anomalous magnetic moments.

[1] J. Trümper *et al.*, Astrophys. J. Lett. **219**, L105 (1978).
[2] W. A. Wheaton *et al.*, Nature (London) **282**, 240 (1979).
[3] A. E. Delsante and N. E. Frankel, Phys. Rev. D **20**, 1795 (1979).
[4] R. C. Duncan and C. Thompson, Astrophys. J. Lett. **392**, L9 (1992); Astrophys. J. **473**, 322 (1996).
[5] R. C. Duncan, ''Physics in Ultra-strong Magnetic Fields,'' astro-ph/0002442.
[6] C. Kouveliotou *et al.*, Nature (London) **393**, 235 (1998).
[7] K. Hurley *et al.*, Nature (London) **397**, 41 (1999).
[8] Sut. Glosh *et al.*, ''Electrical Conductivity at the Core of a Magnetar,'' astro-ph/0106153.
[9] D. Lai and S. Shapiro, Astrophys. J. **383**, 745 (1991).
[10] C.-L. Ho, V. R. Khalilov, and C. Yang, Mod. Phys. Lett. B **10**, 1141 (1996).
[11] A. Broderick, M. Prakash, and J. M. Lattimer, Astrophys. J. **537**, 351 (2000).
[12] In-Saeng Suh and G. J. Mathews, Astrophys. J. **546**, 1126 (2001).
[13] M. Chaichian, S. Masood, C. Montonen, A. Pérez Martínez, and H. Pérez Rojas, Phys. Rev. Lett. **84**, 5261 (2000).
[14] A. P. Martínez, H. P. Rojas, and H. J. M. Cuesta, ''Magnetic Collapse of a Neutron Gas: No Magnetar Formation,'' hep-ph/0011399.
[15] S. Chakrabarty, D. Bandopadhyay, and S. Pal, Phys. Rev. Lett. **78**, 2898 (1997).
[16] D. Bandopadhyay, S. Chakrabarty, and S. Pal, Phys. Rev. Lett. **79**, 2176 (1997).
[17] S. T. Shapiro and S. A. Teukolsky, *Black Holes, White Dwarfs, and Neutron Stars: The Physics of Compact Objects* (Wiley, New York, 1983).
[18] J. M. Lattimer, C. J. Pethick, M. Prakash, and P. Haensel, Phys. Rev. Lett. **66**, 2701 (1991).
[19] I. M. Ternov, V. G. Bagrov, and V. Ch. Zhukovskii, Moscow Univ. Phys. Bull. **21**, 21 (1966).
[20] Yu. M. Lostukov and V. P. Leventuiev, Izv. Akad. Nauk SSSR, Ser. Fiz. **34**, No. 6, 152 (1969).
[21] J. Schwinger, Phys. Rev. **73**, 416L (1948); Proc. Natl. Acad. Sci. U.S.A. **37**, 152 (1951); **37**, 455 (1951).
[22] I. M. Ternov *et al.*, J. Phys. A **11**, 739 (1978); V. R. Khalilov, *Electrons in Strong Electromagnetic Fields* (Gordon & Breach, Amsterdam, 1996).
[23] E. M. Lifshitz and L. P. Pitaevskiĭ, *Statistical Physics* (Pergamon, Oxford, 1980).
[24] A. D. Balaev *et al.*, Zh. Eksp. Teor. Fiz. **113**, 1877 (1998) (in Russian); D. Ebert, K. G. Klimenko, M. A. Vdovichenko, and A. S. Vshivtsev, Phys. Rev. D **61**, 025005 (2000).
[25] E. M. Lifshitz and L. P. Pitaevskiĭ, *Statistical Physics: Theory of Condensed State* (Nauka, Moscow, 1978) (in Russian).
[26] L. D. Landau, E. M. Lifshitz, and L. P. Pitaevskiĭ, *Electrodynamics of Continuous Media*, 2nd ed. (Pergamon, New York, 1984).
[27] Ch. Kittel, *Introduction to Solid State Physics*, 4th ed. (Wiley, New York, 1978).